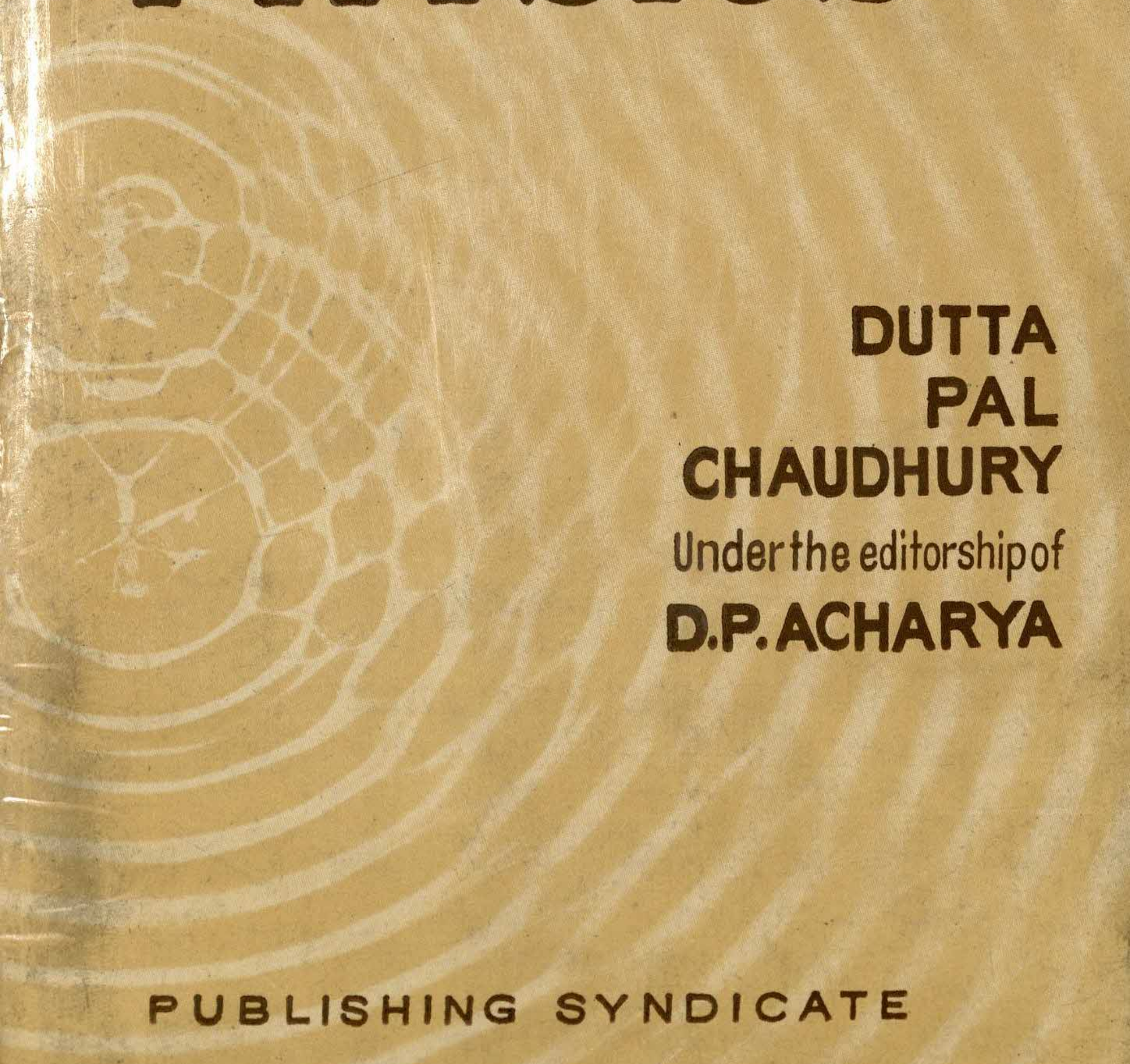
ELEMENTS OF HIGHER SECONDARY PHYSICS



DUTTA
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Under the editorship of
D.P. ACHARYA

PUBLISHING SYNDICATE

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Written according to the New Curriculum & Syllabuses of West Bengal Council of Higher Secondary Education for Classes XI & XII

ELEMENTS
OF
HIGHER SECONDARY PHYSICS

[PART I]

[Including Mechanics, General Properties of Matter, Heat, Vibrations and Waves]

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PREFACE TO THE SEVENTH REVISED EDITION

West Bengal Council of Higher Secondary Education introduced the Syllabus for its plus two course in 1976. With the introduction of this new pattern of syllabus there was a pressing need for an appropriate text book in Physics—a book that might bridge the gap in the syllabus between the Secondary and Higher Secondary stages and also present the fundamental aspects of physics in correct sequence at the Higher Secondary stage. Our text "Elements of Higher Secondary Physics" was written in this context.

This year the old syllabus has been revised. In presenting to our readers this new edition of the text, a critical review of the entire content has been thoroughly done and a large number of articles have been rewritten. Also this opportunity has been taken to introduce short answer type questions, separately from essay type questions to lay stress on the objective of the revised syllabus where marks for the objective questions are mentioned separately. We regard the solving of numerical problems by students as an essential part of obtaining a full grasp on the basic physical concepts. To that end, a number of problems have been included in the exercise. Thus, as with the earlier text, we sincerely hope that the book will serve the purpose intended.

We acknowledge with much pleasure our debts to colleagues and friends whose advice over the last decade aided in the preparation of this work. We thank our beloved publisher who has contributed far beyond the requirement of duty to expedite the completion of the work. Comments, suggestions and indication of errors will be thankfully received from the readers for future improvement of the book.

Calcutta.
1st October, 1985.

D. Datta.
B. Pal
B. Chaudhuri

PREFACE TO THE FIRST EDITION

This book has been written to cover the Higher Secondary Syllabus on Physics of the West Bengal Council of Higher Secondary Education. Our primary objective is to provide the students with an understanding of physics as a quantitative science based on a sequence of mathematical logic and simple class demonstrations.

The text is so designed as to consist of materials suitable for students whose mathematical knowledge goes no further than algebra and elementary trigonometry. For the benefit of those who are simultaneously learning calculus in the Higher Secondary courses, alternative treatments based on calculus are included. It is hoped that it will be valuable to the inquisitive students, and that it will facilitate their entry into wider areas of higher courses. Emphasis has been laid on the fundamental principles and their practical applications ; the details of experimental determinations have been given a place of secondary importance. Numerous illustrative problems are worked out in the body of the text. All physical quantities are expressed particularly in C.G.S and M.K.S. units, the S.I. system being briefly described in the Appendix.

If we may claim any success at all, we humbly submit that this is greatly due to Prof. D.P. Acharya, who has been kind enough to edit the entire manuscript, offering many valuable suggestions.

We shall deem our labours amply rewarded, if the students for whom it is intended, find it useful. We shall be glad to receive suggestions from teachers and professors for further improvement of the text.

We are grateful to Prof. R.K. Guchhait of Gurudas College, Calcutta, for the kind encouragement we have received from him.

23. 9. 1976

D. Dutta
B. Pal
B. Chaudhuri

CONTENTS

Subject *Page*

Units and Measurements (i)

1. MECHANICS

CHAPTER 1 : Vectors 1
CHAPTER 2 : Motion 17
CHAPTER 3 : Newton's Laws of motion : Force 43
CHAPTER 4 : Friction 82
CHAPTER 5 : Dynamics of rotational motion 97
CHAPTER 6 : Statics 129
CHAPTER 7 : Work, energy and power 150

2. GENERAL PROPERTIES OF MATTER

CHAPTER 8 : Gravitation 177
CHAPTER 9 : Elasticity 213
CHAPTER 10 : Hydrostatics 231
CHAPTER 11 : Surface tension 277
CHAPTER 12 : Viscosity 282

3. HEAT

CHAPTER 13 : Heat and temperature 291
CHAPTER 14 : Thermal expansion of solids 299
CHAPTER 15 : Thermal expansion of liquids 321
CHAPTER 16 : Expansion of gases 341
CHAPTER 17 : Calorimetry 365
CHAPTER 18 : Change of state : Properties of Vapours : Hygrometry 381
CHAPTER 19 : Mechanical equaivalent of heat 417
CHAPTER 20 : Kinetic theory of gases 436
CHAPTER 21 : Transmission of heat 450

4. VIBRATIONS AND WAVES

CHAPTER 22 : Vibrations 477
CHAPTER 23 : Waves 501
CHAPTER 24 : Sound waves 551
CHAPTER 25 : Light waves 578
APPENDIX 585
COUNCIL OF H.S. EDUCATION QUESTIONS ... ... [i]
INDEX ... ... [viii]

THE COUNCIL OF HIGHER SECONDARY EDUCATION

SYLLABUS

PHYSICS

Paper—I

Full Marks—80

1. MECHANICS

Marks 20

Particle Dynamics :

Rest and motion : reference frame ; displacement, velocity and acceleration ; momentum ; kinematical equations (in one dimension) ; elementary problems.

Scalars and vectors. Composition and resolution of vectors. Representation of vector by co-ordinates. Addition and subtraction of vectors by geometrical and analytical methods. Relative velocity and acceleration.

Newton's law of motion ; inertia ; units of force ; impulse and impulsive forces ; conservation of linear momentum, elastic collisions of particles moving in the same line ; jets and rockets ; Friction ; Static and kinetic friction ; co-efficient of friction.

Statics :

Centre of mass, centre of gravity, conditions of equilibrium of a system of forces.

Dynamics of Rotational Motion :

Rotational motion of a particle angular velocity ; angular acceleration ; relation between angular velocity and linear velocity ; Angular momentum ; moment of a force about a point and about an axis ; couple ; torque ; relation between angular acceleration and torque (statement only), centripetal force ; centrifugal force (as a pseudo-force).

Work, energy and power :

Definition of work, relevant units. Work done by and against a force. Mechanical energy—kinetic and potential. Conservation of energy ; the case of a freely falling body. Power—definition, units.

2. GENERAL PROPERTIES OF MATTER

Marks 20

Gravitation :

Newton's law of universal gravitation (statement and mathematical relation). Constant of gravitation (Definition & value with units, no experimental determination). Examples of gravitational attraction between heavenly bodies (e.g. the Sun & the Earth, the Earth & the Moon). Gravitational attraction of the Earth, relation between G & g, variation of g. Motion of planets & satellites (Essential points regarding nature of the orbits should be mentioned avoiding Kepler's laws). Weightlessness in orbiting satellites.

Elastic properties of matter :

Definitions, statements and explanation of the terms stress, strain, elastic limit, Hooke's law, modulii of elasticity : Young's modulus, Bulk modulus, rigidity modulus, poisson's ratio, principle of measurement of Y (No experimental details).

Hydrostatics :

Density & Specific gravity (Methods of determination not required). Archimedes' principle—Statement & demonstration. Floatation, Sums on Archimedes' principle. Pressure of fluids. Pressure inside a liquid, unit of pressure. Transmission of fluid pressure, Pascal's law, principle of multiplication of force, Hydraulic press (principle only & problems, no descriptive details). Air pressure & its measurement, Torricelli's & Fortin's barometer, siphon, basic principles of lift pump, compression pump, vacuum pumps. (No mathematical deductions).

3. HEAT *Marks 24*

Thermal Expansion :

Thermal expansion of Solids simple demonstrations ; coefficients of linear, superficial and cubical expansion of solids ; Their relations.

Application of expansion of Solids ; Real and apparent expansions of Liquids ; Relation between expansion co-efficients ; Anomalous Expansion of water ; Effect on marine life.

Expansion of Gases :

Boyle's Law ; Charle's Law ; Equation of state of an ideal gas ; Volume and pressure co-efficients ; Absolute Scale of Temperature.

Calorimetry :

Preliminary definitions ; Principle of calorimetry (no questions on measurements to be set) ; calorimetric problems.

Change of state :

Latent heat ; brief discussion on determination. Evaporation and Boiling ; Effect of pressure on Melting point and Boiling point : Vapour pressure ; Relative Humidity ; Dew, Fog and Cloud.

Mechanical Equivalent of Heat :

Heat as a form of Energy ; Relation between calorie and erg ; Determination of Mechanical Equivalent of Heat, First Law of Thermodynamics ; Isothermal and Adiabatic expansion of gases ; (brief discussion). Specific Heats of Gases ; (elementary ideas).

Kinetic Theory of Gases :

Evidence of molecular structure of matter and of random molecular motion ; Brownian movement (qualitative description), Basic assumptions of kinetic theory of ideal gases. Pressure of an ideal gas (mention of the formula, derivation not required), concept of temperature from kinetic theory.

Transmission of Heat :

Conduction of Heat : Thermal conductivity. Convection of Heat ; Radiation : Radiation as a form of energy.

4. VIBRATIONS AND WAVES

Vibrations : *Marks 16*

Periodic motion ; oscillations and its characteristics, periodic time, frequency amplitude. Simple harmonic motion, relation between S.H.M. and uniform circular motion ; displacement, velocity and acceleration of S.H.M. Energy in S.H.M. ; characteristics of S.H.M. Time period, frequency, amplitude and phase. Graphical and mathematical representation of S.H.M. Simple pendulum, Superposition of two S.H.M. of same frequency in the same direction (a) in phase (b) in opposite phase—graphical treatment. Free vibrations—longitudinal and transverse. Forced vibration ; resonance. Damped vibration (qualitative discussion with example).

Waves :

Elastic Waves—longitudinal and transverse ; characteristics of propagating waves, nature of the medium, wave length, amplitude of wave, time period, frequency, velocity of wave and their relation. Properties of Waves : Laws of reflection and refraction of wave with reference to sound wave ; formation of echo, determination of depth of sea with the help of echo.

Sound Wave : Longitudinal elastic wave, velocity of sound wave ; statement and explanation of Newton's formula and Laplace's correction.

Superposition of waves—graphical representation only, (i) two S.H. waves of nearly equal frequency and amplitude and moving in the same direction (beat); determination of unknown frequency from beating frequency.

(ii) Two S.H. waves of equal amplitude and frequency travelling in opposite direction (standing or stationery waves) ; characteristics of standing waves and comparison with progressive wave.

Transverse standing waves on stretched wire ; law's of transverse vibration of strings fundamental harmonic ;

Longitudinal standing waves in air column.

(i) Vibration of air column in a tube closed at one end. Determination of velocity of sound or the frequency of a tuning fork by resonance with vibration of air column in a tube closed at one end.

(ii) Vibration of air column in a tube open at both ends.

Effect of temperature and humidity on the frequency of air column. Qualitative discussion.

Musical Sound and noise—characteristics of musical sound—Loudness and intensity, pitch and frequency, quality (qualitative discussion only).

Light as wave phenomenon ; finite velocity of light. Mention characteristics of wave phenomena associated with light.

Wave (without going through details). Transverse characteristic of light wave.

ELEMENTS
OF
HIGHER SECONDARY
PHYSICS

PART—1

UNITS AND MEASUREMENTS

[*Recapitulation*]

1. Introduction.

The basic conception of physical quantities, their fundamental units and measurements have already been dealt with, in detail, in the secondary stage. A short discourse is included here as recapitulation of these fundamental ideas.

Physics is an exact science ; the relations are developed by proper reasoning and by rigorous mathematical procedures and the various physical quantities we shall encounter, can be measured with a high degree of accuracy. Hence the importance of accurate measurement and mathematical analysis to physicists.

2. Units.

To measure any physical quantity, we must first have a unit of measurement. ***By* unit *of a quantity we mean a definite and convenient amount of that quantity.*** The magnitude of a physical quantity is obtained by comparing it with the unit. ***The factor by which a quantity is bigger or smaller than the corresponding unit is called the* numerical measure *of that quantity.*** Thus the statement that a room is 5 metre long, means that the physical quantity being measured is ***length***, the unit chosen is ***metre*** and the numerical measure is 5.

In the previous para we have seen that the unit of length is a metre. But what is a metre ? To define it we evidently require a standard length. Similarly we require a standard for every other unit. A ***standard*** is the physical embodiment of a unit. It fixes up and preserves the size of the corresponding unit. A unit may be of same size as that of the standard or may be a multiple or submultiple of it. Thus International Proto-type Kilogram is the standard of mass, the units of mass being kilogram (kg.), metric ton (10^3 kg.), gram (10^{-3} kg.) etc.

3. Fundamental Quantities and their units.

Each of the various physical quantities is expected to have its own unit and a standard. But, it is found that nearly all physical quantities (except temperature, electric current and luminous intensity**) can be derived from the units of three fundamental quantities, namely, length, mass and time. The units of the three fundamental quantities are independent of each other, and are termed as ***fundamental units***. The units of all other physical quantities can be derived from one or more fundamental units and are called ***derived units***. Density of a substance is expressed in gm./c.c. which is derived from gm. and cm., the fundamental units of mass and length respectively. Setting up of standards of only the fundamental units is thus necessary.

*** This chapter is not included in the syllabus.**

**** See Appendix—S. I. System.**

4. Standards.

A question still remains whether there is a unique standard for a particular fundamental quantity. Unfortunately, the answer is 'no'. World is still divided into two major portions using two different standards for each of length and mass.

In ***metric system*** the standard of length is taken as the distance between two fine lines engraved on a platinum-irridium bar kept at ***International Bureau of Weights and Measures*** in Paris maintained at 0°C. and called ***International Proto-type Metre***. Originally ***metre*** was intended to be one ten-millionth (10^{-7}) of the distance along the meridian through Paris, from the equator to the north pole, and the said bar was constructed on the basis of this definition. The bar was taken as the standard metre and accurate copies of it were distributed all over the world. Recently in 1960, the International Bureau of Weights and Measures decided to define the ***metre*** by optical method, the most accurate method of measuring lengths. According to this definition a metre is exactly 1,650,763·73 wave-lengths of the orange-red light emitted by Krypton-86.

International Proto-type Kilogram is the standard of mass in the metric system and is defined as the mass of a platinum-irridium cylinder kept together with proto-type metre in Paris, copies being distributed all over the world. It was also intended to be the mass of 1000 c.c. of water at its maximum density (i.e., at nearly 4°C).

In British or F.P.S. system the standard of length is the ***British Imperial Yard*** which is defined as the distance, at 62°F, between specified lines ruled on two gold plugs set into a certain bronze bar preserved in the Standards Department of the Board of Trade, London.

The standard of mass in the same system is the mass of certain piece of platinum marked 'P.S. 1844, 1 lb'. preserved along with Imperial Yard and is called *Pound Avoirdupois*.

The standard of time called ***mean solar second*** (or simply ***second***) is same in both systems and is defined as $\frac{1}{86400}$ of the average length of a solar day (mean solar day) throughout the year. This definition of a second, although satisfactory for our daily routine, fails to satisfy modern scientists since the mean solar day varies slightly from year to year and century to century. Hence, in 1946, International Bureau of Weights and Measures defined the second as the time in which a cesium atom (which has only one isotope) undergoes 9,192,631,770 internal vibrations.

5. Systems of Units.

A complete set of units, both fundamental and derived, for all physical quantities is called a ***system of units***. There are now mainly three systems in common use, each of which derives its name from the initial letters of the fundamental units upon which it is based. These are :

(1) C.G.S. system, having centimetre, gram and second as fundamental units,
(2) M.K.S. system, in which fundamental units are metre, kilogram and second,

(3) F.P.S. system based on foot, pound and second as fundamental units.

A fourth system of unit, namely S.I., introduced in 1960, is described in the appendix.

The first two, namely C.G.S. and M.K.S. systems, are also called metric system and obviously metric standards of length and mass are used in them. The F.P.S. system, on the other hand, is based on British standards.

The metric system has a definite advantage over F.P.S. system. Calculations are much easier, since the various multiples and sub-multiples of unit are related by simple powers of 10. The following prefixes are generally used to denote these relationships. The letter in the parenthesis indicates the respective abbreviation.

Deka (da)	$=10^{1}$	1	Dekametre	$=10$ m
Hecto (h)	$=10^{2}$	1	Hectometre	$=10^{2}$ m
Kilo (k)	$=10^{3}$	1	Kilometre	$=10^{3}$ m
Mega (M)	$=10^{6}$	1	Megadyne	$=10^{6}$ dyne
Deci (d)	$=10^{-1}$	1	Decigram	$=10^{-1}$ gm
Centi (c)	$=10^{-2}$	1	Centimetre	$=10^{-2}$ m
Milli (m)	$=10^{-3}$	1	Milligram	$=10^{-3}$ gm
Micro (μ)	$=10^{-6}$	1	Micro second	$=10^{-6}$ sec
Nano (n)	$=10^{-9}$	1	Nanometre	$=10^{-9}$ m
Pico (p)	$=10^{-12}$	1	Picofarad	$=10^{-12}$ farad

The F.P.S. system has, however, no such advantage. The relation between the various subdivisions of a unit are rather obscure and inconvenient, such as 1 foot=12 inches, 1 yard=3 ft., 1 mile=1760 yds. Even an inch is divided into 8ths, 16ths, and 64ths to measure very small lengths. Similar inconvenient relations exist between the subunits of mass.

The students are advised to memorise the following chart for a ready conversion of units of one system to the other :

1 inch (in.) =2·54 centimetre (cm)=0·0254 metre
1 foot (ft.) =30·48 centimetre=0·3048 metre
1 metre (m.) =39·37 inch
1 kilometre (km.) =0·6215 mile
1 mile (mi.) $=1{\cdot}609\times10^{3}$ metre=1·609 kilometre
1 pound (lb.) =453·6 gram (gm.)
1 kilogram (kg.) =2·205 pound.

6. Units of volume of liquids.

The units of volume of liquids, though derived ones, should be mentioned especially, since these have different names. Thus, in metric system unit of volume is ***litre*** which is defined as the volume of 1 kg. of water, at normal atmospheric pressure and at the temperature of its maximum density (at 4°C). Actually 1 litre (lt.)=1000·028 cubic centimetre (c.c.). For all practical purposes, it is taken to be 1000 c.c. Hence 1 millilitre (ml.)=1 c.c.

In F.P.S. system the unit of volume of liquids is ***gallon*** which is defined as the volume of 10 lbs. of water at 62°F. and is equivalent to 277·42 cubic inches (in^{3}.). 1 gallon=4·546 litres.

7. Dimension of a physical quantity.

Having defined the units of fundamental quantities, we now briefly mention the concept of dimensions of other physical quantities.

The* dimension *of a physical quantity is characterised by the powers of the fundamental units that enter into the derived unit suitable for its measurement.

Speed has the derived units—*cm./sec.*, *km./hr.*, *mi./hr.* etc. The most common feature in them is that they are all derived by dividing a unit of length by a unit of time. So the dimension of speed will be L/T or LT^{-1}, where L stands for length and T for time.

Similarly, different units of acceleration e. g., $cm./sec.^2$, $ft./sec.$,^{2} $mi./hr.^2$ etc. represent a unit of length divided by square of a unit of time. Hence its dimension will be represented by L/T^2 or LT^{-2}.

Following the same reasoning, force, which is the product of mass (M) and acceleration (LT^{-2}), has the dimension MLT^{-2}.

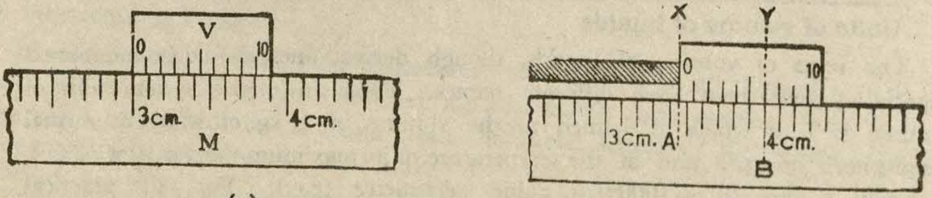
Dimensions of some other physical quantities are enlisted in the following table and students are expected to check them as an exercise.

Quantity	*Dimension*	*Quantity*	*Dimension*
Area	L^2	Work	ML^2T^{-2}
Volume	L^3	Power	ML^2T^{-3}
Density	ML^{-3}	Surface Tension	MT^{-2}
Pressure	ML^{-1}-T^{-2}	Viscosity	$ML^{-1}T^{-1}$
Momentum	MLT^{-1}	Young's Modulus	$ML^{-1}T^{-2}$

8. Measuring Instruments.

As the measurement of length, mass, volume and time was thoroughly taken into consideration in the secondary course, we shall not go through it any more. In this article we shall rather present a brief description of some appliances for measurement of length which enables greater accuracy to be obtained than is possible by mere eye-estimation.

Vernier : It consists of a short auxilliary scale V, called the ***vernier scale***, which slides along the edge of an ordinary scale M, called the ***main scale***.



(*a*) (*b*)

Fig. 1 : Vernier Scale

For the same length of these scales, the number of divisions in the vernier scale is usually greater by one than those in the main scale. Fig. 1 (*a*) shows such a vernier scale with 10 divisions of total length same as that of 9 main scale divisions. Each main scale division is taken as 1 mm. Hence the difference in length between a scale division and a vernier division is 0·1 mm. or 0·01 cm. This difference is the smallest distance measurable with the vernier and is termed as ***vernier constant***. So a vernier of this type will read accurately upto tenth of a mm.

One end of a rod whose length is to be determined is made coincident with the zero mark on the scale. The vernier is then slided along the scale until its zero mark just touches the other end of the rod. Fig. 1 (*b*) shows such a position of the vernier.

From the main scale it is clear that the length of the rod is slightly greater than 3·4 cm. To find this excess, look for the vernier scale mark which perfectly coincides with a main scale mark. In the diagram [Fig. 1 (*b*)] 6th vernier division mark (Y) coincides with the main scale mark B. Now, if A be the main scale mark just before the end of the rod,

$$AB = 6 \times \text{value of 1 main scale division} = 6 \times 0{\cdot}1 \text{ cm., and}$$

$$XY = 6 \times \text{value of 1 vernier division} = 6 \times 0{\cdot}09 \text{ cm.}$$

$$\therefore \quad \text{Excess } AX = AB - XY = 6\,(0{\cdot}1 - 0{\cdot}9) = 6 \times 0{\cdot}01 \text{ cm.}$$

$$= 6 \times \text{vernier constant} = 0{\cdot}06 \text{ cm.}$$

Hence, the length of the rod=3·4+0·06=3·46 cm.

Thus the excess is obtained by simply noting the number of vernier division mark coinciding with a main scale mark and multiplying it by the vernier constant. This gives the length correct to second decimal place.

Slide Callipers : These are also termed as ***vernier callipers*** and essentially consist of a steel scale (S) with a fixed jaw (J_1) at one end and a sliding jaw (J_2) capable of moving along the length of the scale (Fig. 2). The sliding jaw carries a vernier (V). When the two jaws are made to touch each other, the vernier zero coincides with the scale zero. If not, correction is necessary. This is known as ***zero error correction***.

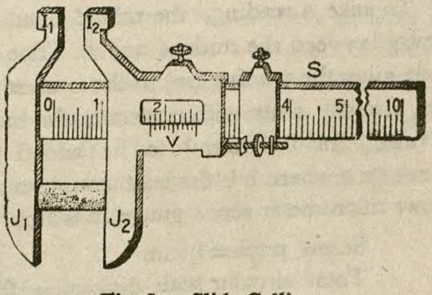


Fig. 2 : Slide Callipers

The body whose length is to be measured is placed between the jaws and the sliding jaw is moved until it touches one end of the body, the other end being already in contact with the fixed jaw. The reading of the scale with that of the vernier gives the perpendicular distance between the jaws, i. e., the length of the body.

In a defective instrument, when the two jaws touch each other, the vernier zero may cross the main scale zero or may not reach it. In both cases, the reading of the vernier scale multiplied by the vernier constant gives the measure of the error. For measuring a length, this error should be added or subtracted from the otherwise measured length, according as the vernier zero has crossed the main scale zero or has not reached it.

The callipers shown in the diagram also contain inside jaws (I_1, I_2) which are used to measure the internal diameter of a tube.

Micrometer Screw Gauge : It is used for measuring the diameter of a wire or similar small lengths so as to get readings accurate upto the 3rd or 4th place of decimal.

The main features of the instrument are shown in Fig. 3. It consists of a U-shaped metallic frame *F* carring a fixed stud *A* at one arm. The other arm carries a hollow cylinder *C* having a linear scale in mm. (or 0·5 mm.) engraved upon it. The most important part is a screwed spindle *S*, fitted with a thimble *T*. It works within the cylinder *C* acting as a nut and terminates into another similar stud *B*. The spindle is made to rotate by the milled head *E*. The distance through which the spindle moves for each complete turn is called the pitch of the screw ; its value is generally 1 mm. (or 0·5 mm). The bevelled edge of thimble is provided with a circular scale with 100 equal divisions. Each division on the circular scale represents a screw travel of 0·01 mm. (or 0·005 mm.), which is obtained by dividing screw pitch by total circular scale divisions. The fraction, being the least distance measurable by instrument, is called the ***least count***.

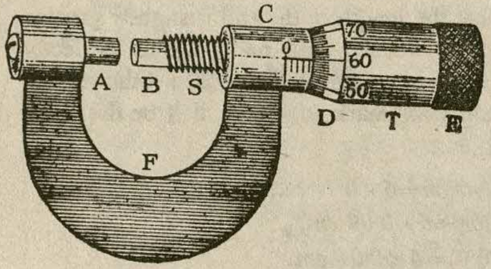


Fig. 3 : Screw Gauge

To take a reading, the milled head is rotated until the body is gripped very gently between the studs *A* and *B*. The position of the bevelled edge on the linear scale gives the reading (*m*) to the nearest mm. (or 0·5 mm). The position (*r*) of the circular scale with respect to the base line of the linear scale gives the fraction of mm. (or 0·5 mm.) to be added to *m*. Therefore, the required length $=m+r\times c$ where *c* is the least count, expressed in the same unit as *m*. The Fig. 3 shows micrometer screw gauge in which

Screw pitch=1 mm.
Total circular scale division=100
∴ Least count=0·01 mm.
Linear Scale reading=4 mm., and
Circular scale reading=59
∴ Length of the object=4+59×0·01=4·59 mm.=0·459 cm.

Instrumental error creeps in when on bringing the studs *A* and *B* in contract, the zero of the circular scale does not coincide with the zero of the linear scale. If the reading is positive (i.e., circular scale zero is yet to reach the linear scale zero) its value should be subtracted from the final reading. If it is negative (i.e., the circular scale zero crosses the linear scale zero), its value should be added to the final reading.

Spherometer : This instrument is designed mainly to measure the radius of curvature of spherical surfaces accurately upto thousandth part of a mm. (10^{-3} mm.)

It consists of a nut *N* at the centre of a triangular frame supported by three pointed legs *A*, *B* and *C*, tips of which form the vertices of an equilateral triangle (Fig. 4). A screw *S* moves through the nut and is provided with a milled head *M* and a circular scale *D*. A linear scale *L* is attached vertically with the frame. The least count of the instrument is measured in the same way as that of a micrometer screw-guage.

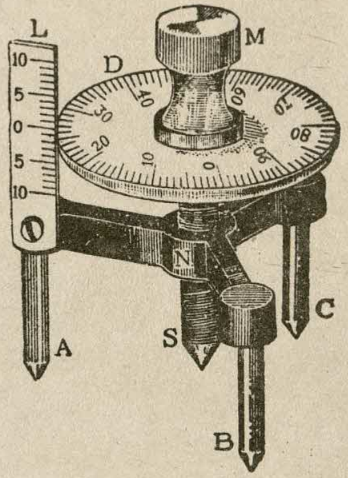


Fig. 4 : Spherometer

The use of spherometer requires a perfectly plane glass plate called the ***basic plate***. After placing it on the base plate, the screw is rotated until its tip just touches the surface. The reading on the linear scale just below the edge of the circular scale plus that on circular scale multiplied by the least count gives the initial position of the screw. The screw is then raised up, if necessary (for convex surface), and the spherometer is placed over the spherical surface. The screw tip is then again made to touch the spherical surface. This is the final position of the screw and the reading is noted. For concave surfaces, on placing the spherometer over the spherical surface, the screw tip is lowered to touch it.

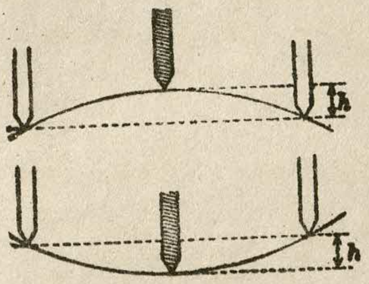


Fig. 5

Let the difference between the two readings be *h* (Fig. 5). If *d* be the length of a side of the equilateral triangle formed by the fixed legs, the radius of curvature *r* of the surfaces is given by

$$r=\frac{d^2}{6h}+\frac{h}{2}$$

d and *h* being expressed in the same unit. The proof of the formula is included in the appendix.

1

MECHANICS

Particle Dynamics

CHAPTER 1

VECTORS

1·1. Scalar and Vector Quantities.

Most of the quantities discussed in physics and in everyday affairs can be broadly classified into two types viz., *scalar* and *vector quantities.*

Let us consider the following—'the rod is 2 metres long', the vessel contains 200 gms. of water' and 'we waited there for five hours.' In these expressions, 2 metres long, 200 gms, five hours, we have some ideas of the magnitude of a length, a mass and a time. But length, mass or time has nothing to do with any direction. Such quantities which gave us an idea of some magnitude but are not associated with any direction are scalar quantities. Volume, density, energy, temperature, speed etc. are similar examples of scalar quantities. Thus, **scalar** *quantities are defined as those quantities which have magnitude but no direction.* Two or more scalar quantities of the same type can be added or subtracted by ordinary arithmetical laws. Thus, 5 sec.+ 3 sec.=8 sec., 20°C—5°C = 15°C. etc.

But when we say that the car was moving with a velocity of 40 km. per hour (note, both magnitude and unit have been given), or Mr. X walked from the hut through a distance of 30 metres, the description is not complete. The question that naturally arises, is—"in which direction ?" Thus, we must say that the car was moving south west or, Mr. X walked due east. Evidently, the velocity of the car and the displacement of Mr. X are not scalar quantities but they belong to some other type. These quantities are called *vector quantities.* Thus to specify completely a vector quantity, it is not sufficient to state its magnitude (a number) and unit, but the direction must also be given.

Thus any quantity which needs both direction in space and magnitude for its description is called a **vector** *quantity.* Other examples are acceleration, force, weight, momentum, etc. These quantities cannot be added or subtracted by simple arithmetical laws like scalars. Thus a man swimming across a river has a velocity which is not simply the arithmetical sum of the velocity of the man relative to water and the velocity of the water itself. The sum is obtained keeping in mind their directions ; the method is discussed, in detail, in Art. 1·3.

1·2. Vectors.

Vectors render themselves to a very simple geometrical representation. *A vector can be represented by an arrow-tipped straight line segment.* The length of the line drawn in a convenient scale represents the **magnitude** of the vector.

The **direction** of the vector quantity is depicted by placing an arrow at the end of the line (Fig. 1·1). Thus, for example, if a particle originally lying at a point O gets displaced to a point, say A, at a distance of 40 metres from O in a direction 60° north of east, this displacement of the particle being a vector quantity, can be represented by the directed line segment OA. The scale conveniently adopted is such that 1 cm. represents a displacement of 10 metres. The tail of the arrow, O, is called the **origin** ; the head, A, is called the **end** or **terminus**.

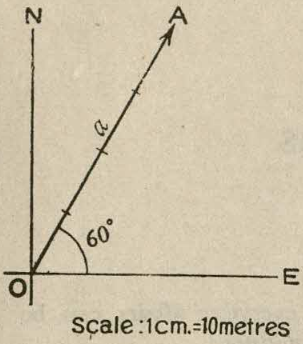


Fig. 1·1 : Representation of a vector

Thus, symbolically we can represent a vector by two letters, the first one indicating the origin and the second one the terminus, with a small arrow placed over them. Thus, in the previous example, the displacement vector can be written symbolically as $\overrightarrow{OA}$. This way of notation, though suitable for the students, is a bit cumbersome. In this book, we shall denote a vector by using a *bold-faced type* to differentiate them from scalar quantities. Thus the displacement vector $\overrightarrow{OA}$ would be represented simply as $\mathbf{a}$.

Two vectors $\mathbf{a}$ and $\mathbf{b}$ are said to be ***equal*** if they have equal lengths and the same direction, whatever their origin may be. This is denoted symbolically by $\mathbf{a}=\mathbf{b}$. Thus, in the Fig. 1·2, $\overrightarrow{OA}=\mathbf{a}$, and $\overrightarrow{O'A'}=\mathbf{b}$ are equal vectors.

The vector having the same magnitude as $\mathbf{a}$, but the opposite direction is defined as the **negative** of $\mathbf{a}$ and is denoted by $-\mathbf{a}$. Thus in Fig. 1·2, the vector $\overrightarrow{O''A''}$ is the negative of the vector $\overrightarrow{OA}$. Evidently, the converse is also true, i.e., the vector $\overrightarrow{OA}$ is the negative of the vector $\overrightarrow{O''A''}$.

Parallel vectors, whatever be their magnitude, are said to be **collinear**. Thus, in the Fig. 1·2, vectors $\overrightarrow{PQ}$, $\overrightarrow{OA}$, $\overrightarrow{O'A'}$, $\overrightarrow{O''A''}$ are all collinear.

Vectors all of which lie in or parallel to the same plane are said to be **coplanar**.

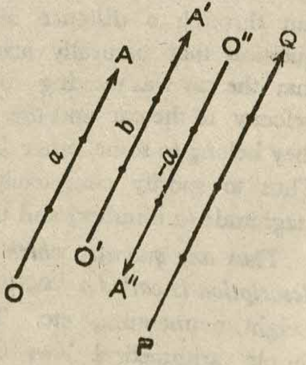


Fig. 1·2 : Illustration of equal negative and parallel vector

1·3. Composition or Addition of Vectors.

As mentioned earlier, vector quantities cannot be added arithmetically ; they are added by a more complicated process called **vector-addition**. In this process the directions as well as the magnitudes of all the vector quantities to be added are taken into account. This process can be carried out in two ways, one employing the ***geometrical*** and the other ***trigonometrical method.***

(a) Geometrical Method :

Let us again pay our attention to Mr. X. In one fine morning, out in a walk, he started from his hut, travelled due east through a distance of 30 metres and arrived on the bank of the river. This displacement can be represented by $\overrightarrow{OA}=\mathbf{a}$ (Fig 1·3). He found that the river was flowing at that spot from north to south. At some distance, he saw a tree on the bank of the river and walked due north through a distance of 40 metres and reached the tree. This displacement is represented by $\overrightarrow{AB}=\mathbf{b}$. Spending a little while there, he then returned to the hut in the same path. Next morning he started from the hut straight in the direction of the tree (well, he measured that the walk will be shorter) and reached the spot. This displacement is represented by $\overrightarrow{OB}=\mathbf{c}$.

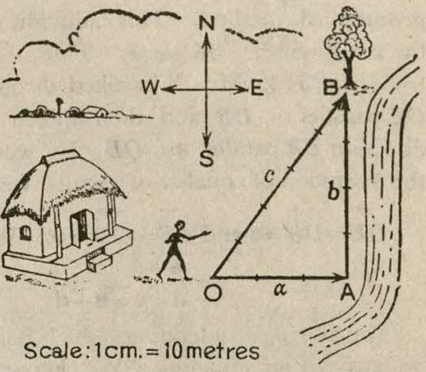


Fig. 1·3 : The resultant of two displacements at right angles to each other

Vectorially, this displacement **c** is equivalent to the displacement **a** plus the displacement **b**. This equivalence may be expressed as an equation :

$$\boxed{\begin{matrix}\text{Displacement}\\ \text{from } O \text{ to } A\end{matrix}} + \boxed{\begin{matrix}\text{Displacement}\\ \text{from } A \text{ to } B\end{matrix}} = \boxed{\begin{matrix}\text{Displacement}\\ \text{from } O \text{ to } B\end{matrix}}$$

or, vectorially, $\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB}$ i.e., $\mathbf{a}+\mathbf{b}=\mathbf{c}$, ... (1·1)

c is the **resultant** of the two displacements **a** and **b**.

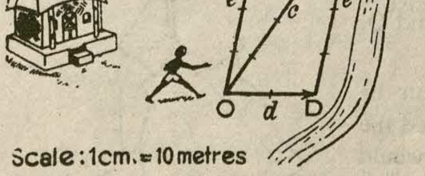


Fig. 1·4 : The resultant of two displacements which are inclined at any angle to each other

It is important to note that the equation (1·1) does not mean that the sum of the magnitudes of the first two vectors is equal to that of the third. Evidently, from the Fig. 1·3, the length *OB* is less than the sum of the lengths of *OA* and *AB*.

For such additions to be performed, it is ***not*** necessary that the two displacements are to be ***at right angles to each other***. Thus, one day Mr. X cut short his walk towards the river by going through a distance of 20 metres [represented by the vector $\overrightarrow{OD}=\mathbf{d}$ in the Fig. 1·4], turned towards the tree and walking through a distance of 41·2 metres in that direction, he reached there. This displacement is represented by $\overrightarrow{DB}=\mathbf{e}$. Evidently, as before $\mathbf{d}+\mathbf{e}=\mathbf{c}$.

We thus see that two displacement vectors can be added by simple geometrical method. This addition can be carried out in the reverse order and the result will be the same. Thus, if instead of moving first towards the river from the hut, Mr. X walked through a distance of 41·2 metres in the direction *OE* parallel to *DB* and then walked through a distance of 20 metres in the direction *EB* parallel to *OD*, he would have arrived at the same point. From the definition of equality of vectors, we have

$\overrightarrow{OE}=\overrightarrow{DB}=\mathbf{e}$ and $\overrightarrow{EB}=\overrightarrow{OD}=\mathbf{d}$. We thus get,

$$\mathbf{e}+\mathbf{d}=\mathbf{c}$$

$$\therefore \quad \mathbf{d}+\mathbf{e}=\mathbf{e}+\mathbf{d} \qquad \ldots \qquad \ldots \qquad (1\cdot2)$$

The above considerations which hold for displacement vectors are also true for any vector quantity. We know that force acting on a body is a vector quantity. *The combined effects of the two forces acting on a body can be represented by the effect of a single force acting on the body, which is called the resultant force*. Two simple examples are cited here.

On a windless day, when sails are ineffective, a boat is usually towed along a canal with the help of long ropes [Fig. 1·5]. The pulls exerted by the two boatmen on the two banks of the canal are represented in direction and magnitude by the vectors **P** and **Q**. The boat advances not along the direction of either **P** or **Q** but along the direction of their resultant force **R**, where **R**=**P**+**Q**.

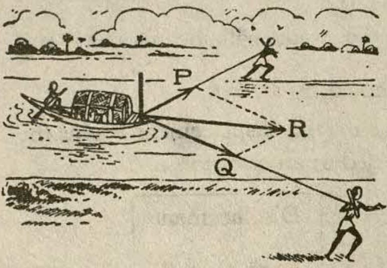


Fig. 1·5 : The resultant of two forces

As another example, let us picture a boy aiming his sling at a bird (Fig. 1·6). He pulls the two rubber cords backwards so that forces **P** and **Q**, along the lengths of the stretched cords, begin to act upon the piece of stone held in the boy's hand. When released, the stone is hurled forward along the direction of the resultant of **P** and **Q**, not along the direction of any one of them.

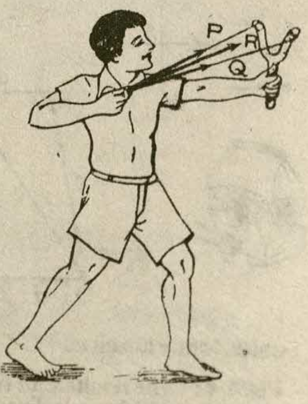


Fig. 1·6 : The stone moves along the direction of the resultant

Thus, we conclude that two vectors can be added to give a third vector, which is called the resultant of the two vectors and which would produce the same effect as that of the original vectors together. The process of finding the resultant is called the **combination** or **composition** or **addition** of vectors. The magnitude of the resultant is not, in general, equal to the sum of the magnitudes of the components. Also, it should be borne in mind that only vectors of the same kind can be combined. As for example, to add a displacement vector to a force vector is absurd.

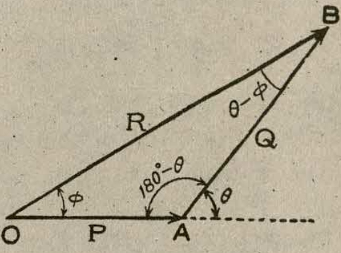


Fig. 1·7 : Triangle law of addition of vectors

To add the two vectors **P** and **Q**, the vectors are drawn with the origin of **Q** coinciding with the terminus of **P**. The vector sum i. e., the resultant is the vector **R** which completes the triangle, drawn from the origin O of **P** to the terminus B of **Q** as shown in the Fig. 1·7. This is called the **triangle law of addition of vectors.**

A rather similar geometric method is to draw the two vectors **P** and **Q** from a common origin as in Fig. 1·8. A parallelogram is drawn. Then the diagonal of the parallelogram passing through the common origin of **P** and **Q** represents, in magnitude and direction, the resultant vector, **R.** This is known as **parallelogram law of vector addition.** Its similarity with triangle law is obvious because the line AB being equal and parallel to OC also represents the vector **Q.**

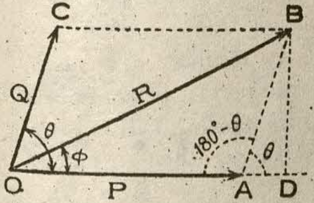


Fig. 1·8 : Parallelogram law of vector addition

When both vectors are in the same direction or in opposite direction, the resultant vector is as shown in the Fig. 1·9 (a) and 1·9 (b). Only in this case, the magnitude of the resultant is equal to the sum (or difference) of the magnitudes of the components.

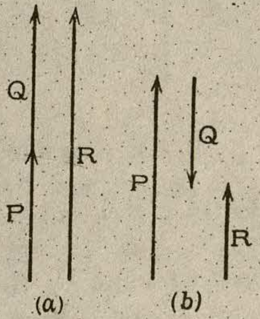


Fig. 1·9 : Addition of vectors which act in the same or opposite direction

(b) Analytical Method :

The magnitude and the direction of the resultant **R** of any two vectors **P** and **Q** can also be determined using geometrical and trigonometrical laws. We should keep in mind that the length of a line-segment representing a vector gives its magnitude. We drop perpendicular BD from B on OA. Let θ be the angle between **P** and **Q** [Fig. 1·8]. Then from the right-angled triangle ODB,

$$OB^2 = OD^2 + DB^2 = (OA + AD)^2 + DB^2$$

$$= OA^2 + AD^2 + 2\,OA.AD + DB^2$$

$$= OA^2 + AB^2 + 2\,OA.\,AB\cos\theta$$

$$[\because\ AD^2 + DB^2 = AB^2 \text{ and } AD = AB\cos\theta\ ].$$

$$\text{or, } R^2 = P^2 + Q^2 + 2\,PQ\cos\theta \quad \ldots \quad \ldots \quad (1\cdot3)$$

To find the direction of **R**, let ϕ be the angle between **R** and **P** ; then,

$$\tan\phi = \frac{BD}{OD} = \frac{AB\sin\theta}{OA + AD} = \frac{AB\sin\theta}{OA + AB\cos\theta} = \frac{Q\sin\theta}{P + Q\cos\theta} \quad \ldots \quad (1\cdot4)$$

from which we can determine the angle ϕ.

Special cases :

(i) If **P** and **Q** are at right-angles to each other, $\theta=90°$.
Then since $\cos 90°=0$.

$$R=\sqrt{P^2+Q^2} \text{ and } \tan\phi=\frac{Q}{P} \quad \cdots \quad \cdots \quad (1{\cdot}5)$$

(ii) If **P** and **Q** are in the same direction, $\theta=0°$. Hence

$$R^2=P^2+Q^2+2PQ \text{ or, } R=P+Q \text{ and } \phi=0 \quad \cdots \quad \cdots \quad (1{\cdot}6)$$

The resultant is thus in the same direction.

(iii) If **P** and **Q** are in the opposite direction, $\theta=180°$.

$R^2=P^2+Q^2-2PQ$

If $P>Q$, $R=P-Q$, $\phi=0$ … … (1·7)

The resultant is thus in the direction of the vector of greater magnitude.

(c) Polygon rule :

The previous method for combining two vectors by placing them end to end can be generalized for any number of vectors. We first find the resultant of any two, then combine this resultant with a third and so on. For example, let us consider a particle on which four forces **a**, **b**, **c** and **d** are acting simultaneously [Fig. 1·10 (*a*)]. We first combine **a** and **b** by the triangle method to get their resultant, **e**=**a**+**d** ; **e** is then combined similarly with **c** giving their resultant, **f**=**e**+**c**=**a**+**b**+**c**.

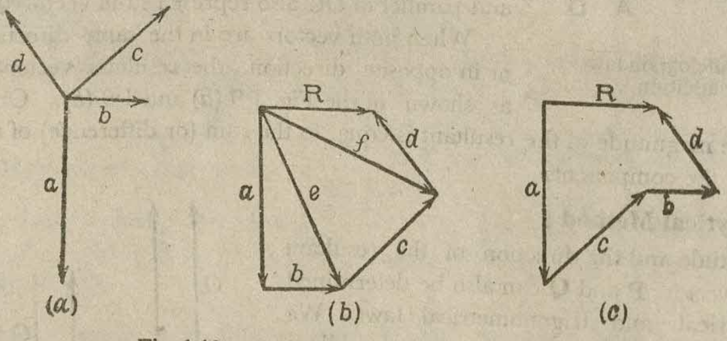


Fig. 1·10 : Polygon rule of addition of vectors

Finally, **f** and **d** are combined to get the final resultant [Fig. 1·10 (*b*)],

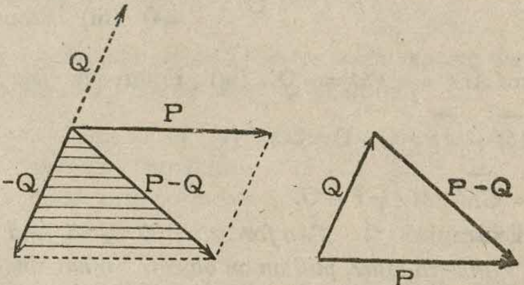
$$\mathbf{R}=\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d} \quad \cdots \quad \cdots \quad (1{\cdot}8)$$

It is unnecessary to draw the intermediate resultants **e** and **f** ; we could have proceeded by drawing the vectors in succession such that the origin of one coincides with the terminus of the previous and then completing the polygon from the origin of the first to the terminus of the last one to get the resultant vector **R**. This is known as the **polygon rule**. Evidently, the shape of the figure gives the name of the rule. The order in which the vectors are drawn is immaterial ; the same resultant will be obtained, even though orders are changed. This can be seen from Fig. 1·10 (*c*).

1·4. Subtraction of vectors.

To subtract two vector, one vector is added to the negative of the second. If **P** and **Q** are two vectors, the vectors difference **P**—**Q** can be written as, **P**+(—**Q**) [just as is done in arithmetic] i. e., it is the vector sum of the vectors **P** and —**Q**. The method is shown in the Fig. 1·11 (*a*). The vectors **P** and **Q** are drawn from a common origin. The vector of same length as **Q** but opposite in direction then represents —**Q**. This vector is also drawn from the same origin. The resultant of **P** and —**Q** is then found by the parallelogram method.

Alternatively, the vector differences can also be found by the triangle method. **P** and **Q** are drawn from a common origin ; the vector from the terminus of **Q** to the terminus of **P**, then represents the vector **P**—**Q**. Comparison of Fig. 1·11 (*b*) with the shaded triangle of Fig. 1·11 (*a*) shows that this is so. Confirmation is also available from Fig 1·11 (*b*), if we note that the vector **P** is the vector sum of the vectors **Q** and **P**—**Q**.

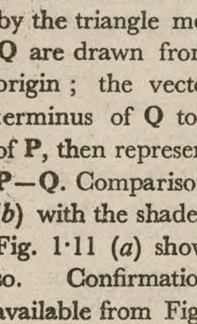


(*a*) (*b*)

Fig. 1·11 : Subtraction of vectors

1·5. Multiplication of a vector by a real number.

From the previous discussion on the addition of vectors, it follows that **a**+**a**+**a** ......to *n* terms, is a vector in the same direction as the vector **a** and with length *n* times that of the vector **a**. This vector is thus represented by *n***a** [Fig. 1·12 (*a*)]. Hence a vector multiplied by a number is also a vector which is parallel to the original vector.



(a) (b)

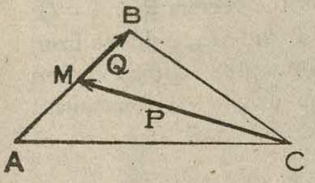
Fig. 1·12 : Multiplication of vector by a real number

This leads us to an important representation of a vector and to the concept of **unit vector**. The direction of any vector **a** may be specified by a vector of unit length in the same direction and sense. This vector is called a unit vector and is represented by the symbol $\mathbf{a}_1$. The vector **a**, then, is represented by multiplying this unit vector $\mathbf{a}_1$ by its magnitude, *a* (printed in italic type). The vector **a** is thus '*a*' times as long as $\mathbf{a}_1$, or, $\mathbf{a}=a\mathbf{a}_1$ [Fig. 1·12 (b), here $a=2$].

The students may be interested to know about the product of two vectors. It can be shown that the product of two vectors can be either a scalar, or a vector. An example of product of two vectors being a scalar has been given in Art. 7·1, while an example of the product being a vector has been given in Art. 5·6.

Example 1.1 *In a triangle ABC, M is the mid-point of the side AB. The line segments CM and MB represent the vectors* **P** *and* **Q** *respectively. Express* *each of the following directed line segments in terms of vectors* **P** *and* **Q** (*i*) *CB*, (*ii*) *AM*, (*iii*) *MA*, (*iv*) *AB*, (*v*) *CA*.

Solution : (i) From the triangle law of addition of vectors,



$$\overrightarrow{CB}=\overrightarrow{CM}+\overrightarrow{MB}=\mathbf{P}+\mathbf{Q}$$

(ii) $\overrightarrow{AM}$ being equal in length to $\overrightarrow{MB}$ and the directions of both being the same $\overrightarrow{AM}=\overrightarrow{MB}=\mathbf{Q}$ (iii) From the definition of negative vector $\overrightarrow{MA}=-\overrightarrow{AM}=-\mathbf{Q}$ (iv) From the law of composition vectors, $\overrightarrow{AB}=\overrightarrow{AM}+\overrightarrow{MB}=\mathbf{Q}+\mathbf{Q}=2\mathbf{Q}$. (v) From the triangle law of addition of vectors, $\overrightarrow{CA}=\overrightarrow{CM}+\overrightarrow{MA}=\mathbf{P}-\mathbf{Q}$.

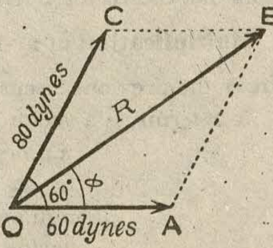
Example 1·2. *Two forces, of 60 dynes and 80 dynes acting at an angle of 60° with each other, pull on an object. What single pull would replace the given force ?*

Solution : The two forces are drawn from a common origin, *O*, making an angle of 60°. *OA* and *OC* represent the forces 60 dynes and 80 dynes respectively. The parallelogram *OABC* is completed. The diagonal *OB* represents the resultant *R*. Then,

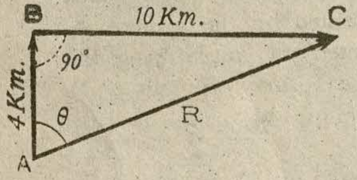
$$R^2=60^2+80^2+2.\ 60.\ 80\ \cos 60^\circ$$
$$=3600+6400+4800=14800$$
$$\therefore\ R=121{\cdot}7 \text{ dynes.}$$

The angle ϕ is given by,

$$\tan\phi=\frac{80\sin 60^\circ}{60+80\cos 60^\circ}$$
$$\therefore\ \phi=34{\cdot}7^\circ.$$



Example 1·3. *What is the resultant of a displacement 4 km. N and one 10 km. E ?*



Solution : $\overrightarrow{AB}$ represents a northward displacement of 4 km. and $\overrightarrow{BC}$ an eastward displacement of 10 km. From the triangle law of addition of vectors, the resultant displacement is given by $\overrightarrow{AC}$. Let θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$\therefore$ The magnitude of the resultant is given by

$$R=\sqrt{10^2+4^2}=\sqrt{116}=10{\cdot}78 \text{ km.}$$

and direction by $\tan\theta=\frac{10}{4}=2{\cdot}5$, $\therefore\ \theta=68{\cdot}2^\circ$

[illegible] ultant displacement is 10·78 km. at 68·2° E of N.

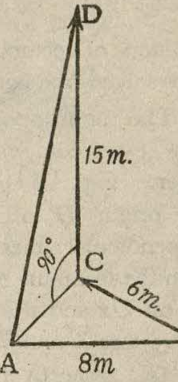
Example 1·4. *What is the angle between two equal forces so that their resultant is one-third of one of the forces ?*

Solution : Let θ be the angle between two equal forces of magnitude P. The magnitude of the resultant is $P/3$. Hence

$$\frac{P^2}{9}=P^2+P^2+2\ P.P\cos\theta$$

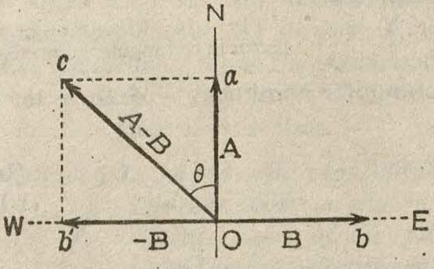
or, $\frac{1}{18}=1+\cos\theta \quad \therefore \cos\theta=-\frac{17}{18}=-0{\cdot}945 \quad \therefore \theta=160^\circ$.

Example 1·5. *At take off, a helicopter rises 15 metres while moving north 8 metres and west 6 metres. How far is it from its starting position ?*



Solution : $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CD}$ represent the northward, the westward and the vertical displacements of the helicopter respectively. From the polygon rule of addition of vectors, $\overrightarrow{AD}$ represents the resultant displacement. Join AC. Then, $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$ and $\overrightarrow{AD}=\overrightarrow{AC}+\overrightarrow{CD}$. Their magnitudes are given by, $AC=\sqrt{AB^2+BC^2}=\sqrt{8^2+6^2}=10$ m. and $AD=\sqrt{AC^2+CD^2}=\sqrt{10^2+15^2}=18{\cdot}03$ m.

Hence the helicopter is at a distance of 18·03 metres from its starting point.



Example 1·6. *Given vector* **A**$=30$ *m./sec. north and vector* **B**$=40$ *m./sec. east, find the vector difference* **A**$-$**B**.

Solution : Vectors **A** and **B** are drawn from a common origin O. The vector $\overrightarrow{Ob'}$, equal in magnitude of **B** but oppositely directed, represent $-$**B**. The diagonal Oc of the parallelogram with sides Oa and Ob' represents the vector difference **A**$-$**B**. The magnitude is given by $Oc=\sqrt{Oa^2-Ob'^2}=\sqrt{30^2+40^2}=50$ m./sec. and its direction, by $\tan\theta=\frac{ac}{Oa}=\frac{Ob'}{Oa}=\frac{40}{30}$ $\theta=53{\cdot}1^\circ$. Hence the vector difference is 50 m./sec. at an angle 53·1° W of **N**.

1·6. Resolution of vectors.

The process of ***resolving a vector*** into its components amounts to determining a set of vectors, whose resultant is the given vector. Each vector in that set is called a ***component*** of the given vector. Thus this ***process of resolution inverse to the process of vector addition*** [with one point of difference, as we shall see later].

A vector can be resolved into any number of components. The components are found by joining them end to end as in vector addition, such that the first one begins at the origin of the given vector and the last one ends at its terminus. Thus in Fig. 1·13 (*a*), the vector **R** has been resolved into four components, while in Fig. 1·13 (*b*), it has resolved into two components.

Thus, in Fig 1·13 (*a*) $\mathbf{R}=\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}$; in Fig. 1·13 (*b*), $\mathbf{R}=\mathbf{e}+\mathbf{f}$.

It should be remembered that these component vectors need not lie in one plane.

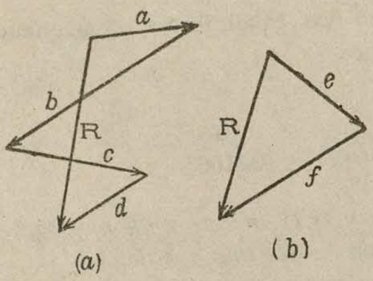


Fig. 1·13 : Resolution of vector

We can now point out the difference of the process of resolution with that of addition of vectors. While addition always gives a unique resultant, a vector can be resolved into components in a number of ways.

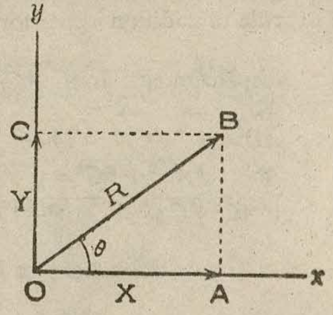


Fig. 1·14 : Resolution of a vector in rectangular components

The most important process is that in which a vector is resolved in one plane into two components at right angles to each other. Fig. 1·14 illustrates this process. From the origin *O* of the vector **R**, two mutually perpendicular lines *Ox* and *Oy* are drawn. Then, from the terminus *B* of the vector, perpendiculars *BA* on *Ox* and *BC* on *Oy* are dropped. Evidently the vectors **X** and **Y** represented by the lines *OA* and *OC* respectively, are the components of the vector **R**. This can be easily understood if we remember that the line *AB*, being parallel and equal to *OC*, also represents the vector **Y** and the simple vector addition of **X** and **Y** then gives the vector **R**. **X** and **Y** are called the **rectangular components** of the vector **R**, the magnitudes of **X** and **Y** are

$$X=R\cos\theta,\ Y=R\sin\theta \qquad \ldots \qquad \ldots \qquad (1{\cdot}9($$

$$\text{so that, } R=\sqrt{X^2+Y^2} \qquad \ldots \qquad \ldots \qquad (1{\cdot}10)$$

where θ is the angle between **R** and **X**.

In solving a problem, we can keep the vector **R** aside and instead carry on with the two components **X** and **Y** only. These two, taken together, are equivalent in all respects to the single vector **R**.

A vector has no component and no effect in a direction perpendicular to its own direction, since for $\theta=90°$, $\cos\theta=0$. Thus **X** has no component along the *y*-axis and **Y** has no component along the *x*-axis. This points to a conclusion of immense importance—***the effects on a body of the two rectangular components can be considered separately.***

As an example, let us consider the case of a ship being towed by a tugboat by means of a line. If the tension in the line be **T**, then this force applied to the ship by the tugboat, is represented by the line segment *OA* in a convenient scale

[Fig. 1·15]. The ship is moving forward in the horizontal direction *OX*. *OY* represents the vertically downward direction. The force vector **T** is resolved into

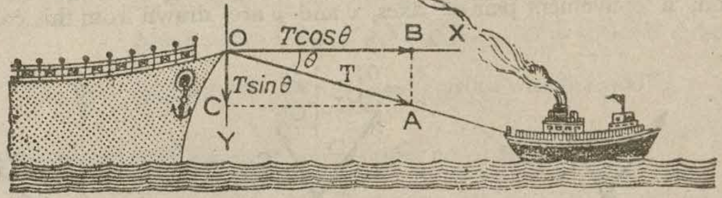


Fig. 1·15 : Illustration of the resolution of a vector in rectangular components

two rectangular components in the said directions. These components **T** cos θ and **T** sin θ are represented by *OB* and *OC* respectively. Obviously in considering the motion of advance of the ship through water, only the component **T** cos θ is taken into account as the effective force. The other component **T** sin θ only produces a downward pull on the bow of the ship and does, in no way, contribute to the forward motion of the ship.

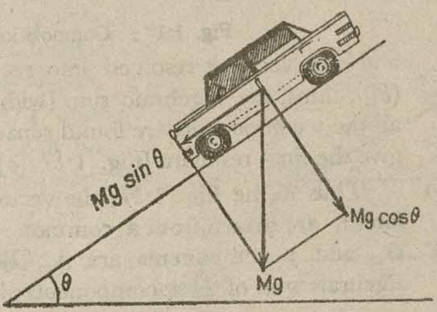
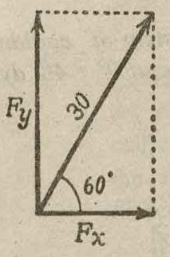


Fig. 1·16 : Illustration of the resolution of a vector in rectangular components

As another example (Fig. 1·16), imagine a motor car moving down an inclined plane with its engine shut. The force acting on it, is its own weight, *Mg*, acting vertically downwards. This force can be resolved into two rectangular components, *Mg* sin θ along, and *Mg* cos θ perpendicular to the surface of the incline. Thus the effective force under whose action the car rolls down the incline is *Mg* sin θ. The other component *Mg* cos θ is balanced by the reaction force of the surface.

Example 1·7. ***A boy pulls a rope attached to a box with a force of 30 dynes. The rope makes an angle of 60° with the ground. Compute the effective value of the pull tending to move the box along the ground and the effective value tending to lift the box vertically.***



Solution : The force tending to move the box along the ground is the horizontal component F_x. The force tending to the box off the ground is the vertical component F_y. Thus

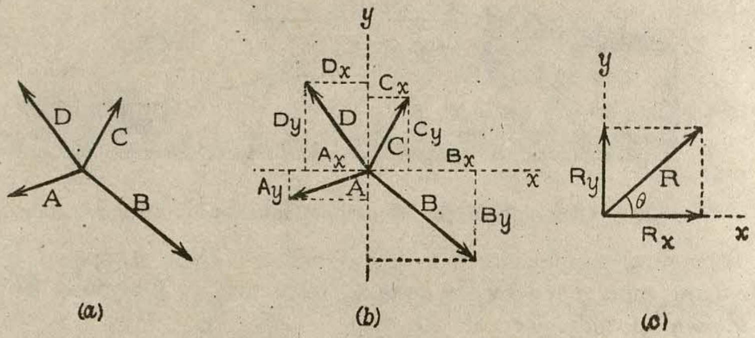
$$F_x = 30 \cos 60° = 15 \text{ dyne.}$$

$$F_y = 30 \sin 60° = 26 \text{ dyne (nearly)}$$

1·7. Composition of vectors by rectangular resolution.

The previously discussed polygon method of composition of a number of vectors is though geometrically satisfactory, but it is analytically a bit awkward, because, in general, this involves working with oblique triangles. A simple and elegant method is provided by employing the conception of resolution of a vector in rectangular components. We shall restrict our discussions to a system of co-planar vectors only. The method is as follows :

First, all the vectors are drawn from a common origin [Fig. 1·17 (*a*)] ; second, a convenient pair of axes, x and y are drawn from this common origin and each vector is resolved into rectangular components along the axes [Fig. 1·17 (*b*)] ; third, the algebraic sum (with proper consideration of signs) of all the x and all the y components are found separately ; and fourth, these sums are combined to give the final resultant [Fig. 1·17 (*c*)].



(*a*) (*b*) (*c*)

Fig. 1·17 : Composition of vector by rectangular resolution

Thus in the Fig. 1·17, the vectors **A**, **B**, **C**, and **D** whose resultant is to be found, are drawn from a common origin. Their x components are $\mathbf{A}_x$, $\mathbf{B}_x$, $\mathbf{C}_x$, $\mathbf{D}_x$, and y components are $\mathbf{A}_y$, $\mathbf{B}_y$, $\mathbf{C}_y$, and $\mathbf{D}_y$ respectively. If we call the algebraic sum of these components along x-axis, $\mathbf{R}_x$, and that along y-axis, $\mathbf{R}_y$ then

$$\left.\begin{aligned}\mathbf{R}_x&=\mathbf{A}_x+\mathbf{B}_x+\mathbf{C}_x+\mathbf{D}_x\\ \mathbf{R}_y&=\mathbf{A}_y+\mathbf{B}_y+\mathbf{C}_y+\mathbf{D}_y\end{aligned}\right\} \quad \cdots \quad (1{\cdot}11)$$

Some of these components may be negative ; thus in the Fig. 1·17 (*b*), $\mathbf{A}_x$, $\mathbf{D}_x$, $\mathbf{A}_y$, and $\mathbf{B}_y$ are negative.

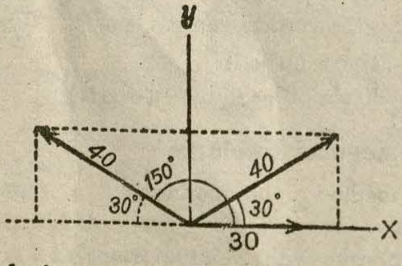
The magnitude of the resultant **R** is obviously given be

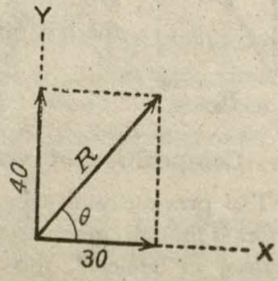
$$R=\sqrt{R_x^2+R_y^2} \quad \cdots \quad \cdots \quad \cdots \quad (1{\cdot}12)$$

and its direction by $\tan\theta=\dfrac{R_y}{R_x}$ or, $\sin\theta=\dfrac{R_y}{R}$ $\quad \cdots \quad \cdots \quad (1{\cdot}13)$

where θ is the angle which it makes with the x-axis.

Example 1·8. *Compute the resultant of the following system of coplanar forces the angles being given with respect to the x-axis : 30 dyne at 0° ; 40 dyne at 30° ; 40 dyne at 150°.*





Solution : Each force is resolved into x- and y-components. The algebraic sum of these components along x-axis gives R_x and that along y-axis R_y.

$$R_x=30+40\cos 30^\circ-40\cos 30^\circ=30 \text{ dyne.}$$

$$R_y=0+40\sin 30^\circ+40\sin 30^\circ=40.\tfrac{1}{2}+40.\tfrac{1}{2}=40 \text{ dyne.}$$

$$\text{Magnitude of } \mathbf{R}=\sqrt{R_x^2+R_y^2}=\sqrt{30^2+40^2}=50 \text{ dyne.}$$

$$\text{and } \tan\theta=\frac{R_y}{R_x}=\frac{40}{30}=\frac{4}{3}, \text{ from which } \theta=53^\circ.$$

1·8. Representation of a vector by co-ordinates.

Let us consider any vector **r**. From the origin O of the vector we draw three mutually perpendicular lines OX, OY and OZ respectively (Fig. 1·18). These are known as right-handed *cartesian system of axes*. Inth e figure, OY and OZ are in the plane of the paper and OX perpendicular to the plane, points towards the reader. Drop perpendicular CB from the terminus C of the vector **r** on the plane passing through OX and OY. Draw perpendicular BA from B on the axis OX. Join OB. Evidently from the scheme of vector addition,

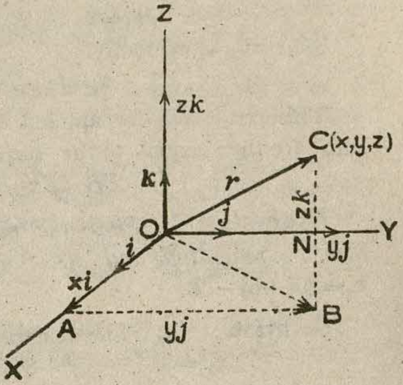


Fig. 1·18 : Representation of a vector by co-ordinates

$$\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB}\;;\quad \overrightarrow{OB}+\overrightarrow{BC}=\overrightarrow{OC}=\mathbf{r}$$

$$\therefore\quad \mathbf{r}=\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{BC}$$

Now, if the co-ordinates of the terminus C of the vector **r** be x, y and z respectively then it is obvious that the magnitude or length of OA is equal to x ; similarly, those of AB and of BC are y and z respectively. Thus if the three unit vectors along the X, Y, Z axes be **i**, **j**, **k** repectively, then

$$\overrightarrow{OA}=x\,\mathbf{i},\ \overrightarrow{AB}=y\,\mathbf{j},\ \overrightarrow{BC}=z\,\mathbf{k}\ ;$$

$$\therefore\ \mathbf{r}=x\,\mathbf{i}+y\,\mathbf{j}+z\,\mathbf{k} \qquad \cdots \qquad \cdots \qquad (1{\cdot}14)$$

Hence, any vector **r** can be represented in this way in terms of the co-ordinates of its terminus with respect to a cartesian system of axes, passing through the origin of the vector and three unit vectors along the three axes. x, y, z are called the **rectangular components**, or **resolutes**, or resolved parts of **r** along those directions.

It is evident from the figure that,

$$OC^2=OB^2+BC^2=OA^2+AB^2+BC^2$$

$$\text{so that, } r^2=x^2+y^2+z^2 \qquad \cdots \qquad \cdots \qquad (1{\cdot}15)$$

Thus, the square of the magnitude of a vector is equal to the sum of the squares of its rectangular components.

If the angles between the vector $\mathbf{r}$ and the axes OX, OY, OZ be α, β, γ respectively, then

$$\cos\alpha=\cos\angle COA=\frac{OA}{OC}=\frac{x}{r}$$

$$\left.\begin{aligned} \therefore\quad \alpha&=\cos^{-1}\frac{x}{r}\\ \text{Similarly,}\quad \beta&=\cos^{-1}\frac{y}{r}\\ \gamma&=\cos^{-1}\frac{z}{r}\end{aligned}\right\} \quad \cdots \quad \cdots \quad (1\cdot16)$$

$\cos\alpha$, $\cos\beta$, and $\cos\gamma$ are called the *direction cosines* of the vector $\mathbf{r}$.

If two vectors are given,

$$\mathbf{r}_1=x_1\,\mathbf{i}+y_1\,\mathbf{j}+z_1\,\mathbf{k} \text{ and } \mathbf{r}_2=x_2\,\mathbf{i}+y_2\,\mathbf{j}+z_2\mathbf{k}$$

their sum is evidently,

$$\mathbf{r}_1+\mathbf{r}_2=(x_1+x_2)\,\mathbf{i}+(y_1+y_2)\,\mathbf{j}+(z_1+z_2)\,\mathbf{k} \quad \cdots \quad (1\cdot17)$$

This result may be applied to any number of vectors ; the components of the sum are then equal to the sum of the components, so that,

$$\Sigma r=\mathbf{i}\Sigma x+\mathbf{j}\Sigma y+\mathbf{k}\Sigma z \quad \cdots \quad \cdots \quad (1\cdot18)$$

Example 1·9. *Find the magnitude of the vector* $\mathbf{r}_1=5\mathbf{i}-6\mathbf{j}+8\mathbf{k}$. *Find also the magnitude and direction of the resultant of this vector and the vector* $\mathbf{r}_2=20\mathbf{i}+6\mathbf{j}-8\mathbf{k}$.

Solution : (*a*) The magnitude of $\mathbf{r}_1$ is given by

$$r_1^2=5^2+(-6)^2+8^2=125 \text{ or, } r_1=11\cdot2 \text{ unit}$$

(*b*) The resultant of the two vectors is

$$\mathbf{r}=\mathbf{r}_1+\mathbf{r}_2=(5+20)\,\mathbf{i}+(-6+6)\,\mathbf{j}+(8-8)\,\mathbf{k}=25\,\mathbf{i}.$$

Hence the magnitude of the resultant is 25 unit and it is directed along X-axis.

● EXERCISE ●

[A] Essay type questions

1. Distinguish between vector and scalar quantities, giving examples from different branches of physics. How could you geometrically represent a vector ?

2. What do you mean by the resultant of a set of vectors ? How can you obtain the resultant of two vectors inclined at an angle θ ? What is the resultant of two collinear vectors which act in the same direction or in the opposite direction ?

3. Explain the difference of scalar addition and vector addition. Write down the triangle law of addition of vectors. Is there any discord between this law and the parallalogram law of vector addition ? Explain.

4. State the parallelogram law of vector addition. With the help of this law, determine the magnitude and direction of the resultant of two vectors acting at the same point and inclined to each other at any angle. What will be the magnitude and direction of the resultant if the two vectors be at right angles to each other ?

5. State the polygon rule of vector addition. Prove this law with the help of triangle law of vector addition.

6. What is the difference between scalar subtraction and vector subtraction ? Explain the method of subtraction of vectors.

7, What do you mean by the component of a vector in ag iven direction ? How can you determine the component of a vector in a given direction? Is it possible to have any number of components for a given vector? Explain.

8. What is resolution of vectors? What do you mean by rectangular components of a vector? Determine their magnitudes.

9. Explain how can you obtain the resultant of a set of vectors by the method of rectangular resolution.

10. What is a unit vector? What is the significance of multiplying a vector by a real number?

11. How can you represent a vector by co-ordinates in three dimensional space? Discuss the method of finding the resultant of two vectors by this method.

[B] Short answer type questions

1. Which of the following quantities are vectors and which are scalars?
Mass, Velocity, Specific gravity, Power, Time, Acceleration, Momentum, Energy, Speed, Temperature, Specific heat, Torque, Moment of a force.

2. Define :—Coplanar vector, Collinear vector, Unit vector, Zero vector, Negative vector.

3. Prove that if three vectors can be represented in magnitude and direction by the three sides of a triangle taken in order, their resultant would be zero.

4. (i) Can the resultant of two vectors inclined to each other at any angle be zero? (ii) Can the magnitude of the resultant of two vectors be smaller than the magnitude of either? Explain your answer.

5. **A**, **B** and **C** are three vectors. Prove that (i) $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$ (ii) $\mathbf{A}+\mathbf{B}+\mathbf{C}=\mathbf{A}+\mathbf{C}+\mathbf{B}$

6. Two vectors **A** and **B** of equal magnitude are at right angles to each other. Prove that the vectors ($\mathbf{A}+\mathbf{B}$) and ($\mathbf{A}-\mathbf{B}$) have equal magnitude and are normal to each other.

7. "The combined action of two or more vectors can be represented by that of a single vector"—Justify the statement with the help of suitable examples. What is the name given to this single vector?

8. Can any one of the components of a vector have a greater magnitude than that of the given vector itself? What would be your answer for the rectangular component of a vector?

9. Can a vector have a component at right angles to itself?

10. "We can consider the effects of the two rectangular components of a vector separately" —Justify the statement with some practical examples.

11. Which one of the following statements is correct? Justify your answer.
(i) A vector has only one component, (ii) A vector has only two components. (iii) A vector has countless components.

12. Two vectors are acting at a point. If one of them be reversed in direction, their resultant turns through an angle of 90°. In this case, which one of the following statements is correct? Explain your answer with suitable reasons.
(i) Two vectors are unequal in magnitude.
(ii) Two vectors are equal in magnitude.
(iii) The magnitude of one vector is double that of the other.

13. The magnitudes of the vectors **P** and **Q** are 5 and 10 unit respectively. The magnitude of their resultant can never be equal to (i) 12 (ii) 5 (iii) 4 (iv) 8 unit. Which one of the above answers is correct? Why?

14. Two vectors **P** and **Q** have respectively the magnitudes 4 and 3 unit. The maximum and the minimum values of the magnitude of their resultant are (i) 12 (ii) 1 (iii) 7 (iv) 5 unit. Point out the two correct answers with suitable reasoning.

Problems.

1. Perform graphically the following vector additions and subtractions.
(a) $\mathbf{A}+\mathbf{B}$, (b) $\mathbf{A}+\mathbf{B}+\mathbf{C}$, (c) $\mathbf{A}-\mathbf{B}$, (d) $\mathbf{A}+\mathbf{B}-\mathbf{C}$.

2. If the sides *AB* and *AD* of a parallelogram *ABCD* represent the vectors **P** and **Q** what are the vectors represented by (i) *AC*, (ii) *CD*, (iii) *BC*, (iv) *BD*.

[*Ans.* (i) $\mathbf{P}+\mathbf{Q}$, (ii) $-\mathbf{P}$, (iii) **Q**, (iv) $\mathbf{Q}-\mathbf{P}$]

3. *CD* is the median of a triangle *ABC*. If *CD* and *DB* represent the vectors **P** and **Q**, what are the vectors represented by (i) *CB*, (ii) *AD*, (iii) *AB* (iv) *CA* ?

[*Ans.* (i) $\mathbf{P}+\mathbf{Q}$, (ii) **Q**, (iii) 2**Q**, (iv) $\mathbf{P}-\mathbf{Q}$.]

4. Find the properties of two vectors **A** and **B** when (a) $\mathbf{A}+\mathbf{B}=\mathbf{C}$ and $A+B=C$, (b) $\mathbf{A}+\mathbf{B}=\mathbf{A}-\mathbf{B}$, (c) $\mathbf{A}+\mathbf{B}=\mathbf{C}$, and $A^2+B^2=C^2$, (b) $\mathbf{A}-\mathbf{B}=\mathbf{C}$ and $A+B=C$. [*Ans.* (a) **A** and **B** are parallel (b) $\mathbf{B}=0$ (c) **A** perpendicular to **B**, (d) **A** and **B** are in opposite direction.]

5. Can two force of magnitude 6 kg. and 10 kg. respectively have a resultant of 12 kg. ? What are the greatest and least values that the magnitude of their resultant can have ? Explain.

[*Ans.* Yes ; 16 kg. and 4 kg.]

6. A vector **r** is the resultant of two vectors **a** and **b** which make angles of 30° and 60° respectively with it on opposite sides. How large are the latter vectors ?

[*Ans.* $\mathbf{a}=0{\cdot}866\mathbf{r}$, $\mathbf{b}=0{\cdot}5\mathbf{r}$]

7. A helicopter flies a straight course of 100 km. in a direction 30° north of west. How far is it then (a) to the north of its take off point, and (b) to the west ?

[*Ans.* (a) 50 km. (b) 86·6 km.]

8. What is the angle between a 4·00 unit vector and a 5·00 unit vector so that their sum is 8·00 units ? [*Ans.* 54·9°]

9. Add the following forces by the component method ; 30 dynes at 30°, 15 dynes at 120° and 10 dynes at 315° to *x*-axis. [*Ans.* 33·01 dynes at 39·3°]

10. Find analytically the resultant of the vectors

$\mathbf{a}=4\mathbf{i}-6\mathbf{j}+12\mathbf{k}$
$\mathbf{b}=-4\mathbf{i}+6\mathbf{j}-12\mathbf{k}$
$\mathbf{c}=6\mathbf{i}-4\mathbf{j}-10\mathbf{k}$
$\mathbf{d}=8\mathbf{j}-5\mathbf{k}$ [*Ans.* $6\mathbf{i}+4\mathbf{j}-15\mathbf{k}$]

11. A car travels at an average of 40 km./hr. After travelling east for 1 hr. it turns and travels north for 45 mins., then east again for 30 mins.. How far is it from the starting point ?

[*Ans.* 67·08 km. ; 26·6° N of E]

12. The resultant of two forces at right angles is 50 gm. wt. If one of the forces makes an angle of 30° with the resultant, calculate the forces. [*Ans.* 43·3 and 25 gm. wt.]

Harder Problems

1. Find the horizontal force and the force inclined at 30° to the vertical, whose resultant is a vertical force *P* dynes. [*Ans.* $P/\sqrt{3}$ and $2P/\sqrt{3}$ dyne]

2. If the resultant of two forces is equal in magnitude to one of them, and perpendicular to it in direction, find the other. [*Ans.* $\sqrt{2}$ times the former, inclined at 135° to it]

3. Determine the resultant of three vectors of magnitude 1, 2 and 3 whose directions are those of the sides of an equilateral triangle, taken in order.

[*Ans.* 1·732 units at an angle of 210° to the first vector]

4. A man walks 2·0 km. directly south west, then 4·0 km. east, and finally 6·0 km. north east. (a) How many kms. did he walk ? (b) Specify the *x*— and *y*— co-ordinates of his final position w.r.t. the staring point. (c) What is his displacement w.r.t. the starting point ?

[*Ans.* (a) 12·0 km. ; (b) 6·83 ; 2·83 ; (c) 7·4 km. ; 22·5° N of E]

5. Two forces *P* and *Q* acting at a point have got a resultant *R* ; if *Q* be doubled, then *R* is doubled. Again if *Q* be reversed in direction then also *R* is doubled. Prove that $P : Q : R=\sqrt{2} : \sqrt{3} : \sqrt{2}$.

6. Two force *P* and *Q* acting on a particle at an angle θ have a resultant $(2k+1)\sqrt{P^2+Q^2}$. When they act an angle 90°—θ, the resultant is $(2k-1)\sqrt{P^2+Q^2}$. Prove that $\tan\theta=(k-1)/(k+1)$.

7. The resultant of two force **P** and **Q** acting at a point is equal to $\sqrt{3}\mathbf{Q}$ and makes an angle 30° with the direction of **P**. Show that either

$$\mathbf{P}=\mathbf{Q} \text{ or } \mathbf{P}=2\mathbf{Q}.$$

8. *OA*, *OB* and *OC* represent in magnitude and direction three vectors **P**, **Q** and **R**. If $\mathbf{P}+\mathbf{Q}=2\mathbf{R}$, prove that *C* is the midpoint of *AB*.

9. The magnitude of the resultant of **P** and **Q** is the equal to the magnitude of **P**. Show that the resultant of 2**P** and **Q** is perpendicular to **Q**.

2 CHAPTER

MOTION

2·1. Body and Particle.

We start this chapter with some elementary definitions of certain terms which we shall come across in our subsequent discussions.

First of all, we explain what is meant by a body. A certain amount of matter limited in all directions and consequently having *a finite size, shape and occupying some definite space* is called a **body**. A body is said to be **rigid** if the distance between any pair of its constituent particles remains unchanged. On the other hand, a **particle** is defined as a portion of matter infinitely small in size so that for the purpose of investigation, the distance between its different parts may be neglected. Thus *a particle has only a definite position, but no dimension.*

It is often convenient to neglect the size and shape of a body while considering its motion and to treat it as a particle. For example in discussing the motion of a train, it is not necessary to give a detailed description of the train in order to find its position or speed.

2·2. Rest and motion ; Reference Frame.

A body is said to be *at rest when it does not change its position with time* ; on the other hand, if *the position of a body continually changes with time, it is said to be in motion.*

Attempts to visualise the state of rest or of motion of a body leads us to the concept of reference frame. Description of the state of a body requires a second object, with respect to which the state must be specified ; otherwise, it would have no sense. The inherent meaning of the above statement is that when we speak of a body in rest or in motion we always say this in comparison with a second body, which is known as the **reference body**. Thus when we say that the car is at rest or in motion, the description seems to be complete. However, this implies that the state of the car is being described with respect to the earth—the reference body in this example.

To locate the position of a body relative to the reference body, a system of co-ordinates fixed on the reference body is constructed. This is known as **reference frame**.

A body may be described as in motion or at rest depending upon the reference body from which it is being observed. If two cars *A* and *B* move side by side in same direction with same speed [Fig. 2·1], it would appear to the passengers of the cars that they are mutually at rest with respect to one another. Obviously, relative to a reference frame on *A*, *B* is at rest. The reverse is also true.

Depending upon the choice of reference body, the description of the state of motion of a body may also vary. As a fast train over-takes a slower one a passenger in the former will have the impression that the latter is going backwards. Actually, the slower train is moving forwards all the time, but as it is not going fast enough to keep up with the first one, it appears to go in a direction opposite to that of the latter.

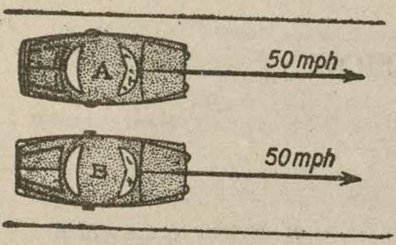


Fig. 2·1

Choice of suitable reference frame.

Choice of a suitable frame for describing the state of a body is important. *That frame is chosen, referred to which the description of motion of the body becomes simpler.* To consider the motion of a man in a railway compartment the reference frame is taken to be on the compartment while the motion of the train is described relative to a reference frame on earth. Earth is also moving in the solar system and to describe its motion, a heliocentric reference frame is chosen.

Absolute rest or Absolute motion is unknown.

It is evident now that *to know whether a body is in a state of absolute rest or of absolute motion, a reference frame absolutely fixed in space is necessary.* But no such stationary frame can be found in the universe. In general to discuss the state of rest or of motion of different terrestrial bodies, earth is taken as the frame of reference. But earth itself spins around its axis and also whirls round the sun at a speed of nearly 30·6 km. per second. The sun is also not fixed ; the whole solar system is moving amongst the galaxy of stars which themselves are always in motion with respect to each other. The whole universe seems to be always expanding. Thus an absolutely stationary reference frame is not realisable in practice and as such, the concepts of absolute rest and absolute motion are in reality meaningless. *The state of rest or of motion of a body is, therefore, always relative.* In fact, a suitable reference frame on a second object is chosen which is taken as fixed. In the following pages unless anything is mentioned, the state of a body will always be described with respect to a reference frame fixed on earth.

2·3. Kinds of Motion.

The motion of a body may be either *translational* or *rotational* or both. The translational motion may again be of two kinds—*rectilinear* and *curvilinear.*

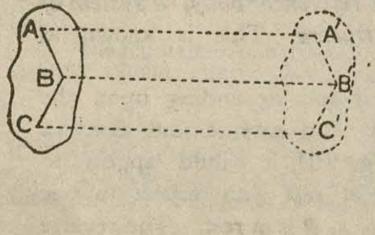


Fig. 2·2 : Rectilinear motion

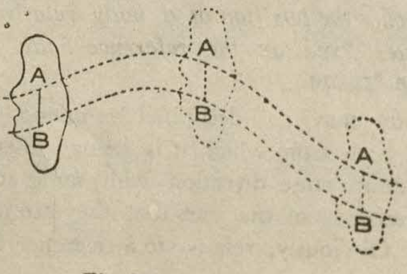


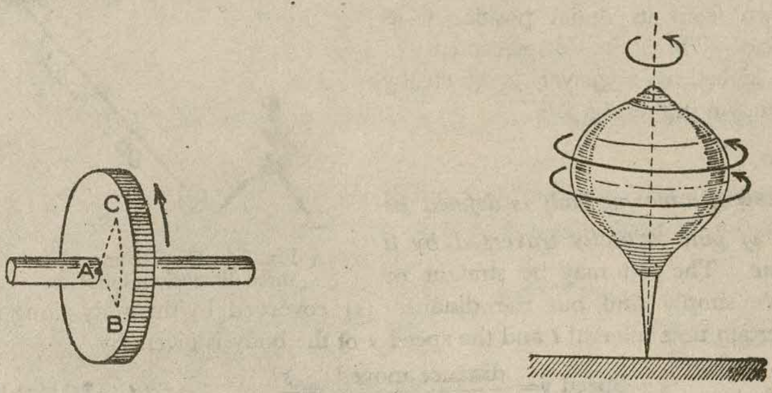
Fig. 2·3 : Curvilinear motion

If the constituent particles of a body move identically so that the line joining *any two points of it always remains parallel to itself, the body is then said to be in* **translational** *motion.*

When the translational motion takes place along a straight line, it is said to be *rectilinear* [Fig. 2·2]. Motion of a freely falling stone, or a train running in a straight track or of a bullet immediately after firing are examples of rectilinear motion.

When the translational motion occurs along a curved path it is called ***curvilinear*** [Fig. 2·3]. An approximate example of curvilinear motion is the motion of arm chairs in a vertical merry-go-round.

If a body revolves about a fixed point or axis, it is said to be in **rotational** motion. As the body rotates, the line joining any two points in it, makes continually changing angle with its initial direction. Motion of a fly-wheel about its axis [Fig. 2·4 (*a*)] or of a rapidly spinning top [Fig. 2·4 (*b*)] are common examples of rotational motion.



(*a*) (*b*)

Fig. 2·4 : Rotational motion

Often, the motion of a body is complex having a combination of both translation and rotation. Typical examples are the motion of a spinning cricket ball or of the wheels of a vehicle or of the earth around the sun etc.

In general, motion of a body occurs in space ; the motion is said to be *three dimensional.* Three co-ordinates (x, y, z) are then necessary to specify the position of the body. If, however, the body is restricted to move on a plane, the motion is said to be *two dimensional.* Two co-ordinates (x, y) are then sufficient to locate its position. When the body moves along a straight line (rectilinear motion), the motion is said to be *one dimensional.* Only one co-ordinate (x) is then sufficient to specify its position.

2·4 Displacement.

The **displacement** *of a moving body in a particular direction during a certain interval of time is defined as its change of position in that interval. It is given by the vector drawn from the initial position of the body to its final position.*

The displacement is independent of the actual path traversed by the body in motion so long as its initial and final position remain the same.

Thus, if in a certain interval of time, a car moves from O to P, its displacement is given by the vector $\overrightarrow{OP}$ [Fig. 2·5]. The car may effect this displacement along various routes (shown by the dotted lines), but its displacement remains the same.

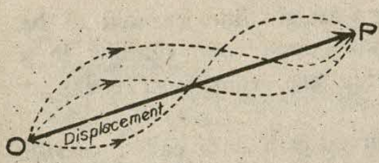


Fig. 2·5 : Displacement

This concept of displacement of a body is also valid when the motion of the body is not confined in a plane ; when the motion occurs in space (three dimensional motion), the displacement at any instant is given as before, by the vector drawn from its initial position to its final position. Thus, the displacement of the football kicked by a player is given by the vector shown in the Fig. 2·6.

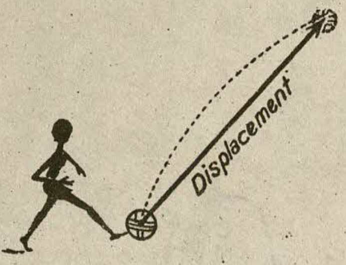


Fig. 2·6 : Displacement of three dimensional motion

2·5 Speed.

The speed of a moving body is defined as the length of path actually traversed by it per unit time. The path may be straight or curved. We simply find out the distance (s) coverved by the body along its path in a certain time interval t and the speed v of the body is given by

$$\text{Speed } v = \frac{\text{distance moved}}{\text{elapsed time}} = \frac{s}{t} \qquad \ldots \qquad (2 \cdot 1)$$

The concept of speed thus involves the idea of magnitude only, not any particular direction. *It is thus a scalar quantity.* The unit of speed is one centimetre per second (cm./sec.) in the C. G. S. system and one metre per second (m./sec.) in the M. K. S. system. In the F. P. S. system, the unit is one foot per second (ft./sec.). Many other units such as mile per hour (mi./hr.), kilometre per hour (km./hr.) are also used.

The speed of a body may be uniform or variable. *The speed is said to be* **uniform** *when the body in motion covers equal lengths of path in equal intervals of time, however, small these intervals may be.* Thus, if a body moving with a uniform speed covers the distance s in time t, then in time $t/4$, the distance moved will be $s/4$; in time $t/10$, the distance will be $s/10$ etc.

When the speed is variable, we introduce the concept of average speed. A simple example will help to grasp the concept easily. Let us consider a car moving along the road OA [Fig. 2·7]. It covers the first 50 miles from O to B in 2 hours, the next 40 miles from B to C in one hour, while the remaining 60 miles from C to A in 2 hours.

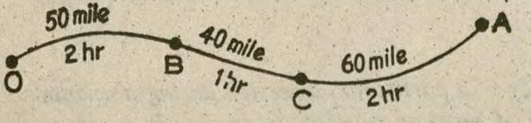


Fig. 2·7 : Average speed

The average speed over the total path is simply obtained by finding out the total distance covered and then dividing this by the time spent in doing so. Thus the average speed of the car, $\bar{v}$, is given by,

$$\bar{v}=\frac{50+40+60}{2+1+2}=\frac{150}{5}=30 \text{ mile per hour.}$$

Thus for a particle moving with a variable speed, if the length of path covered in time t be s,

$$\text{average speed, } \bar{v}=\frac{\text{distance covered}}{\text{elapsed time}}=\frac{s}{t} \quad \ldots \quad \ldots \quad (2{\cdot}2)$$

Comparison with equation (2·1) makes it obvious that *the* **average speed** *$\bar{v}$ represents the value of that constant speed at which the same object could have traversed the same distance s in the same time interval t.*

The value of the average speed depends on the length of the time interval chosen. In the foregoing example, if the average speed is computed over the last 3 hours i. e., for the journey of the car from B to A, it comes out to be $\frac{40+60}{2+1}=\frac{100}{3}=33\frac{1}{3}$ mile/hour.

When the speed is variable, we may be interested in finding out the speed of the moving particle at a certain instant of time or at a particular point of its path, called its instantaneous speed. In doing so, a logical difficulty arises. An instant of time has no duration and hence a body cannot cover any distance precisely at any instant.

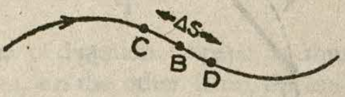


Fig. 2·8

To avoid this difficulty, we calculate the average speed of the particle over a small length of path $CD=\Delta s$,* which includes the point B at which the instantaneous speed is required [Fig. 2·8]. This is given by $\Delta s/\Delta t$, where Δt is the small interval of time during which the path CD is described. Now, if gradually still smaller lengths of path always including the point B, are chosen, the points C and D approach B and more nearly the average speed over this length of path will equal the instantaneous speed at B. *In the limit when the path length becomes infinitesimally small, the average speed over it equals the instantaneous speed at B.*

Thus, *the* **instantaneous speed** *at a point is defined as the limit of the average speed over a path length that tends to become zero but always includes the point.* We should note that as the path Δs becomes increasingly smaller, the corresponding time interval Δt also gets smaller. Their ratio is, however, finite.

The instantaneous speed v is thus given by $v=\underset{\Delta t\to 0}{Lt}\frac{\Delta s}{\Delta t}$

In the calculus notation, this is written as $v=\frac{ds}{dt}$ ... (2·3)

Henceforth, by the speed of a particle we shall always imply its instantaneous speed, unless otherwise mentioned.

---

* Read this as 'delta ess'. This must not be taken to mean 'Δ times s'. This placing of the symbol Δ before s means "a very small change in the value of s".

Acc. no - 16557

2·6 Velocity.

The **velocity** *of a moving body is defined as its rate of displacement.* Alternatively, it can also be defined as the change of position of the moving body in a definite direction, per unit time. Thus, the distance covered by a body in a definite direction in one second is a measure of its velocity. *Velocity is thus a vector quantity.* Units of velocity in different systems are the same as those of speed.

The velocity of a body may be uniform or variable. To be uniform, its magnitude and also its direction must remain the same. *The velocity of a moving body is said to be* **uniform**, *when moving in a particular direction along a straight line, it covers equal lengths of path in equal intervals of time, however small these intervals may be.*

When the velocity is variable, the average velocity of the moving body is obtained by dividing its displacement by the length of the time interval during which the displacement takes place. Considering again the example for the moving car discussed earlier, its displacement $\mathbf{s}$ is given by the vector $\overrightarrow{OA}$, its actual path being shown by the dotted line [Fig. 2·9 (*a*)]. Thus its average velocity $\overline{\mathbf{v}}$ is given by

$$\mathbf{v} = \frac{\mathbf{s}}{t} \qquad \ldots \qquad (2{\cdot}4)$$

where t is the time spent in making the trip. The average velocity $\overline{\mathbf{v}}$ is represented by the vector $\overrightarrow{PQ}$ in Fig. 2·9 (*b*). The length of the vector $\overrightarrow{PQ}$ is a measure of the magnitude of the average velocity. While its direction is parallel to that of displacement vector $\overrightarrow{OA}$.

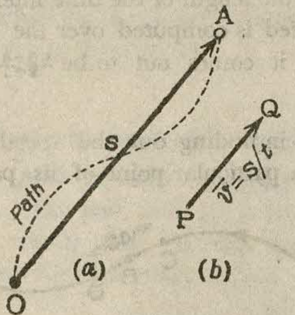


Fig. 2·9 : Average velocity

To find out the instantaneous velocity of a particle moving with non-uniform velocity, we again consider the path of a particle shown in Fig. 2·8. Measured from its initial position O, the displacement $\mathbf{s}_1$ of the particle at the position C is given by the vector $\overrightarrow{OC}$, while its displacement $\mathbf{s}_2$ at the position D is given by the vector $\overrightarrow{OD}$ [Fig. 2·10 (*a*)]. The change in displacement $\Delta\mathbf{s}$ of the particle as it moves from C to D is given by the vector $\overrightarrow{CD}$. From the conception of vector difference [vide Article 1·4],

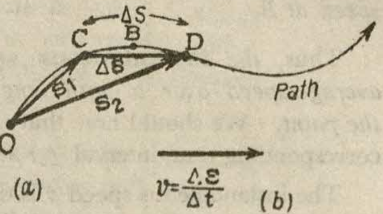


Fig. 2·10 : Instantaneous velocity

$$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} \qquad \text{i. e.,} \quad \Delta\mathbf{s} = \mathbf{s}_2 - \mathbf{s}_1$$

The average velocity of the particle during the interval of time Δt as the particle moves from C to D, is $\overline{\mathbf{v}} = \frac{\Delta \mathbf{s}}{\Delta t}$ which is represented by the arrow shown in Fig. 2·10 (*b*). As in the discussions on instantaneous speed, if the displacement $\Delta \mathbf{s}$ is made progressively smaller, then in the limit when it approaches zero, the average velocity over the interval equals the instantaneous velocity at the point B. Thus we can define *the* **instantaneous velocity** *at a point at a certain instant of time as the limit of the average velocity over a displacement that tends to zero but always includes the point.* Its symbol is **v**. Thus,

$$\mathbf{v} = \underset{\Delta t \to 0}{Lt} \; \frac{\Delta \mathbf{s}}{\Delta t}$$

In calculus notation, this is written as

$$\mathbf{v} = \frac{ds}{dt} \quad \ldots \quad \ldots \quad (2\cdot5)$$

Obviously, the magnitude of the instantaneous velocity vector **v** is the instantaneous speed v and its direction is along the tangent to the path drawn at the corresponding point [Fig. 2·11].

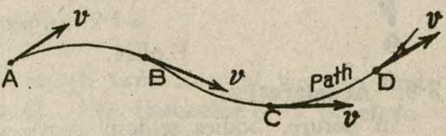


Fig. 2·11 : Velocity is tangent to the path

From now on, unless referred otherwise, the term velocity will mean the instantaneous velocity.

2·7. Distinction between speed and velocity.

The definition of speed does not involve the idea of direction. Speed is thus a scalar quantity. The definition of velocity involves, on the other hand, the idea of both magnitude and direction. Velocity is thus a vector quantity. Speed is the magnitude of velocity. Both speed and velocity are, however, expressed in the same units.

A moving car with its speedometer constantly showing 60 km./hr. obviously has a uniform speed. But the direction of motion may be constantly changing, the road being curved and thus its velocity is not uniform but is changing.

Example 2·1. *A train travelling at an average speed 60 km. p. h. takes 20 min. to travel from a station A to a station B. Find the distance between the two stations.*

Solution : Speed v=60 km./hr.=1 km./min.

Distance $s = v.t = 1 \times 20 = 20$ km.

Example 2·2. *A car covers the first half of the distance between two places at a speed of 40 km./hr. and the second half at 60 km./hr. What is the average speed of the car ?* *[I.I.T. '74]*

Solution : Let the distance between the two places be $2x$ km.

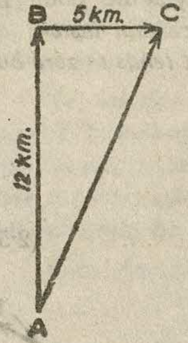
∴ Time taken by the car for the first half on the journey$=\frac{x}{40}$ hr.

Also, the time taken for the second half$=\frac{x}{60}$ hr.

The total time of the journey$=\frac{x}{40}+\frac{x}{60}=\frac{5x}{120}$ hr.

Average speed$=\frac{\text{distance}}{\text{time}}=\frac{2x}{5x/120}=48$ km./hr.

Example 2·3. *A cyclist moves 12 km. due north and then 5 km. due east in 3 hr. Find (a) his average speed, (b) average velocity in m./sec.*



Solution : In the Fig. A shows the initial and C the final position of the cyclist. The total distance covered by the cyclist is $AB+BC=12+5=17$ km.

$\therefore$ Its average speed$=\frac{17}{3}$ km./hr.$=1{\cdot}57$ m./sec.

Its displacement is $\overrightarrow{AC}$ and the magnitude is given by

$AC=\sqrt{AB^2+BC^2}=\sqrt{12^2+5^2}=13$ km.

$\therefore$ Its average velocity$=\frac{13}{3}$ km./hr.

$=1{\cdot}2$ m./sec. along $\overrightarrow{AC}$, i.e., at $\tan^{-1}\frac{5}{12}$ or, 22·6° E of N.

2·8. Acceleration.

In nature, bodies seldom move with a uniform velocity. In most cases, velocity of a body changes in magnitude or in direction or in both ; the motion is said to be accelerated or the body is said to have an acceleration.

Acceleration *of a moving body is defined as its rate of change of velocity.* Since velocity is a vector quantity, *acceleration is also a vector quantity.* Units of acceleration in C. G. S. and M. K. S. systems are one centimetre per second per second (cm./sec^2) and one metre per second per second (m./sec.^{2}) respectively. In F. P. S. system, the unit is one foot per second per second (ft./sec^2).

Like velocity, acceleration of a particle may also be uniform or variable. ***Acceleration is said to be uniform** when the velocity of the particle changes by equal amounts in equal intervals of time, however small the intervals may be.*

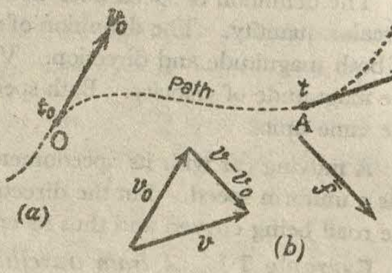


Fig. 2·12 : Average acceleration

In analogy with previous discussions, we introduce the concept of average acceleration and instantaneous acceleration of a particle moving with non-uniform acceleration. The average acceleration of a moving particle during a certain interval of time is defined as the ratio of its change in velocity to the length of the time interval. If the velocity of a particle at a point O on its path be $\mathbf{v}_0$, corresponding time being t_0 and the velocity at a later time t be $\mathbf{v}$, when the particle is at A, the change in the velocity during this interval $(t-t_0)$ is $\mathbf{v}-\mathbf{v}_0$ [Fig. 2·12(*a*).

Hence the average acceleration $\bar{\mathbf{f}}$ is given by $\bar{\mathbf{f}}=\frac{\mathbf{v}-\mathbf{v}_0}{t-t_0}$

If time is counted from the instant when the particle is at O, the expression reduces to

$$\bar{\mathbf{f}}=\frac{\mathbf{v}-\mathbf{v}_0}{t} \quad \ldots \quad \ldots \quad (2{\cdot}6)$$

The difference $(\mathbf{v}-\mathbf{v}_0)$ of the two velocities can be found by the triangle method for finding vector difference [Fig. 2·12(*b*)]. The average acceleration vector will be parallel to this.

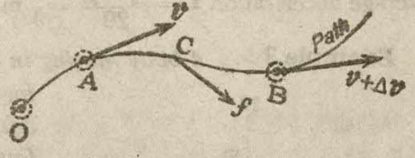


Fig. 2·13 : Instantaneous acceleration

If the time interval is chosen to be very small, given by Δt, the corresponding change in the velocity will also be very small denoted by $\Delta \mathbf{v}$ [Fig. 2·13]. The position of the particle at the beginning and at the end of the interval are denoted by *A* and *B*. The average acceleration over the interval is thus given by

$$\bar{\mathbf{f}}=\frac{\Delta \mathbf{v}}{\Delta t}$$

As before, by considering an infinitesimally small time interval we can find out the instantaneous acceleration at the point *C*. *The* **instantaneous acceleration** *of a particle at a point at a certain instant of time is defined as the limit of the average acceleration over a displacement that tends to zero, but always includes the given point.* Thus it is given by $\mathbf{f}=\underset{\Delta t\to 0}{Lt}\frac{\Delta \mathbf{v}}{\Delta t}$

In calculus notation,

$$\mathbf{f}=\frac{d\mathbf{v}}{dt} \quad \ldots \quad \ldots \quad (2{\cdot}7)$$

Since $\mathbf{v}=\frac{d\mathbf{s}}{dt}$, $\quad \mathbf{f}=\frac{d}{dt}\left[\frac{d\mathbf{s}}{dt}\right]=\frac{d^2\mathbf{s}}{dt^2} \quad \ldots \quad (2{\cdot}8)$

Henceforth, unless otherwise specified, the term 'acceleration' will mean the 'instantaneous acceleration' of the particle.

We can resolve the acceleration vector $\mathbf{f}$ into two components, one along the path and the other perpendicular to it. Change in the velocity of a particle can be brought about by changing its magnitude or direction. *The component of acceleration parallel to the path changes the magnitude of the velocity while the component normal to the path changes its direction.*

Though the term speed signifies the magnitude of the velocity, no such corresponding word is there to describe the magnitude of acceleration. Hence the word acceleration is used either to express the vector quantity or its magnitude.

2·9. Negative Acceleration.

When a moving body is slowing down, its final velocity is less than its initial velocity and its acceleration is then negative. *The rate of decrease of the velocity is called* **retardation** *or* **deceleration**. For example, a body thrown upwards, a moving car after the application of brakes etc., execute retarded motion.

Example 2·4. *An athlete running in a straight line increases his velocity from 12 km./hr. to 20 km./hr. in 20 sec. What is his average acceleration ?*

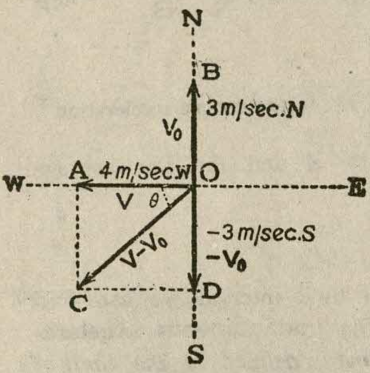
Solution :

$$20 \text{ km./hr.}=\frac{20\times 1000}{3600} \text{ m./sec.}=\frac{50}{9} \text{ m./sec.}$$

$$12 \text{ km./hr.} = \frac{12 \times 1000}{3600} \text{ m./sec.} = \frac{10}{3} \text{ m./sec.}$$

average acceleration $\bar{f} = \dfrac{\frac{50}{9} - \frac{10}{3}}{20} = \dfrac{1}{9}$ m./sec^2 = 11·1 cm./sec.^{2} (approx.)

Example 2·5. *A body moving in a curved path possesses a velocity 3 m./sec. towards north at any instant of its motion. After 10 sec., the velocity of the body was found to be 4 m./sec. towards west. Calculate the average acceleration during this interval.*



Solution : To solve this problem the vector nature of velocity must be taken into account. In the figure, the initial velocity $\mathbf{v}_0$ and the final velocity $\mathbf{v}$ are drawn from a common origin. The vector difference of them is found by the parallelogram method (vide Art. 1·4).

The magnitude of the difference is

$$|\mathbf{v} - \mathbf{v}_0| = OC = \sqrt{OA^2 + AC^2}$$
$$= \sqrt{4^2 + 3^2} = 5 \text{ m./sec.}$$

The direction is given by $\tan\theta = \frac{3}{4} = 0{\cdot}75$ $\quad\therefore\quad \theta \simeq 37°$

$\therefore$ Average acceleration $= \dfrac{\mathbf{v} - \mathbf{v}_0}{t} = \frac{5}{10} = 0{\cdot}5$ m./sec.^{2} at 37° S of W.

2·10 Rectilinear Motion with Uniform Acceleration.

The simplest example of accelerated motion is that of a particle moving in a straight line with uniform acceleration. In this case three simple relations connecting the distance, velocity, acceleration and time can be obtained as follows :

If a particle moves along a straight line with a uniform acceleration f and if u and v are its velocities at the beginning and at the end of any time interval t during its motion, and s the distance traversed during the said interval of time, then

(i) $v = u + ft$
(ii) $s = ut + \frac{1}{2} ft^2$
(iii) $v^2 = u^2 + 2fs$

(i) To prove the relation $v = u + ft$

Let u be the initial velocity of the particle and v its final velocity for the interval of time t. Thus the change in velocity during this time $t = v - u$.

Hence the rate of change of velocity $= \dfrac{v-u}{t}$

But this is, by definition, the uniform acceleration f of the body. Therefore,

$$f = \frac{v-u}{t} \quad \text{or,} \quad v = u + ft \quad \ldots \quad \ldots \quad (2{\cdot}9)$$

If the body starts from rest, then $u = 0$; thus

$$v = ft \quad \ldots \quad \ldots \quad (2{\cdot}10)$$

(ii) To prove the relation $s=ut+\frac{1}{2}ft^2$

When the body is moving with a uniform acceleration, its velocity is continually changing. The average velocity during an interval of time t in which the body describes the distance s is given by eqn. (2·4)

$$\bar{v}=\frac{s}{t} \qquad \text{or,} \quad s=\bar{v}t$$

Now from eqn. (2·9), the velocity of the body at a time t' after the beginning of the interval $t(t'\leqslant t)$ is $u+ft'$ and the velocity at a time t' before the end of the interval t is $u+f(t-t')$

$\therefore$ Mean of these two velocities$=\frac{u+ft'+u+f(t-t')}{2}$

$$=\frac{u+u+ft}{2}=\frac{u+v}{2}$$

We can divide the whole time interval t in such pairs of instants equally spaced in time after the initial and before the final instants. For each such pair of instants, the average velocity can be calculated as above and the result will always be $\frac{u+v}{2}$. Hence the average velocity $\bar{v}$ over the time interval t is given by

$$\bar{v}=\frac{u+v}{2} \quad \dots \quad \dots \quad (2\cdot11)$$

$\therefore$ the distance s traversed in time t is given by

$$s=\bar{v}t=\frac{u+v}{2}t=\frac{u+u+ft}{2}t \qquad \text{from eqn. (2·9)}$$

$$=ut+\tfrac{1}{2}ft^2 \quad \dots \quad \dots \quad (2\cdot12)$$

For bodies starting from rest, $u=0$; then

$$s=\tfrac{1}{2}ft^2 \quad \dots \quad \dots \quad (2\cdot13)$$

(iii) To prove the relation $v^2=u^2+2fs$

To get this relation, we eliminate t between the eqns. (2·9) and (2·12).

Thus from eqn. (2·9), $v^2=(u+ft)^2=u^2+2uft+f^2t^2$

$$=u^2+2f(ut+\tfrac{1}{2}ft^2)$$

$$=u^2+2fs \quad \dots \quad \dots \quad (2\cdot14)$$

If the body starts from rest, then $u=0$

$$v^2=2fs \quad \dots \quad \dots \quad (2\cdot15)$$

Alternative proof of the three relations using calculus :

The above equations can be obtained in a simple and elegant manner using calculus definitions of velocity and acceleration. We have from equation (2·7),

$$f=\frac{dv}{dt} \quad \text{or,} \quad dv=fdt$$

Acceleration being uniform, f is constant. Integrating,

$$\int dv=\int fdt+c_1 \quad \text{or,} \quad v=ft+c_1$$

where c_1 is a constant of integration. If u be the initial velocity $v=u$, when $t=0$; then $u=0+c_1$

$\therefore$ $v=u+ft$ which is equation (2·9)

But $v=\frac{ds}{dt}$ $\therefore$ $ds=vdt=udt+ftdt$

Integrating, $\int ds=\int u\,dt+\int ft\,dt+c_2$

or, $s=ut+\frac{1}{2}ft^2+c_2$

where c_2 is another constant of integration. Now at the start $s=0$, and $t=0$. Hence $c_2=0$.

$\therefore\ s=ut+\frac{1}{2}ft^2$ which is equation (2·12).

Now, we can write $f=\frac{dv}{dt}=\frac{dv}{ds}\cdot\frac{ds}{dt}=v\frac{dv}{ds}$ or, $v\,dv=f\,ds$

Integrating, $\int v\,dv=\int f\,ds+c_3$ or, $\frac{v^2}{2}=fs+c_3$

where c_3 is the constant of integration. Now at the start, $s=0$ and $v=u$; hence

$$\frac{u^2}{2}=0+c_3$$

$\therefore\ \frac{v^2}{2}=fs+\frac{u^2}{2}$ or, $v^2=u^2+2fs$ which is equation (2·14).

2·11. Distance traversed by a particle in any particular second of its motion.

To calculate the distance moved by a particle in any particular second, say third, or sixth, or nth in general, we proceed as follows :

The distance traversed in the nth second=the distance traversed in n seconds −the distance traversed in $(n-1)$ seconds.

$$=un+\tfrac{1}{2}fn^2-[u(n-1)+\tfrac{1}{2}f(n-1)^2]$$
$$=u+\tfrac{1}{2}f(2n-1) \qquad \cdots \qquad \cdots \qquad (2·16)$$

Example 2·6. *A body moves from rest with constant acceleration 10 m./sec.² Find (a) the speed v at the end of 10 sec. ; (b) the average speed $\bar{v}$ for the 10 sec. interval ; (c) the distance s covered in 10 sec. from rest.*

Solution : Since the body starts from rest, its initial velocity $u=0$.

(a) From eqn. (2·9), $v=u+ft=0+10\times10=100$ m./sec.

(b) From eqn. (2·11), $\bar{v}=\frac{1}{2}(u+v)=\frac{1}{2}(0+100)$ m./sec.=50 m./sec.

(c) From eqn. (2·12), $s=ut+\frac{1}{2}ft^2=(0+\frac{1}{2}\times10\times10^2)=500$ m.

Alternatively, $s=\bar{v}t=50\times10=500$ m.

Example 2·7. *A car is travelling at 20 m./sec. along a straight road. The driver sees a 36 km./hr. speedlimit sign 100 m. ahead. What constant retardation is required to reduce the speed to 36 km./hr. in this distance ? Find also the time taken by the car to reach the speedlimit sign.*

Solution : $u=20$ m./sec. ; $v=36$ km./hr.=10 m./sec. ; $s=100$ m.

Using the equation (2·14), we have,

$$f=\frac{v^2-u^2}{2s}=\frac{10^2-20^2}{2\times100}=-1·5 \text{ m./sec.}^2$$

Now, $v=u+ft$, $\therefore\ 10=20+(-1·5)\,t$ or, $t=6·67$ sec. (nearly)

Example 2·8. *A car starts from rest and is accelerated uniformly at the rate of 3 m./sec.² for 10 sec. It then maintains a constant speed for 20 sec. The brakes are then applied and the vehicle uniformly retarded to rest in 5 sec. Find the maximum speed reached in km./hr. and the total distance covered in metre.*

Solution : $u=0$ m./sec , $f=3$ m./sec.^{2}, $t=10$ sec. Hence from eqn. (2·10), we get $v=ft=30$ m./sec.$=108$ km./hr, , the maximum speed reached.

Now applying eqn. (2·13), $s_1=\frac{1}{2}\times 3\times 10^2=150$ m., the distance traversed in first 10 secs. ... ... (1)

For the next 20 secs, the distance traversed is $s_2=vt=30\times 20=600$ m. (2)

When brakes are applied, the car is moving with a speed of 30 m./sec. and being retarded comes to rest in 5 sec. Its acceleration is thus negative and is given by

$$f=\frac{0-30}{5}=-6 \text{ cm./ sec.}^2$$

Hence from eqn. (2·14) we have, $s_3=\dfrac{0^2-30^2}{2\times(-6)}=75$ m. ... (3)

Thus the total distance traversed is found out by adding (1), (2) and (3) and thus $s=s_1+s_2+s_3=150+600+75=825$ m. and the maximum speed reached$=$ 108 km./hr.

Example 2·9. *A train moving with uniform retardation takes 20 sec. and 30 sec. to travel two successive quarter kilometre. How much further will it travel before coming to rest ?*

Solution : For the first quarter kilometre, we have from the equation (2·12) since $\frac{1}{4}$ km.$=250$ m.

$$250=u\times 20+\frac{f}{2}\times 20^2 \quad \text{or,} \quad 2u+20f=25 \qquad \dots \qquad (1)$$

For both path lengths taken together, total time spent and the corresponding distance traversed are 20 sec.$+$30 sec.$=$50 sec. and $\frac{1}{4}$ km.$+\frac{1}{4}$ km.$=\frac{1}{2}$ km.$=500$ m respectively.

Hence applying eqn. (2·12) once again,

$$500=u\times 50+\tfrac{1}{2}f\times 50^2 \quad \text{or,} \quad u+25f=10 \qquad \dots \qquad (2)$$

Solving (1) and (2) for u and f, we get

$$u=\tfrac{85}{6} \text{ m./sec.}, f=-\tfrac{1}{6} \text{ m/sec.}^2$$

Now if s be the total distance travelled by the train before it stops, then from eqn. (2·14),

$$0=(\tfrac{85}{6})^2+2(-\tfrac{1}{6})s \quad \text{or,} \quad s=602{\cdot}1 \text{ m.}$$

Thus the train will move through a further distance of (602·1$-$500)$=$102·1 m.

Example 2·10. *A particle moving with uniform acceleration described in its last second of motion one quarter of the whole distance. If it started from rest, how long was it in motion and through what distance did it move, if it described 10 m. in the first second ?*

Solution : In the 1st. second of its motion, the particle described 10 m. from rest.

Hence from eqn. (2·13), $10=\frac{1}{2}f\times 1^2$ · or, $f=20$ m./sec.^{2}

Now suppose that the particle starting from rest described the whole length of its path, s in t sec. ; then from equation (2·13),

$$s=\tfrac{1}{2}\times 20\times t^2=10t^2 \qquad \dots \qquad (1)$$

Again, the particle moved $\frac{1}{4}$th of the path in the t-th sec. Hence from eqn. (2·16),

$$\tfrac{1}{4}\,s=\tfrac{1}{2}f(2t-1)=\tfrac{1}{2}\times 20\times(2t-1)$$
$$=20t-10$$
$$\therefore\ s=80t-40 \qquad \ldots \qquad (2)$$

Eliminating s from (1) and (2) we get, $t^2-8t+4=0$, the solution of which is $t=4\pm2\sqrt{3}$.

If the two values of t are t_1 and t_2, we have $t_1=7{\cdot}464$ sec. and $t_2=0{\cdot}536$ sec.

The second value t_2 is, however, impossible and hence it is rejected. Thus the motion lasted for 7·464 sec. From equation (2) the distance traversed

$$s=80\times7{\cdot}464-40=557{\cdot}1 \text{ m.}$$

Example 2·11. *A particle moving in a straight line with uniform acceleration moves through distances of 80 m. and 100 m. in two successive seconds. Find the velocity at the beginning of the first second and the acceleration. Find also the distance moved in the next second.*

Solution : Let u be the velocity of the particle at the beginning of the first second. Then from equation (2·16), if its acceleration be f

$$80=u+\tfrac{1}{2}f \qquad \ldots \qquad (1)$$

For the next second, putting $n=2$ in the same equation, we get

$$100=u+\tfrac{3}{2}f \qquad \ldots \qquad (2)$$

Solving (1) & (2), we get $u=70$ m./sec. and $f=20$ m./sec.^{2}

The distance moved in the next second i.e., the third second is given by

$$s=70+\tfrac{1}{2}\times20(2\times3-1).$$
$$=70+10\times5=120 \text{ m.}$$

Note : The problem can be solved without assuming eqn. (2·16). This is shown in the Ex. 2·12.

Example 2·12. *When a car is moving at 66 m./sec. brakes are applied and it slows uniformly to 22 m./sec. in 5 second. Determine (a) the acceleration and (b) the distance s_5 covered during the 5th second.*

Solution : (*a*) From equation (2·9), we have

$$f=\frac{v-u}{t}=\frac{22-66}{5}=-8{\cdot}8 \text{ m./sec.}^2$$

(*b*) From equation (2·12),

the distance covered in 5 sec.$=66\times5+\frac{1}{2}\times(-8{\cdot}8)\times5^2=220$ m.

and the distance covered in 4 sec.$=66\times4+\frac{1}{2}(-8{\cdot}8)\times4^2=193{\cdot}6$ m.

Hence, the distance covered in the 5th sec. is $s_5=220-193{\cdot}6=26{\cdot}4$ m.

Alternatively, we have directly from equation (2·16),

$$s_5=66+\tfrac{1}{2}(-8{\cdot}8)\,(2\times5-1)=26{\cdot}4 \text{ m.}$$

2·12. Graphical Representation.

A useful and convenient method of studying the motion of a body is to construct graphs in which the magnitudes of displacement, velocity or acceleration are plotted as ordinates and the time as abscissa.

Distance-time graph : A body moving with uniform velocity traverses equal distances in equal intervals of time ; hence the graph of distance against time will be a straight line viz,, OA in Fig. 2·14.

$$\text{The gradient or slope of the line} = \frac{AB}{OB} = \frac{\text{Distance}}{\text{Time}} = \text{Velocity}.$$

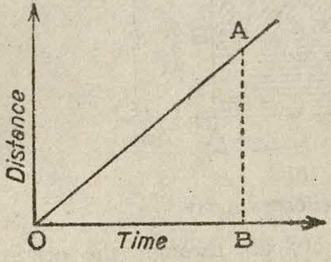


Fig. 2·14 : Distance-time graph for uniform velocity

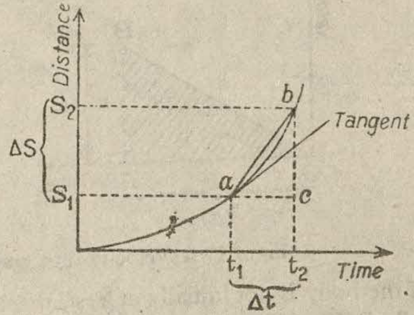


Fig. 2·15 : Distance-time graph for non-uniform velocity

Hence the uniform velocity of the particle is given by the slope of the line.

For a body moving with a variable velocity, the graph will be a curved line [Fig. 2·15]. Corresponding to the time interval $t_2-t_1=ac$, the distance moved by the body is $S_2-S_1=bc$. Thus the average velocity $\bar{v}$ over the interval is given by,

$$\bar{v}=\frac{S_2-S_1}{t_2-t_1}=\frac{bc}{ac}=\text{slope of the chord } ab$$

To get the instantaneous velocity at time t_1, as the time interval is made vanishingly small, the point b approaches the point a. In the limit, the slope of the chord ab equals the slope of the tangent to the curve at the point a. The instantaneous velocity v at a point is thus given by the slope of the tangent to the graph at that point.

Velocity-time graph : For a body moving with uniform velocity, the plot of velocity against time will obviously be a straight line parallel to the time axis, viz, line CB in Fig. 2·16. The area of the rectangle $OABC$ between the velocity-time curvé and the time axis is given by,

area $OABC=OC\times OA=vt$, which is the distance s through which the body moves.

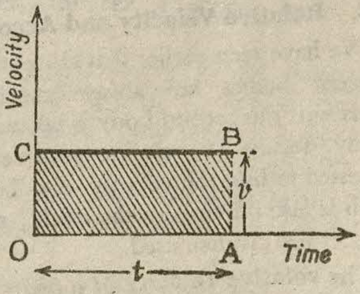


Fig. 2·16 : Velocity-time graph for uniform velocity

If the body moves in a straight line with uniform acceleration, the velocity-time curve will be a straight line. If the body starts from rest, the line passes through the origin, viz. line OB in Fig. 2·17 (a). When it has an initial velocity u, the line DC will be as shown in Fig. 2·17 (b). In these cases also, the distance traversed by the body is numerically equal to the area between the velocity-time curve and the time axis. To prove this, from Fig. 2·17 (*a*) the area of the traingle,

$$OAB=\tfrac{1}{2}.OA.AB=\tfrac{1}{2}t.v=\tfrac{1}{2}t.ft=\tfrac{1}{2}ft^2=\text{Distance moved.}$$

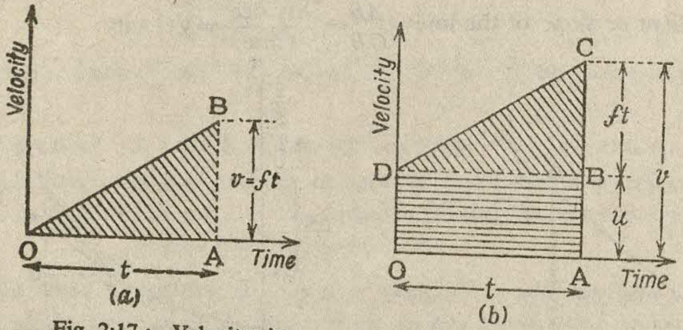


Fig. 2·17 : Velocity-time graph for non-uniform velocity

If the body is not initially at rest, the line does not pass through the origin [Fig. 2·17 (*b*)]. The area $OACD$

$$=\text{Area of the rectangle } OABD+\text{Area of the traingle } BCD$$
$$=OD.OA+\tfrac{1}{2}.DB.BC$$
$$=ut+\tfrac{1}{2}t.ft=ut+\tfrac{1}{2}ft^2=\text{Distance moved.}$$

For non-uniform acceleration, the plot of velocity against time will be a curved line [Fig: 2·18]. Proceeding exactly as before, we can show that the average acceleration over the interval (t_2-t_1) is given by the slope of the chord ab. The instantaneous acceleration at a point is given by the slope of the tangent to the curve at that point.

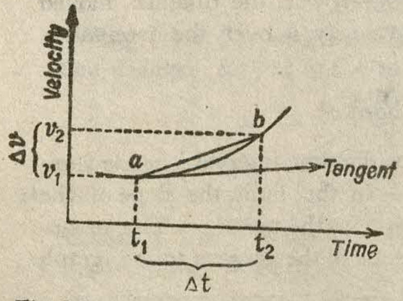


Fig. 2·18 : Velocity-time graph for non-uniform acceleration

Applying the methods of calculus, it can be shown that the distance moved in this case is also given by the area included between the curve and the time axis.

2·13. Relative Velocity and Acceleration.

We have seen earlier that all motions are relative and thus the velocities of different bodies are always specified relative to some other body. In our discussions this second body is taken as the earth so that the term "the velocity of a body" means its velocity relative to the earth. In some problems we are interested to find the velocity of a body (A) with respect to another body (B) which is also moving on the earth ; the terminology "relative velocity of A with respect to B" is then used.

The **relative velocity** *of a body with respect to a second which is also moving* (*relative of the earth*) *is the vector difference of their velocities* (*relative to the earth*). Thus if the velocities of the two bodies A and B be $\mathbf{v}_A$ and $\mathbf{v}_B$ respectively, the velocity of A relative to B is

$$\mathbf{v}_{AB}=\mathbf{v}_A-\mathbf{v}_B \quad \ldots \quad \ldots \quad (2\cdot17)$$

and the velocity of *B* relative to *A* is

$$\mathbf{v}_{BA}=\mathbf{v}_B-\mathbf{v}_A$$

$$\text{Obviously,}\quad \mathbf{v}_{AB}=-\mathbf{v}_{BA}$$

It follows that if two bodies are moving *in the same straight line along same direction,* the relative velocity of *A* with respect to *B* is given by the simple expression $v_{AB}=v_A-v_B$. If $v_A>v_B$, then v_{AB} possesses identical direction as that of $\mathbf{v}_A$ and $\mathbf{v}_B$, so that when observed from *B, A* would appear to be moving forward with the velocity v_{AB}. Obviously v_{BA} would have opposite direction; hence when observed from *A, B* would seem to be moving backwards with velocity $v_{BA}(=-v_{AB})$.

If the two bodies are moving *in the same straight line but in opposite directions,* we may take the velocity of one, say v_A, as positive and that of the other v_B as negative. The relative velocity of *A* with respect to *B* is then obtained from the simple relation $v_{AB}=v_A-(-v_B)=v_A+v_B$. So when observed from *B, A* would seem to be approaching *B* with the velocity v_{AB}. Now $v_{BA}=-v_B-v_A=-(v_A+v_B)$. Hence when observed from *A, B* would appear to be approaching *A* with the velocity $v_{BA}(=-v_{AB})$.

Thus if two cars *A* and *B*, approach each other from opposite direction [Fig. 2·19], with velocities, say, 50 km./hr. and 60 km./hr. to the drivers of each car, the other car seems to be moving towards him much faster. In this example, they will seem to be approaching each other at the rate of 50+60=110 km./hr. which is the relative velocity of one with respect to the other. However, if at the same time another car *C* overtakes *A*, say with a velocity of 70 km./hr. the driver of *A* would think that car *C* is moving forward much slower, viz., at the rate of 70−50=20 km./hr., the relative velocity of *C* with respect to *A*. To the driver of the car *C*, however, the car *A* appears to be travelling backwards with the same velocity.

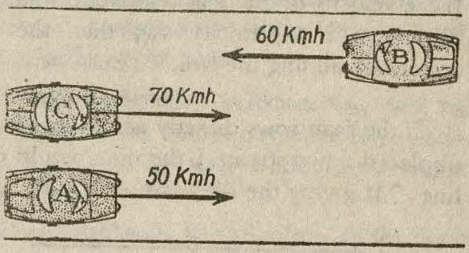


Fig. 2·19 : **Relative velocity**

To find the relative velocity of two bodies which are ***not moving in the same straight line,*** the arrows representing their velocities viz, $\mathbf{v}_A$ and $\mathbf{v}_B$ are to be drawn from a common point. Now to obtain the velocity of *A* relative to *B* i.e., $\mathbf{v}_{AB}$, the arrow representing $-\mathbf{v}_B$ is drawn from the same point. $\mathbf{v}_{AB}$ is then found by adding $\mathbf{v}_A$ with $-\mathbf{v}_B$ by the parallelogram law.

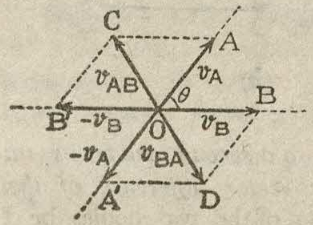


Fig. 2·20

For illustration, let us suppose that two cars *A* and *B* are moving with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ respectively, the angle between their direction of motion being θ. In Fig. 2·20 $\mathbf{v}_A$ is represented by the directed line segment $\overrightarrow{OA}$ and $\mathbf{v}_B$ by $\overrightarrow{OB}$. The line segment $\overrightarrow{OB'}$ drawn equal and opposite to $\overrightarrow{OB}$ from the same point O, therefore, represents $-\mathbf{v}_B$. The parallelogram $OACB'$ is completed. Its diagonal $\overrightarrow{OC}$, therefore, completely represents $\mathbf{v}_{AB}$.

Thus the driver of the car B sees the car A to be moving with a velocity v_{AB} in the direction OC. The magnitude and direction of $\mathbf{v}_{AB}$ can be calculated by using equations (1·3) and (1·4).

The line segment $\overrightarrow{OD}$ drawn qual and opposite to $\overrightarrow{OC}$ will represent $\mathbf{v}_{BA}$. It may also be obtained by the addition of $\mathbf{v}_B$ and $-\mathbf{v}_A$ by the above method. Therefore, the driver of the car A sees the car B moving with a velocity v_{BA} $(=v_{AB})$ in the direction OD.

Now equation (2·17) may be written as

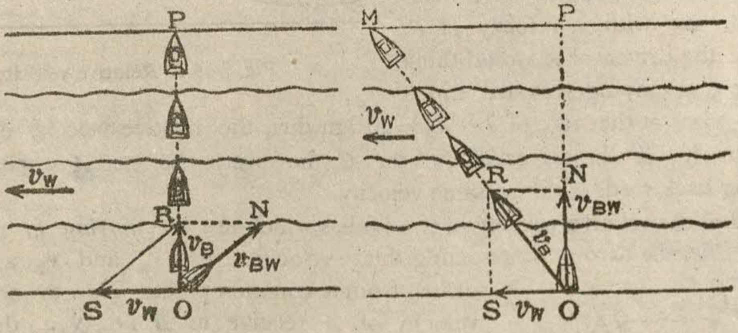
$$\mathbf{v}_A=\mathbf{v}_B+\mathbf{v}_{AB} \qquad \ldots \qquad \ldots \qquad (2{\cdot}18)$$

Thus, when two bodies are moving relative to each other the velocity of the first is the vector sum of the velocity of the second and the velocity of the first relative to the second.

A man in a boat trying to cross a river to reach the directly opposite point (P) on the other bank, must direct his boat in the direction ON shown in Fig. 2·21(*a*). If the velocity of the boat relative to water be $\mathbf{v}_{BW}$, and the water of the river itself flowing with a velocity $\mathbf{v}_W$, then the velocity of the boat $\mathbf{v}_B$ is obtained by vectorially adding the two. Thus

$$\mathbf{v}_B=\mathbf{v}_W+\mathbf{v}_{BW} \qquad \ldots \qquad \ldots \qquad (2{\cdot}19)$$

If the man rows directly across the river [Fig. 2·21 (*b*)], the boat would be displaced down stream ; the man would reach the other bank at the point M, the line OM giving the direction of $\mathbf{v}_B$, the velocity of the boat.



(*a*) (*b*)

Fig. 2·21 : Relative velocity

The **relative acceleration** *of a body with respect to a second one is its rate of change of velocity relative to the second. It is the vector difference of their accelerations (relative to the earth).* If the acceleration of the two bodies be $\mathbf{f}_A$ and $\mathbf{f}_B$, the acceleration of A relative to B is

$$\mathbf{f}_{AB}=\frac{d\mathbf{v}_{AB}}{dt}=\mathbf{f}_A-\mathbf{f}_B \qquad \ldots \qquad \ldots \qquad (2{\cdot}20)$$

Example 2·13. *A straight river 250 m. wide flows at the rate of 3 km./hr. A man who can row at the rate of 5 km./hr. in still water wishes to cross the stream at right angles to the bank. In what direction must he row and how long will he take to cross ?*

Solution : Refer to the Fig. 2·21 (*a*), where $\overrightarrow{OP}$ represents the direction in which the man intends to move. His resultant velocity along *OP* is thus compounded of (i) the velocity of the stream $v_W=3$ km./hr. and (ii) his velocity with respect to still water $v_{BW}=5$ km./hr. This is drawn along the direction in which the man rows.

Now, ∠RON is given by, $\sin\angle RON=\frac{NR}{ON}=\frac{v_W}{v_{BW}}=\frac{3}{5}=0{\cdot}6$

$\therefore\ \angle RON=37°$

Thus the man must row upstream making an angle 37° to the direction *OP*. The magnitude of his resultant velocity v_B along *OP* is given by

$$v_B=OR=\sqrt{ON^2-NR^2}=\sqrt{v_{BW}^2-v_W^2}$$
$$=\sqrt{5^2-3^2}=4 \text{ km./hr.}$$

Hence the time taken to cross the river of width 250 m. i.e., 0·25 km. is

$$=\frac{0{\cdot}25}{4}=\frac{1}{16}\text{ hr.}=3\text{ min. }45\text{ sec.}$$

Example 2·14. *In the previous problem, if the boat is rowed straight across the river, how long will the crossing take and at what point on the opposite bank will the boat reach ? Calculate also the velocity of the boat.*

Solution : Refer to figure 2·21 (*b*). Since the two velocities $\mathbf{v}_{BW}$ and $\mathbf{v}_W$ are perpendicular to each other, their effects can be found out separately. Displacement across the river is effected with the velocity $\mathbf{v}_{BW}$ while displacement along the river is effected with the velocity $\mathbf{v}_W$.

$\therefore$ time necessary for crossing the river $=\frac{0{\cdot}25}{5}=\frac{1}{20}$ hr. $=3$ min. In this time the boat is carried $3\times\frac{1}{20}$ km. down stream by the current. Hence the boat reaches the point *M* on the opposite bank such that $MP=\frac{3}{10}$ km. $=150$ m.

The magnitude of the velocity of the boat is given by

$v_B=OR=\sqrt{ON^2+NR^2}=\sqrt{v_{BW}^2+v_W^2}=\sqrt{5^2+3^2}=5{\cdot}83$ km/hr.

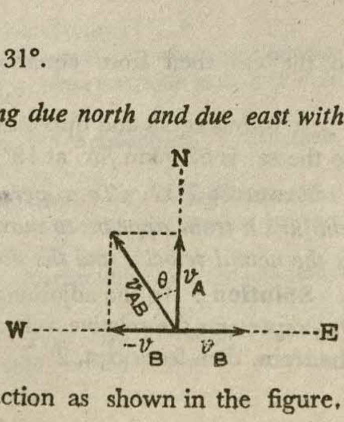
Its direction is given by $\angle RON$, where

$$\tan\angle RON=\frac{RN}{ON}=\frac{3}{5}=0{\cdot}6 \text{ or, } \angle RON=31°.$$

Example 2·15. *Two cars A and B are moving due north and due east with the velocities 80 km./hr. and 60 km./hr. respectively. Calculate (a) the relative velocity of A w.r.t. B and (b) that of B w.r.t. A.*

Solution : (a) Let $\mathbf{v}_A$ and $\mathbf{v}_B$ be the velocities of the cars *A* and *B* respectively. From eqn. (2·17), the velocity of *A* relative to *B* is

$$\mathbf{v}_{AB}=\mathbf{v}_A-\mathbf{v}_B$$



$\mathbf{v}_{AB}$ is obtained by the method of vector subtraction as shown in the figure.

Obviously $\mathbf{v}_{AB}$ is the resultant of $\mathbf{v}_A$ and $-\mathbf{v}_B$ and is obtained by parallelogra method. Its magnitude is given by,

$$v_{AB}=\sqrt{v_A^2+v_B^2}=\sqrt{80^2+60^2}=100 \text{ km/hr.}$$

The angle θ is given by

$$\tan\theta=\frac{v_B}{v_A}=\frac{60}{80}=0{\cdot}75 \quad \therefore\ \theta=36{\cdot}87^\circ$$

Hence the relative velocity of the car *A* w.r.t. *B* is 100 km./hr. at 36·87° W. of N.

(b) Here, the relative velocity of the car *B* w.r.t. *A* is

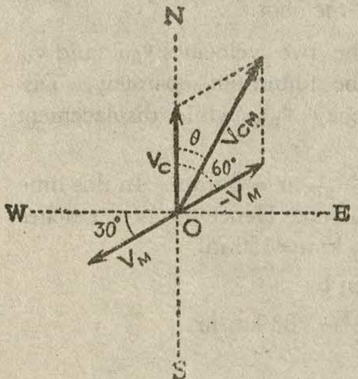
$$\mathbf{v}_{BA}=\mathbf{v}_B-\mathbf{v}_A=-\mathbf{v}_{AB} \text{ from eqn. (2·17)}$$

It is thus equal and opposite to $\mathbf{v}_{AB}$. Hence the relative velocity of the car *B* w.r.t *A* is 100 km./hr. at 36·87° E. of S.

Example 2·16. *A car leaves a certain place and travels due N. at 40 km./hr., whilst a motor cyclist leaves the same place at the same time and travels in a direction 30° S. of W. at 20 km./hr. Find (i) the velocity of the car relative to the cyclist, (ii) the velocity of the cyclist relative to the car.*

Solution : $\mathbf{v}_C$ and $\mathbf{v}_M$ represent the velocities of the car and the motor cycle respectively. The velocity of the car relative to the cyclist is from equation (2·17)

$$\mathbf{v}_{CM}=\mathbf{v}_C-\mathbf{v}_M$$



$\mathbf{v}_{CM}$ is thus obtained by the method of vector subtraction It is obvious that $\mathbf{v}_{CM}$ is the resultant of $\mathbf{v}_C$ and $-\mathbf{v}_M$ which is obtained by the parallelogram method. Hence its magnitude is given by

$$v_{CM}^2=40^2+20^2+2.40.20.\cos 60^\circ=2800$$

$$\therefore\ v_{CM}=52{\cdot}9 \text{ km./hr.}$$

The angle θ is given by $\tan\theta$

$$=\frac{20\sin 60^\circ}{40+20\cos 60^\circ}=0{\cdot}346$$

$$\therefore\ \theta=19^\circ$$

Thus the velocity of the car relative to the cyclist is 52·9 km./hr. at 19° E. of N.

If $\mathbf{v}_{MC}$ be the velocity of the cyclist relative to the car then from equation (2·17),

$$\mathbf{v}_{MC}=\mathbf{v}_M-\mathbf{v}_C=-\mathbf{v}_{CM}$$

It is thus equal and opposite to $\mathbf{v}_{CM}$. Hence the velocity of the cyclist relative to the car is 52·9 km./hr. at 19° W. of S.

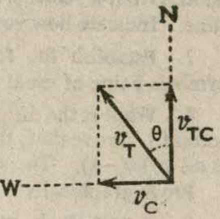
Example 2·17. *To a person moving due west in a car with a velocity 40 km./hr., a train appears to move due north with a velocity 40$\sqrt{3}$ km./hr. What is the actual velocity and the direction of motion of the train ?*

Solution : In the adjoining figure, $\mathbf{v}_C$ represents the velocity of the car and $\mathbf{v}_{TC}$ represents the relative velocity of the train w.r.t. car. If $\mathbf{v}_T$ be the velocity of the train, then from eqn. 2·17,

$$\mathbf{v}_{TC}=\mathbf{v}_T-\mathbf{v}_C$$

$$\text{or,}\quad \mathbf{v}_T=\mathbf{v}_{TC}+\mathbf{v}_C$$

$\mathbf{v}_T$ is thus obtained by adding $\mathbf{v}_{TC}$ and $\mathbf{v}_C$ vectorially ; obviously $\mathbf{v}_T$ is the resultant of $\mathbf{v}_{TC}$ and $\mathbf{v}_C$ obtained by the parallelogram method. Hence its magnitude is given by



$$v_T=\sqrt{v_{TC}^2+v_C^2}$$
$$=\sqrt{(40\sqrt{3})^2+(40)^2}=80 \text{ km./hr.}$$

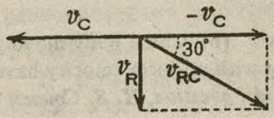
The angle θ is given by

$$\tan\theta=\frac{v_C}{v_{TC}}=\frac{40}{40\sqrt{3}}=\frac{1}{\sqrt{3}} \quad \therefore\ \theta=30°.$$

Hence, the actual velocity of the train is 80 km./hr. at 30° W. of N.

Example 2·18. *Rain is falling vertically. But it hits the windscreen of a car making an angle of 30° with the horizontal when the car is moving at 30 km. p.h. on a horizontal road. What is the actual velocity of the rain drops through the air ?*

Solution : In the adjoining figure, $\mathbf{v}_C$ and $\mathbf{v}_R$ represent the actual velocity of the car and the rain respectively. If $\mathbf{v}_{RC}$ be the velocity of the rain relative to the car, then from eqn. 2·17.



$$\mathbf{v}_{RC}=\mathbf{v}_R-\mathbf{v}_C$$

Hence $\mathbf{v}_{RC}$ is drawn by the method of vector subtraction. Obviously $\mathbf{v}_{RC}$ is the resultant of $\mathbf{v}_R$ and $-\mathbf{v}_C$. According to the question $\mathbf{v}_{RC}$ makes an angle 30° with the horizontal as shown in the Fig.

$$\tan 30°=\frac{v_R}{v_C}=\frac{1}{\sqrt{3}} \qquad \therefore\quad v_R=\frac{v_C}{\sqrt{3}}=\frac{30}{\sqrt{3}}=10\sqrt{3} \text{ km./hr.}$$

● EXERCISE ●

[A] Essay type questions

1. Explain the terms 'motion' and 'rest'. Discuss the importance of the reference frame in describing the motion of an object ? "Absolute rest or Absolute motion is unkown" Explain.

2. Define translational motion. How many kinds of such motion are possible ? Name them and discuss with examples. Distinguish between translational motion and rotational motion.

What kind of motion is taking place in each of the following examples ?

(i) arm chairs of rotating vertical merry-go-round. (ii) fly-wheel (iii) wheels of a moving vehicle. (iv) the earth rotating round the sun (v) a car driven along a straight path (vi) axle of an electric motor.

3. Define :—(i) Displacement (ii) Speed (iii) Velocity (iv) Acceleration. Name their units. Why does time occur twice in the unit of acceleration ?

4. Distinguish between the following :—(i) speed and velocity (ii) average speed and instantaneous speed (iii) average velocity and instantaneous velocity (iv) acceleration and retardation (v) average acceleration and instantaneous acceleration.

Which one is more important in dynamics—average acceleration or instantaneous acceleration ?

5. Define and explain giving at least one example in each case—(i) uniform motion (ii) uniformly accelerated motion (iii) non-uniform accelerated motion. How do you distinguish between rectilinear and curvilinear motion ?

6. A body has an initial velocity and a uniform acceleration. Explain how can you deduce from a velocity-time graph (i) the average velocity (ii) the distance travelled in a given time. Indicate how you can get the instantaneous velocity from the above graph.

7. Establish the following equations :—(i) $v=u+ft$ (ii) $s=ut+\frac{1}{2}ft^2$ (iii) $v^2=u^2+2fs$, the symbols being of usual significance.

8. What is the distinction between the distance traversed by a particle in t second and t-th second ? Prove that, the distance traversed by a uniformly accelerated particle in t-th second is $u+\frac{1}{2}f(2t-1)$. The symbols are of usual significance.

Prove graphically $s=ut+\frac{1}{2}ft^2$

9. A particle is moving along a straight path. Draw its velocity-time graph for the following cases :—

(i) When the particle moves with uniform acceleration
(ii) When the acceleration of the particle increases
(iii) When the acceleration of the particle decreases
(iv) When the distance traversed by the particle obeys the relation $s=4+5t+2$
(v) When the acceleration of the particle is given by $f=12\cos(6t)$

10. What are meant by 'Relative velocity' and 'Relative acceleration' ?

Deduce the expressions for the relative velocity and accleration of a body with respect to a second body which is also in motion.

[B] Short answer type questions

1. (a) Can a body have an acceleration with zero velocity ? (b) Can a body moving with uniform speed have variable velocity ? (c) Can a body moving with uniform velocity have variable speed ?

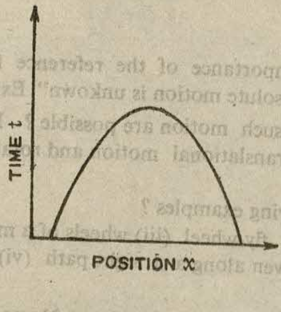
[*Model question, H. S. Council*]

2. (i) Can an object have an eastward velocity while experiencing west-ward acceleration ? (ii) Can the direction of the velocity of a body change when its acceleration is constant ?

[*Model question, H, S. Council*]

3. A man walks round the perimeter of a square shaped garden, each side of it being of length 100 m. When he returns to the starting point, his displacement is (i) 100 m (ii) 400 m (iii) zero. Which one is the correct answer ?

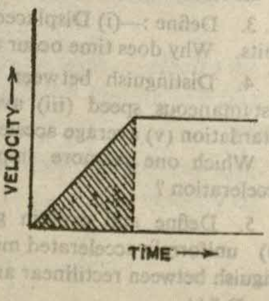
4. What is the nature of the motion expressed by the equation $s=s_0+ut+\frac{1}{2}ft^2$



5. Is the time variation of position, shown in the figure observed in nature. Answer with reasons.

[*I. I. T. '79*]

6. In the given velocity-time graph, the area of the shaded triangle represent which one of the following quantities ? (i) average speed (ii) average acceleration (iii) distance traversed by the uniformly accelerated body (iv) instantaneous acceleration.



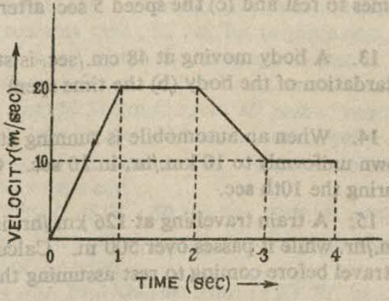
7. The figure represents the velocity-time graph of a particle moving in a straight line. Answer from the figure the following questions :—

(i) Did the body ever move with uniform velocity ?

(ii) Did the body ever move with uniform acceleration ?

(iii) What is the distance traversed by the body in 4 second ?

(iv) Did the body ever move with retardation ?



8. A train is moving along a straight track. Explain what would be the nature of its velocity-time graph in the following cases :—(i) It is moving with a uniform acceleration (ii) Its acceleration is increasing. [*H. S. '78*]

9. What could be the nature of the path of a moving body if its velocity and acceleration be always directed along the path ?

10. A boy sitting on a rail road car moving with a constant velocity drops a coin while leaning over the car. Describe the path of the coin as seen by (i) the man on the train (ii) a person standing on the ground near the rail. [*Model question, H. S. Council*]

11. During rains, the rain-drops falling vertically appear to come down obliquely to a person sitting in a running train. Explain. [*H. S. '79*]

Problems.

1. How long would a train travelling at a steady speed of 40 km. p. h. take to travel through a tunnel, which is 6 km. long ? [*Ans.* 9 min.]

2. A car covers first quarter of its journey at a speed of 30 km./hr., second quarter at 40 km./hr. and the rest at 60 km./hr. What is the average speed of the car during its journey ? [*Ans.* 43·64 km./hr.]

3. A motor car moves 40 km. due east and then 30 km. due north in 6 hr. Calculate its average (i) speed, (ii) velocity in m./sec.

[*Ans.* (i) 3·24 m./sec., (ii) 2·31 m./sec., 36·9° north of east]

4. A car starts from rest with constant acceleration of 9 m./sec.^{2}. Find its (a) instantaneous speed at the end of 10 sec., (b) the average speed for the 10 sec. interval and (c) the distance covered in 10 sec. from rest. [*Ans.* (a) 90 m./sec. (b) 45 m./sec. (c) 450 m.]

5. A train starts from a station and acquires a velocity of 60 km. p. h. in 3 min. Calculate the acceleration and the distance moved in 3 min. from the station.

[*Ans.* 9·26 cm./sec.^{2}, 1·5 km.]

6. A car moves from rest with a uniform acceleration of 1·5 m./sec^2. After what time and distance will its speed be 36 km./hr. ? [*Ans.* 6·67 sec. ; 33·33 m.]

7. Before leaving the ground, an aeroplane travelling with constant acceleration makes a run on a field of 600 m. in 20 sec. from rest. Calculate (a) the acceleration (b) the speed in km./hr. with which the plane leaves the ground and (c) the distance travelled during the first and the last second of its run. [*Ans.* (a) 3 m./sec.^{2} (b) 216 km./hr. (c) 1·5 m. ; 58·5 m.]

8. A motor car increases its speed from 30 km./hr. to 60 km./hr. in 30 sec. while travelling north. What is the acceleration ? [*Ans.* 27·8 cm./sec.^{2} towards North]

9. A train's speed increases uniformly from 15 km./hr. to 60 km./hr. in 15 sec. Determine (a) the average speed (b) the acceleration and (c) the distance travelled during the period.

[*Ans.* (a) 37·5 km./hr. (b) 83·33 cm./sec.^{2} (c) 156·25 m.]

10. A body moving at 5 cm./sec. is being accelerated uniformly and passes over next 300 cm. in 20 sec. Calculate (a) the acceleration of the body, (c) the speed of the body at the end of 20 sec. [*Ans.* (a) 1 cm./sec.^{2} (b) 25 cm./sec.]

11. A motor car starts with a velocity of 50 km./hr. and moves under a constant retardation of 20 cm./sec^2. Calculate (i) the velocity after 5 sec., (ii) the time when it stops, and (iii) the distance moved before coming to rest. [*Ans.* (i) 46·4 km./hr., (ii) 69·45 sec., (iii) 482·2 m.]

12. A train moving with a speed of 48 km./hr. is brought to rest in 20 sec. by the application of brakes. Calculate (a) the retardation (b) the distance travelled before the train comes to rest and (c) the speed 5 sec. after the application of brakes.

[*Ans.* (a) 0·67 m./sec.^{2} (b) 133·33 m. (c) 36 km./hr.]

13. A body moving at 48 cm./sec. is stopped within a distance of 30 cm. Calculate (a) the retardation of the body (b) the time spent to stop the body.

[*Ans.* (a) 38·4 cm./sec.^{2} (b) 1·25 sec.]

14. When an automobile is running at 30 km./hr., the brakes are applied and it slows down uniformly to 10 km./hr. in 10 sec. Calculate the acceleration and the distance covered during the 10th sec.

[*Ans.* −55·6 cm./sec.^{2} ; 305·2 cm.]

15. A train travelling at 126 km./hr. is checked by signals and its speed is reduced to 54 km./hr. while it passes over 500 m. Calculate (a) the acceleration. (b) How much further will it travel before coming to rest assuming the same acceleration ?

[*Ans.* (a) −1 m./sec.^{2} (b) 112·5 m.]

16. A car starts from rest and accelerates uniformly to 40 km./hr. in 27 sec. It then continues to move at uniform speed for the next 3 min. The brakes are then applied to bring the car at rest in 18 sec. Calculate the total distance travelled by car.

[*Ans.* 2·25 km.]

17. A train travelling at 36 km./hr. has its speed reduced to 9 km./hr. in 10 sec. Assuming the brakes exert a constant retarding force, find how far the car will travel before coming to rest from a speed of 54 km./hr. ?

[*Ans.* 150 m.]

18. A bullet leaves the barrel of a gun with a velocity of 600 m./sec. If the barrel is 150 cm. long, find (a) the average acceleration of the bullet within the barrel and (b) time required by the bullet to travel the length of the barrel.

[*Ans.* (a) 120 km./sec.^{2} (b) 5×10^{-3} sec.]

19. A car has speeds of 18 km./hr. and 54 km./hr. at the beginning and end of a measured half-kilometre distance. If the car possesses uniform acceleration, how much further must it travel before it attains a speed of 90 km./hr. ?

[*Ans.* 1 km.]

20. A body moving with uniform acceleration describes 22 m. in the 3rd sec. and 34 m. in 5th sec. Calculate (a) the initial velocity (b) the acceleration and (c) the distance moved in the 10th sec.

[*Ans.* (a) 7 m./sec. (b) 6 m./sec.^{2} (c) 64 m.]

21. A train starts from a station *A*, accelerates uniformly at the rate of 1·5 m./sec.^{2} and reaches its maximum speed in 20 sec. It then continues to move at this speed for 10 min. and then retards uniformly to rest at the station *B* after further 30 sec. Calculate (*a*) the maximum speed of the train (*b*) the retardation and (*c*) the distance between the stations *A* and *B*.

[*Ans.* (a) 108 km./hr. (b) 1 m./sec.^{2} (c) 18·75 km.]

22. When a batsman strikes a cricket ball its velocity changed from 30 m./sec. due north to 30 m./sec. due south. What was the change in velocity ? If the hit lasts for 0·02 sec., calculate the acceleration produced.

[*Ans.* 60 m./sec. ; 3×10^3 m./sec.^{2}]

23. A swimmer whose speed is 2 km. p. h. in still water requires to cross a river 1 km. wide. The river flows at 1 km. p.h. (*a*) Find the direction he must take and the time he will require to reach a point on the other bank directly opposite his starting point. (*b*) If he swims straight across the river, find how long will the crossing take and at what point on the opposite bank will he reach ?

[*Ans.* (*a*) At 60° with the bank upstream, 34·64 min. (*b*) 30 min. ; 0·5 km. downstream from a point directly opposite his starting point.]

24. The pilot of an aeroplane which flies at 260 km./hr. when there is no wind wishes to reach a point 120 km. due south when a 100 km./hr. wind is blowing due east. In what direction must he point the nose of the machine ? Also find the time the aeroplane takes to reach the point.

[*Ans.* 22°37′ W. of S. ; 0·5 hr.]

25. A boat crosses a river, 500 m. wide and flowing at 3 km./hr., from one bank to a point directly opposite on the other bank in 7·5 min. Find the velocity of the boat in still water and the direction in which it is actually steered.

[*Ans.* 5 km./hr. ; 53°8′ with bank upstream]

26. A car travelling due east at 60 km./hr. is overtaking a loaded truck travelling in the same direction at 20 km./hr. on a straight road. Find (*a*) the velocity of the car relative to

the truck and (*b*) the velocity of the truck relative to the car. (*c*) Repeat your calculation for the car travelling in the opposite direction.

[*Ans.* (*a*) 40 km./hr. towards east ; (*b*) 40 km./hr. towards west ; (*c*) 80 km./hr. towards west ; 80 km./hr. towards east.]

27. A car moves due north at 50 km./hr., while a wind blows from the north-east at 30 km./hr. Find the speed and direction of the wind as they appear to a man in the car.

[*Ans.* 74·31 km./hr. ; 16°35′ east of north]

28. A motor car is travelling due north along a straight road at 40 km./hr. and a cyclist is riding due east along another straight road at 30 km./hr. Find the velocity of the car relative to the cyclist and also the velocity of the cyclist relative to the car.

[*Ans.* 50 km./hr., 53°8′ N. of W. ; 50 km./hr., 53°8′ S. of E.]

29. To a cyclist riding towards north at 10 km. p.h. the wind appears to be blowing from the north-west at 12 km. p. h. Find the true velocity of the wind.

[*Ans.* 8·62 km./hr. from 79°53′ W. of S.]

30. A man rows directly across a flowing river in time t_1 and rows an equal distance down the stream in time t_2. If u be the speed of the man in still water and v that of the stream, show that

$$t_1 : t_2 = \sqrt{u+v} : \sqrt{u-v}$$

31. A train 100 m. long is travelling at 40 km. p.h. along a straight track. Another train of length 75 m. travels at 50 km. p.h. along a parallel track in opposite direction. Find the velocity of one train relative to the other. Also find the time taken for them to cross each other. [*Ans.* 90 km. p.h. ; 7 sec]

32. A man is walking at a velocity of 9 km./hr. parallel to railway track. A train of length 90 m. runs past the man from behind with a velocity 63 km./hr. Find (a) the velocity of the train relative to the man and (b) the time taken for the train to cross the man.

[*Ans.* (a) 54 km./hr. (b) 6 sec.]

33. A car A travels at 40 km. p. h. towards east starting from a point. Another car B travels at 80 km. p. h. towards north starting simultaneously from the same point. Find the time at which the distance between A and B will be 3 km. [*Ans.* 120·75 sec]

34. A particle passes a point A with a velocity of 30 m./sec. in the direction AB. After 8 sec. it passes the point B. If the distance between AB is 200 m., (i) find its velocity and displacement at the end of 30 sec. (ii) when does the particle reaches A again and (iii) if the acceleration ceases after 60 sec. what will be its displacement at the end of 75 sec. ?

[*Ans.* (i) −7·5 m./sec., 337·5 m. (ii) 48 sec. (iii) −1125 m.]

35. Two points A and B are 25 m. apart. A particle P starts from B with a velocity 2 m./sec. in the direction AB and has an acceleration 4 m./sec^2 in the same direction AB. Simultaneously with P another particle Q starts from A with a velocity 11 m./sec in the direction AB and has an acceleration of 2 m./sec.^{2} in the same direction AB. Prove that Q will never be able to catch P and the minimum distance between them will be 5 m.

36. The points A, B and C lie on the same straight line the distance AB being 60 m. A particle crosses the point A with a velocity 2 m./sec. and moves with uniform acceleration along the straight line. After crossing the point B, it takes 10 sec. to reach C with velocity 5 m./s. Find the acceleration of the particle. [*Ans.* 0·1 m./sec^2]

Harder Problems

1. A car possesses a velocity 5 m./sec. towards south and then the velocity of the car changes to 8 m./sec. towards east in 20 sec. Find the average acceleration during this interval.

[*Ans.* 0·47 m./sec.^{2} at 32° N. of E.]

2. In a cricket match when the batsman hits the ball its velocity changed from 60 m./sec. due north to 80 m./sec. due west. If the time of striking the ball is 10^{-2} sec, calculate the acceleration produced. [*Ans.* 10^4 m./sec.^{2} at 53·06° W. of S.]

3. A body travels 200 cm. in the first two seconds and 220 cm. in the next four seconds. What will be the velocity at the end of seventh second from the start ? [*I. I. T.* '64]

[*Ans.* 10 cm./sec.

4. A motor cycle moving with uniform retardation takes 10 sec. and 20 sec. to travel successive quarter kilometre. How much further will it travel before coming to rest?

[*Ans.* 10·42 m.]

5. A car *A* travelling at 18 km./hr. along a straight road observes a car *B*, 100 m. ahead, travelling at 54 km./hr. *A* accelerates uniformly and overtakes *B*, after *B* has travelled 600 m. Calculate *A*'s acceleration and also its speed at the instant it overtakes *B*.

[*Ans.* 62·5 cm./sec.^{2}; 108 km./hr.]

6. A uniformly accelerated body travels during the first two intervals of 6 sec. distances of 30 m. and 80 m. Find the distance travelled by the body at 20th sec. [*Ans.* 27·92 m.]

7. Robbers in a car are moving with a uniform speed of 72 km./hr. They pass a policeman on a motor cycle at rest. The policeman immediately starts chasing the robbers. He accelerates at 4 m./sec^2. for 9 sec. and then travels at a constant speed. When and where will he overtake the robbers?

[*Ans.* 10·125 sec.; 202·5 m.]

8. A mailtrain travelling with a speed of 72 km./hr. is 200 m. behind a goods train which is also moving on the same track in the same direction with a speed of 18 km./hr. The driver of the mail train immediately pulls the brakes so as to produce a constant retardation of 2 m./sec^2. If the goods train continues with the same constant speed, calculate and find whether there will be any collision between the two trains.

[*Ans.* No collision]

9. A man on the road is just 9 m. behind the entrance door of a train when the train begins to take motion from rest with a uniform acceleration of 2 m./sec^2. The man immediately starts and runs with a uniform speed to get into the train. He is just able to get in. Find the speed of his running.

[*Ans.* 6 m./sec.] [*Jt. Entrance* '72]

10. A car *A* is travelling on a straight level road with a uniform speed of 60 km./hr. It is followed by another car *B* which is moving with a speed of 70 km./hr. When the distance between them is 2·5 km., the car *B* is given a deceleration of 20 km./hr^2. After what distance and time will *B* catch up with *A*?

[*Ans.* 32·5 km., 0·5 hr.] [*I. I. T.* '66]

11. The maximum speed of a bus is 72 km. p.h. It accelerates uniformly at the rate of 1 m./sec^2. and retards uniformly at the rate 4 m./sec^2. Find the least time in which it can do a journey of 1 km.

[*Ans.* 62·5 sec.]

12. What velocity must be imparted on a cricket ball moving at 15 m./sec. in order to make it travel at 20 m./sec. towards the square leg i.e, in a direction at right angles to its original course? If the change in velocity is effected in $\frac{1}{5}$ sec. find the acceleration.

[*Ans.* 25 m./sec., 126°52′ with the original direction; 125 m./sec.^{2}]

13. A swimmer whose velocity in still water is 4 km./hr. wishes to cross a river 1 km. wide flowing at 3 km./hr. He can swim only for 15 min. Find (a) the direction he must take (b) the distance he covers.

[*Ans.* (a) Perpendicular to the bank (b) 1·25 km.]

14. To cross a river 500 m. wide a boat capable of travelling at 10 km. p. h. in still water, is steered upstream at 45° to the bank. If the river flows at 1 km. p.h. find how much time the boat is wasting.

[*Ans.* 1·24 min.]

[*Hints.* Find the time for the quickest trip across (when steered normally to the bank) and compare it with the one the boat is actually taking.]

15. A man is walking on a horizontal road at 3 km./hr. The rain appears to him to come down vertically at the rate of 3 km./hr. Calculate the actual velocity and the direction of the rainfall.

[*Ans.* $3\sqrt{2}$ km./hr.; at 45° with the vertical]

16. Rain is falling vertically at 3 km./hr. At what angles to the vertical should a man walking at 1·5 km./hr. hold his umbrella so as to obtain the maximum protection?

[*Ans.* 26°34′ to vertical]

17. A train starts from a station *A* and covers a distance of 4 km. to reach the next station *B*. If the maximum possible acceleration of the train is 1·5 m./sec.^{2} and its maximum retardation is 6 m./sec.^{2}, find the least time in which the train can undertake the journey from *A* to *B*.

[*Ans.* 81·65 sec.]

18. A body moving in a straight line with uniform acceleration describes three successive equal distances in time intervals t_1, t_2 and t_3 respectively. Show that

$$\frac{1}{t_1}-\frac{1}{t_2}+\frac{1}{t_3}=\frac{3}{t_1+t_2+t_3}$$

18. A mail train and a goods train are moving on the same track in the same direction with velocities 108 km./hr. and 36 km./hr. respectively. The driver of the mail train sights the goods train when the latter is only 175 m. ahead of him and immediately puts on brakes to get a deceleration of 0·65 m./sec.^{2} At the same instant the driver of the goods train apprehending danger from behind makes his train pick up an acceleration of 0·5 m./sec.^{2} Determine whether a collision can be averted.

[*Ans.* yes]

3
CHAPTER
NEWTON'S LAWS OF MOTION : FORCE

3·1. Newton's Laws of Motion.

In the preceding chapter, the motions of bodies have been discussed without going into the laws governing the motion. In this chapter, these laws will be discussed. As we shall see, such discussion will lead us to the conception of the quantities, mass and force.

Sir Isaac Newton first introduced these concepts systematically. In his celebrated work "Principia", he enunciated three fundamental laws governing motion, which constitute the foundation of machanics. The laws are axiomatic ; no theoretical proof of them is available. But their real 'proof' lies in the astonishing success with which they can be applied to explain various problem in physics and engineering. The laws state :

1. **First Law**—*Every material body persists in its state of rest or of uniform motion in a straight line if and only if it is not compelled by any external force to change that state.*

2. **Second Law**—*The rate of change of the momentum of body is proportional to the impressed force and takes place in the direction in which the force acts.*

3. **Third Law**—*To every action there is an equal and opposite reaction.*

3·2. Discussion of the First Law.

The first law of motion provides two very important concepts viz., (1) *the inertia of a body* and (2) *the definition of force.*

Inertia of a body—The first part of the law indicates that left to itself, a body, if as rest, has a tendency to remain in that state of rest or if moving, it always continues to move in a straight line with uniform velocity. This tendency of all material bodies to maintain their state of rest or of motion is a fundamental property of matter and is called inertia. For this reason, Newton's first law of motion is often called *the law of inertia.*

The **inertia** *of a body is defined as that property of it, which tends to resist a change in its state of rest or of motion. More massive bodies have greater inertia. Hence mass is defined as a quantitative measure of inertia.*

Obviously the property of inertia may be of two kinds. viz., (1) Inertia of rest and (2) Inertia of motion.

Inertia of rest—*This is defined as the property of material bodies to persist in their state of rest.* This seems obvious to us. It is a common experience that a body remains at rest unless something is done to change that state. The body does not suddenly move out of its own accord.

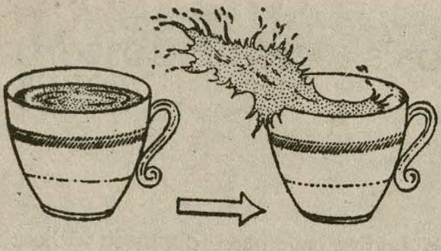


Fig. 3·1

Demonstration Experiments :

(1) A cup is filled up with water and then pulled sharply [Fig. 3·1]. Some water will splash behind because its inertia of rest tries to keep it in the same place.

(2) A ball *B* is put on a piece of card board *R* just above a hollow cup *H* fixed on a vertical stand *V* [Fig. 3·2]. A stiff metal spring *S* is fixed on the same base. This can be swung aside and held there by clamp *C*. When the spring is released, it sharply strikes the card board piece *R* and imparts a sudden velocity to it. The card flies away while the ball due to its inertia of rest does not move forward but drops inside the cup. However, if the card is given a slow push, the ball will also move forward and will fall outside the cup.

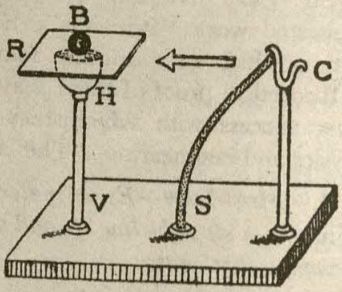


Fig. 3·2

(3) A large mass *M* is suspended by a piece of thread *A*. It can be pulled downward by another thread *B* [Fig 3·3]. If the pull *P* is applied slowly and steadily, the string *A* snaps ; but if the pull *P* is applied suddenly, the string *B* snaps. In the first case, the slow and steady pull brings the mass *M* downwards, thereby stretching the string *A* ; ultimately it snaps. But when the pull is sudden, the force *P* is momentarily very large. Due to its inertia, the mass *M* does not have time to move down so that the lower thread *B* gets snapped under the action of *P*.

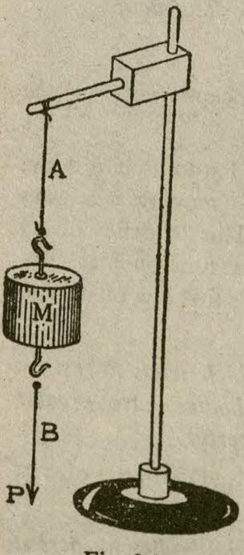


Fig. 3·3

Practical Illustrations : When a vehicle starts suddenly and rapidly gains speed, a passenger standing or sitting unmindfully leans backwards. The lower part of his body being in contact with the vehicle, shares its motion but the upper part due to inertia of rest, tends to stay at rest. The man thus leans backwards.

To clean a garment from dust particles, it is suddenly set into motion. The loose dust particles tending to remain at rest, fall off. If the horse suddenly starts galloping, the rider leans backwards. A stone thrown at a window pane smashes it but a bullet fired against the pane makes a clean hole. All these illustrate the inertia of rest.

Inertia of Motion—*The property of a body to continue in its uniform motion along a straight line is called its inertia of motion.* This aspect of first law does not seem obvious. It is a common experience that a moving body left to itself gradually stops. But a simple point is to be noted here. The force of friction due to ground or air or due to other causes always acts on the body. The action of these external forces brings the motion to a stop. These external forces can be minimized but can not be altogether eliminated. Thus a marble rolling on ground comes quickly to rest ; but the same marble rolling on a marble floor or glass plate continues in its state of motion for much longer time. The nearest approach to resistance free motion is the motion of the stellar bodies in space where there is virtually no material medium. The law is thus obtained as an inference from experimental observations.

Demonstration Experiments . (1) A cup filled up with water is steadily slided towards a barrier. When it hits the barrier, the cup stops but the water continues moving in forward direction and thus splashes out [Fig. 3·4].

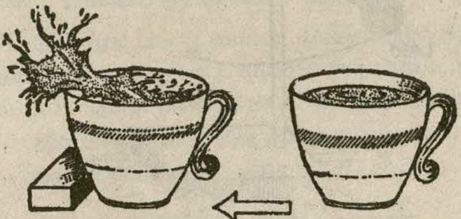


Fig. 3·4

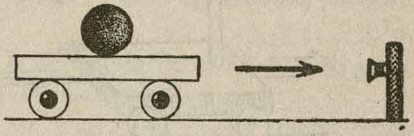


Fig. 3·5

(2) A spherical ball is placed on a toy car [Fig. 3·5]. If the car moves steadily the ball will share the motion. When the car stops after colliding with the buffer at the end of the track, the ball will continue its forward motion due to inertia and will jump over the barrier.

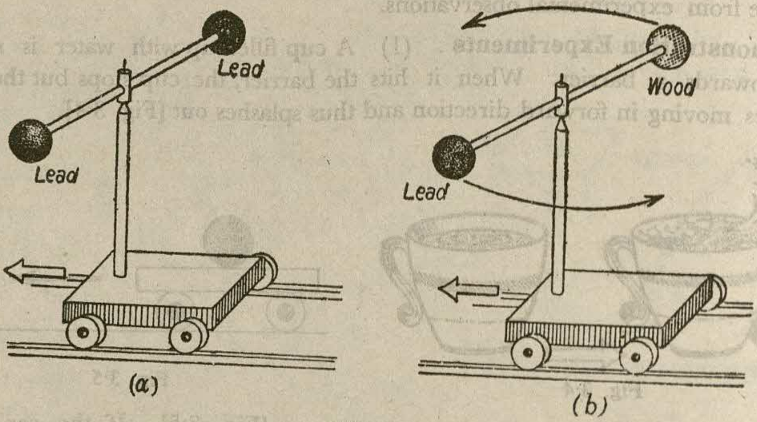
Practical Examples : When a moving vehicle suddenly stops, a passenger standing or sitting unmindfully falls forward. His lower part being in contact with the vehicle stops with it but the upper part of the body continues moving forward so that the person leans forward. Similarly, a person jumping out of a moving vehicle is thrown forward. In the long jump event, an athlete always runs through a little distance. The inertia of motion helps him to cover a long distance in air.

If a person on a moving vehicle throws a ball vertically upwards, it again comes back to his hand after a short interval of time, the motion of the vehicle remaining the same in that interval of time. Though the person has meanwhile advanced through a certain distance, the ball also retains the same forward speed due to inertia and thus ultimately comes down to his hand. However, this does not happen when the motion of the vehicle changes during the interval. The ball drops behind him if the vehicle accelerates and it drops in front of him if the vehicle retards.

Inertia and mass—We know that mass of a body is the measure of the quantity of matter in it. Actually, the meaning of the word inertia carries with it the essence of the concept of mass. It is a common experience that greater efforts are necessary to change the state of rest or of motion of a body having a larger mass. Larger masses thus have greater inertia. It is easy to kick a football and set it moving, but to put into motion a lead ball of similar size is very difficult. Similarly a moving football can be easily stopped, but attempts to stop a lead ball of same size and moving with same speed would send one spinning.

Mass is thus a measure of inertia. Both are measured in the same units.

Demonstration Experiment : Two balls are mounted at the ends of a thin uniform bar. The bar is attached to a vertical stand mounted on a toy car. The bar with the balls can rotate freely about a vertical axis [Fig. 3·6].



(*a*) (*b*)

Fig. 3·6

At first two identical lead balls are chosen. The car is pushed in a particular direction. The bar is placed perpendicular to this intended direction of motion. Now as the car suddenly moves, starting from rest, the bar will be found to remain in its original direction. If the car suddenly stops, then also the bar remains undisturbed [Fig. 3·6 (*a*)].

Now, we replace one of the lead balls by a wooden ball of the same size. As the car is suddenly moved from rest, the bar is found to be rotating. The lead ball moves backward while the wooden one goes forward [Fig. 3·6 (*b*)]. Similarly if the moving car is suddenly stopped, the lead ball is found to be moving forward, the wooden one going backward.

This experiment nicely demonstrates the effect of inertia and the dependence of inertia on mass. In the first case, inertias of both the balls having the same mass are the same. Hence both of them have the same tendency to remain at rest or in motion. Thus no effect is observed on suddenly moving or stopping the car. In the latter case, the mass and consequently, the inertia of the lead ball is greater than the wooden one. The lead ball thus has the greater tendency to remain at rest or in motion than the wooden ball. Hence as the car suddenly moves or stops, the lead ball moves further backward or forward than the wooden one.

Force—The second part of the first law provides the conception of force. If we see that an object originally at rest begins to move or if during its motion it speeds up, slows down or changes the direction of motion, we conclude that some external cause, termed force, is acting upon the body. Change of state of rest or of motion of a body thus results only under the action of an external force.

It is to be noted, however, that if there be two or more external forces acting on a body so that the resultant force is zero, there is evidently no effect on the body. As a matter of fact no object can be found upon which there are no external forces. Two men can push a table in opposite directions but with forces of same magnitude. Since the net unbalanced force is zero, the table remains at rest. The word "force" in the law means an *unbalanced force*. Thus we define:

Force *is that which changes or tends to change the state of rest or of uniform motion of a body along a straight line.*

Since a change in speed or direction of motion means a change in velocity of a body, this may be taken as the criterion for the presence of an unbalanced force acting on it.

3·3. Discussion of the second law.

The first law of motion provides the concept of force ; the second law, as we shall see now, gives a quantitative definition of force. Before proceeding further, the concept of *momentum* must be explained.

Momentum : *The momentum of a body is defined as the product of its mass m and its velocity* **v**. Employing the symbol p for momentum,

$$p=mv \quad \ldots \quad \ldots \quad (3{\cdot}1)$$

Since it is the product of a scalar quantity (mass) and a vector quantity (velocity), momentum is also a vector quantity. Its direction is represented by an arrow drawn in the same direction as that of the velocity. The unit of momentum in C.G.S. system is gm. cm./sec., in M.K.S. system kg. m/sec. and in F.P.S. system lb. ft./sec.

Newton used the word 'motion' as the concept of momentum. According to him a fast moving body has more motion than when it is moving slowly. Also a heavily loaded truck, moving with the same velocity as a car, has more motion in it than the car, because the mass of the truck is much greater. Greater effort i.e., greater force will be required to stop the truck within a certain interval of time than the force required to stop the car within the same interval. Thus, to get a measure of the quantity of motion in the body, we must take into consideration both its mass and its velocity i.e., its momentum. This explains the terrific impact of a rifle bullet ; though its mass is very small, but its velocity is very high and the small mass multiplied by the high velocity gives it a large momentum.

Measurement of Force.

Let us consider a body of mass m moving initially with a velocity u. If a constant force P acts on it for a time t, its velocity will change. After the time t has elapsed, let its final velocity be v. Then the initial momentum of the body$=mu$ and the final momentum$=mv$. So the change of momentum in time $t=mv-mu$. Therefore, the rate of change of momentum$=\dfrac{m(v-u)}{t}=mf$ [$\because\ v=u+ft$.] where f is the acceleration produced by the action of the force.

From second law, the force P is proportional to the rate of change of momentum. Hence,

$$P \propto mf$$

or, $P = kmf$ where k is the constant of proportionality.

If we now choose unit force to be one that acting continuously upon unit mass produces unit acceleration, then $P=1$, when $m=1$ and $f=1$. In that case $k=1$. We thus arrive at the relation.

$$P = mf \quad \dots \quad \dots \quad \dots \quad (3{\cdot}2)$$

i.e., *force=mass × acceleration.*

Since acceleration is a vector quantity, mass being a scalar, *force is also a vector quantity. Obviously the directions of force vector and acceleration vector are the same.*

Alternative Method—The above relation can be obtained simply by using the method of calculus. If p be the momentum of a body of mass m moving with a velocity v, then

$$p = mv$$

The rate of change of momentum$=\frac{dp}{dt}=\frac{d}{dt}(mv)$

From second law, force P acting on the body is proportional to the rate of change of momentum. Hence,

$$P \propto \frac{dp}{dt} \quad \text{or,} \quad P = k.\frac{dp}{dt} = k.\frac{d}{dt}(mv)$$

$$= km.\frac{dv}{dt} \quad \text{Since mass of the body remains constant.}$$

$$= kmf \quad \text{where } f = \frac{dv}{dt} = \text{acceleration}$$

As before, the constant of proportionality k may be made equal to unity by the choice of suitable units. Hence

$$P = \frac{dp}{dt} = mf \quad \dots \quad \dots \quad (3{\cdot}3)$$

Unit of Force.

There are two systems of unit of force viz., (*a*) **absolute** (*b*) **gravitational**. *In the absolute system, the unit force is defined as that force which acting on a unit mass produces unit acceleration of it. In the gravitational system, the force with which the earth attracts a unit mass is defined to be the unit force.* [vide Art. 8·4].

The absolute units of force are so called because their values are independent of the place at which observation is made. The values of the gravitational units however vary from place to place, since the force on a body due to earth's gravitational attraction varies, though slightly, from place to place. For most practical purposes this variation is negligibly small; it is very gradual and does not exceed 0·6 percent over the whole surface of the earth. But for accurate scientific measurements, the place of the earth to which the unit is referred must be stated explicitly.

(a) Absolute units of Force.

In the C. G. S. system, the absolute unit of force is called a **dyne**. *A dyne is defined to be that force which acting on a mass of one gram produces in it an acceleration of one centimetre per second per second.*

The absolute unit of force in the F. P. S. system is called a **poundal** *which is the force that acting on a mass of one pound produces in it an acceleration of one foot per second per second.*

In the M. K. S. system, the absolute unit of force is called a **newton**. *If a force produces an acceleration of one metre per second per second acting on a mass of one kilogram, it is called newton.*

Relation among a newton, a dyne and a poundal.

By definition, $1 \text{ newton} = 1 \text{ kg.} \times 1 \text{ metre/sec.}^2$

$$= 1000 \text{ gm.} \times 100 \text{ cm./sec.}^2$$

$$= 10^5 \text{ dyne.}$$

Similarly, $1 \text{ poundal} = 1 \text{ lb.} \times 1 \text{ ft./sec.}^2$

$$= 453{\cdot}6 \text{ gm.} \times 30{\cdot}48 \text{ cm./sec.}^2$$

$$= 13825{\cdot}728 \text{ dyne} = 1{\cdot}3826 \times 10^4 \text{ dyne}$$

$$= 0{\cdot}13826 \text{ newton.}$$

(b) Gravitational units of force.

In the C. G. S. system, the gravitational unit of force is one **gram-weight** (gm. wt.). *It is defined as the weight of one gram i.e., the force with which the earth attracts a mass of 1 gm.*

Under the action of the force of earth's attraction, a body falls downward with an acceleration 'g'. Hence from eqn. (3·2),

$$1 \text{ gm. wt.} = 1 \text{ gm.} \times g \text{ cm./sec}^2 = g \text{ dyne.}$$

In the C. G. S. system, the value of g is taken to be 981 cm./sec.^{2} for most practical purposes. Hence,

$$1 \text{ gm. wt.} = 981 \text{ dyne.}$$

In the F. P. S. system, the gravitational unit of force is one **pound weight** (lb. wt.) *which is the weight of one pound. It is thus the force with which the earth attracts a mass of one pound.* **As before,**

$$1 \text{ lb. wt.} = 1 \text{ lb.} \times g \text{ ft./sec.}^2$$

$$= g \text{ poundal}$$

$$= 32{\cdot}2 \text{ poundal}$$

the value of g being taken to be 32·2 ft./sec.^{2} in the F.P.S. system.

The gravitational unit of force in M.K.S. system is one **Kilogram weight** (kg. wt.) *which is the weight of one kilogram. It is thus defined as the force with which the earth attracts a mass of one kilogram.* So,

$$1 \text{ kg. wt.} = 1 \text{ kg.} \times g \text{ metre/sec}^2 = g \text{ newton.}$$

Since in this system, the value of g is 9·81 metre/sec.^{2},

$$\therefore \quad 1 \text{ kg. wt.} = 9{\cdot}81 \text{ newton.}$$

Example 3·1. *A certain force acts on a body of mass 100 gm. and gives it a velocity of 50 cm./sec. in 25 sec. Find the magnitude of the force.*

Solution : Here, $m=100$ gm , $u=0$, $v=50$ cm./sec. and $t=25$ sec.

$$\text{Acceleration } f=\frac{v-u}{t}=\frac{50}{25}=2 \text{ cm./sec.}^2$$

$$\therefore \text{ Force } P=mf=100\times 2=200 \text{ dyne.}$$

Example 3·2. *A force acts on a 10 kg. mass and increases its velocity from 5 m./sec. to 10 m./sec. in 5 sec. Find the force (a) in dyne, and (b) in newton.*

Solution : Here, $m=10$ kg. $=10^4$ gm. ; $u=500$ cm/sec.
$v=1000$ cm./sec. ; $t=5$ sec.
$\therefore$ From equation (3·2), $P=mf$

$$=m\frac{v-u}{t}=10^4\times\frac{(1000-500)}{5}$$

$$=10^4\times 10^2=10^6 \text{ dyne } =10 \text{ newton.}$$

Example 3·3. *A body of 196 gm. is subjected to a force of 20 gm. wt. Find the (a) acceleration and (b) the time taken to move 1 m. from rest. If the force ceases to act, (c) how far will the body move in the next 5 sec. ? Given $g=980$ cm./sec.^{2}*

Solution : (a) $P=20$ gm. wt. $=20\times 980=19600$ dyne ; $m=196$ gm.

$$\therefore \ f=\frac{P}{m}=100 \text{ cm./sec.}^2$$

(b) $s=1$ m. $=100$ cm. ; $u=0$; $f=100$ cm./sec.^{2} Hence from the equation $s=ut+\frac{1}{2}ft^2$, we get, $t=1{\cdot}414$ sec.

(c) To find the velocity acquired by the body at $t=1{\cdot}414$ sec. starting from rest, we apply the equation $v=u+ft$.

$$\therefore \ v=0+100\times 1{\cdot}414=141{\cdot}4 \text{ cm./sec.}$$

When the force ceases to act, the body will describe uniform motion ; hence $f=0$, and in this case the displacement, $s=vt=141{\cdot}4\times 5=707$ cm. $=7{\cdot}07$ m.

Example 3·4. *A body of mass 1 kg. is moving with a velocity of 100 cm./sec. It is brought to rest in 10 sec. by applying a force against its motion. What distance it moved through before coming to rest ? What is the magnitude of the retarding force ?*

Solution : Here, $m=1$ kg. $=1000$ gm., ; $u=100$ cm./sec. $v=0$; $t=10$ sec.

$\therefore$ Acceleration $f=\dfrac{v-u}{t}=-\dfrac{100}{10}=-10$ cm./sec. (*−ve* sign denotes retardation)

$\therefore$ Retarding force $P=m.f=1000\times 10=10^4$ dyne. (neglecting the *−ve* sign)

Again, if s be the distance through which the body moved before coming to rest, we have

$$v^2=u^2+2fs \quad \text{or,} \quad 0=100^2+2(-10)s$$

$$\therefore \ s=\frac{100\times 100}{2\times 10}=500 \text{ cm.}$$

Example 3·5. *A motor car of mass 1 metric ton is running with a speed of 36 km./hr. It is brought to rest within a distance of 20 metre. Find the average force of resistance in newton and the time required to stop the car.*

Solution : Here, $m=1$ metric ton$=1000$ kg.

$$u=36 \text{ km./hr}=\frac{36\times 1000}{60\times 60}=10 \text{ m/sec.}$$

$$v=0 \text{ and } s=20 \text{ m.}$$

$$v^2=u^2+2fs$$

$$\therefore\quad 0=10^2+2f\times 20$$

$$\therefore\quad f=-\frac{10\times 10}{2\times 20}=-2\cdot 5 \text{ m./sec.}^2 \text{ (retardation)}$$

$\therefore$ Average force of resistance $P=mf=1000\times 2\cdot 5=2500$ newton. Now if t be the time required to stop the car, then

$$\text{since } v=u+ft$$

$$\therefore\quad 0=10+(-2\cdot 5)t \qquad \text{or,}\quad t=\frac{10}{2\cdot 5}=4 \text{ sec.}$$

Example 3·6. *A bullet of mass 10 gm. is moving with a velocity of 100 m./sec. After passing through a mud wall 50 cm. thick, its velocity reduces to 20 m./sec. Calculate the average resistance of the wall in dyne and in newton.*

Solution : Here $m=10$ gm. ; $n=100$ m/sec.$=10^4$ cm/sec.

$$v=20 \text{ m./sec}=2\times 10^3 \text{ cm./sec. ; } s=50 \text{ cm.}$$

$$\because\quad v^2=u^2+2fs$$

$$\therefore\quad (2\times 10^3)^2=(10^4)^2+2f\times 50$$

$$\therefore\quad f=-\frac{(10^4)^2-(2\times 10^3)^2}{2\times 50}=-9\cdot 6\times 10^5 \text{ cm./sec.}^2 \text{ (retardation)}$$

$\therefore$ Average resistance $P=mf=10\times 9\cdot 6\times 10^5=9\cdot 6\times 10^6$ dyne$=96$ newton.

Example 3·7. *A hammer weighing 5 kg. descending from a height of 0·5 m. struck a nail and stopped in $\frac{1}{10}$th sec. Find the force, both in dyne and in gm. wt. exerted on the nail.*

Solution : Let the velocity of the hammer just before striking the nail be v. Since it falls from rest through a height of 0·5 m. i.e, 50 cm, we have

[vide Art. (8·8)]

$$v=\sqrt{2gh}=\sqrt{2\times 981\times 50}=313\cdot 2 \text{ cm./sec.}$$

Hence the momentum of the hammer$=mv=5\times 10^3\times 313\cdot 2$

$$=15\cdot 66\times 10^5 \text{ gm. cm/sec.}$$

Since it comes to rest in $\frac{1}{10}$th sec.,

$$\text{Rate of change of momentum}=\frac{15\cdot 66\times 10^5}{1/10} \text{gm. cm./sec.}^2$$

Thus from Newton's second law, the force exerted on the hammer

$$=15\cdot 66\times 10^6 \text{ dyne}=\frac{15\cdot 66\times 10^6}{981} \text{ gm. wt.}=1\cdot 596\times 10^4 \text{ gm. wt.}$$

This force acting upwards brings the hammer to rest. From Newton's third law, an equal downward force is exerted on the nail. Hence the downward force on the nail due to impact is $15\cdot 66\times 10^6$ dyne or $1\cdot 596\times 10^4$ gm. wt.

The weight of the hammer also acts downwards on the nail. Thus the total downward force on the nail

$$=(1\cdot 596\times 10^4+5000) \text{ gm. wt.}=2\cdot 096\times 10^4 \text{ gm. wt.}$$

$$=2\cdot 096\times 10^4\times 981 \text{ dyne}=2\cdot 056\times 10^7 \text{ dyne.}$$

3·4. Newton's first law as a special case of the second.

A closer scrutiny of Newton's laws enables us to deduce the first law from the second. From the relation $P=mf$ (deduced from the second law) it is evident that if $P=0$ we have $f=0$, i.e. $\frac{dv}{dt}=0$. Or, in other words, in absence of a force P, $\frac{dv}{dt}=0$ i.e. the velocity v is a constant. If this constant is zero, without any force a body's state of rest remains unchanged ; if the constant has any other value, without any force a body's state of uniform motion in a straight line does not change. Obviously, this is nothing but the first law of motion.

The concept that mass is a measure of inertia, is also obtained from the second law of motion. Inertia of a body is really the property of the body by which the body resists any change in its state of rest or of uniform motion in a straight line. The greater the inertia of a body, the greater is the force necessary to produce the same change in the condition of the body. Now, from the relation $P=mf$ it is evident that when f is constant P is proportional to m, i.e., the force P necessary to produce the same acceleration f, depends upon the mass of the body. The greater is the mass of the body the greater is the force required to produce the same acceleration, i.e., to produce the same change in the condition of the body. Thus mass of a body is really a measure of the intertia of the body.

3·5. Impulse.

The **impulse** *of a constant force acting on a body is defined as the product of the magnitude of the force and the time during which it acts on the body.*

Thus if a constant force P acts on a body of mass m for a time t then the impulse J of the force is given by

$$J=Pt \qquad \cdots \qquad \cdots \qquad (3\cdot4)$$

If the action of the force produces an acceleration f of the body so that its velocity changes from u to v during the said interval, then from eqn. (3·2),

$$\begin{aligned} J&=Pt=mft \\ &=m.\frac{v-u}{t}.t \qquad [\because\ v=u+ft\ ] \\ &=m(v-u) \\ &=mv-mu \qquad \cdots \qquad \cdots \qquad (3\cdot5) \\ &=p_2-p_1 \end{aligned}$$

where $p_1=mu$ and $p_2=mv$ give the initial and final momentum of the body.

Hence, *the change in momentum of a body equals the impulse of the force acting on it*. The relation is known as impulse-momentum relation.

Effect of a given impulse is thus to produce a change in momentum which is independent of the mass of the body. For example, a force of one dyne acting for one second on a toy car or a marble will produce, in each case, a momentum change of one *gm. cm./sec*. Of course, the change in velocity of the body depends on its mass.

Using calculus method, the above relation can be obtained directly.

Thus from eqn. (3·3), $P=\frac{dp}{dt}$

$$\text{or, } Pdt=dp \qquad \ldots \qquad \ldots \qquad \ldots \qquad (3{\cdot}6)$$

The left hand side of the above equation gives the impulse for the interval dt while the right hand side represents the change in momentum in that interval.

Since force is a vector quantity, *impulse is also a vector having the same direction as that of the force*. It is measured in units of force × time, which are the same as the units in which momentum is measured. Thus units of impulse in different systems are *gm. cm./sec.*, *lb.ft./sec.* and *kg. m./sec.*

Impulse of varying force.

If the force acting on a body for a time t be variable, then in order to find the impulse of the force, we divide the time t into a large number of very short intervals of time $\triangle t_1$, $\triangle t_2 \ldots \triangle t_n$. The force during each such interval may be taken as constant so that the impulse during each of these intervals can be calculated. Adding we get the total impulse for the finite interval t. Thus if P_1, P_2, P_n represent the values of the forces for the said intervals and v_1, $v_2 \ldots$ are the velocities at the end of corresponding intervals, then from eqn. (3·5), for the first interval $\triangle t_1$.

$P_1 \triangle t_1=mv_1-mu$, u being the initial velocity.

Similarly, for the other time intervals,

$$P_2\triangle t_2=mv_2-mv_1$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$P_n\triangle t_n=mv-mv_{n-1}$, v being the final velocity.

So that on adding, the total impulse is given by,

$$J=P_1\triangle t_1+P_2\triangle t_2+\ldots\ldots P_n\triangle t_n$$
$$=\Sigma P\triangle t=mv-mu$$

In the calculus notation, the above summing up process is nothing but integration. Thus

$$J=\int_o^t Pdt=\int_o^t \frac{dp}{dt}dt \qquad \text{[from eqn. (3·3)]}$$

$$=\int_{p_1}^{p_2} dp=p_2-p_1=mv-mu \quad \ldots \qquad (3{\cdot}7)$$

Graphical Representation of Impulse.

If the force acting on a body is constant, the plot of force against time will obviously be a straight line parallel to the time axis (shown in the Fig. 3·7 by a heavy line). The area of the rectangle formed by the line and the ordinates at $t=0$ and $t=t$ evidently gives the impulse of the force during that interval.

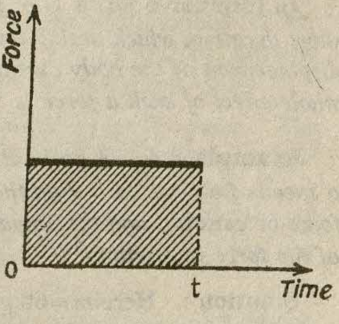


Fig. 3·7 : Impulse of a constant force

If however the force varies, the curve may be of any shape depending upon the nature of the variation [Fig. 3·8]. The above consideration still holds. We can divide the whole interval t into a large number of small intervals $\triangle t$ during each of which the force remains practically constant. The product $P \triangle t$ is then given by the area of the shaded strip. On evaluating the product $P\triangle t$ i.e., the impulse over each such interval and then summing up, we get the total impulse of the force. Obviously this is represented by the area under the force curve between the time $t=0$ and $t=t$. Hence we can write,

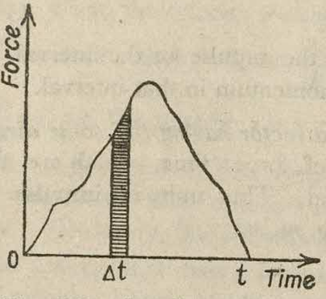


Fig. 3·8 : Impulse of a varying force

Impulse of a force$=J=$Area under the P—t curve.

3·6. Impulsive force.

Frequently we encounter a particular type of forces which are of very large magnitude but of very short duration. Such forces are called impulsive forces. These forces are of the nature of blows, collisions, explosions etc. Examples are : the collision of the striker with the pieces in the carom board, hitting a cricket ball with the bat, a ball bouncing on the floor, hitting a nail with the hammer, striking the strings in some musical instrumênts etc., etc.

The time of action of an impulsive force is supposed to be so small that the particle under application of the force has no time for an appreciable displacement ; the displacement during the action of the impulsive force is, therefore, neglected. However the forces being very large, a sudden change of velocity and hence of momentum takes place. Neither the magnitude of the force can be determined nor it is necessary. The whole effect of the force is measured by knowing the change of momentum produced i. e., by knowing the impulse of the force. Hence the name of such type of forces. We thus formally define an impulsive force as—

An **Impulsive force** *is a force of very large magnitude but of extremely short duration, which acting on a body produces a finite change of its momentum, displacement of the body during the action of the force being negligible. The whole effect of such a force is measured by its impulse.*

Example 3·8. *A golf ball of mass 60 gm. is hit by club. The ball acquires a speed of 50 m./sec. Find the impulse of the force exerted on the ball. If the force be constant and the impact lasts for 3×10^{-3} sec., calculate the magnitude of the force in newton.*

Solution : Here $m=60$ gm. ; $u=0$;

$v=50$ m./sec.$=5\times 10^3$ cm./sec.

From eqn. (3·5), the impulse of the force

$$J=m\,(v-u)=60\times 5\times 10^3=3\times 10^5 \text{ gm. cm./sec.}$$

Since $J=Pt$, where P is the average force exerted by the club on the ball during the impact which lasts for t sec.

$$\therefore \quad P=\frac{J}{t}=\frac{3\times10^5}{3\times10^{-3}}=10^8 \text{ dyne}=10^3 \text{ newton.}$$

Example 3·9. *A ball of mass 20 gm. moving with a velocity 30 cm./sec. is hit by a bat which causes it to move in reverse direction with velocity 60 cm./sec. What is the impulse of the force? If the blow lasts for 0·01 sec., find the average force in newton exerted on the ball.*

Solution : Since the direction of motion is reversed after impact the change in velocity, $v-u=60$ cm./sec.$-(-30$ cm./sec.$)=90$ cm./sec., the direction of initial velocity being taken as negative.

Hence from the eqn. (3·5), impulse of the force

$$\begin{aligned} J&=m(v-u)\\ &=20\times90\\ &=1\cdot8\times10^3 \text{ gm. cm./sec.}\end{aligned}$$

But $J=Pt$, P being the average force acting on the ball for a duration of t sec. Hence,

$$\begin{aligned} P=\frac{J}{t}&=\frac{1\cdot8\times10^3}{0\cdot01}=1\cdot8\times10^5 \text{ gm. cm./sec.}^2\\ &=1\cdot8\times10^5 \text{ dyne}=1\cdot8 \text{ newton.}\end{aligned}$$

3·7. Discussion of the third law.

The terms 'action' and 'reaction' in the statement of the third law simply stand for forces. Keeping this in view, the law can be restated as : **Whenever one body exerts a force on another, which is called action, the second body always exerts on the first a force, called reaction, which is equal in magnitude but opposite in direction to the action.** Thus if $\mathbf{F}_{AB}$ be the force exerted by a body A on another body B, the body B exerts a force $\mathbf{F}_{BA}$ on A such that

$$\mathbf{F}_{AB}=-\mathbf{F}_{BA} \qquad \ldots \qquad \ldots \qquad (3\cdot8)$$

The principle holds rigorously, no matter what the state of motion of the bodies may be. Whether the bodies exert forces on each other by physical contact or act on each other from a distance, the law always holds true.

The law thus asserts that *in nature forces always occur in pairs.* The relationship between them is perfectly reciprocal and either of them may be termed the action force and the other the reaction. It should be understood that one of these forces does not begin to play before the other, in the nature of cause and effect. Rather, the two forces arise at the same time. Consequently, when one vanishes, the other also vanishes at the same instant.

A question might be asked that, "if whenever a force acts on a body, there is always an equal and opposite reaction, then why should a body move at all ?" The answer to this apparent puzzle lies in the fact that the two forces always act on different bodies, not on the same body. Due to this reason, it will not be correct to compute the resultant of a pair of action-reaction forces and put it as zero. Two equal and opposite forces neutralise each other only when they act on the same body. Some simple examples will clarify the above discussion :

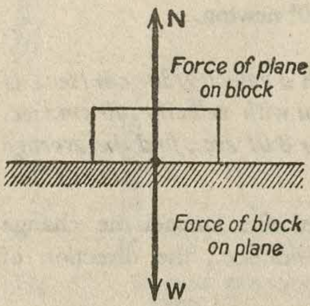


Fig. 3·9

(1) Consider a block resting on a plane. Due to its weight **W**, it will exert a force on the plane in the downward direction (Fig. 3·9). From third law, the plane will exert an equal and opposite force **N** on the block. **W** is thus the force of action on the plane and **N** the force of reaction on the block. Hence $\mathbf{W} = -\mathbf{N}$.

(2) If the plane is removed, the block falls downwards under the action of its own weight **W**, which is the force with which the earth attracts the block. From third law, the block will attract the earth with an equal and opposite force **W′**. Here **W** is the action force and **W′** the reaction force (Fig. 3·10). Hence $\mathbf{W}' = -\mathbf{W}$.

One might then be prompted to ask "why does the block move towards the earth and not the latter towards the former ?" Since the mass of the earth is very large compared to that of the block, the force acting on earth produces an imperceptibly small acceleration of it [$f = P/m$]. Any resulting movement of the earth is thus negligible. This would not remain true if the mass of the block becomes comparable to that of the earth.

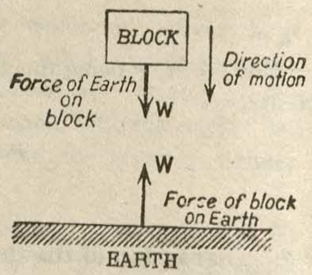


Fig. 3·10

(3) As a magnet attracts a piece of iron, the iron also attracts the magnet with an equal and opposite force. It is a common experience that if a magnet is placed near an iron piece, the latter moves towards the magnet. However, if we keep the piece of iron fixed, the magnet being free will move towards the iron.

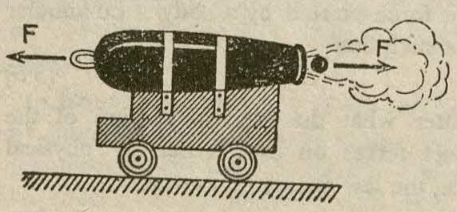


Fig. 3·11

(4) When a shell is fired from a cannon, the force **F** applied on the shell hurls it with a tremendous velocity in the forward direction. The shell in turn applies an equal and opposite reaction force **F** on the cannon (Fig. 3·11), under whose action the cannon moves backwards. This is known as recoil phenomenon.

(5) Another interesting example is that of a man pulling a rickshaw. As the man pulls the rickshaw forwards with a force **P** applied through the handle, the rickshaw also pulls the man backwards with an equal and opposite force **P** (Fig. 3·12). How is it that the rickshaw with the puller goes forward ?

To draw the richshaw forward, the puller strikes the ground obliquely with his feet and in doing so he exerts a force on the ground. Hence the ground applies an equal and opposite reaction force **N** on the man. This can be resolved in two components, **H** in the horizontal direction and **V** in the vertical direction. The vertical component **V** is neutralised by the weight of the man, acting downwards. When the horizontal component **H** becomes greater than the force **P** applied by the rickshaw in the backward direction, the man moves forward, under the action of the force **H—P**.

Let us now consider the rickshaw separately. If **f** be the total resisting force on it due to friction of the ground, then it will move provided $\mathbf{P} > \mathbf{f}$; its motion thus occurs under the action of the force $\mathbf{P} - \mathbf{f}$.

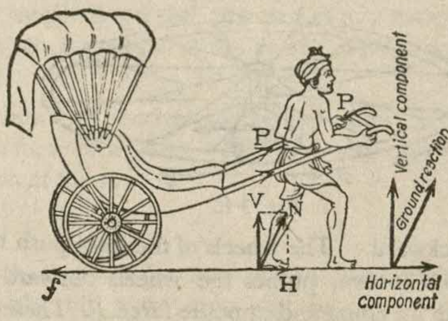


Fig. 3·12

If they move with no acceleration i.e., with uniform velocity, then from first law, net forces acting on them must be zero. Hence,

$$\mathbf{H} - \mathbf{P} = 0 \text{ and } \mathbf{P} - \mathbf{f} = 0 \text{ ; i.e., } \mathbf{P} = \mathbf{H} = \mathbf{f}.$$

The above considerations also apply in the case of an animal driven carriage.

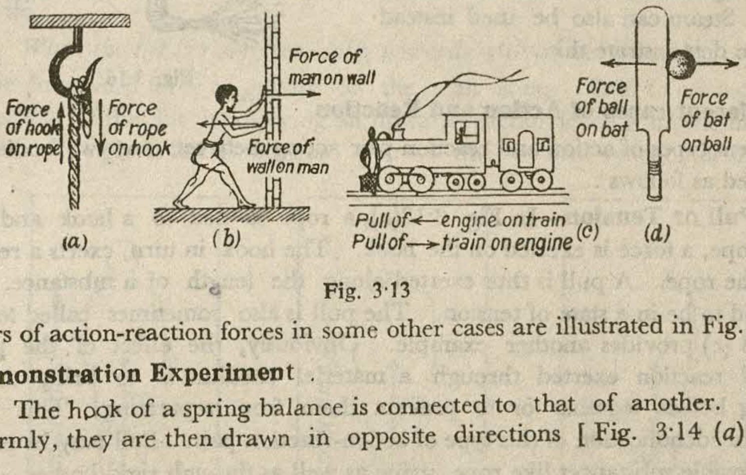


Fig. 3·13

Pairs of action-reaction forces in some other cases are illustrated in Fig. 3·13.

Demonstration Experiment .

(1) The hook of a spring balance is connected to that of another. Holding them firmly, they are then drawn in opposite directions [Fig. 3·14 (*a*)]. The

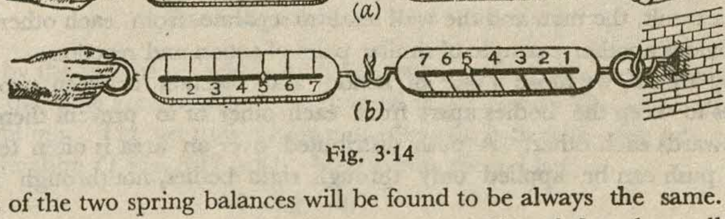


Fig. 3·14

readings of the two spring balances will be found to be always the same. If one of them is fastened to a firm support e.g., a wall, and the other pulled with a certain force [Fig. 3·14 (*b*)], it will be found again that both the spring balances are exhibiting the same reading. This simple experiment conclusively proves that action and reaction are equal and opposite.

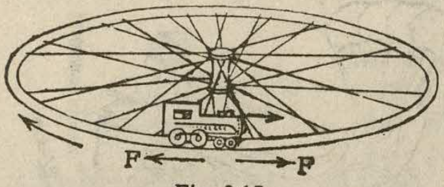


Fig. 3·15

(2) A beautiful experiment for demonstrating the third law of motion is illustrated in Fig. 3·15. A toy train is mounted on a track which is also free to move. It will be found that as the train moves forward, the track begins to move backward. The wheels of the train push back on the track with a force *F* and the track in turn, pushes the wheels forward with an equal and opposite force *F*. These two form an action and reaction pair.

(3) Rotary lawn sprinkler provides a simple example of practical application of the third law (Fig. 3·16). Water escaping from the nozzles reacts in a backward direction against them, thus producing rotation. Steam can also be used instead of water to demonstrate this.

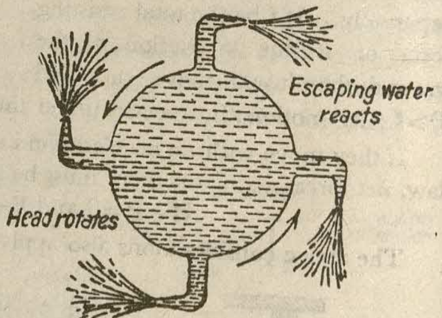


Fig. 3·16

3·8. Different cases of Action and Reaction.

Different types of action and reaction pair acting between any two bodies may be classified as follows :

(1) **Pull or Tension**—In Fig. 3·13(*a*), a rope is tied to a hook and as we pull the rope, a force is exerted on the hook. The hook, in turn, exerts a reaction through the rope. A pull is thus exerted along the length of a substance. The rope is said to be in a state of tension. The pull is also sometimes called tension. Fig. 3·13 (*c*) provides another example. Obviously, the effect of the pair of action and reaction exerted through a material medium is to bring the two interacting bodies together or to prevent them from separating. This is the criterion for identification of this type of action-reaction pair. Pull may be exerted through flexible substances like rope, string as well as through rigid bodies.

(2) **Push**—In Fig. 3·13(*b*), a man is seen to press a wall with his hands. In doing so he applies a force on the wall which in turn applies a reaction on the man. As a result, the man and the wall tend to separate from each other. Fig. 3·13(*d*) provides another example of similar pairs of action and reaction.

Thus by a push, we mean a pair of action and reaction between two bodies which tries to keep the bodies apart from each other or to prevent them from moving towards each other. A push distributed over an area is often termed a thrust. A push can be applied only through rigid bodies, not through flexible substances.

(3) **Attraction and repulsion**—When the pair of action and reaction between two bodies separated by a distance attempts to bring them together,

attraction between them is said to exist. When it tends to keep them apart, repulsion between them occurs. Gravitational, electrical, magnetic forces are of these nature. These types of forces exist irrespective of the presence of any material medium between the two bodies, though the magnitude of the forces may depend upon the nature of the intervening medium.

(4) **Friction**—As a body tends to slide over another, the second body exerts a reaction on the first which is called the force of friction. It always acts opposite to the direction of motion tending to prevent the motion.

3·9. Reaction in a moving lift.

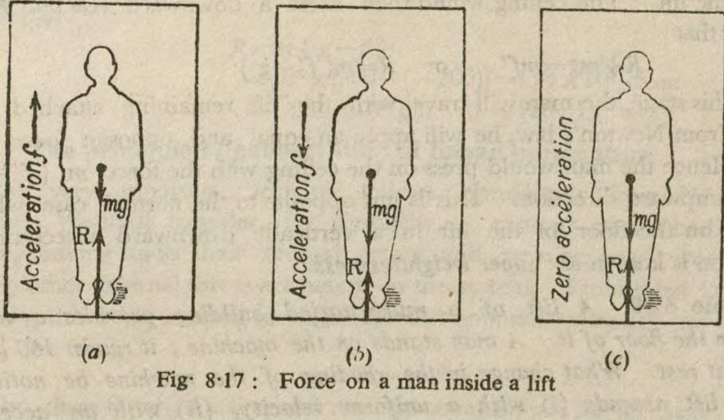
Perhaps all of us are familiar with the peculiar feelings experienced while travelling in a modern swift lift. When the lift starts from rest in the upward direction with an acceleration, we feel a gain in weight till the lift attains a steady velocity. When the lift decelerates to rest, we experience the sensation of lightness. When the lift is descending just the reverse feelings are experienced during starting and stopping of the lift.

We shall now give an explanation of the above phenomena in a quantitative manner.

Let us imagine a man of mass m inside the lift. The following different cases may arise :

(*i*) *When the lift is rising vertically upwards with an acceleration f.*

The downward force of gravity on the man is mg. Due to contact with the lift, the floor of the lift is applying an upward reaction force R on the man. The sensation of weight is felt by the man due to this vertical reaction acting on him applied by the surface on which he is standing. As the man rises along with the lift with the same acceleration f, the upward reaction force R must be greater than his weight mg. The net upward force on the man is thus $R-mg$, which produces the acceleration f [Fig. 3·17 (*a*)]. Hence from second law,



(*a*) (*b*) (*c*)

Fig· 8·17 : Force on a man inside a lift

$$R-mg=mf$$

$$\text{or,}\quad R=m\,(g+f) \qquad \dots \qquad \dots \qquad (3{\cdot}9)$$

The man has, therefore, a feeling of increase in weight.

(*ii*) ***When the lift is going vertically downwards with an acceleration*** f ($f<g$).

In this case, as the man descends along with the lift with the same acceleration f, the downward force on him must be greater than the upward one. Hence R will be less than mg so that the net downward force is $mg-R$ [Fig. 3·17 (*b*)]. Therefore, from second law,

$$mg-R=mf \quad \text{or,} \quad R=m(g-f) \qquad \cdots \qquad \cdots \qquad (3{\cdot}10)$$

The man thus feels lighter.

(*iii*) ***When the lift is falling freely*** ($f=g$).

If the lift is allowed to fall freely (as may be the case, when the lift rope snaps), the lift will descend with an acceleration g. In this case,

$$f=g, \quad \therefore \quad R=0$$

So the reaction on the man vainishes. Hence he feels completely weightless.

(*iv*) ***When the lift is at rest or moving uniformly.***

If the lift be at rest or moves upwards or downwards with a uniform velocity, then $f=0$. Hence $R=mg$ i.e., the reaction is equal to the weight of the man [Fig. 3·17 (*c*)]. So the man feels the same as when at rest on earth.

(*v*) ***When the lift is moving downwards with an acceleration*** f' ***greater than*** g.

If the lift descends with an acceleration f' greater than g (which is possible only if the lift is driven downwards with a powerful engine), then as soon as the lift starts, the acceleration g with which the man falls downwards being smaller than the downward acceleration f' of the lift, the man gets detached from the floor of the lift. So as time passes on, while both the lift and the man are moving downwards, within the lift the man rises upwards and will ultimately reach the ceiling of the lift. The ceiling would then exert a downward reaction R on the man, such that

$$R+mg=mf' \quad \text{or} \quad R=m(f'-g)$$

From this stage, the man will travel with the lift remaining attached with its ceiling. From Newton's law, he will apply an equal and opposite force on the ceiling. Hence the man would press on the ceiling with the force $m(f'-g)$ in a vertically upward direction. This is just opposite to the normal case when his weight acts on the floor of the lift in a vertically downward direction. This phenomenon is known as "*super weightlessness.*"

Example 3·10. *A lift of a multi-storied building provides a weighing machine on the floor of it. A man stands on the machine ; it reads 160 lb. when the lift is at rest. What change in the reading of the machine be noticed (a) while the lift ascends (i) with a uniform velocity, (ii) with an acceleration 6 ft./sec.², and (b) while it descends with (i) uniform velocity, (ii) with an acceleration 6 ft./sec.², (iii) with an acceleration 32 ft./sec.² ?*

Solution : (a) (i) No change.

(ii) $f=6$ ft./sec.², $g=32$ ft./sec.², $m=160$ lb.

Hence applying equation (3·9) we get,

$$R=m\times(g+f)=160\times(32+6)=160\times38 \text{ poundal.}$$

Hence the reading of the weighing machine $=\frac{160\times 38}{82}=190$ lb.

∴ The reading of the machine increases by $(190-160)=30$ lb.

(b) (i) No change.

(ii) Applying equation (3·10) we get,

$$R=m\times(g-f)$$
$$=160\times(32-6)=160\times 26 \text{ poundal}$$

∴ The reading of the machine $=\frac{160\times 26}{32}=130$ lb.

∴ The reading of the machine decreases by $(160-130)=30$ lb.

(iii) In the case $f=32$ ft./sec^2.

$$\therefore\ R=m(g-f)$$
$$=160\,(32-32)=0$$

∴ Machine thus reads zero i.e., the man is in the weightlessness condition.

Example 3·11. *A lift in the coal pit begins to descend with an acceleration of 100 cm./sec.^{2} until it attains a speed of 1 m./sec. Later it retards at the rate of 200 cm./sec.^{2} Find in each case the force exerted on the floor of the lift by the occupants whose total weight is 1000 kg.*

Solution : $f=100$ cm./sec^2., $g=980$ cm./sec.^{2}, $m=10^3$ kg.$=10^6$ gm. ; Hence applying equation (3·10) we obtain that the force exerted on the floor of the lift is

$$R=m\times(g-f)=10^6\times(980-100)$$
$$=88\times 10^7 \text{ dyne}=8{\cdot}98\times 10^5 \text{ gm. wt.}=898 \text{ kg. wt.}$$

When the lift moves with the speed of 1 m./sec. uniformly, it has no acceleration. Evidently, in this case, the force exerted $=1000$ kg. wt.

For its downward retarded motion, $f=-200$ cm./sec.^{2} ; hence equation (3·10) gives,

$$R=m\,(g-f)$$
$$=10^6\times[980-(-200)]=118\times 10^7 \text{ dyne}$$
$$=1{\cdot}204\times 10^6 \text{ gm. wt.}=1204 \text{ kg. wt.}$$

3·10. The principle of conservation of linear momentum.

This very important principle follows immediately from Newton's laws of motion. We shall consider, for simplicity, a system* of two bodies of mass m_1 and m_2 moving under their mutual action and reaction only. We shall assume that no other external forces are acting on the system. From third law, the force F_1 on m_1 due to m_2 is equal in magnitude but oppositely directed to the force F_2 on m_2 due to m_1

$$\text{or, } F_1=-F_2 \quad \ldots \quad \ldots \quad (3{\cdot}11)$$

Also, these forces of action and reaction obviously act for the same time. The impulses of these two forces are, therefore, equal and opposite. Thus,

---

* By system of bodies we mean any well-defined collection of matter. The system may consist of any combination of particles, rigid bodies, gases, liquids etc., but the term well-defined implies that we have to point out exactly which material is a part of the system : all other matter in the universe is considered to be external to the system. The forces which the bodies of the system exert on one another are called internal forces. External forces are forces exerted on the bodies of the system by bodies external to the system. When we assume that no external forces act on a system, it is said to be isolated.

$$\text{impulse of } F_1 \text{ on } m_1 = -\text{impulse of } F_2 \text{ on } m_2 \quad \ldots \quad (3{\cdot}12)$$

If the velocity of m_1 before interaction be u_1 and velocity after interaction be v_1, then its change of momentum due to interaction is $(m_1v_1 - m_1u_1)$. Similarly, the change of momentum of m_2 is $(m_2v_2 - m_2u_2)$ where u_2 and v_2 are its velocities before and after interaction.

Now, since the change in momentum of a body equals the impulse of the force acting on it, we have from (3·12),

$$m_1v_1 - m_1u_1 = -(m_2v_2 - m_2u_2)$$

$$\text{or,} \quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \ldots \quad \ldots \quad (3{\cdot}13)$$

The left hand side of equation (3·13) is the total linear momentum of the system before interaction, the right side is the total linear momentum after the interaction. Total linear momentum of the system therefore remains constant. Anything which remains constant in a process is said to be conserved. We thus arrive at the principle of conservation of linear momentum for two bodies. It can be shown that this is true whatever be the number of bodies in the system. The principle then says : *If there is no external force acting upon a system of bodies, the total linear momentum of the system remains constant.*

The principle can be obtained much more simply from equation (3·11) using the calculus method. Thus if p_1 and p_2 be the momenta of m_1 and m_2 respectively, then from second law,

$$F_1 = \frac{dp_1}{dt} \quad \text{and} \quad F_2 = \frac{dp_2}{dt}$$

From equation (3·11), $\dfrac{dp_1}{dt} = -\dfrac{dp_2}{dt}$ or, $\dfrac{d}{dt}(p_1 + p_2) = 0$

$$\text{or,} \quad p_1 + p_2 = \text{constant} \quad \ldots \quad (3{\cdot}14)$$

which shows that the total linear momentum of the system remains constant throughout.

It should be noticed that the principle holds regardless of the character of the interactions, no assumption has been made regarding the nature of the forces F_1 and F_2. Also, since momentum is a vector quantity, the principle requires that the total momentum of the system should remain constant in both magnitude and direction. The principle also implies that the total momentum of a system can only be changed by external forces acting on the system. This principle together with the principle of conservation of energy [vide Art. 7·9] provides a very powerful tool to the physicists in solving complex problems on interaction between different bodies.

As a man jumps from a boat, the boat moves in the backward direction. Initially both of them were at rest, so that initial momentum of the system is zero. Thus while jumping, the forward momentum of the man must be equal to the backward momentum of the boat, so that their sum is zero. This is found to be true ; momentum is conserved.

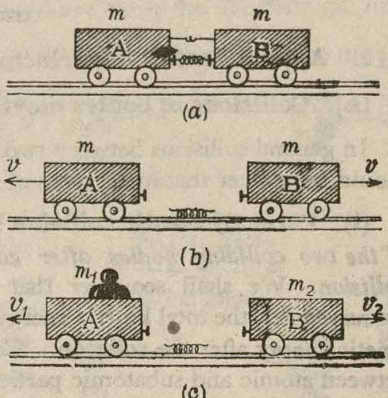
The recoil of a gun provides another example. Before explosion, the momentum of the system consisting of the gun and the shell is zero. When fired, the shell acquires a forward momentum ; hence the gun recoils with an equal backward momentum so that the sum remains zero.

During its forward motion if the shell explodes in mid-air, the fragments will fly off in all directions. If the momenta of the fragments at any instant is summed vectorially, it will be found that the total momentum vector remains always the same in both magnitude and direction to the initial momentum vector of the shell.

The motion of rockets and jet planes provides examples of this principle. We shall discuss such motion in detail in a later article.

Demonstration Experiments.

(1) Two small cars A and B of equal mass $m_1=m_2=m$, are placed on a horizontal track. A compressed spring, not fixed to any of the car, is placed between them which are held together by a string. [Fig. 3·18 (*a*)]. If the string is now burnt, the cars fly apart in opposite directions under the influence of the spring which drops down. If the velocities of the two cars are measured, they will be found to be equal [Fig. 3·18 (*b*)]. The measurement is done by noting, with a stop watch, the time necessary for a car to cover the distance between two previously marked places on its path. Dividing the distance by the time spent we get the magnitude of the velocity.



(*a*)

(*b*)

(*c*)

Fig. 3·18

The observation illustrates conservation of momentum. Initially the two cars are at rest and the total momentum is thus zero. With motions to the right taken as positive, velocity of B is positive while that of A is negative. Hence the total momentum of the system when the cars are in motion is ($mv-mv$)$=0$, the same as the initial value.

If the masses of the cars be different, say m_1 and m_2 respectively, which can be done simply by placing a weight on one of the cars [Fig. 3·18 (*c*)], the cars fly apart with unequal velocities. The velocity of the lighter car will be found to be greater than that of the heavier. On measuring the velocities v_1 and v_2 of the cars, it will be found that the product m_1v_1 equals the product m_2v_2. Since the velocities are oppositely directed, the total momentum is again zero.

(2) Another interesting experiment to illustrate this principle may be performed with six or seven billiard balls or glass or steel marbles. The mass of the balls should be same. They are placed in a line on a smooth table, each ball being in contact with the next (Fig. 3·19). If a ball, of the same mass is shot

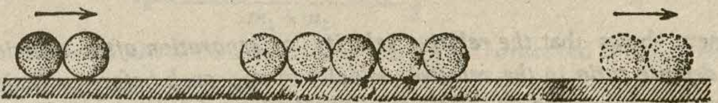


Fig. 3·19

against one end of the series, say from the left end, it will be stopped by collision with the others and the one on the extreme right hand end will roll out with the same velocity. When two balls are shot as shown in the figure, two balls at the other end of the line will move out. If three balls are shot, three will roll out at the other end.

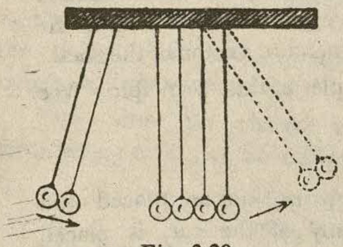


Fig. 3·20

The experiment may be performed by suspending the balls by strings so that each ball remains in contact with the next, as illustrated in Fig. 3·20.

3·11. Applications of the principle of conservation of linear momentum.

[A] Collisions of bodies moving in the same line.

In general collisions between two bodies are divided into three categories. We should remember that in all types of collisions, *momentum is always conserved.*

(i) Perfectly elastic collision : In this type of collision, *the relative velocity of the two colliding bodies after collision equals their relative velocity before collision.* We shall see later that in this type of collision *the kinetic energy is conserved* i.e., the total kinetic energy before the collision is equal to the total kinetic energy after the collision. Collision of this kind is an ideal one. Collisions between atomic and subatomic particles are regarded as perfectly elastic. Very near approximations to perfectly elastic collisions are those between ivory or glass or steel balls.

(ii) Perfectly inelastic collision : This type of collision is characterized by the fact that *after the two bodies have collided, they stick to each other and move as one unit i.e., the relative velocity after collision is zero.* Kinetic energy is not conserved. Some kinetic energy is lost during collision being converted into other forms of energy, chiefly heat energy.

This type of collision is also an ideal one. Collision between two balls of clay provides a close example.

(iii) Partly elastic collision : *In such collisions the colliding bodies separate after collision* ; *however, the relative velocity of separation is smaller than the relative velocity of approach.* In this type of collision, also, some kinetic energy is lost during collision.

Partly elastic collisions are the most general of three cases and usually occur in practice.

Experiment shows that *the relative velocity of separation after collision (say, v) bears a definite ratio to the relative velocity of approach before collision (say, V).* This ratio is called the **co-efficient of restitution** (*e*) for the bodies. Hence

$$e=\frac{v}{V} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (3{\cdot}15)$$

$$\text{Co-efficient of restitution}=\frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

The value of e depends upon the nature of the materials of which the bodies are made of and also to a small extent on the magnitude of V. Approximate value of the co-efficient of restitution for glass is about 0·96 and for lead about 0·16.

It follows from the definition that for partly elastic collisions, the value of e is less than unity $(0<e<1)$. Perfectly elastic and perfectly inelastic collisions are the two extreme cases and no such collisions are found to occur in practice within material bodies. For the former, $e=1$ ($\because$ $v=V$) and for the latter, $e=0$ ($\because$ $v=0$).

We shall now consider the different types of collision quantitatively. In doing so, we shall limit ourselves only to the study of collision between two bodies in which the velocities both before and after collision are along the same straight line.

Perfectly elastic collision : In the Fig. 3·21, two bodies of masses m_1 and m_2 and velocities u_1 and u_2 travelling along the same line collide exactly "head on" in a perfectly elastic manner. During impact, the bodies exert oppositely directed action and reaction forces P on each other. The force P acting on each body brings about a change in its momentum. The force is obviously an impulsive one. The instant after the bodies separate, they move along the same straight line with velocities v_1 and v_2. All the velocities are supposed to be in the same direction and obviously if the bodies are to collide, $u_1>u_2$ and if they are to separate after collision, $v_2>v_1$. Hence

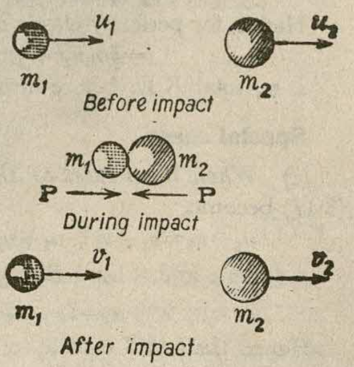


Fig. 3·21 : Elastic collision between two particles

the relative velocity of approach$=u_1-u_2$

the relative velocity of separation$=v_2-v_1$

From the definition of perfectly elastic collision, we have

$$u_1-u_2=v_2-v_1 \qquad \ldots \qquad \ldots \qquad (3\cdot16)$$

From the principle of conservation of linear momentum, total momentum before impact=total momentum after impact

$$\text{i.e.,}\ m_1u_1+m_2u_2=m_1v_1+m_2v_2 \qquad \ldots \qquad \ldots \qquad (3\cdot17)$$

From these two equations, we can solve for v_1 and v_2 and get

$$\left.\begin{aligned} v_1&=\frac{(m_1-m_2)u_1+2m_2u_2}{m_1+m_2} \\ v_2&=\frac{2m_1u_1+(m_2-m_1)u_2}{m_1+m_2} \end{aligned}\right\} \qquad \ldots \qquad \ldots \qquad (3\cdot18)$$

We shall now show that the kinetic energy is conserved in such collisions. So

$$\text{total K.E. before impact}=\frac{1}{2}\,m_1u_1^2+\frac{1}{2}\,m_2u_3^2$$

$$=\frac{1}{2(m_1+m_2)}\Big[\,(m_1+m_2)\,(m_1u_1^2+m_2u_2^2)\,\Big]$$

$$=\frac{1}{2(m_1+m_2)}\Big[\,m_1^2u_1^2+m_2^2u_2^2+m_1m_2(u_1^2+u_2^2)\,\Big]$$

$$=\frac{1}{2(m_1+m_2)}\Big[m_1^2u_2^2+m_2^2u_2^2+2m_1m_2u_1u_2$$
$$+m_1m_2(u_1^2+u_2^2-2u_1u_2)\Big]$$

$$=\frac{1}{2(m_1+m_2)}\Big[(m_1u_1+m_2u_2)^2+m_1m_2(u_1-u_2)^2\Big]$$

$$=\frac{1}{2(m_1+m_2)}\Big[(m_1v_1+m_2v_2)^2+m_1m_2(v_1-v_2)^2\Big]$$

[from eqn. 3·16 and 3·17]

$$=\frac{1}{2(m_1+m_2)}\Big[(m_1+m_2)\,(m_1v_1^2+m_2v_2^2)\Big] \text{ in a similar way}$$

$$=\tfrac{1}{2}m_1v_1^2+\tfrac{1}{2}m_2v_2^2=\text{total K.E. after impact.}$$

Hence for perfectly elastic collision,

$$=\tfrac{1}{2}m_1u_1^2+\tfrac{1}{2}m_2u_2^2=\tfrac{1}{2}m_1v_1^2+\tfrac{1}{2}m_2v_2^2 \quad \ldots \quad \ldots \quad (3{\cdot}19)$$

i. e., total K.E. before impact=total K.E. after impact.

Special cases.

(*i*) *When the masses of the bodies are equal, $m_1=m_2$.* Equation (3·16) and (3·17) become,

$$u_1-u_2=v_1-v_2\ ;\ u_1+u_2=v_1+v_2$$

Addition and substraction give,

$$u_1=v_2\ ;\ u_2=v_1 \quad \ldots \quad \ldots \quad (3{\cdot}20)$$

Hence the final velocity of the second particle is equal to the initial velocity of the first particle and vice-versa. *The particles exchange velocities during impact.*

If the *second particle is initially at rest,* then $u_2=0$.

∴ From eqn. (3·20), we get, $v_1=0$ and $v_2=u_1$ i.e., after collision, the first particle stops and the second particle moves forward with velocity u_1 i.e., the velocity of first particle before collision.

(*ii*) *If the particle of mass m_2 is originally at rest,* $u_2=0$. From equations (3·16) and (3·17),

$$u_1=v_2-v_1 \text{ and } m_1u_1=m_1v_1+m_2v_2$$

Solving for v_1 and v_2 we get,

$$v_1=\frac{m_1-m_2}{m_1+m_2}u_1\ ;\ v_2=\frac{2m_1}{m_1+m_2}u_1 \quad \ldots \quad \ldots \quad (3{\cdot}21)$$

If $m_2 \gg m_1$, $v_1 \approx -u_1$ and $v_2 \ll u_1$.

Thus the second body remains practically at rest, while the first body rebounds in the opposite direction with approximately the same velocity.

The condition is nearly achieved when a ball bounces off a wall. The wall is of very large mass compared to that of ball ; it remains at rest and the ball rebounds with approximately the same velocity with which it strikes the wall.

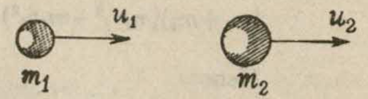
Perfectly inelastic collision : Fig. 3·22 shows the perfectly inelastic collision of two bodies of masses m_1 and m_2 and velocities u_1 and u_2 travelling along the same line. After collision, the bodies stick together and let us suppose that the combined body travels in the same direction with the velocity v. Then from the principle of conservation of linear momentum,

total momentum before collision=total momentum after collision.

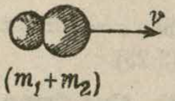
$$\therefore\quad m_1u_1+m_2u_2=(m_1+m_2)v$$

$$\text{or, } v=\frac{m_1u_1+m_2u_2}{m_1+m_2} \qquad \cdots \qquad (3\cdot22)$$

Some kinetic energy is wasted during collision, which may be calculated as follows :



Before impact



After impact

Fig. 3·22
Perfectly inelastic collision

Before the collision, the total K.E. $=\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2$

After collision, the total K. E. $=\frac{1}{2}(m_1+m_2)v^2$

Hence, loss in K. E. $=\frac{1}{2}(m_1u_1^2+m_2u_2^2)-\frac{1}{2}(m_1+m_2)v^2$

$$=\tfrac{1}{2}(m_1u_1^2+m_2u_2^2)-\tfrac{1}{2}\,\frac{(m_1u_1+m_2u_2)^2}{m_1+m_2}$$

$$=\frac{m_1m_2}{2(m_1+m_2)}(u_1-u_2)^2 \qquad \cdots \qquad \cdots \quad . \qquad (3\cdot23)$$

But (u_1-u_2) is the relative velocity of approach before the collision. So the ***loss in K. E. is proportional to the square of the relative velocity of approach.***

Partly elastic collision : We again refer to the Fig. 3·21 but here the collision is not perfectly elastic. We see that from the definition, the relative velocity of separation i.e., (v_2-v_1) is smaller than the relative velocity of approach i.e., (u_1-u_2) and we have for the coefficient of restitution,

$$e=\frac{v_2-v_1}{u_1-u_2}$$

From the principle of conservation of linear momentum

$$m_1u_1+m_2u_2=m_1v_1+m_2v_2 \qquad \cdots \qquad \cdots \qquad (3\cdot24)$$

But, $e(u_1-u_2)=v_2-v_1$

Multiplying by m_2, we get

$$em_2u_1-em_2u_2=m_2v_2-m_2v_1 \qquad \cdots \qquad \cdots \qquad (3\cdot25)$$

Subtracting (3·25) from (3·24), we get

$$(m_1-em_2)u_1+(1+e)m_2u_2=(m_1+m_2)v_1$$

$$\therefore\quad v_1=\frac{(m_1-em_2)u_1+(1+e)m_2u_2}{m_1+m_2} \qquad \cdots \qquad (3\cdot26)$$

$$\text{Similarly, }\; v_2=\frac{(m_2-em_1)u_2+(1+e)m_1u_1}{m_1+m_2} \qquad \cdots \qquad (3\cdot27)$$

Loss in K.E. $=(\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2)-(\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2)$

$$=\frac{1}{2(m_1+m_2)}[(m_1+m_2)(m_1u_1^2+m_2u_2^2)-(m_1+m_2)(m_1v_1^2+m_2v_2^2)]$$

$$\text{But,}\quad (m_1+m_2)(m_1u_1^2+m_2u_2^2)=m_1^2v_1^2+m_2^2u_2^2+m_1m_2(u_1^2+u_2^2)$$

$$=(m_1u_1+m_2u_2)^2+m_1m_2(u_1-u_2)^2$$

Similarly, from eqn. (3·24) and the expression for e,

$$(m_1+m_2)(m_1v_1^2+m_2v_2^2)=(m_1v_1+m_2v_2)^2+m_1m_2(v_1-v_2)^2=(m_1u_1+m_2u_2)^2 + m_1m_2e^2(u_1-u_2)^2$$

Hence,

$$\text{Loss in K.E.}=\frac{m_1m_2}{2(m_1+m_2)}(1-e^2)(u_1-u_2)^2 \quad \ldots \quad (3{\cdot}28)$$

when $e=1$, the r.h.s. reduces to zero. When $e=0$, eqn. (3·28) reduces to eqn. (3·23).

[B] Gun firing a shell.

This example was discussed earlier (Fig. 3·11). We shall now deduce an expression for the velocity of the gun.

Before firing, initial momentum of the system consisting of the gun and the shell is zero. After firing let V be the velocity of the gun of mass M, the velocity of the shell of mass m being v. Thus total momentum of the system after firing $=MV+mv$. From principle of conservation of momentum,

$$MV+mv=0$$

$$\text{or,}\quad V=-\frac{m}{M}v.$$

The negative sign shows that the two velocities are oppositely directed i.e., as shell goes forward, the gun recoils.

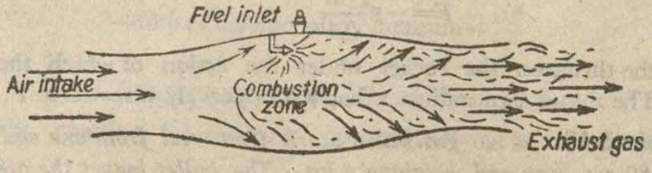
[C] Jets and Rockets.

Everyone of us is now familiar with the sight of flying jet planes and rockets, driven forward by the jet of rapidly moving gas ejected from them. The principle of propulsion is based on the principle of conservation of linear momentum.

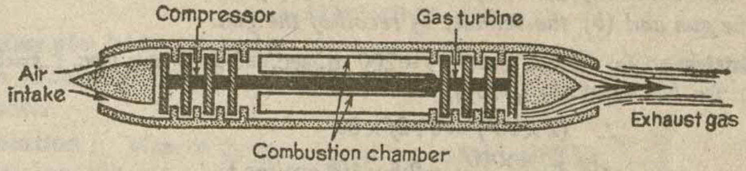
In essence, a jet or a rocket is simply a combustion chamber in which solid or liquid fuel is burnt. The combustion chamber has an opening or nozzle at its rear. As the fuel is ignited, the gaseous products of combustion formed at a high pressure, flows out of the chamber with a very high velocity through the rear opening. This stream of exhaust gases thus possesses a large momentum in the backward direction. Since momentum is conserved, the jet or the rocket develops an equal forward momentum, and thus moves forward with a very high velocity.

In jet engines, the oxygen necessary for burning the fuel is obtained by sucking in air at the front of the engine. Depending on the nature of construction, jets are classified in two types—(i) ram jet (ii) turbo jet. The ram jet is the simplest type of jet engines [Fig. 3·23 (a)]. This has no moving parts and consists essentially of a long tube. As the plane flies through air, air is forced into the combustion chamber. Thus a **ram jet** can only work if the machine is already moving rapidly.

In **turbo jets**, the hot gas from the combustion chamber before escaping through the rear nozzle rotates a turbine [Fig. 3·23 (b)]. This turbine in turn drives a compressor at the front of the engine which forces air into the combustion chamber.



(a) Ram jet



(b) Turbo jet

Fig. 3·23 : Principle of jet propulsion

A **rocket**, however carries its own oxygen, usually in the frozen liquid form. A rocket can therefore fly outside the earth's atmosphere where there is no air. Due to the consequent absence of air resistance, its motion is facilitated. At present it is the only means of propulsion in space. Fig 3·24 shows the section of a rocket. The fuel and the oxidizer are kept in separate tanks, *F* and *O*, respectively. They are pumped into the combustion chamber and sprayed in the form of fine jets to form the combustible mixture. Usually liquid hydrogen, kerosene, ethyl alcohol etc. are used as fuels. Liquid oxygen, ozone, hydrogen peroxide etc. are used as oxidizers. The payload (*P*) viz. artificial satellite, warhead etc. is placed at the front of the rocket.

Multiple-stage rockets are used in space flights. Each stage contains the propulsion system and a part of the total propellant. The first stage is called the booster stage. When all the fuel in the first stage is consumed, it gets separated from the rocket. The second stage is then fired. The rocket now being lighter attains larger acceleration and hence greater speed.

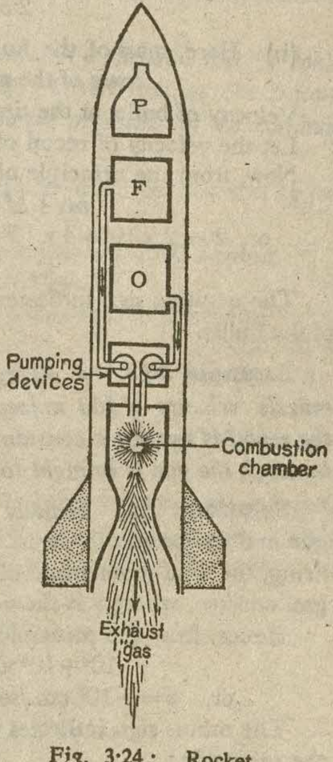


Fig. 3·24 : Rocket

To calculate the thrust on the rocket, let the mass of fuel consumed in time Δt be Δm. Then $\Delta m/\Delta t$ gives the rate of discharge of hot gases i.e. the mass of the gas ejected per second from the rocket. If v be the velocity of the exhaust gas relative to the rocket, then the change of momentum per second of the exhaust gas is $v\ \Delta m/\Delta t$. Hence the reaction force developed on the rocket is

$$F=-v\frac{\Delta m}{\Delta t} \qquad \ldots \qquad \ldots \qquad (3{\cdot}29)$$

This is the thrust on the rocket under the action of which the rocket goes forward. The minus sign indicates that v is oppositely directed to F.

Example 3·12. *A 20 gm. bullet starts from rest from one end of the barrel of the gun 80 cm. long and weighing 4 kg. The bullet leaves the other end of the barrel with a velocity 200 m./sec. Calculate (a) the time taken by the bullet to leave the gun and (b) the velocity of recoil of the gun.*

Solution : (a) Here $u=0$; $v=200$ m./sec.$=2\times10^4$ cm./sec. ; $s=80$ cm.

$$\because\ v^2=u^2+2fs$$

$$\therefore\ (2\times10^4)^2=0+2f\times80$$

$$\therefore\ f=\frac{4\times10^8}{160}=2{\cdot}5\times10^6\ \text{cm/sec.}^2$$

Again $\because\ v=u+ft$

$$\therefore\ 2\times10^4=0+2{\cdot}5\times10^6\times t$$

$$\therefore\ t=\frac{2\times10^4}{2{\cdot}5\times10^6}=8\times10^{-3}\ \text{sec.}$$

(b) Here, mass of the bullet $m=20$ gm.

mass of the gun $M=4$ kg.$=4\times10^3$ gm.

Velocity of bullet at the time of leaving the gun$=2\times10^4$ cm./sec.

Let the velocity of recoil of the gun$=V$ cm./sec.

Now, from the principle of conservation of momentum

$$mv+MV=0 \quad (\because\ \text{Initial momentum of the system is zero})$$

or, $20\times2\times10^4+4\times10^3V=0$

$$\therefore\ V=-100\ \text{cm./sec.}$$

The negative sign indicates that the gun moves is a direction opposite to that of the bullet.

Example 3·13. *A 10 kg. gun fires a projectile of mass 20 gm. with a muzzle velocity of 500 m./sec. Find out (a) the recoil velocity of the gun. If the recoil is against a constant resisting force of 20 kg. wt., calculate (b) the time in which the gun is brought to rest and (c) the distance it recoils.*

Solution : (a) Initially at the instant of firing, the system comprising the gun and the projectile is at rest, and hence it has no momentum. But after firing, the total momentum of the system is given by $(20\times500\times10^2+10\times10^3\times v)$ gm. cm./sec, where v is the velocity of recoil of the gun in cm./sec.

Hence, from the principle of conservation of momentum,

$$10^6+10^4\times v=0$$

or, $v=-10^2$ cm./sec.$=-1$ m./sec.

The minus sign indicates that the gun moves in direction opposite to that of the projectile.

(b) The resisting force $P=20$ kg. wt.$=196\times10^5$ dyne

$$m=10\ \text{kg.}=10^4\ \text{gm.}$$

If the gun comes to rest in time t, its momentum change during this interval of time$=m(v-0)=10^4\times 10^2=10^6$ gm. cm./sec.

From Newton's second law,

$$P=\text{rate of change of momentum}=\frac{10^6}{t}$$

$$\therefore \quad t=\frac{10^6}{196\times 10^5}=\frac{1}{19{\cdot}6}=0{\cdot}05 \text{ sec.}$$

(c) The resisting force being constant, the gun's motion is uniformly retarded.

$\therefore$ distance traversed=average velocity × time.

$$=\frac{100+0}{2}\times 0{\cdot}05=2{\cdot}5 \text{ cm.}$$

Example 3·14. *A rocket consumes 0·2 kg. of fuel in one sec. and ejects it with a velocity of 200 m./sec. Calculate the force exerted by the ejected gas on the rocket.*

Solution : Mass of the gas ejected per second=0·2 kg.

Velocity of the ejected gas=200 m./sec.

$\therefore$ Change in momentum per sec.$=0{\cdot}2\times 200$ kg. m./sec.^{2}
$=40$ kg. m./sec.^{2}

which is the force exerted on the ejected gas [from second law of Newton].

Hence from third law, the force exerted on the rocket in the forward direction =40 newton.

Example 3·15. *Find the minimum rate at which a rocket must be consuming fuel in order to be able to take off vertically. Given that, at the instant it takes off, the mass of the rocket is 3000 kg. and the products of the combustion are ejected at a velocity of 300 m./sec.*

Solution : Let the required mass of the fuel consumed per sec.$=m$ kg./sec.

Velocity of the ejected gas=300 m./sec.

Change in momentum per second of the ejected gas$=m\times 300$ newton.

From second law, this is the force on the ejected gas.

$\therefore$ force on the rocket acting vertically upwards$=m\times 300$ newton.

[from 3rd law]

In order to take off vertically, this upward force must be just equal to the weight of the rocket which acts downwards.

$\therefore \quad m\times 300=3000\times 9{\cdot}81 \qquad [\because \quad g=9{\cdot}81 \text{ m./sec}^2]$

$\therefore \quad m=98{\cdot}1$ kg./sec.

Example 3·16. *At some instant of time a rocket of mass m_0 is travelling with a velocity u_0. A little later its mass decreases to m and velocity increases to u. If the velocity of the exhaust gas relative to the rocket be v, then prove that*

$$u=u_0+v\log_e m_0/m$$

Solution : From relation [3·2(*a*)], $F=-v\dfrac{dm}{dt}$

$$\text{But } F=mf=m\frac{du}{dt}$$

$$\therefore \quad du=-v\frac{dm}{m}$$

Hence,

$$\int_{u_0}^{u} du = -v \int_{m_0}^{m} \frac{dm}{m}$$

or, $u = u_0 + v \log_e m_0/m$

Example 3·17. *A 2 kg. ball moving at 3 m./sec. collides centrally with another ball at rest weighing 0·5 kg. Find the velocity in m./sec. of each after impact, if (a) they stick together, and if (b) the collision is perfectly elastic.*

Solution : (a) Since after collision the two balls stick together, the collision is completely inelastic. In this case velocities of both the balls after collision are equal.

In eqn. (3·17) we put $v_1 = v_2 = v$; also $u_2 = 0$, because one of the balls is at rest before collision.

$$\therefore \quad (2 \times 10^3) \times (3 \times 10^2) + 0 = (2 + 0{\cdot}5) \times 10^3 \times v$$

Hence $v = 240$ cm./sec. $= 2{\cdot}4$ m./sec.

(b) Since in this case the collision is elastic, we have from eqn. (3·16),

$$3 \times 10^2 - 0 = (v_2 - v_1)$$

$$\therefore \quad v_2 - v_1 = 300 \quad \ldots \quad \ldots \quad \text{(i)}$$

Again from eqn. (3·17),

$$(2 \times 10^3) \times (3 \times 10^2) + 0 = (2 \times 10^3 \times v_1) + (0{\cdot}5 \times 10^3) \times v_2$$

$$\therefore \quad v_1 + 0{\cdot}25 v_2 = 300 \quad \ldots \quad \ldots \quad \text{(ii)}$$

Solving (i) and (ii) we get

$v_1 = 180$ cm./sec. $= 1{\cdot}8$ m./sec. ; $v_2 = 480$ cm./sec. $= 4{\cdot}8$ m./sec.

Thus, after collision both the balls move in the same direction.

Example 3·18. *A sphere of mass 500 gm. moving with a velocity 200 cm./sec. collides centrally with another sphere of mass 100 gm. and velocity 100 cm./ sec. coming from opposite direction. Find the velocity of each after impact, if (a) they stick together and if (b) the collision is perfectly elastic. (c) Also find the loss in K. E. in the former case.*

Solution : Mass of the first sphere $m_1 = 500$ gm.

Velocity of the first sphere before collision $u_1 = 200$ cm./sec.

$\therefore$ Momentum of the first sphere before collision

$$= m_1 u_1 = 500 \times 200 = 10^5 \text{ gm. cm./sec.}$$

Mass of the second sphere $m_2 = 100$ gm.

Velocity of the second sphere before collision $u_2 = 100$ cm./sec.

$\therefore$ Momentum of the second sphere before collision

$$= m_2 u_2 = 100 \times 100 = 10^4 \text{ gm. cm./sec.}$$

Since the spheres are moving in opposite direction, their total momentum before collision $= m_1 u_1 - m_2 u_2 = 10^5 - 10^4 = 9 \times 10^4$ gm. cm./sec.

(*a*) When they stick together after collision, let v be the velocity of the composite unit. Then from principle of conservation of momentum,

$$9 \times 10^4 = (500 + 100)\, v$$

$$\therefore \quad v = 150 \text{ cm./sec.}$$

the direction being that of the first sphere before collision.

(*c*) The loss in K.E. $= \frac{1}{2}.500.200^2 + \frac{1}{2}.100.100^2 - \frac{1}{2}.600.150^2$

$$= 375 \times 10^4 \text{ ergs} = 0{\cdot}375 \text{ joules,}$$

(*b*) When the collision is perfectly elastic, then from eqn. 3·16, $200-(-100) = v_2 - v_1$; or $v_2 - v_1 = 300$ and from eqn. 3·17, $500v_1 + 100v_2 = 9\times10^4$ or, $5v_1 + v_2 = 900$

Solving for v_1 and v_2, we get,

$$v_1 = 100 \text{ cm./sec.}, \quad v_2 = 400 \text{ cm./sec.}$$

The direction of both being that of the first sphere before collision.

Example 3·19. *A steel ball of radius 6 cm. is initially at rest on a horizontal frictionless surface. It is struck head on by another steel ball of radius 3 cm. travelling with a velocity of 450 cm./sec. Calculate the velocity of two balls after collision. Assume the collision to be perfectly elastic.*

Solution : Mass of the stationary ball$=k\times6^3$

Mass of the moving ball$=k\times3^3$

where k is a constant.

From eqn. 3·16, $v_2 - v_1 = 450$

Again from eqn. 3·17,

$$k\times3^3\times450 = k\times3^3\times v_1 + k\times6^3\times v_2$$

$$\text{or, } v_1 + 8v_2 = 450$$

Solving we get $v_1 = -350$ cm./sec., $v_2 = 100$ cm./sec.

So the moving ball rebounds in the opposite direction and the stationary ball moves forward.

Example 3·20. *A ball falls through a vertical height H on a massive horizontal slab. After impact, it rebounds to a height h. Find the coefficient of restitution.*

Solution : Let the velocity of the ball just before impact be V. Since it falls freely through a height H, we have

$$V = \sqrt{2gH}$$

where g is the acceleration due to gravity.

As the slab remains at rest, V is the relative velocity of approach before impact.

Let the velocity of the ball just after impact be v. This is also the relative velocity of separation. As the ball rises to a height h, hence,

$$v = \sqrt{2gh}$$

$\therefore$ Coefficient of restitution, $e = \dfrac{v}{V} = \sqrt{\dfrac{h}{H}}$

Example 3·21. *Two spheres moving in the same direction collide. The masses are 5 kg. and 2 kg. respectively and the velocities are 10 and 5 metres per sec. respectively. The coefficient of restitution is 0·6. Find the velocities after impact. Find also the energy wasted.*

Solution : From the principle of conservation of momentum

$$5\times10 + 2\times5 = 5v_1 + 2v_2 \text{ or, } 5v_1 + 2v_2 = 60$$

Also $v_2 - v_1 = 0{\cdot}6(10-5) = 3$

Hence, $v_1 = 7{\cdot}7$ m./sec., $v_2 = 10{\cdot}7$ m./sec.

Both being in the same direction as before impact.

From eqn 3·28, Loss in K. E.

$$= \frac{5\times2}{2(5+2)}(1-0{\cdot}6^2)(10-5)^2 = 11{\cdot}43 \text{ joule}$$

This may also be obtained from the first principle.

Example 3·22. *A 20 gm. bullet is fired horizontally into a 5 kg. block of wood suspended by a long string, and the bullet remains embedded in the wood. Calculate the velocity of the bullet if the impact causes block to swing 15 cm. above its initial level.*

Solution : Let the velocity of the bullet just before impact be u_1 and the velocity of (bullet + block) just after impact be v_1. The block is initially at rest. Then from the principle of conservation of momentum,

$$20u_1=(20+5\times10^3)v_1$$

To find v_1, we note that the system starting with this velocity rises through a vertical distance of 15 cm.

$$\text{Hence, } v_1=\sqrt{2gh} \qquad [\because \text{ K.E.}=\text{P.E. i.e. } \tfrac{1}{2}mv_1^2=mgh\,]$$

$$=\sqrt{2\times980\times15}=171{\cdot}5 \text{ cm./sec.}$$

$$\therefore \quad u_1=\frac{5020}{20}\times171{\cdot}5=43046{\cdot}5 \text{ cm./sec.}=430{\cdot}465 \text{ m./sec.}$$

● EXERCISE ●

[A] Essay type questions

1. (a) What do you understand by inertia of rest and inertia of motion ? Explain with examples. Explain the statement—'mass provides a measure of inertia'. Show how this can be deduced from the second law.

(b) State Newton's second law of motion and explain how the unit of force is derived therefrom. Define the absolute and gravitational units of force in the Metric and the British systems.

2. State Newton's laws of motion. Explain how the first law gives the definition of force and the second law a method of measuring the force. How do you obtain the absolute units of force ? Why they are so called ?

3. A boy sitting on a vehicle moving at a constant speed throws a ball straight up into the air. Will the ball fall in front of him, into his hand or behind him ? If the vehicle accelerates in the meantime where will the ball fall ?

4. Define momentum and state its c. g. s. unit. What do you understand by the impulse of a force ? Prove that the impulse of the force acting on a body is equal to its change in momentum.

What is impulsive force ? Explain with suitable examples.

5. State Newton's third law of motion and explain with examples.

Explain : (i) Two teams having a tug-of-war must always pull equally hard on one another. (ii) The team that pushes against the ground harder wins.

6. Establish the relations of dyne with poundal, Newton, gram-weight and pound-weight respectively. Which ones among these are absolute units of force ?

7. 'Action and reaction are equal and opposite'—explain with an experiment. Can action and reaction balance each other ? Explain with particular reference to the following example —A body at rest is set into motion on application of a force.

8. What is a perfectly elastic collision ? Two bodies of equal masses moving in the same straight line in the same direction collide in a perfectly elastic manner. Prove that the bodies exchange velocities during impact.

9. State the principle of conservation of linear momentum and explain with examples. How can you prove it from Newton's third law of motion ?

10. (i) Is Newton's first law valid for all frames of reference ? Discuss.

(ii) According to Newton's law, only an outside force impressed by another body can alter the state of motion of a given body. Then what outside force brings a car or any other self-moving vehicle to a stop under breaking ? [*Model questions, H. S. Council*]

11. Distinguish among perfectly elastic, partly elastic and perfectly inelastic collisions. Define coefficient of restitution. What is its value for the above type of collisions ? Prove that for elastic collisions of two bodies moving in the same line, the K. E. is conserved.

12. Prove that the K. E. is not conserved for both partly elastic and perfectly inelastic collisions. Calculate the loss of K. E. in each case.

13. A perfectly elastic collision took place between a moving body and a body at rest. Prove that (i) if the masses of the two bodies are equal, then after collision, the moving body stops and the previously stationary body moves forward with the same velocity as that of the moving body before collision, (ii) if the mass of the stationary body be much larger than that of the moving one, then after collision the first body remains practically at rest, while the second body rebounds in the opposite direction with approximately the same velocity.

14. The masses of a gun and a bullet are M and m, respectively. The gun is fired. If E be the energy released by the explosion of the gun powder, calculate the velocity with which the bullet moves forward.

15. Apply the laws of conservation of energy and momentum to show that for elastic collision of a body of mass M with another of mass KM, the energy lost by the former for a head-on collision is a maximum when $K=1$. [*Jt. Entrance, 1979*]

16. Two perfectly elastic flat discs A and B, one K times as massive as the other rest on a smooth horizontal table. The disc A is made to move towards B with a velocity U and makes a head-on collision.

Apply the conservation laws to find out an expression for the fraction of K. E. of A transferred to B. Also show that the value of this fraction is the same whether B is K times as massive as A or vice versa.

17. How can you obtain the principle of conservation of linear momentum from Newton's third law of motion. A gun recoils when fixed. Why ? The masses of a gun and a bullet are M and m respectively. After firing, the bullet flies off with a velocity v. Determine the direction and magnitude of the velocity of the gun.

18. A body of mass m moving with a velocity V in the X-direction collides with another body of mass M moving in the Y-direction with a velocity v. They coalesce into one body during the collision. Calculate—(i) the direction and magnitude of the momentum of the final body. (ii) the fraction of initial kinetic energy transferred into heat during the collision, in terms of the two masses.

19. (a) Explain the working principle of a jet or a rocket. What is the essential difference between a jet engine and a rocket ?

(b) Jet planes generally fly at higher altitudes, while propeller planes fly at lower altitudes. Why ? [*I. I. T. '76*]

(c) 'A rocket is the only means of propulsion in space'.—Justify the statement.

20. In a moving lift, a person sometimes feels himself (i) heavier (ii) lighter (iii) of the same weight. Explain these phenomena with proper reasons.

A lift is going downwards. Determine the conditions under which a person inside the lift will experience the feeling of (i) weightlessness (ii) super-weightlessness.

[B] Short answer type questions

1. A man is standing inside a bus moving in a straight line with uniform speed. What will happen under the following circumstances :—(i) the bus suddenly accelerates in the forward direction (ii) the bus decelerates (iii) the bus rounds a curve.

2. A heavy block of mass m is supported by a chord from the ceiling. A chord B is attached to the bottom of the block. What happens when (i) a sudden jerk is given to B (ii) a steady pull is given on to B?

3. Why an athlete in the long jump event always runs through a little distance before jumping?

4. A boy sitting in a train moving with a constant velocity throws a ball straight up into the air. Will the ball fall in front of him, into his hand or behind him?

If in the mean time the train (i) accelerates or decelerates (ii) begins to round a curve, where will the ball fall?

5. Show that Newton's first law can be obtained from the second law.

6. "Equal forces acting on a billiard ball or on a carrom piece for the same time produce same change in their corresponding momenta but do not produce same change in their velocities"—Justify the statement.

7. "We know that when a force acts on a body, it gets accelerated. As every body is attracted by the earth, so no terrestrial body can remain at rest or can move with uniform velocity." Do you agree with this statement? Justify your answer.

8. Answer the following questions briefly :

(i) It is not possible for a person sitting on a chair to lift the chair by its handle, however strong he might be, unless he lets his feet touch the ground. Why?

(ii) A horse pulls a cart. According to Newton's third law, the cart also pulls the horse with an equal force and in the opposite direction. How do the horse and the cart move at all? [*H. S.* '*68*

(iii) On a stationary sail-boat air is blown at the sails from a fan attached to the boat· Will the boat move?

(iv) The passengers sitting inside a stationary car are pushing it from inside. Will the car move?

(v) A cyclist can't stop the cycle by pulling the handle backwards. Why?

(vi) When a vehicle starts suddenly, the passengers lean backwards but when a moving vehicle suddenly stops, they fall forward. Why?

(vii) Why is a man hit harder when he falls on a paved floor than when he falls on the sand from the same height? [*Model questions, H. S. Council*]

9. A uniform rope of length L, resting on a frictionless horizontal surface, is pulled at one end by a force F. What is the tension in the rope at a distance l from the end where the force is applied? [*I. I. T.* '*78*]

10. Two men stand facing each other on two boats floating in still water at a short distance apart. A rope is held at its ends by both. The two boats are found to meet always at the same point whether each man pulls separately or both pull together. Why? Will the time taken to meet be different in the two cases? (Neglect friction). [*I. I. T.* '*74*]

11. A metoerite burns in the atmosphere before it reaches the earth's surface. What happens to its momentum? [*I. I. T.* '75]

12. A body of mass ω is placed on another one of mass W. Under what conditions the action-reaction between the two bodies will be (*i*) equal to the weight ω, (ii) greater than the weight ω (iii) less than the weight ω (iv) zero? Determine the reaction in each case.

13. Explain with reasons—(i) A stone thrown at a window pane smashes it but a bullet fired against the pane makes a clean hole.

(ii) A passenger alighting carelessly from a moving vehicle falls forward.

(iii) A man is standing inside a lift. (*a*) He feels heavier when the lift ascends with an acceleration, (*b*) He feels lighter when the lift descends with an acceleration.

(iv) Due to the earth's attraction, an apple descends to the ground but under its reaction the earth does not move forward towards the apple.

(v) To clean a garment from dust particles, it is suddenly set into motion.

(vi) A gun appear to have greater 'kick' when fired with the butt held loosely against the shoulder than when held tightly.

14. A Jet engine works on the principle of (i) conservation of mass (ii) conservation of energy (iii) conservation of linear momentum (iv) conservation of angular momentum—which one is the correct answer ? [*I. I. T.* '75]

15. 'A rocket moves forward by pushing the surrounding air backwards'—Is the statement true or false ? Give reasons in brief. [*I. I. T.* '80]

16. A bullet is fired from a rifle. If the rifle recoils freely, determine whether the kinetic energy of the rifle is greater than, equal to or less than that of the bullet. [*I. I. T.* 78]

17. Answer the following questions briefly :—(i) Which one will impart a greater impulse on the wall—a golf ball or a blob of mud, both having the same mass and velocity ?

(ii) Which of the two physical quantities may be chosen to measure the linear acceleration of a particle—(a) mass and velocity (b) distance and force (c) mass and force (d) velocity and density ?

(iii) Is there any force acting on a vehicle when it is moving on a rough road in a straight line with uniform velocity ?

(iv) A man carrying a suitcase in his hand is standing inside a lift which is descending with an acceleration. Will the suitcase appear lighter to him ?

(v) Two stones of equal mass rest on the pans of an equal-arm balance in a lift. Will they remain balanced when the lift accelerates upwards ?

18. An eye with a bird is hung from a spring balance. Will the reading of the spring balance differ in the following two cases—(i) when the bird is sitting inside the cage (ii) when it is flying inside the cage ?

19. A boy holding a spring balance in his hand suspends a weight of 100 gm from it. The balance slips from his hand and falls down. What will be the reading of the balance while it is in air ? [*Model Questions, H. S. Council*]

20. A sand glass is weighed in a sensitive balance. Will its weight in the following two cases be different ?—(i) When all the sands are in the lower compartment (ii) When the sand particles are falling from the upper to the lower compartment.

21. Consider the following two cases :—(i) Two boys pull at the ends of a rope each with the force F in opposite directions, (ii) A boy ties one end of a rope to a tree and pulls at the other end with a force $2F$. In which case the tension in the rope will be greater ?

22. A truck and a car are moving with the same momentum. When brakes are applied, they stop after covering the distances s_1 and s_2 respectively, the braking forces in both the cases being same. Select with reasoning the correct answer from among the following : (i) $s_1<s_2$ (ii) $s_1>s_2$ (iii) $s_1=s_2$ (iv) the values of s_1 and s_2 depend upon the power of the engine.

[C] Simple Problems.

1. Velocity of a body weighing 1 kg. changes from 5 cm./sec. to 20 cm./sec., in 5 sec. Find the force acting. (*H. S.* '66) [*Ans.* 3000 dyne]

2. A force of 200 newton acts on a body for 10 sec. and gives it a velocity of 40 m./sec. Find the mass of the body. [*Ans.* 50 kg.]

3. A body of mass 250 gm. traverses in successive seconds distances of 100, 110 and 120 cm. respectively. What force is acting on it ? [*Ans.* 2500 dyne]

4. A force acts for 20 sec. on a body of mass 5 kg. and then ceases to act. The body moves uniformly and describes 10 metres in the next 5 sec. Find the magnitude of the force. [*Ans.* 0·5 newton]

5. A body of mass 100 gm. has a momentum of 200 gm. cm. per sec. What is its velocity ? If the above momentum was acquired from rest in 10 secs., what were the acceleration and the force acting on the body ? [*Ans.* 2 cm./sec., 0·2 cm./sec.^{2}, 20 dyne]

6. What shall be the upward force in newton exerted on a 50 kg. body so that it causes the body to fall with an acceleration of 7·8 m./sec.^{2} ? [*Ans.* 100 newton]

7. A force of 100 dyne acts on a mass 10 gm. for 5 sec. Find the change of momentum and the distance travelled during the time. [*Ans.* 500 c. g. s. unit, 125 cm.]

8. A motor car of mass 400 lb. is running at a velocity of 30 mile/hr. It was brought to rest within a distance of 40 ft. by applying brakes uniformly. What force acted on the car ? [*H. S.* '71 (*comp.*)] [*Ans.* 9680 poundal]

9. A motorist rushing at 36 km./hr. finds a man 26 m. ahead. He stops the car within 1 m. of the man by applying brakes. Calculate (a) the time required to stop the car and (b) the retarding force that acted on the car. Given, the total mass of the car and the passengers= 2500 kg. [*Ans.* (a) 5 sec. (b) 5000 newton]

10. A motor car moving with a velocity of 14 m./sec. can be stopped by applying brakes in 20 metre. Prove that the resistance provided by the brakes to the car's motion is half of the weight of the car.

11. A bullet of mass 10 gm. moving with velocity of 100 m./sec. is stopped within 10 cm. of the target. Find the resistance offered by the target in newton. [*Ans.* 500 newton]

12. A 1 kg. hammer travelling at a speed of 500 cm./sec. strikes a nail and drives it 2 cm. into a block of wood. Find (a) the acceleration of the hammer, and (b) the constant resisting force. [*Ans.* $-6{\cdot}25\times10^4$ cm./sec.^{2} ; $6{\cdot}25\times10^7$ dyne]

13. A body of mass 200 gm. is moving with a velocity of 100 cm./sec. (a) Calculate the magnitude of the force required to stop it in 10 sec. (b) Also calculate the distance it moves before coming to rest. [*Ans.* (a) 2000 dyne (b) 500 cm.]

14. A bullet of mass 25 gm. is fired from a gun with a velocity of 400 m./sec. After passing through a target 20 cm. thick, its velocity drops to 100 m./sec. Calculate the resistance offered by the target. [*Ans.* 9375 newton]

15. When a golf ball of mass 62 gm. is hit by club, it acquires a speed of 80 m./sec. The impact lasts 2×10^{-3} sec. Assuming the force that the club exerts on the ball is constant, what is its magnitude ? [*Ans.* $2{\cdot}48\times10^8$ dyne]

16. A cricket ball of mass 100 gm. and moving with a velocity of 400 cm./sec. is brought to rest by a player in 0·01 sec. Find (a) the impulse and (b) the average force in newton applied by the player. [*Ans.* (a) 4×10^4 gm. cm./sec. (b) 40 newton]

17. A cricket ball of mass 156 gm. travelling at 1·5 m./sec. is moving towards a fieldsman who fumbles the catch, but succeeds in reducing the speed of the ball to 0·5 m./sec. What average force did the fieldsman exert on the ball, if the ball was in contact with his hands for 0·25 sec. ? [*Ans.* $6{\cdot}24\times10^4$ dyne]

18. A man weighing 60 kg. is standing on a lift. Find his weight as recorded by a weighing machine when (a) the lift is ascending with a uniform velocity of 3 m./sec., (b) the lift is ascending with a uniform acceleration of 3 m./sec.^{2}, (c) the lift is descending with a uniform acceleration of 3 m./sec.^{2}. [*Ans.* (a) 60 kg. (b) 78·38 kg. (c) 41·64 kg.]

19. A man of 150 lb. causes a lift to move downwards with an acceleration of 3 ft./sec.^{2}. What thrust does the man exert on the floor ? If the lift moves upwards with the same acceleration, what thrust will he experience ? Calculate the acceleration of the lift for which the man feels weightless. (*Jt. Entrance* '74) [*Ans.* 4350 poundal, 5250 poundal, 32 ft./sec.^{2}]

20. A man weighing 70 kg. is standing on a lift. Find his weight as recorded by a weighing machine when the lift is (a) stationary (b) descending with a uniform velocity of 4 m./sec. (c) descending under the action of gravity. [*Ans.* (a) 70 kg. (b) 70 kg. (c) zero]

21. A man whose weight is 70 kg., enters a lift at the eighth floor of a twenty storied building and stands on a spring weighing machine. When the lift starts, he finds that the scale reads 65 kg. for a short period and then shows a steady reading of 70 kg. Just before the lift stops, the scale reads 73 kg.

Was he ascending or descending ? Find the acceleration of the lift both on starting and stopping. [*Ans.* Descending ; 0·7 and 0·42 m./sec.^{2}]

22. A 800 kg. gun fires a projectile of mass 20 kg. with a velocity of 400 m./sec. (a) Find the recoil velocity of the gun. If the recoil is against a constant resisting force of 1200 kg. wt. calculate (b) the time in which the gun is brought to rest and (c) the distance it recoils. [*Ans.* (a) 10 m./sec. (b) 0·68 sec. (c) 3·4 m.]

23. A bullet of mass 100 gm. leaves the gun with a velocity of 500 m./sec. The velocity of recoil of the gun is 10 m./sec. Calculate the mass of the gun. [*Ans.* 5 kg.]

24. A rocket burns 0·25 kg. of fuel per second and ejects it as a gas with a velocity of 1 km./sec. Calculate the force exerted by the ejected gas on the rocket. [*Ans.* 250 newton]

25. A 40 kg. ball moving at 20 m./sec. strikes centrally another ball at rest weighing 50 kg. Find the velocity of each after impact, if (a) they stick together and (b) if the collision is perfectly elastic. (c) Calculate also the loss in K. E. of the balls due to impact in the former case.
[*Ans.* (a) 8·89 m./sec. (b) $v_1=-2{\cdot}22$ m./sec. and $v_2=17{\cdot}78$ m./sec. (c) 4444·4 Joule]

26. A body of mass 1 kg. moving with a velocity of 10 m./sec. towards right makes head on collision with another body of mass 3 kg. and velocity 2 m./sec. coming from opposite direction. Calculate the velocity of each after impact, if (a) they stick together and (b) the collision is perfectly elastic. (c) Also calculate the loss in K. E. of the bodies due to impact in the former case. [*Ans.* (a) 1 m./sec. towards right ; (b) 8 m./sec. towards left ; 4 m./sec. towards right ; (c) 54 joule]

27. Two spheres of masses 6 gm. and 8 gm. are moving towards each other in the same line with velocities 10 cm./sec. and 6 cm./sec. respectively. Find their velocities after collision if the coefficient of restitution is 0·75. Calculate also the loss of energy.
[*Ans.* −6 cm./sec., 6 cm./sec., 192 erg.]

28. A balloon of total mass 1000 kg. floats motionless over the earth's surface. If 100 kg. of sand ballast are thrown overboard, with what acceleration the balloon starts to rise ?
[*Ans.* 1·09 m./sec.^{2}]

29. A 2000 kg. lift asends with an acceleration of 2 m./sec. Find the tension in the rope. Repeat your calculation for the lift descending with the same acceleration.
[*Ans.* 23620 newton ; 15620 newton]

30. An inflated balloon contains 4 gm. of air which is allowed to escape from a nozzle at a speed of 5 m./sec. Assuming that the balloon deflates at a steady rate in 0·5 sec., what is the force exerted on the balloon ? [*Ans.* 0·04 N]

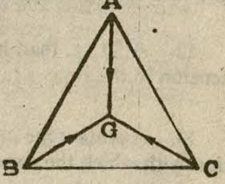
31. A body of mass 1·5 kg. is moving along a straight line *AB* with a velocity of 5 m./sec. A force of 6 Newton acts on it at right angles to *AB* for 3 sec. What will be the velocity of the body at the end of 3 sec ? [*Ans.* 13 m./sec. at an angle $\tan^{-1}(\frac{12}{5})$ to *AB*]

32. A particle of mass 4·5 gm is moving in a straight line *AB* with uniform velocity 2 cm./sec. When it crosses the point *A*, a force of 0·3 dyne begins to act on it, the direction of the force being always perpendicular to *AB*. 30 sec. later the particle is at *C* such that *BC* is perpendicular to *AB*. If *M* be the midpoint of *AB*, show that the direction *MC* is the direction of motion of the particle when it is at *C*. Find the displacement *AC*. [*Ans.* 67·08 cm.]

33. A ball of mass 2 kg. is suspended by a string from the ceiling of a car. If the car moves horizontally with a constant acceleration of 1·5 m./sec, find the angle which the string makes with the vertical and also the tension in the string. [*Ans.* 8·7° ; 19·83 N]

34. A shot of mass 250 gm. is fired from a gun of mass 20 kg. with a velocity 75 m./sec. relative to the gun. Find the actual velocity of the shot and the velocity of recoil of the gun.
[*Ans.* 74·074 m./sec ; −0·926 m./sec.]

35. Three particles *A*, *B* and *C* of equal mass move with equal speed *V* along the medians of an equilateral triangle as shown in the figure. They collide at the centroid *G* of the triangle. After collision *A* comes to rest, *B* retraces its path with speed *V*. What is the velocity of *C* ? (*I. I. T.* '82) [*Ans.* *V* along $\overrightarrow{BG}$]



36. A body of mass 1 kg. initially at rest, explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. The two pieces of equal mass fly off perpendicularly to each other with a speed of 30 m./sec. each. What is the velocity of heavier fragment ? (*I. I. T.* '81)
[*Ans.* 14·14 m./sec directed opposite to the bisector of the right angle]

[D] Harder Problems

1. A particle of mass 5 gm. is initially moving along a straight line with a velocity of 10 cm./sec. and then a force of $10\sqrt{2}$ dyne is applied to it at an angle of 45° to the initial direction of motion. Calculate the change in its kinetic energy during the first second.
(*I. I. T. '65*) [*Ans.* 120 erg]

2. Two bodies of different masses have the same kinetic energy. Prove that the heavier body has momentum greater than that of the lighter one.

3. A rifle bullet moving with a velocity of 400 m./sec. is just able to pierce a plank 8 cm. thick. What should be the velocity of the bullet so that it can just pierce a plank 13·52 cm. thick. [*Ans.* 520 m./sec.]

4. A 60 kg. man standing in a lift holds a spring balance with a load of 4 kg. suspended from it. What would be the reading of the balance when the lift is descending with an acceleration of 5 m./sec.^{2} ? [*Ans.* 1·959 kg.]

5. A rope of negligible mass can support a load of M kg. Prove that the mass of the greatest load which can be raised is equal to $M/\left(1+\frac{2h}{gt^2}\right)$kg. where g is the acceleration due to gravity and h is the height through which the said load rises from rest with uniform acceleration in time t.

6. A steel ball of radius 2 cm. is initially at rest on horizontal frictionless surface. It is struck head on by another steel ball of 4 cm. radius, travelling with a velocity of 81 cm./sec. Calculate the velocities of the two balls after collision.
(*I. I. T. '64*) [*Ans.* 144 cm./sec. ; 63 cm./sec.]

7. A neutron makes a perfectly elastic head on collision with a stationary carbon nucleus. Show that, the neutron losses 0·284 of its initial kinetic energy as a result of the impact.

8. Two bodies A and B of masses 100 gm. and 400 gm. respectively are moving towards each other with speeds 100 cm./sec. and 10 cm./sec. They suffer a head on collision and stick together. (i) In which direction will the combined mass move after the collision, (ii) what will be the distance travelled by the combined mass after 10 sec., (iii) assuming the two bodies to be originally at the same temperature and specfic heat of both the bodies to be 0·1, calculate the rise in temperature of the combined mass after the collision. [*I. I. T.* '71]
[*Ans.* (i) In the direction of motion of A. (ii) 120 cm. (iii) $2{\cdot}3\times10^{-4}$°C.]

9. Find the minimum rate at which a rocket must be consuming fuel in order to be able to take off vertically, given that, at the instant it takes off, the mass of the rocket is 5000 kg. and the products of the combustion are ejected at a velocity of 250 m./sec. [*Ans.* 196·2 kg./sec.]

10. A space vehicle of mass m while moving towards the moon with velocity u is divided into two parts by an explosion. The smaller part (mass $m/3$) is then found to move towards the moon with velocity v, while the larger part (mass $2m/3$) has its velocity reduced to $u/2$. Find (a) the velocity v in terms of u, (b) the K. E. of the whole system before and after explosion and (c) energy in joule to be supplied during explosion, if $m=1000$ kg. and $u=4$ km./sec.
[*Ans.* (a) $v=2u$; (b) $\frac{1}{2}mu^2$ and $\frac{3}{4}mu^2$; (c) 4×10^9 joule]

11. A bolloonist estimates the motion of the balloon by hanging a 700 gm. weight at the end of a spring balance. The spring balance reads 800 gm. wt. How is the ballon moving ?
[*Ans.* With an acceleration 140 cm./sec.^{2} upwards]

12. A 50 kg. load hangs at the end of a rope. Find the acceleration of the load if the tension in the rope is (a) 50 kg. wt. (b) 40 kg. wt. and (c) 75 kg. wt.
[*Ans.* (a) Zero (b) 1·96 m./sec.^{2} downwards (c) 4·9 m./sec.^{2} upwards]

13. A crane rope can withstand a tension of 1000 kg. wt. What is the greatest acceleration with which the rope can raise a crate weighing 800 kg. ? [*Ans.* 2·45 m./sec.^{2}]

14. A machine gun placed directly in front of a target fires 500 bullets per minute into the target. The bullets have velocity 600 m./sec. and mass 50 gm. Calculate the average force exerted on the target. [*Ans.* 250 newton]

15. Find the force exerted on the sail of a boat by a 30 km. p. h. wind, the area of the ail being 6 sq. m. Density of air=1·29.gm./lt. at N.T.P. [*Ans.* 537·5 newton]

16. What force is exerted on a vertical wall when a horizontal jet of water 5 cm. in diameter and having speed 10 m./sec. impinges on it ? Given, after striking the wall, the water is moving parallel to the wall and density of water$=1000$ kg./m^3. [*Ans.* 196·25 newton]

17. A gun is mounted on a platform having perfectly frictionless wheels. The mass of the platform with the gun, projectiles and operator is M. The mass of a projectile is m. The gun is fired in a horizontal direction. The initial velocity of the projectile is v. What will be the speed of the platform after Nth shot ? [*Ans.* $V_N = Nmv/(M-Nm)$]

18. A man of mass 70 kg. stands on a stationary horizontal platform of mass 35 kg. which has almost frictionless wheels and can move along the horizontal floor. Explain mathematically what happens (a) if he starts to walk steadily across the platform, moving at 3 m./sec. relative to the platform, (b) if he then brings himself to rest relative to the platform again.
[*Ans.* Velocity of the platform$=2$ m./sec. (b) the platform comes to rest]

19. Two boys of masses 30 kg. and 50 kg. sit facing one another on light frictionless trolleys holding the ends of strong taut cord between them. The lighter boy tugs the cord and acquires a velocity of 40 cm./sec. What is the initial velocity of the other boy ? What happens to their motion when they collide ? [*Ans.* 24 cm./sec.]

20. In still water, a boat is placed in a perpendicular direction to the bank. The distance of the nearest end of the boat from the bank is 0·6 m. A man walked along the boat from this nearest end to the other. If the masses of the boat and the man be 200 kg. and 50 kg. respectively and if the length of the boat be 2·5 m., then will the boat be able to reach the bank ?

What should be the minimum length of the boat in order that it can reach the bank ?
[*Ans.* No ; 3 m.]

21. A small metal ball is dropped vertically on a horizontal fixed plane of the same metal from a height of 8 metre. If the coefficient of restitution is 0·5, find the heights of the first, second and the third rebounds. If the mass of the ball is 2 gm. how much energy is wasted during these impacts ? [*Ans.* 2 m., 0·5 m. ; 0·125 m.,; 0·154 Joule]

22. For a partly elastic collision of two bodies, show that

$$(u_1-v_1)/(u_1-u_2)=m_2(1+e)/(m_1+m_2)$$

the symbols having their usual meaning.

If the masses of the two bodies are equal and the coefficient of restitution is 0·5, show that exactly half the energy is lost in collision if

$$(1-\sqrt{2})\,u_1=(1+\sqrt{2})u_2$$

23. Two small equal spheres hang by vertical strings 1·6 m. and 2 m. long respectively so that they just touch each other with their centres at the same level. The former sphere is drawn aside till its string subtends an angle of 60° with the vertical and is then released. After the impact, the second sphere is found to swing through an arc of 45°. Find the coefficient of restitution. (Take $g=10$ m./sec.^{2}). [*Ans.* 0·7115]

24. An engine of mass 100 tonnes is coupled to and pulls a carriage of mass 35 tonnes. The resistance to the motion of the engine is $\frac{1}{80}$ of its weight and the resistance to the motion of the carriage is $\frac{1}{175}$ of its weight. Find the tension in the coupling if the tractive force exerted by the engine is 27710 N. [*Ans.* 5460 N]

25. An engine of mass 200 tonnes is pulling a train of 10 carriages, each of mass 60 tonnes. The engine is exerting its maximum pull of 2×10^5 N when the train is moving at a constant speed of 36 km./hr. Due to the disengagement of the coupling between the 5th and 6th carriages, the rear five carriages get separated and the engine hauls the first five carriages exerting the same pull. Assuming that the tractive resistance is proportional to the weight, find how far ahead the front half of the train is when the rear half comes to rest ?
[*Ans.* 400 m.]

26. A shell of mass 30 kg. is moving vertically upwards. When its velocity is 150 m./sec. it bursts into two pieces. If the lower piece of mass 5 kg. is instantaneously stopped by the explosion find (a) how much higher the upper piece will ascend and (b) energy supplied during explosion. [*Ans.* (a) 1653·06 m. (b) 67500 Joule]

4
CHAPTER

FRICTION

4·1. Friction.

Whenever a body in contact with a second one, moves or tends to move, forces opposing the motion come into play. They are tangential to the surface along which motion occurs or tends to occur. These forces are called **forces of friction** *or* **frictional forces** *between the two surfaces in contact.*

As such forces always act on a body in a direction opposite to that in which it moves, reversal of the direction of motion also reverses the direction in which these forces act. Frictional forces may also act when there is actually no relative motion but there is tendency for such relative motion.

For example, in trying to push a book lying on a table, the push applied may not be sufficient to set the book in motion. The applied force is then balanced by an equal frictional force exerted on the book by the table. When the applied force is able to overcome this frictional force, the book slides, say from right to left (Fig. 4·1). A frictional force to the right acts on the book, trying always to bring the book to rest.

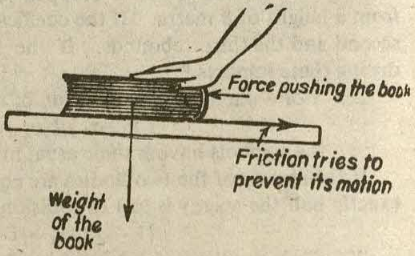


Fig. 4·1 : Friction opposes motion

Often friction is very useful. The brakes of a moving vehicle work by friction. In driving a car friction is essential. In absence of friction, the wheels would spin around but the car would not move. Friction makes it possible for us to walk; otherwise we would slip as soon as we try to walk. Friction is useful in transmitting power from a motor to a machine by means of a belt. When work is done against frictional forces, heat is produced. This is useful in striking a match. On feeling cold, we rub our hands together to make them warm.

On the other hand, *in many other cases friction is a disadvantage.* Friction reduces the efficiency of a machine, thus necessiating greater supply of energy for its operation. It causes wear and tear of the moving parts, produces heat leading to more damage, and thereby shortens the life of the machine. Hence the need of minimizing friction. This is done by using wheels, bearings, rollers and lubricant. To reduce air friction, high speed automobiles and aeroplanes are built in a special shape, called streamlined shape. On re-entering earth's atmosphere, a space-craft may become red hot due to the same cause. A special heat shield is fitted to protect the astronauts.

4·2. Types of Friction.

All forms of friction may be classified into following three categories : *(a) sliding friction (b) rolling friction (c) fluid friction*. The first two categories may be of two kinds—(i) *static* and *(ii) kinetic*.

If the surface of one body slides or tends to slide over that of another, forces of friction opposing such motion are known as **sliding friction.**

Frictional forces opposing the motion of a body rolling or tending to roll on another are called **rolling friction.**

Sliding and rolling friction are usually present between solid surfaces. Rolling friction is found to be much smaller than sliding friction. It is for this reason that wheels are used on vehicles.

When there is a tendency of motion between two surfaces in contact, but actually no relative motion, the existing frictional force is called **static friction**.

When the surface of a body in contact with another is in relative moiton, the frictional force that exists is known as **kinetic** *or* **dynamical friction**.

When a gas or a liquid (i. e. a fluid) flows across a stationary object or when an object moves through a previously stationary fluid, the friction produced is known as **fluid friction**. Such forces exist when a ship plies through water or when any object e.g. car, aeroplane, space ship etc., moves through air. Fluid friction is found to be much less than two types of friction involving solid surfaces.

4·3. Sliding Friction.

On closer examination, under suitable magnification, any surface though apparently smooth will be found to be full of irregularities. These irregularities in the surfaces of two bodies in contact, tend to interlock (Fig. 4·2) and offer resistance to motion giving rise to the force of sliding friction. If the surfaces are made smoother, friction diminishes.

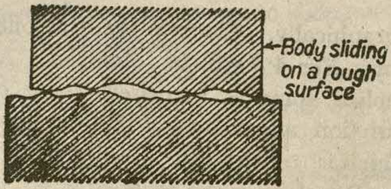


Fig. 4·2 : Origin of sliding friction

Within a few percent accuracy, certain simple observations are found to be true for nearly all cases of sliding friction. These can be illustrated by a simple experiment.

A block is placed on a horizontal table. A string attached to the block passes over a smooth pulley attached at one end of the table. Weights can be placed on a pan tied to the other end of the string [Fig. 4·3].

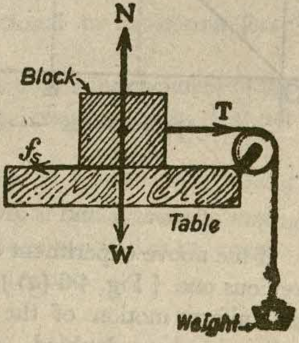


Fig. 4·3 : Experiment on sliding friction.

When the weight pan is detached, the tension in the string will be zero. The block is then in equilibrium under the action of its weight *W* acting vertically downwards and equal and opposite reaction force *N* applied on the block by the table.

If a small mass is now placed on the pan, the block will not move. The horizontal force *T* on the block due to tension in the string (which is equal to the weight of the pan+small mass) is then balanced by the frictional force f, that comes into play in the opposite direction. This is a force of static friction.

The tension in the string is then gradually increased by adding small masses on the pan. All the while frictional forces also go on increasing balancing the tension. But there is a limit to such increase of frictional forces. When that limit is crossed, with a further small increase in the tension, the block breaks away from the surface and begins to slide. ***Frictional force is thus a self-adjusting force,*** and so long as the body is at rest, this equals the applied force. ***The maximum value of the force of static friction when the body is just on the point of sliding is called*** **limiting friction.**

When the body begins to slide, it is found that application of a force slightly smaller than the limiting friction is sufficient to maintain the body in uniform motion. Since during uniform motion no resultant force acts on the body, the applied force must be balanced by the opposing force of friction. We thus conclude that when a body actually slides, the force of friction falls to a slightly smaller value ; it then remains practically constant even if the sliding motion be increased. This new frictional force f_k, is called the **Kinetic** or **Dynamic** friction [Fig. 4·4].

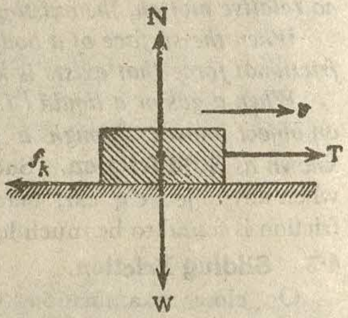


Fig. 4·4 : Kinetic friction

All these observations can be represented graphically. Plotting the frictional force against the applied force, the graph shown in Fig. 4·5 will be obtained. As the applied force is increased, force of static friction also increases at the same rate which is represented by the straight portion OA of the curve inclined at 45° to the axes. When the applied force reaches a certain value OD, the body is on the verge of sliding. The static friction is then maximum given by AD, which is the limiting friction. On further increasing the applied force, the body begins to move and the slightly lower value of kinetic friction coming into play, remains constant afterwards and is given by CE.

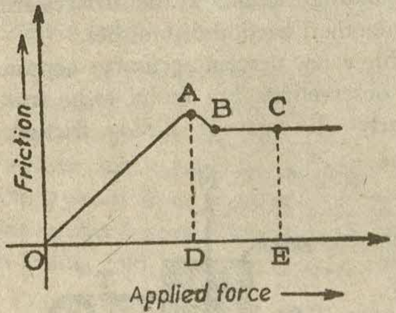


Fig. 4 5

If the above experiment by repeated by placing another similar block on the previous one [Fig. 4·6 (a)], the force necessary to just initiate sliding or to maintain uniform motion of the system and hence the opposing force of friction, will be found to have doubled. On adding another block, the force will be found to have tripled and so on. Frictional force thus increases proportionally with the total downward force or the reaction force pressing the two bodies together.

If the second block is tied behind the first [Fig. 4·6 (b)] or if any one of the first blocks is placed with its smaller face on the table [Fig. 4·6 (*c*)], the forces of friction will be found to be the same to the previous values. This shows that the frictional force is independent of the area of contact.

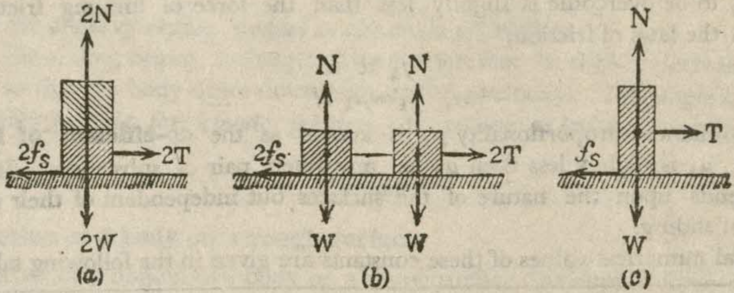


Fig. 4·6

If a large glass sheet is placed on the table and the experiment is repeated with the block on the glass sheet, the force of friction will be found to be lower. Hence friction depends upon the nature of the surfaces in contact.

4·4. Laws of Friction.

The above observations can be written in a generalized manner, known as the *laws of friction*, which are given below :

(1) The force of friction between two surfaces in contact is parallel to the surfaces sliding or tending to slide over each other and acts always opposite to the direction of motion.

(2) The force of friction depends upon the material (i.e. iron, glass, etc.) and the condition (i.e., on polishness, roughness, wetness etc.) of the surfaces in contact.

(3) The force of friction is independent of the area of the surface of contact.

(4) The maximum force of static friction or the limiting friction is proportional to the normal force which one surface exerts on another.

(5) Limiting friction is slightly greater than kinetic friction.

(6) The force of kinetic friction is also proportional to the normal force between the surfaces.

(7) Over a wide range, the force of kinetic friction is independent of the relative velocity of the two surfaces, provided that the resulting heat does not affect the condition of the surfaces.

(8) Other things remaining equal, sliding friction is greater than rolling friction.

Co-efficient of friction : If the normal force between two bodies in contact be *N*, then from the law of friction, the force of limiting friction

$$f_s \propto N$$

$$\text{or,}\quad f_s = \mu_s N \qquad \ldots \qquad \ldots \qquad (4{\cdot}1)$$

where μ_s is the constant of proportionality. μ_s is called the **co-efficient of limiting (or static) friction**. It is obvious that the value of this constant depends upon the materials of the surfaces in contact and also on their condition. Its value is ordinarily less than unity.

When one of the bodies is sliding uniformly over another, the force of kinetic friction f_k to be overcome is slightly less than the force of limiting friction f_s. Also from the laws of friction,

$$f_k \propto N$$

$$\text{or,} \quad f_k = \mu_k N \quad \ldots \quad \ldots \quad (4{\cdot}2)$$

The constant of proportionality μ_k is known as the **co-efficient of kinetic friction.** μ_k is slightly less than μ_s for any given pair of substances. Its value also depends upon the nature of the surfaces but independent of their relative velocity of sliding.

Typical numerical values of these constants are given in the following table.

Material	μ_s	μ_k
Steel on steel	0·15	0·09
Leather on wood	0·5	0·4
Glass on glass	0·94	0·4
Rubber tire on dry concrete road	1·0	0·7
Rubber tire on wet concrete road	0·7	0·5

Angle of friction : When the force of friction is limiting, the normal reaction N and the force of limiting friction f_s may be compounded into a single force R, called the resultant reaction [Fig. 4·7]. The angle, α, which this resultant R makes with the normal reaction N, is called the **angle of friction.** Now $N = R \cos \alpha$ and $f_s = R \sin \alpha$

$$\text{so that } \frac{f_s}{N} = \frac{R \sin \alpha}{R \cos \alpha} = \tan \alpha$$

$$\text{But } f_s = \mu_s N \quad \therefore \quad \mu_s = \tan \alpha \quad \ldots \quad (4{\cdot}3)$$

The co-efficient of limiting friction is thus equal to the tangent of the angle of friction.

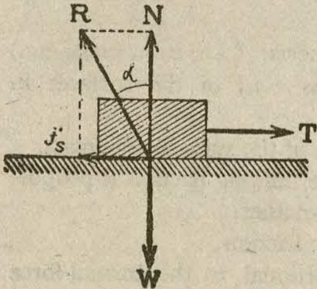


Fig. 4·7 : Angle of friction

4·5. Angle of Repose.

A block of weight W is placed on an inclined plane whose inclination may be varied. If the inclination of the plane be very small, the frictional force would prevent the downward motion of the block. On increasing the angle of inclination, the limiting point will be reached when the body just begins to slide downwards. This particular angle is known as the **angle of repose.** Thus it may be defined as *the maximum angle of inclination of a plane such that a body resting on the plane, is on the point of sliding downwards under the action of its own weight.* From Fig. (4·8), if θ be the angle of repose, then $W \sin \theta = f_s$, $W \cos \theta = N$.

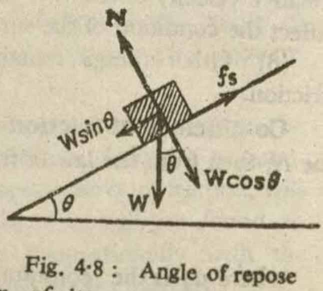


Fig. 4·8 : Angle of repose

$$\text{Hence, } \mu_s=\frac{f_s}{N}=\frac{W\sin\theta}{W\cos\theta}=\tan\theta$$

But $\mu_s=\tan\alpha$, where α is the angle of friction.

$$\therefore \quad \theta=\alpha \qquad \ldots \qquad \ldots \qquad (4·4)$$

Thus *the angle of repose is equal to the angle of friction.*

After the sliding begins, the angle of inclination may be slightly decreased to a value θ', so that the body slides down with uniform velocity. *This angle is known as the limiting angle for kinetic friction.* Proceding as before, it can be shown that

$$\mu_k=\tan\theta' \qquad \ldots \qquad \ldots \qquad (4·5)$$

4·6. Motion of a body on a rough surface.

For producing motion of a body on a rough surface, the applied force must be large enough to overcome friction. If the applied force be P and the force of kinetic friction opposing the motion be f_k, then the resultant force ($P-f_k$) is effective in producing the acceleration f of the body. From Newton's second law, the equation of motion is given by

$$P-f_k=mf \qquad \ldots \qquad \ldots \qquad (4·6)$$

where m is the mass of the body.

Since, $f_k=\mu_k N$, we get $P=\mu_k N+mf$ ... ... (4·7)

Example 4·1. *A 150 gm. block rests on a horizontal floor. A minimum horizontal force of 30 gm. wt. is required to start it moving ; it is then found that a horizontal force of 25 gm. wt. keeps it moving with uniform velocity. (a) Find the co-efficient of static friction and the co-efficient of kinetic friction. (b) What is the force of friction when a force of 15 gm. wt. acts on the block ?*

Solution : (a) The normal reaction force is equal to the weight of the block. Hence, $\mu_s=\frac{30}{150}=0·2$; $\mu_k=\frac{25}{150}=0·17$.

(*b*) The force of friction is 15 gm. wt. directed opposite to the applied force ; the block remains at rest.

Example 4·2. *The angle of inclination of a plane can be varied. A body resting on the plane, just begins to slide down when the angle of inclination is 30°. It is found that it keeps moving down without acceleration, if the angle of inclination is lowered to 25°. Find the co-efficient of static friction and the co-efficient of kinetic friction.*

Solution : From article (4·5),

$$\mu_s=\tan 30°=0·58 \; ; \; \mu_k=\tan 25°=0·47.$$

Example 4·3. *A horizontal force, equal to the weight of 20 gm. acts for 5 sec. on a block of mass 50 gm. placed on a rough horizontal floor. Calculate the velocity of the block at the instant the force ceases to act. Given that the co-efficient of kinetic friction between block and floor is 0·2.*

Solution : Since the normal reaction force acting on the block is 50 gm. wt., frictional force opposing its motion is

$$f_k=0·2\times 50=10 \text{ gm. wt.}=10\times 981 \text{ dyne.}$$

Since the force acting on the block in the direction of motion is 20×981 dyne, we have from equation (4·6)

$$20 \times 981 - 10 \times 981 = 50\, f$$

where f is the acceleration of the body.

$$\therefore \quad f = \tfrac{981}{5} \text{ cm./sec}^2.$$

Since the block starts from rest, its velocity at the end of 5 sec. is given by, $v = u + ft = 0 + \tfrac{981}{5} . 5 = 981$ cm./sec.

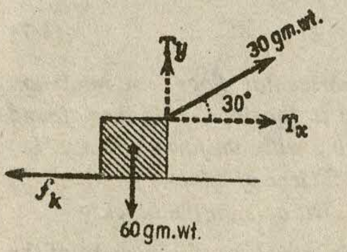
Example 4·4. *A block of mass 60 gm. is pulled along a horizontal surface by means of a rope making an angle of 30° above the horizontal surface, so that starting from rest, the block gains a speed of 4·9 metre/sec. in 5 sec. If the tension in the rope is 30 gm. wt., what is the co-efficient of kinetic friction ? [Take g=980 cm./sec.^{2}]*

Solution : Velocity of the block at the end of 5 sec. = 490 cm./sec.

∴ Acceleration of block $= f = \frac{490 - 0}{5} = 98$ cm./sec.^{2}

The horizontal component of tension in the rope is

$T_x = 30 \cos 30° = 30 \times 0{\cdot}866 = 25{\cdot}98$ gm. wt. $= 25{\cdot}98 \times 980$ dyne. This force overcomes the force of friction and gives the block a horizontal acceleration.



The vertical component of tension in the rope is, $T_y = 30 \sin 30° = 15$ gm. wt. $= 15 \times 980$ dyne. Since the weight of the block acts downwards, the total vertical force on the block $= 60 \times 980 - 15 \times 980 = 45 \times 980$ dyne.

∴ the force of friction opposing the motion $= f_k = \mu_k \times 45 \times 980$ dyne.

Hence from equation (4·6),

$$T_x - f_k = mf$$

$$\text{or,} \quad 25{\cdot}98 \times 980 - \mu_k \times 45 \times 980 = 60 \times 98$$

$$\text{or,} \quad \mu_k = 0{\cdot}44.$$

Example 4·5. (*a*) *What is the acceleration of a block sliding down an inclined plane of inclination 60°, the co-efficient of friction being 0·2 ?* (*b*) *What force acting parallel to the plane can just prevent the motion of the block of mass 100 gm. ?* (*c*) *What force parallel to the plane is required to keep the block moving up the plane at constant speed ? [Take g=980 cm./sec.^{2}].*

Solution : Refer to Fig. 4·8.

(*a*) The normal reaction force $N = W \cos 60° = mg/2$

$$\therefore \text{ frictional force } f_k = \mu_k N = 0{\cdot}2 \times \frac{mg}{2} = 0{\cdot}1\, mg.$$

Downward component, along the plane, of the weight of the block

$$= W \sin 60° = \frac{\sqrt{3}}{2} mg.$$

∴ If the body slides down with an acceleration f

$$\frac{\sqrt{3}}{2} mg - 0{\cdot}1\, mg = mf$$

$$\text{or, } f=\left(\frac{\sqrt{3}}{2}-0{\cdot}1\right)g=750{\cdot}7 \text{ cm./sec.}^2$$

(*b*) To prevent motion, an upward force P along the plane and equal to the downward force mf, must be applied on the block.

$$\therefore\ P=\frac{\sqrt{3}}{2}mg-0{\cdot}1\ mg$$

$$=0{\cdot}766\times100\times980 \text{ dyne}=76{\cdot}6 \text{ gm. wt.}$$

(*c*) Since force of friction always opposes the motion, in this case of motion up the plane, friction acts down the plane. Since the block has no acceleration, the applied upward force P must be equal to the total downward force along the plane.

$$\therefore\ P=mg\sin60^\circ+f_k$$

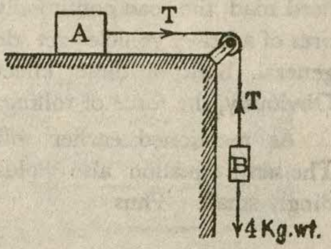
$$=\frac{\sqrt{3}}{2}mg+0{\cdot}1mg=0{\cdot}966\times100\times980 \text{ dyne}$$

$$=96{\cdot}6 \text{ gm. wt.}$$

Example 4·6. *A block A of 10 kg is pulled along a table, by a light inextensible string passing over a weightless pulley and carrying a load B of 4 kg at the other end. Find the tension in the string and the acceleration of the system. Neglect the friction in the pulley and the friction between block A and the table.*

Solve the problem if the co-efficient of kinetic friction between the block A and the table be 0·1 [Take g=980 cm./sec.²].

Solution : Let T be the tension in the string and α be the acceleration of the system.



1st part : Net force acting on the system= mass × acceleration of the system

$$\text{or, } 4\times10^3\times980-T+T=(10+4)\times10^3\times\alpha$$

$$\therefore\ \alpha=280 \text{ cm./sec.}^2$$

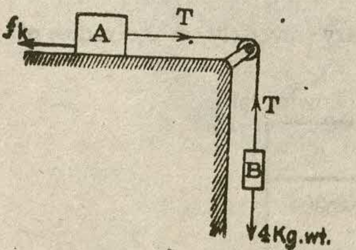
Again, net force acting on B=mass × acceleration of B.

$$\text{or, } 4\times10^3\times980-T=4\times10^3\times280$$

$$\therefore\ T=2{\cdot}8\times10^6 \text{ dyne}$$

Or, net force acting on A=mass × acceleration of A

$$\text{or, } T=10\times10^3\times280=2{\cdot}8\times10^6 \text{ dyne}$$



2nd part : Retarding force of friction on the block A is

$$f_k=0{\cdot}1\times10\times10^3\times980 \text{ dyne}=9{\cdot}8\times10^5 \text{ dyne}$$

Now, net force acting on the system= mass × acceleration of the system.

$$\text{or, } 4\times10^3\times980-T+T-9{\cdot}8\times10^5=(10+4)\times10^3\times\alpha$$

$$\therefore\ \alpha=210 \text{ cm./sec.}^2$$

Again, net force acting on B=mass × acceleration of B.

$$\text{or, } 4\times10^3\times980-T=4\times10^3\times210$$

$\therefore \quad T=3{\cdot}08\times10^6$ dyne

Or, net force acting on A = mass × acceleration of A

or, $T-9{\cdot}8\times10^5=10\times10^3\times210$

$\therefore \quad T=3{\cdot}08\times10^6$ dyne.

4·7. Rolling Friction.

When a body rolls on another, one or both of them are usually deformed at the place of contact. In the case of a hard wheel rolling over a soft road, the deformation of the road's surface is clearly visible [Fig. 4·9 (*a*)]. The weight of the wheel depresses the surface at the place of contact and a mound develops in front of the wheel. As a result, the applied force always pull the wheel over a minute hill developed in the ground. On the other hand, if a soft wheel moves over a hard road, the road continually pushes the wheel out of shape [Fig. 4·9(*b*)]. The tires of a heavy vehicle get deformed at the place of contact with the road. In general, both of these effects are present, which give rise to rolling friction. Obviously, the force of rolling friction will be smaller, the harder the surfaces.

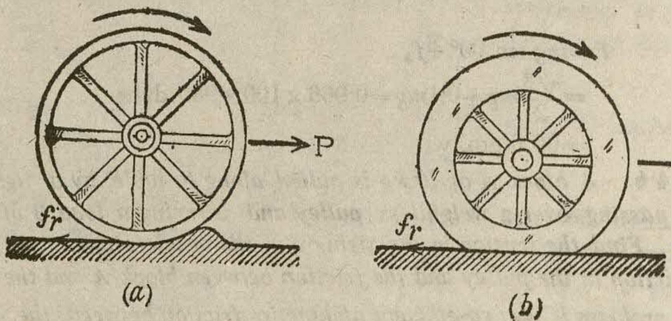


Fig. 4·9 : Origin of rolling friction

As mentioned earlier rolling friction is much smaller than sliding friction. The same equation also holds for rolling friction but the co-efficients are exceedingly small. Thus

$$f_r=\mu_r N \qquad \cdots \qquad \cdots \qquad (4{\cdot}8)$$

where f_r = the force of rolling friction,

N = the normal reaction between the surfaces,

μ_r = co-efficient of rolling friction.

The force of rolling friction is found to vary inversely as the radius of the roller.

Some values of the co-efficients are given in the following table :

Material	μ_r
Cast iron on rails	0·004
Rubber tires on concrete	0·03
Ball Bearing on steel	0·002

Pushing or pulling a roller : Pushing a roller is found to be more difficult than pulling it. As the roller is pushed with a force P, its vertical component acts downwards increasing the normal pressure on the ground [Fig. 4·10]. Hence the normal reaction by the ground also increases and therefore the frictional force opposing the motion increases proportionally. If, however, the roller is pulled with the same force P, the vertical component of P acts upwards, thereby decreasing the effective normal pressure. The opposing frictional force thus also decreases. **Pulling a roller is, therefore, easier than pushing it.**

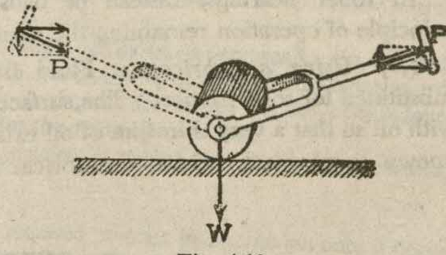


Fig. 4·10

4·8. Methods of minimising friction.

Several methods of reducing friction are in common use. This is important from the view point of manufacturing or operating machineries. One or more of the following methods are adopted :

(*i*) ***Polishing the bearings*** : The contact surfaces of moving parts are polished to a high degree. A hard material is selected, so that it does not wear away rapidly.

(*ii*) ***Using antifriction metals*** : When steel slides over an alloy of lead and antimony, the co-efficient of friction is less than when steel slides on steel. The alloy is known as antifriction metal. The process is called babbitting, after the inventor Babbitt. Bearings are packed with this alloy.

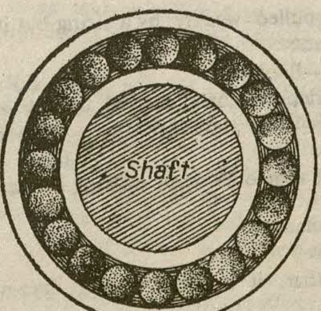


Fig. 4·11 : Ball Bearing

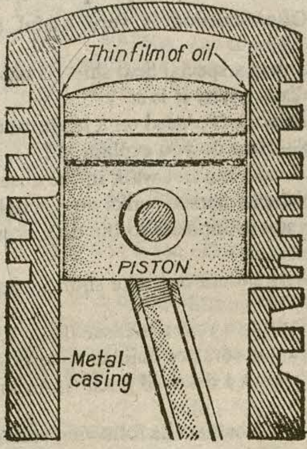


Fig. 4·12 : Lubricated piston

(*iii*) ***Using ball bearings or roller bearings*** : The fact that rolling friction is much less than sliding friction is employed. In ball bearings, a number of hard steel balls are placed loosely in a metal case round the shaft (Fig. 4·11). With the rotation of the shaft, the balls also roll in the groove, called a 'race'. There is thus little or no possibility of sliding.

In roller bearings, instead of balls, a number of metal rollers are placed, principle of operation remaining the same.

(iv) Using a Lubricant : Fluid friction, which has a much less value, is substituted for solid friction. The surfaces moving across each other are lubricated with oil so that a very thin film of oil exists between them [Fig. 4·12]. The oil is known as lubricant. Common lubricants are oils, grease, graphite and wax.

● EXERCISE ●

[A] Essay type questions

1. Name several types of mechanism in which friction is essential for proper operation. What becomes of the energy expended against friction ? 'The operation in a leath e machine is impossible unless lubricants are used'—Explain the statement.

2. Define (i) Sliding and rolling friction (ii) Static and kinetic friction. Deduce the equation of motion of a body on a rough surface.

3. State the laws of static friction. Define coefficient of static friction. Establish the relation between the angle of repose and the coefficient of friction. [*H. S.* '80]

4. What is friction ? Explain why the force of friction increases with increase in normal reaction. Define angle of friction and angle of repose. Establish the relation between them.

5. Write a short account of :—(i) Limiting friction (ii) Rolling friction. Write briefly the methods of minimising friction.

6. What is the difference between coefficient of static friction and coefficient of kinetic friction ?

A body is placed on a plane inclined at an angle θ with the horizontal. Show that the body can not remain at rest on the inclined plane if $\theta > \tan^{-1} u$ where u is the coefficient of static friction.

[B] Short answer type questions

1. 'Often friction is very useful but in many other cases friction is a disadvantage'.—Discuss.

2. Unless a special heat shielf is used in a space craft, the astronaut can never be returned back alive. Why it is so ?

3. 'A block is placed on a horizontal table and is pulled weakly by a string but it does not move'—How do you explain it ?

4. 'A man can not walk in the absence of friction'—Explain.

5. 'When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion'—Is the statement true ? Give reasons in brief.

6. Why should one take short steps rather than the long ones when walking on ice or on slippery roads ? [*I. I. T.* '81]

7. 'Pulling a roller is easer than pushing it.—Explain.

8. Racing cars are built in streamlined shape. Why ?

9. To shift a drum of pitch from one place to other, it is easier to roll it than pull it. Why ?

10. State whether the following statements are correct :

(i) If the weight of a body is kept constant, the limiting friction is independent of the area of surface in contact (ii) If the area of the surface in contact is kept constant, the limiting friction is independent of the weight of the body (iii) The direction of the frictional force is independent of the direction of motion of a body on a rough plane (iv) Normal reaction goes on increasing as the inclination of the plane increases.

11. 'Friction is a self-adjusting force'—Explain.

12. 'In the absence of friction, a nail can't be fixed on the wall of a room'.—Is the statement true ? Explain in brief.

13. A block is placed on an inclined plane whose inclination is very small. On gradually increasing the angle of inclination, a time comes when the block begins to slide downwards. After the sliding begins, the angle of inclination is slightly decreased ; yet the body is found to continue its motion. Give scientific reasons for the above observations.

14. When a car gets bogged in mud, it fails to advance though its wheels go on rotating. Why ?

15. Why wheels are used in carriages ?

16. While pulling a roller, a large force is required to start its motion but once it begins to roll, comparatively smaller force is found to be sufficient to maintain its motion. Explain giving reasons, in brief.

17. You are marooned on a frictionless horizontal plane and cannot exert any horizontal force by pushing against the surface. How would you get off by (i) jumping (ii) rolling your body or (iii) sneezing ? Explain you answer.

[C] Simple Problems

1. A boy pushes a box resting on a horizontal floor with a force of 5 kg. wt., but does not move it. What frictional force is called into play ?
[*Ans.* 5 kg. wt.]

2. A 50 kg. block rests on a horizontal floor. A minimum horizontal force of 22 kg. wt. is required to start it in motion and a minimum horizontal force of 18 kg. wt. keeps the block moving with uniform speed. Find (*a*) the co-efficient of static and kinetic friction, (*b*) the frictional force in the above cases and also when a force of 12 kg. wt. is applied on the block.
[*Ans.* (*a*) 0·44 ; 0·36 (*b*) 22, 18 and 12 kg. wt.]

3. What horizontal forces would be necessary to make a wooden box of weight 15 kg. begin to slide along (*a*) a wooden floor, (*b*) a stone floor ? Given, co-efficient of static friction of wood on wood=0·5, wood on stone=0·4.
[*Ans.* (*a*) 7·5 kg. wt. (*b*) 6 kg. wt.]

4. A block requires a force of 200 gm. wt. to start it moving on a horizontal floor. (*a*) What is the mass of the block ? (*b*) Find also the normal reaction between the two surfaces. Given, μ_s=0·25.
[*Ans.* (*a*) 800 gm. (*b*) 800 gm. wt.]

5. A block of mass 50 gm. can slide on a rough horizontal surface, the co-efficient of friction between the block and the surface being 0·6. Find the least force of pull which acting on the block at an angle of 30° to the upward drawn vertical, will cause the block to slide.
[*Ans.* 29·43 gm. wt.]

6. A 10 kg. body is pulled 5 m. along a floor at constant speed. If the co-efficient of kinetic friction is 0·2, calculate the horizontal force required and the work done.
[*Ans.* 2 kg. wt. ; 98 Joule]

7. A 50 kg. body rests on a rough horizontal floor. A horizontal force of 20 kg. wt. acts on the block for 5 sec. Calculate the velocity of the block at the end of 5 sec. The co-efficient of kinetic friction between the block and the floor is 0·25.
[*Ans.* 7·35 m./sec]

8. A stone pushed over the floor with the velocity 2 m./sec. slides over a distance of 5 m. before it stops. Find the co-efficient of friction between the stone and the floor. [*Ans.* 0·041]

9. A 200 kg. sledge is drawn along a rough horizontal floor by a 100 kg. wt. force making an angle 30° above the floor. How far will the sledge move from rest in 10 sec. Given μ_k=0·2.
[*Ans.* 138·67 m.]

10. A body weighing 100 gm. is pulled through 10 m. along a horizontal surface for which the co-efficient of kinetic friction is 0·45. The applied force is 50 gm. wt. directed 30° above the horizontal. Calculate its velocity at the end of the journey if it starts from rest.
[*Ans.* 4·33 m./sec.]

11. A body placed on a horizontal plank just starts to slide when the plank is tilted upwards to an inclination of 15°. It keeps moving down without acceleration, if the angle is then lowered to 12°. Find the co-efficients of static friction and kinetic friction from the above data.
[*Ans.* μ_s=0·27 ; μ_k=0·21]

12. What is the acceleration of a block sliding down a 30° incline, the co-efficient of friction being 0·2. [*Ans.* 320·3 cm./sec.^{2}]

13. What force parallel to an inclined plane of height 3 m. and base 4 m. can just support a block of mass 500 gm. on the plane ? The co-efficient of friction=0·25. [*Ans.* 200 gm. wt.]

14. A 50 kg. block rests on a rough plane inclined at 30° with the horizontal. Calculate the force parallel to the plane required to keep the block moving up the plane at constant speed. Given $\mu_k=0{\cdot}25$. [*Ans.* 35·82 kg. wt.]

15. If a force of 10 kg. wt. parallel to a plane of inclination 60° is required to keep a 10 kg. block moving up the plane with constant speed, find the co-efficient of kinetic friction. [*Ans.* 0·27]

16. A block weighing 100 kg. is placed on an inclined plane of height 6 m. and base 8 m. The co-efficient of friction is 0·3. (a) Would the block slide down the plane due to its own weight ? If so, how far it will move in 1 sec. starting from rest ? (b) What force parallel to the plane must be applied to just support the block on the plane ? (c) What force parallel to the inclined plane is required to keep the block moving up the plane at constant velocity ? (d) If an upward force of 94 kg. wt. parallel to the plane is applied to the block what will be its acceleration ? (e) How far will the block move in 1 sec. starting from rest ? (f) What will happen if an upward force of 50 kg. wt. parallel to the plane is applied ? (g) If an upward force of 26 kg. wt. parallel to the plane is applied what will happen ? (h) How far will the block move in 1 sec. starting from rest ?

[*Ans.* (a) Yes, 176·4 cm. (b) 36 kg. wt. (c) 84 kg. wt. (d) 98 cm./sec.^{2} upwards. (e) 49 cm. (f) remains at rest, (g) slides down with acceleration 98 cm./sec.^{2} (h) 49 cm.]

17. A body is placed on a plane inclined at 45° with the horizontal. Starting from rest, the body acquired a velocity of 2 m./sec. after sliding through a distance of 36·4 cm. Find the co-efficient of friction between the body and the plane. [*Ans.* 0·21]

18. A weightless pulley is attached to the edge of a horizontal table. A mass of 15 kg. is pulled along the table by a light inextensible string passing over the pulley and carrying another mass of 3 kg. at the other end. If the co-efficient of friction between the 15 kg. mass and the table be 0·1, find (a) acceleration of the system, (b) the tension of the string. Neglect the friction in the pulley. [*Ans.* (a) 81·67 cm./sec.^{2} (b) $2{\cdot}595\times10^6$ dyne]

19. A rope so lies on a table that a part of it hangs over. When the length of the hanging part is 25 per cent of the entire length, the rope begins to slide. Find the co-efficient of friction between the rope and the table. [*Ans.* 0·33]

20. A man holds a 2 kg. block from falling by pushing it horizontally against a vertical wall. If μ_s is 0·20, what force must he exert ? [*Ans.* 10 kg. wt.]

21. A man lifts a 16 kg. can of oil by pressing his two hands towards each other against the smooth sides. If the co-efficient of static friction is 0·25, what force must he apply with each hand ? [*Ans.* 64 kg. wt.]

22. The minimum force required to move a block up a rough plane inclined at a angle of 30° to the horizontal was found to be five times the minimum force needed to move the block down the plane. If the forces act parallel to the plane, find the co-efficient of friction between the plane and the block. [*Ans.* 0·866]

23. A man weighs 70 kg. What is the greatest weight he can pull along a horizontal floor by a horizontal rope if the co-efficient of friction between his shoe soles and the floor is 0·4 and that between the weight and the floor is 0·35 ? Calculate also the tension in the rope in this case. [*Ans.* 80 kg ; 274·4 N]

(**Hints :** Force of friction on the man causes him to move forward)

24. A block of mass 1 kg. lies on a horizontal surface in a truck. The co-efficient of static friction between the block and the surface is 0·6. If the acceleration of the truck is 5 m./sec.^{2}, find the frictional force acting on the block. [*Ans.* 5N]

[D] Harder Problems

1. The driver of a train applied brakes and the speed of the train changed uniformly from 47·5 km./hr. to 30 km./hr. during the time of 3·3 sec. At what highest value of the co-efficient of friction between a suit-case and a rack will the suit-case begin to slide over the rack during retardation.

[*Ans.* 0·15]

2. A tram-car moves with the acceleration of 49 cm./sec.^{2} Find the co-efficient of friction if 50 per cent of the motor power is spent to overcome the frictional forces and 50 per cent to increase the speed.

[*Ans.* 0·05]

3. A block is projected up a plane inclined at 30° to the horizontal. It returns to its starting point with $\frac{1}{3}$ of its initial velocity. Calculate the co-efficient of kinetic friction between the block and the plane.

[*Ans.* 0·4619]

4. A car weighs 1000 kg. Its driver begins to brake it 25 m. from an obstacle on the road. The force of friction of the brake shoes is constant and equal to 3840 newton. What is the maximum speed from which the car be stopped just before the obstacle ? Neglect the friction of the wheels against the road.

[*Ans.* 49·9 km./hr.]

5. A cart weighing 20 kg. can roll without friction on a horizontal path. A box weighing 2 kg. rests on the cart. The co-efficient of friction between the box and the cart is 0·25. A force of 200 gm. wt. is applied to the box in horizontal direction parallel to the path. Find (a) the acceleration of the box and that of the cart, (b) the force of friction between the box and the cart.

Also, solve the problem when the force applied is 2 kg. wt.

[***Ans.* 1st part** : (a) Acceleration of the system=8·9 cm./sec.^{2} (b) 181·8 gm. wt.

2nd part : (a) Acceleration of the box=7·35 m./sec.^{2}, of cart=0·24 m./sec.^{2}, (b) 0·5 kg. wt.]

6. A skater weighing 70 kg. stands on ice. He throws a stone weighing 3 kg. with a velocity of 8 m./sec. in a horizontal direction. Find the distance through which the skater moves back. Given, $\mu_k=0{\cdot}02$.

[*Ans.* 0·3 m.]

7. A tractor pulls a sledge on an icy road at a uniform speed of 15 km./hr. At what speed could the tractor pull the sledge over a concrete road if the power developed by the motor in both the cases be same. The co-efficient of friction between the sledge and the icy road=0·01 and that between the sledge and the concrete road=0·15.

[*Ans.* 1 km./hr.]

8. A trolly weighing 10^4 kg. stands on rails. A cannon weighing 5×10^3 kg. is fastened on it from which a shell is fired along the rails. The shell weighs 100 kg. and its initial velocity with respect to the cannon is 500 m./sec. Find the distance through which the trolley moves after the shot if (a) it is at rest, (b) it is moving with a velocity of 18 km./hr. and the shell is fired in the direction of its motion, (c) it is moving with a velocity of 18 km./hr. and the shell is fired in the opposite direction. The co-efficient of friction between the trolley and the rails is 0·002.

[*Ans.* (a) 283·4 m. (b) 73·75 (c) 1786 m.]

9. Two bodies *A* and *B* each weighing 1 kg. are linked by a thread which passes over a frictionless pulley as shown in the Fig. (a) The body *B* is placed on an inclined plane forming an angle of 30° with the horizon. Find (a) the acceleration with which the bodies move, (b) the tension in the thread. Neglect the weight of the pulley and the friction between the body *B* and inclined plane.

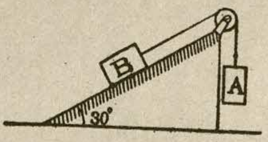


Fig (a) Problem 9

Also solve the problem if the co-efficient of friction between the body *B* and the inclined plane be 0·1. [*Ans.* **1st part** : (a) 2·45 m./sec.^{2} (b) 7·35 newton. **2nd part** : (a) 2·026 m./sec.^{2} (b) 7·774 newton]

10. Two inclined planes forming angles 30° and 45° with the horizon are hinged together as shown in Fig. (b). A weightless pulley is attached to the hinge. Two bodies *A* and *B* each weighing 1 kg. are linked together by a rope passing over the pulley. Find (a) the acceleration of the system, (b) the tension of the rope. Neglect the friction in the pulley and the friction between the bodies *A* and *B* and the inclined planes.

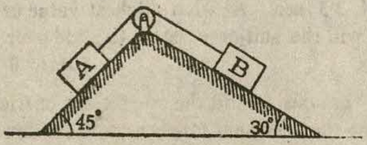


Fig. (b) Problem 10

Solve the problem if the co-efficients of friction between the bodies *A* and *B* and the inclined planes are $\mu_1=\mu_2=0{\cdot}1$.

[*Ans.* 1st. part : (a) 1·015 m./sec.^{2} (b) 5·9 newton ; 2nd part ; (a) 0·244 m./sec.^{2} (b) 5·99 newton]

11. A weightless pulley is attached at the edge of a horizontal table. A load is suspended from a light inextensible thread which passes over the pulley and is attached to a block, whose mass is half that of the load, resting on the table. The load is released from rest and then stopped after it has fallen 180 cm. The block comes to rest after sliding a total distance of 300 cm. Find the co-efficient of kinetic friction between the block and the table.

[*Ans.* $\mu_k=0{\cdot}67$]

12. A hollow sphere has an internal radius of 1 m. At what maximum height from the bottom of the hollow sphere can a particle be supported by friction if the co-efficient of friction is $1/\sqrt{3}$? [*Ans.* 13·4 cm.]

13. Two equal weights tied by a string are placed on two inclined planes equally rough. The planes have equal altitude and are placed back to back with inclination to the horizontal of 60° and 30° respectively. Show that the weights are on the point of sliding when $\mu=2-\sqrt{3}$.

14. A block is laid gently on a horizontal conveyor belt moving at a speed of 5·5 m./sec. If the co-efficient of friction between the block and the belt is 0·45, find how far the block will move on the belt before coming to rest on it ? [*Ans.* 3·43 m.]

15. A body is projected with a speed of 10 m./sec. up a plane inclined at $\tan^{-1}\frac{12}{5}$ to the horizontal. The co-efficient of friction between the body and the plane is 0·45. Find (a) the maximum distance travelled up the plane (b) velocity on reaching the point of projection and (c) time taken to reach the point of projection. [*Ans.* (a) 4·65 m. (b) 8·27 m./sec. (c) 2·05 sec]

16. A block *P* of mass 3 kg. rests on another block *Q* of mass 7 kg. The co-efficient of friction between the blocks *P* and *Q* is 0·5. The block *Q* rests on a smooth (frictionless) horizontal table.

(a) What is the maximum force that can be applied to block *P* so that both *P* and *Q* move without any relative motion ?

(b) What is the maximum force that can be applied to the block *Q* so that both *P* and *Q* move without any relative motion ?

(c) What is the force of friction between *P* and *Q* in case (a) ?

(d) What is the force of friction between *P* and *Q* in case (b) ?

[*Ans.* (a) 14·7 N (b) 49·0 N (c) 10·29 N (d) 14·7 N]

5
CHAPTER

DYNAMICS OF ROTATIONAL MOTION

5·1. Rotational motion of a particle.

As stated in article (2·3), the motion of a body may be translational, rotational or a combination of both. So far we have considered only translational motion with constant acceleration along a straight line. But in nature, rotational motion is of much more common occurence than the translational one. The motion of the earth and of the planets round the sun and also about their own axes, the motion of electrons round the nucleus of an atom and the rotation of machine wheels etc. provide a few examples of rotational motion.

When a rigid body is set into rotation, its motion is generally described with respect to the axis about which it rotates. This axis of rotation may pass through the body or may lie outside. In case of a machine wheel, for example, the axis of rotation, which is normal to the plane of rotation, passes through the geometrical centre. For a stone whirling at the end of a string, the axis is at the other end of the string far from stone itself.

This axis of rotation itself may be either fixed in space or not. Consideration of the rotatory motion of a body about an axis which is itself not fixed is beyond the scope of this text. For a body rotating about a fixed axis, the particles of the body, except those lying on the axis, rotate in concentric circles around the axis and in planes perpendicular to it. The particles situated on the axis, however, remain stationary. Hence such a motion about a fixed axis is also called *circular motion*. The description of the rotational motion of a rigid body about a fixed axis can thus be given by considering the circular motion of one of its constituent particles and then summing over all such particles. Our discussion will thus remain confined to the motion of a particle round a circle.

5·2. Angular displacement.

Let us consider a particle undergoing circular motion in the plane of the paper, O being the centre of the described circle. Hence, the axis of rotation will be a line passing through O and perpendicular to the plane of the Fig. 5·1. The position P of the particle at any instant t will be completely specified by the angle θ made by the radius OP with some direction OX fixed in space. OX is called *the reference line*, and the angle θ, the *angular displacement* of the particle, time being counted from the instant at which the particle crosses the reference line;

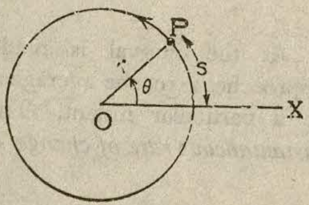


Fig. 5·1 Angular displacement

Thus the **angular displacement** *of a particle undergoing circular motion, during a certain interval of time, is defined as the angle swept out by the radius during that interval.*

It is generally considered to be positive when the rotation is counter clockwise, and negative when clockwise. This displacement θ in circular motion corresponds to the displacement s in linear motion.

Angular displacement is generally expressed in degress, in revolutions (1 rev. =360°) or in radians (rad). In general, all angles are expressed in radian because this being a pure number leads to the simplification of the equations of motion. The relations among the different measures of angle are

$$2\pi \text{ rad.} = 1 \text{ rev.} = 360^\circ$$

$$1 \text{ rad.} = 57{\cdot}3^\circ \text{ (approx.)}$$

Expressed in radian measure, very simple relation exists between the angle θ and the length of the arc s traversed by the particle, viz.

$$\theta = \frac{s}{r} \quad \ldots \quad \ldots \quad (5{\cdot}1)$$

where r is the radius of the circle.

Example 5·1. *The bob of a pendulum of length 90 cm. swings through a 15 cm. arc. Find the angle θ, in radians and degrees, through which it swings.*

Solution :

$$\theta = \frac{s \text{ (length of arc)}}{r \text{ (radius)}} = \frac{15}{90} = \frac{1}{6} \text{ radian.}$$

$$\theta = \frac{1}{6} \times \frac{180}{\pi} = \frac{30}{3{\cdot}14} = 9{\cdot}55^\circ.$$

5·3. Angular velocity.

The angular motion of a particle may be uniform or accelerated, similar to the case of linear motion. For non-uniform angular motion, the **average angular velocity** of a particle, denoted by $\bar{\omega}$ *is defined as the ratio of the angular displacement to the elapsed time.* The sign convention is similar to that adopted for angular displacement.

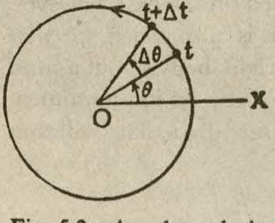


Fig. 5 2 : Angular velocity

If the particle describes an angle $\Delta\theta$ in a small interval of time Δt (Fig. 5·2), the average angular velocity during this small interval is given by

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \quad \ldots \quad \ldots \quad (5{\cdot}2)$$

As the interval is made progressively smaller, then in the limit when Δt approaches zero, the average angular velocity over the interval equals the value at a particular instant. Thus the **instantaneous angular velocity** ω *is the instantaneous rate of change of angular displacement* and is given by

$$\omega = \underset{\Delta t \to 0}{Lt} \; \frac{\Delta\theta}{\Delta t}$$

In calculus notation, this is written as

$$\omega=\frac{d\theta}{dt} \qquad \cdots \qquad \cdots \qquad (5\cdot3)$$

From now on, unless otherwise mentioned, the term angular velocity will mean the instantaneous angular velocity.

If the angular velocity remains constant, the particle is said to be executing uniform circular motion. For such uniform motion, if the angle described be θ in time t, the angular velocity ω is given by

$$\omega=\frac{\theta}{t}$$

$$\text{or,} \quad \theta=\omega t \qquad \cdots \qquad \cdots \qquad (5\cdot4)$$

This equation is analogous to the equation

$$s=vt$$

for uniform linear motion of a body.

For a rigid body, any line drawn perpendicular from any of its particles to the axis of rotation, rotates through the same angle in the same time. Thus *the angular velocity is the same for all the particles* and is therefore a characteristic of the body as a whole.

The angular velocity is generally expressed in *radian per second*; however, in engineering, a common practice is to express it in another unit, called the *revolution per minute* (*rpm.*).

Let T be the time in second for one complete revolution, called the *time period.* Then since in one complete revolution the angle described is 2π radian, we have from equation (5·4),

$$\omega=\frac{2\pi}{T} \qquad \cdots \qquad \cdots \qquad (5\cdot5)$$

Now, $\frac{1}{T}$ gives the number of revolutions per second, which is called the ***frequency***, and is denoted by the letter 'n'

$$\therefore \quad \omega=2\pi n \qquad \cdots \qquad \cdots \qquad (5\cdot6)$$

If N be the number of revolutions per minute, then evidently $\frac{1}{T}=\frac{N}{60}$

From eqn. 5·5,

$$\omega=2\pi.\frac{N}{60}=\frac{N\pi}{30} \qquad \cdots \qquad \cdots \qquad (5\cdot7)$$

Example 5·2. *A body executes 300 revolutions in 6 sec. Find its* (*i*) *frequency* (*ii*) *angular speed,* (*iii*) *time period and* (*iv*) *time required to turn through 60°.*

Solution : (i) Frequency, $n=\frac{300}{6}=50$ per sec.

(ii) angular speed, $\omega=2\pi n=2\times3\cdot14\times50=100\pi$ rad./sec.

(iii) time period, $T=\frac{1}{n}=\frac{1}{50}$ sec.

(iv) $\because$ $180°=\pi$ radian, $\therefore$ $60°=\frac{\pi}{3}$ rad.

Hence, the time required to turn through $60°=\frac{\pi/3}{100\pi}=\frac{1}{300}$ sec.

Example 5·3. *Calculate the angular speed of the earth around the sun assuming the orbit to be circular. Given, 1 year=365 days.*

Solution : $T=1$ yr. $=365\times24\times60\times60$ sec.

$$\omega=\frac{2\pi}{T}=\frac{2\times3{\cdot}14}{365\times24\times60\times60}=1{\cdot}99\times10^{-7}\text{ rad./sec.}$$

5·4. Angular acceleration.

Often a particle in circular motion speeds up or slows down ; its angular velocity thus changes. When a particle suffers a change in angular velocity, it is said to have an angular acceleration.

The **average angular accleration** *of a particle in circular motion during a particular interval is defined as the rate of change of its angular velocity over that interval.*

Thus for a small interval of time $\triangle t$, if the change in angular velocity be $\triangle\omega$, the average angular acceleration is given by

$$\overline{\alpha}=\frac{\triangle\omega}{\triangle t} \qquad \cdots \qquad \cdots \qquad (5{\cdot}8)$$

In the limit, **instantaneous angular acceleration** is given by

$$\alpha=\underset{\triangle t\to0}{Lt}\ \frac{\triangle\omega}{\triangle t}$$

In calculas notation, $\alpha=\dfrac{d\omega}{dt}$ $\qquad \cdots \qquad \cdots \qquad (5{\cdot}9)$

We have, from eqn. (5·3), $\omega=\dfrac{d\theta}{dt}$

$$\therefore\quad \alpha=\frac{d}{dt}\left(\frac{d\theta}{dt}\right)=\frac{d^2\theta}{dt^2} \qquad \cdots \qquad \cdots \qquad (5{\cdot}10)$$

Henceforth, unless otherwise mentioned, the term angular acceleration will imply the instantaneous value.

The unit of angular acceleration is *radian per second per second* or simply *rad/sec.*^{2}

For a particle rotating with a constant angular acceleration, its instantaneous angular acceleration becomes equal to the average value computed over any interval. In this case, we have the simple relation $\alpha=\dfrac{\omega}{t}$.

5·5. Angular velocity and angular acceleration as vectors.

Like linear velocity and linear acceleration of a particle, the angular velocity and angular acceleration are also vector quantities. Both possess magnitude as well as direction.

The vector representing angular velocity (or acceleration) is drawn along the axis of rotation. Its length, in some arbitrary chosen scale, represents the magnitude of the angular velocity (for acceleration). The sense of the vector is that in which a right handed screw would advance when turned in the direction of the rotation of the particle [Fig. 5·3].

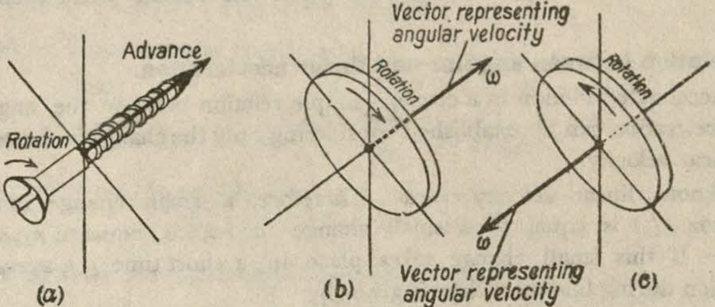

Advance
Rotation
(a)
Vector representing angular velocity
Rotation
ω
(b)
Rotation
ω
Vector representing angular velocity
(c)

Fig. 5 3 : Vector representation of angular velocity and angular acceleration

5·6. Relation between angular and linear velocity.

For a particle executing circular motion, a very simple relationship exists between its angular velocity and linear velocity. For simplicity, we shall first assume that the particle is in uniform circular motion round a point O [Fig. 5·4]. Let the particle describe an arc of length s in time t, its angular displacement, measured from the reference line OX, being θ. Then if r be the distance of the particle from the centre of rotation,

$$s = r\theta$$

where θ is expressed in radians.

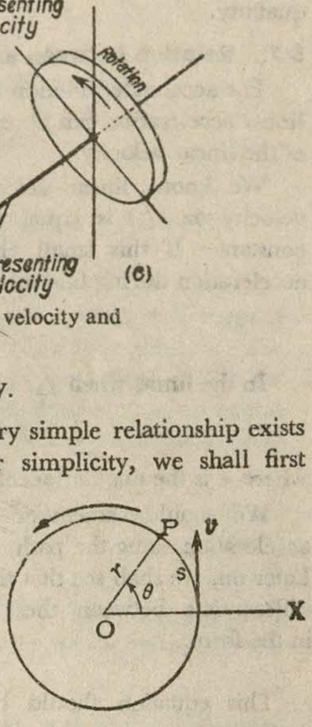


Fig. 5·4

Hence the linear velocity is given by, $v = \frac{s}{t} = \frac{r\theta}{t}$

But $\frac{\theta}{t}$ is the angular velocity ω.

$$\therefore \quad v = \frac{r\theta}{t} = r\omega \quad \ldots \quad \ldots \quad (5\cdot11)$$

For non-uniform motion in a circle, the above relation still holds good for the instantaneous values of the corresponding quantities. To show this, calculus method must be employed. Thus from equation (2·5),

$$v = \frac{ds}{dt}$$

Since $s = r\theta \quad \therefore \quad v = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt}$ [$\because$ r remains constant]

$= r\omega$ from the equation (5·3)

As discussed in the article (2·6), the linear velocity is always directed along the tangent to the path. To bring out this point sharply, equation (5·11) is also written as

$$v_T = r\omega \quad \ldots \quad \ldots \quad (5\cdot12)$$

In passing, we should note that the equation (5·12) is in effect a vector equation. The three quantities involved are all vector quantities. The equation then provides an example of the product of the vectors being itself a vector quantity.

5·7. Relation between angular and linear acceleration.

For accelerated motion in a circle, a simple relation between the angular and linear acceleration can be established considering only the change in the magnitude of the linear velocity.

We know, linear velocity $v=r\omega$. Therefore, a small change in the linear velocity viz., Δv is equal to a small change in $r\omega$, i.e., equal to $r\Delta\omega$, r being constant. If this small change takes place in a short time Δt, average linear acceleration during this short time is given by,

$$\overline{f}=\frac{\Delta v}{\Delta t}=r\frac{\Delta\omega}{\Delta t}$$

In the limit, when Δt is extremely small

$$f=r\frac{d\omega}{dt}=r\alpha \quad \ldots \quad \ldots \quad (5\cdot13)$$

where α is the angular acceleration.

We should remember that this equation gives the magnitude of the linear acceleration along the path, i. e., the tangential component of the acceleration. Later on, we shall see that there is an acceleration directed normal to the path. To differentiate between these two accelerations, the above equation is often written in the form.

$$f_T=r\alpha \quad \ldots \quad \ldots \quad (5\cdot14)$$

This equation should be compared with equations (5·1) and (5·12). The similarity is more conspicuous when the equations are written together :

$$\left.\begin{aligned} s&=r\theta \\ v_T&=r\omega \\ f_T&=r\alpha \end{aligned}\right\} \quad \ldots \quad \ldots \quad (5\cdot15)$$

Example 5·4. *Calculate the angular speed of a car which rounds a curve of radius 8 m. at 50 km./hr.*

Solution : Angular speed, $\omega=\frac{v}{r}$

where, linear speed, $v=\frac{50\times1000}{60\times60}$ m./sec. and radius, $r=8$ m.

$$\therefore \quad \omega=\frac{50\times1000}{60\times60\times8}=1\cdot74 \text{ rad./sec. (approx).}$$

Example 5·5. *The moon is $3\cdot84\times10^5$ km. from the earth and makes one revolution in 27·3 days. Find its angular and linear speed in its orbit. Assume the orbit to be circular.*

Solution : $T=27\cdot3$ days$=27\cdot3\times24\times60\times60$ sec.

$$\omega=\frac{2\pi}{T}=\frac{2\times3\cdot14}{27\cdot3\times24\times60\times60}=2\cdot661\times10^{-6} \text{ rad./sec.}$$

$$v_T=r\omega=3\cdot84\times10^5\times2\cdot661\times10^{-6}=1\cdot022 \text{ km./sec.}$$

Example 5·6. *A flywheel starts from rest and speeds up uniformly to 1200 rpm in 20 sec. Calculate the angular acceleration in rad./sec.^{2} and linear acceleration in m./sec.^{2}, of a point 75 cm. from the axis.*

Solution : Initial angular speed, $\omega_0=0$.

Final ,, ,, $\omega=\frac{1200}{60}\times 2\pi=40\pi$ rad./sec.

$\therefore$ Angular acceleration, $\alpha=\dfrac{\omega-\omega_0}{t}=\dfrac{40\pi}{20}=2\pi$ rad./sec.^{2}

and linear acceleration, $f=r\alpha=75\times 2\pi$

$=150\pi$ cm/sec.$^2=1{\cdot}5\pi$ m./sec.^{2}

5·8. Rotation with constant angular acceleration.

The similarity of the expressions which define angular velocity and angular acceleration with those of linear velocity and linear acceleration is obvious.' For uniformly accelerated angular motion, proceeding as in the article (2·10) for uniformly accelerated linear motion, we can establish the following relationships connecting the angular displacement θ, the angular velocity ω and the angular acceleration α. The relations are

$$\omega=\omega_0+\alpha t \quad \ldots \quad \ldots \quad (5{\cdot}16)$$

$$\theta=\omega_0 t+\tfrac{1}{2}\alpha t^2 \quad \ldots \quad \ldots \quad (5{\cdot}17)$$

$$\omega^2=\omega_0^2+2\alpha\theta \quad \ldots \quad \ldots \quad (5{\cdot}18)$$

where, ω_0 is the initial angular velocity, ω is the final angular velocity, i.e., the angular velocity after a time t, α is the constant angular acceleration, and θ is the angular displacement during the time t.

Example 5·7. *A body revolving with a constant angular acceleration 3 rad./sec.^{2} possesses an angular velocity 6 rad./sec. at an instant $t=0$. A line in the body is horizontal at the instant $t=0$. (i) Find the angle that line makes with the horizontal line at the instant $t=4$ sec. (ii) Also find the angular speed at the said instant.*

Solution : Here $\alpha=3$ rad./sec^2, $\omega_0=6$ rad./sec., $t=4$ sec.

(i) $\theta=\omega_0 t+\frac{1}{2}\times\alpha t^2=6\times 4+\frac{1}{2}\times 3\times 4^2$

$=48$ radian$=\dfrac{48}{2\pi}$rev.$=7{\cdot}64$ rev.

(ii) $\omega=\omega_0+\alpha t=6+3\times 4=18$ rad./sec.

Alternatively, $\omega^2=\omega_0^2+2\alpha\theta=6^2+2\times 3\times 48=324$ rad.^{2}/sec^2.

$\omega=18$ rad./sec.

Example 5·8. *A wheel rotating at 400 rpm increases its speed to 600 rpm in 4 sec. Calculate (a) the angular acceleration of the wheel and (b) the number of revolutions it makes in this time.*

Solution : $\omega_0=400$ rpm$=\dfrac{400\times 2\pi}{60}$ rad./sec.

$\omega=600$ rpm$=\dfrac{600\times 2\pi}{60}$ rad./sec.

$$\therefore \quad \alpha=\frac{\omega-\omega_0}{t}=\frac{200\times 2\pi}{60\times 4}=\frac{5\pi}{3}\text{ rad./sec.}^2$$

$$\theta=\frac{\omega^2-\omega_0^2}{2\alpha}=\frac{(\omega+\omega_0)\,(\omega-\omega_0)}{2\alpha}=\frac{1000\times 2\pi}{60}\times\frac{200\times 2\pi}{60}\times\frac{3}{2\times 5\pi}$$

$$=\frac{200\pi}{3}\text{ rad.}$$

$$\therefore \quad \text{No. of revolutions}=\frac{\theta}{2\pi}=\frac{200\pi}{3\times 2\pi}=33{\cdot}3$$

5·9. Uniform Circular Motion : Centripetal Acceleration.

Let us consider a particle moving in a circle with a uniform speed v. Its velocity, however, is not uniform because though the magnitude of the velocity remains the same, the direction being always tangential to the path, changes continually. The particle thus possesses an acceleration. This acceleration gives rise to a change in the direction of motion but not in speed. The direction of this acceleration should, therefore, always be normal to the path of the particle, i.e., towards the centre of the circular path. Otherwise the presence of the acceleration component along the direction of motion would signify a change in speed. This acceleration is known as **centripetal acceleration** (Lat., *peto.*, to seek).

So we can formally define that

A particle, moving in a circular path with uniform speed, possesses an acceleration which is always directed towards the normal to the path of the particle i.e., towards the centre of the circle in which the particle moves. This acceleration is termed as the centripetal acceleration.

At any instant, let the particle be at some point P of the circular path. Its velocity is along the tangent to the path at P and is represented by the vector $\mathbf{v}_1$ [Fig. 5·5(a)]. After a small interval Δt, the particle has moved to Q and its velocity vector is $\mathbf{v}_2$. Since the particle is moving with a uniform speed v, the magnitudes (i.e., lengths) of both $\mathbf{v}_1$ and $\mathbf{v}_2$ are the same being equal to v. But the directions of $\mathbf{v}_1$ and $\mathbf{v}_2$ are slightly different.

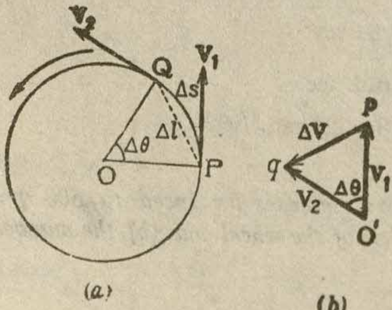


Fig. 5·5

The consequent change in the velocity $\Delta\mathbf{v}$ is found by the triangle method of finding the difference of two vectors [Fig. 5·5 (b)]. The vectors representing the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ are drawn from a common point O' and the triangle is completed.

From the triangle $O'pq$, $\mathbf{v}_1+\Delta\mathbf{v}=\mathbf{v}_2$

Hence, $\Delta\mathbf{v}=\mathbf{v}_2-\mathbf{v}_1$

Since, the veloctity is always tangent to the circle and hence perpendicular to the radius connecting the particle to the centre of the circle, the angle $\Delta\theta$ between the two radii in Fig. 5·5 (a) is the same as the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ in

Fig. 5·5 (*b*). The isosceles triangles *OPQ* and *O′pq* are therefore similar. Hence considering the magnitudes,

$$\frac{pq}{O'p}=\frac{\text{chord } PQ}{PO}$$

or, $$\frac{\Delta v}{v}=\frac{\Delta l}{r}$$

The average acceleration is therefore given by

$$\bar{a}=\frac{\Delta v}{\Delta t}=\frac{v\Delta l}{r\Delta t} \qquad \dots \qquad \dots \qquad (5\cdot19)$$

In the limit when Δt is vanishingly small, the chord Δl joining P and Q becomes indistinguishable from Δs, the arc of the circle between these two points. The magnitude of the instantaneous acceleration at P is then given by

$$a=\underset{\Delta t\to 0}{Lt}\frac{\Delta v}{\Delta t}=\frac{v}{r}\underset{\Delta t\to 0}{Lt}\frac{\Delta s}{\Delta t}$$

But $$\underset{\Delta t\to 0}{Lt}\frac{\Delta s}{\Delta t}=v$$

Hence, $$a=\frac{v^2}{r} \qquad \dots \qquad \dots \qquad (5\cdot20)$$

Since $v=\omega r$, where ω is the angular velocity of the particle,

$$a=\omega^2 r \qquad \dots \qquad \dots \qquad (5\cdot21)$$

Since acceleration is obtained by dividing the change in velocity by the time interval, a scalar, the direction of acceleration must be the same as $\Delta\mathbf{v}$. But as the time interval is made infinitesimally small, in the Fig. 5·5(*b*), $\Delta\theta$ approaches zero and $\Delta\mathbf{v}$ becomes more nearly perpendicular to $\mathbf{v}$. Therefore in the limit as $\Delta\theta\to 0$, $\Delta\mathbf{v}$ is directed towards the centre of the circle. Hence the acceleration is also directed towards the centre of the circle.

Thus as a particle moves in a circular path, it is always subjected to an acceleration at right angles to its velocity i.e., the acceleration is directed towards the centre of the circle. The equation (5·20) shows that this centripetal acceleration increases as the speed is increased and for a given speed, it is greater for a shorter radius.

5·10. Centripetal force.

According to Newton's first law of motion, a moving body, because of its inertia, should exhibit its natural tendency to move in a straight line with uniform velocity. This means that the direction of motion cannot change unless a force is impressed on the body. Thus, it is obvious that for a body undergoing circular motion in which its direction of motion continuously changes, some force must be acting on it.

In the last article, we have seen that when a body of mass m moves in a circle of radius r with a velocity v, it is always subjected to an acceleration $\frac{v^2}{r}$ directed towards the centre. From Newton's 2nd law, a force must be acting on the body to produce this acceleration. This force is obviously directed towards the centre and is equal to the product of mass and acceleration. i.e., equal to $\frac{mv^2}{r}$. If the force ceases to act, the body flies off in a direction tangential to its path and moves with uniform velocity along a straight line.

The force which keeps a body moving with uniform speed in a circular path being always directed inwards along the normal to the path i.e., towards the centre of the circle is called the **centripetal force**.

Now, since the centripetal force acts at right-angles to the direction of motion, there is no displacement in the direction of the centripetal force ; the centripetal force, therefore, does not perform any work. *Centripetal force is thus a 'no-work force'* [vide Art. 7·1].

It should be borne in mind that when an object is seen to move in a circular path, a centripetal force is always exerted on the object by an external agency. The rotating object, in turn, exerts an equal and opposite reaction force on the external agent. The direction of this force is thus outwards, away from the centre of the circle and is called the **centrifugal reaction** (Lat., *fugo*, to fly).

Let us consider the very common example of a stone tied to a string being whirled round a circular path [Fig. 5·6]. The stone is always acted upon by a centripetal force $\mathbf{F_c}$ which is supplied by the tension in the string. If the string suddenly snaps, $\mathbf{F_c}$ ceases to act and the stone flies off in a direction tangential to the circle with uniform velocity. As the stone rotates, it applies an equal and opposite centrifugal reaction, $\mathbf{F_R}$, which is felt by the man as an outward pull on his hand. Like any other action-reaction pair, $\mathbf{F_c}$ and $\mathbf{F_R}$ do not act on the same body, but on two different ones—here action on the stone and reaction on the hand respectively. Both the forces disappear simultaneously when the string snaps.

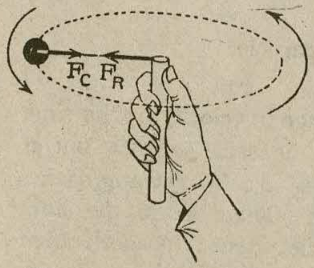


Fig. 5·6 : Centripetal force and centrifugal reaction

The centripetal force may be provided by various other means. The planets in the solar system revolve round the sun. The centripetal force is then due to the gravitational attraction of the sun on the planets. For electrons rotating round the nucleus inside an atom, electrical force of attraction between the nucleus and the electrons provides the required centripetal force. When a car takes a bend on a level road, the centripetal force is provided by friction between the tires and the road.

5·11. Centrifugal force (as a pseudo-force).

We have seen in the last article that every body rotating in a circular path is always acted on by a centripetal force directed towards the centre of the circular path. Take, for instance, the case of the earth rotating round the sun. Here the attraction of the sun provides the necessary centripetal force. The question naturally arises—"What is it that counter-balances this centripetal force ? What is it that prevents the earth from going straight to the sun ?" *Apparently* there is another force acting on the earth away from the sun, equal and opposite to the attraction of the sun. This *apparently existent* force is called centrifugal force. The centrifugal force is thus equal and opposite to the centripetal force. It must, however, be clearly understood that the existence of the centrifugal force is only apparent. The centripetal force is not actually counter-balanced by any real force.

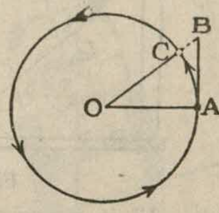


Fig. 5·7

A body rotating in a circular path tends to fly off tangentially, as is obvious from the fact that as soon as the string whirling a stone snaps, the stone flies off in a tangential direction. Let us suppose that at any instant a body in its circular path is at some point A [Fig. 5·7]. If there were no centripetal force towards the centre O, after a short time the body would have gone to B along the tangent. But due to the force acting on it towards O, it moves somewhat towards O, so that its final position is at C on the circular path. This occurs at every instant and the body is constrained to move in a circular path. The effect of the centripetal force is thus always present. This force is not counter-balanced by any other real force. The centrifugal force is thus only *apparent*. It is therefore called a pseudo-force (pseudo means fictitious).

We may now define the **centrifugal force** *as one which is equal and opposite to the centripetal force, acting radially outwards on a body rotating in a circle.* This force should not be confused with the centrifugal reaction discussed earlier. Centrifugal force is not the reaction to the centripetal force as action and reaction cannot act on the same body. Its magnitude is equal to that of the centripetal force i.e., mv^2/r or $mr\omega^2$.

5·12. Demonstration Experiments.

(1) A lighted candle protected from air currents by a lamp chimney is placed on a revolving table at some distance from the axis. The flame will be found to point towards the centre of the table [Fig. 5·8].

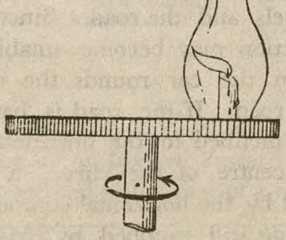


Fig. 5·8

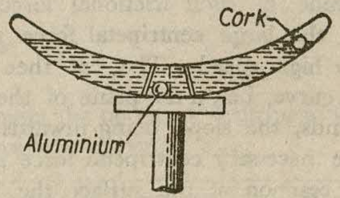


Fig. 5·9

(2) A bent glass tube is partly filled up with water. Two balls, one of aluminium and the other of cork, are placed inside it and the tube is sealed. The cork floats in one arm while the aluminium rests at the bottom. [Fig. 5·9]. But when the tube is revolved, the aluminium is thrown to the outer end of one arm, while the cork floats on the under side of the water in the other arm.

(3) A long piece of tin or aluminium is taken and is then bent to form a track of the shape shown in the Fig. 5·10. A ball is rolled down the track from one side. If it possesses sufficient velocity, it will be found to describe the complete path around the loop without falling down. The centrifugal force balances its weight. The ball loops the loop. An analogous simple experiment is to rotate a glass or a bucket of water in a vertical plane without spilling it.

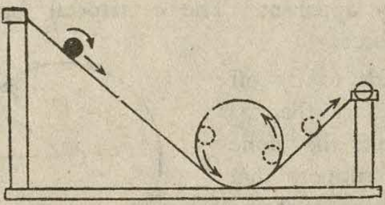
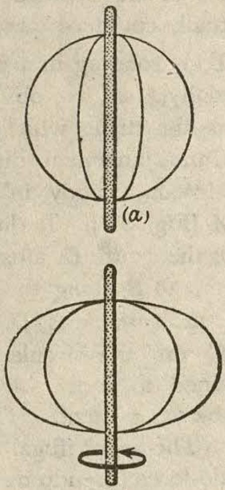


Fig. 5·10

(4) To the lower part of a rod four thin metallic strips are fixed. The other ends of the strips are fixed to a collar which can slide up and down the rod [Fig. 5·11(*a*)].



(*a*)

(*b*)

Fig. 5·11

As the rod is rotated rapidly each particle of the strips experiences a centrifugal force. As a result, the collar slides down and the strips get deformed, bulging out in the middle [Fig. 5·11(*b*)]. This experiment demonstrates why the earth is not a perfect sphere, but an oblate spheroid, i.e., a slight flattened sphere. This has been discussed in detail later on.

5·13. Practical illustrations.

(1) **Banking of roads** : Let us consider a car moving round a bend in the road. On a level road, the centripetal force necessary for effecting the turn is supplied only through friction between the wheels and the road. Since the magnitude of such frictional force is small, friction may become unable to supply the large centripetal force necessary when the car rounds the curve with a high speed. The car then skids off the road. If the road is banked at the curve, i.e., if the plane of the road is kept inclined to the horizontal at the bends, the slope being inwards towards the centre of the curve, a part of the necessary centripetal force is then supplied by the horizontal component of the reaction of the surface, the remaining being still supplied by friction. For a proper angle of banking, this horizontal component of the reaction provides the necessary centripetal force ; frictional force is then no longer necessary.

The forces acting in that case is the reaction R of the road acting normal to the surface and the weight W of the car directed vertically downwards [Fig. 5·12]. If θ is the angle of inclination of the road bed with the horizontal, called the *angle of banking*, then the horizontal component of R i.e., $R\sin\theta$ supplies the necessary centripetal force, while its vertical component $R\cos\theta$ balances the weight. If r be the radius of curvature of the track at that point and m be the mass of the car having a speed v,

$$R\sin\theta=\frac{mv^2}{r}$$

$$R\cos\theta=W=mg$$

Dividing, $$\tan\theta=\frac{v^2}{rg} \quad \ldots \quad \ldots \quad (5{\cdot}22)$$

which gives the proper angle of banking. Since this depends upon v, a road can be ideally banked only for one speed. At any higher speed the force of friction must be depended upon to prevent skidding.

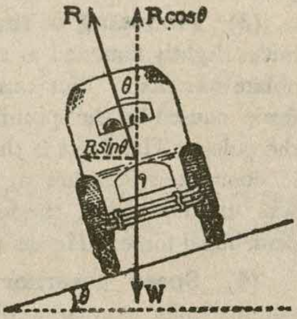


Fig. 5·12. : Banking of roads

All the modern highways are now banked at the curves to prevent the cars from skidding out of the road. The railway tracks are also banked at the bends, the outer rail being kept at a greater elevation than the inner one. The ideal speed proper to the banking is always specified and the drivers are always warned not to exceed this speed limit to minimise the chance of accident.

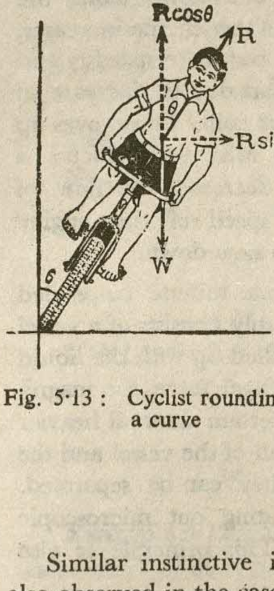


Fig. 5·13 : Cyclist rounding a curve

The case of a cyclist rounding a curve may also be similarly treated. While taking the turn, the cyclist always leans inwards instinctively (Fig. 5·13). The cyclist thus presses the ground obliquely and the reaction of the ground is, therefore, inclined to the horizon. The horizontal component of the reaction provides the necessary centripetal force. Proceeding as above, the proper angle through which he must lean inwards from the vertical in order to keep his balance, will be seen to be given by

$$\tan\theta=\frac{v^2}{rg}\,*$$

Similar instinctive inward leaning to obtain the necessary centripetal force is also observed in the case of a man running around a curve.

---

* As a cyclist leans inwards while rounding a curve, he is prevented from slipping by the friction of the road. If the coefficient of friction between the tyre and the ground be μ, then the maximum possible value of $R\sin\theta$ is μmg. If the cyclist leans inwards further, he will slip. So, for no slipping

$$\mu mg > R\sin\theta, \quad \text{i.e. } \mu mg > \frac{mv^2}{r}$$

$$\text{or, } \mu > \frac{v^2}{rg} \qquad \text{or, } v^2 < \mu rg$$

(2) **Motion of the planets** : As the planets describe orbital motion around the sun, the centripetal force necessary to keep them in motion is supplied by the gravitational force of attraction exerted by the sun on the planets.

(3) **Flattening of the earth** : The earth, we know, is not a perfect sphere, but is slightly flattened at the poles, bulging out at the equator. The earth is an oblate spheriod. This can be explained as due to the action of the centrifugal force caused by the spinning motion of the earth around its axis passing through the poles. The force is thus maximum at the equator and decreasing gradually, it completely vanishes at the poles. Since at the time of formation, the earth was in a fluid state, the force of cohesion was small and could not balance the centrifugal force. Hence the earth bulged out at the equator.

(4) **Speed governor** : Invented by James Watt, this device is used for automatically regulating the speed of an engine. It consists of a spindle *S* which rotates with the shaft of the flywheel of the engine [Fig. 5·14]. Two heavy metal balls *M*, *M* connected to two arms, hinged at the top of the spindle. Two metal strips *L*, *L* called links, connect the arms with a collar *C* which can move up and down along the spindle. As the speed of the engine increases, the spindle rotates more and more speedily and the balls move further apart due to increase in the centrifugal force. The collar thus moves up and, in turn, operates a valve with which it is connected. The valve decreases the flow of steam and hence the speed of the engine diminishes. The reverse occurs when the engine begins to slow down.

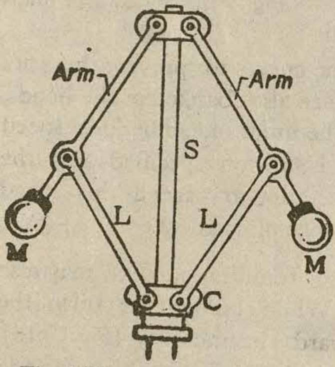


Fig. 5·14 : Speed governor

(5) **Centrifuge** : This instrument is used to separate minute suspended particles of different densities in a liquid. In essence, it simply consists of a vessel which can revolve with a high speed. If the vessel is filled up with the liquid and set into rotation, the particles will experience a centrifugal force, the magnitude of which increases with their masses. Hence after a certain interval heavier particles will be found to be congregated near the outer wall of the vessel and the lighter ones near the wall closer to the centre, whence they can be separated. It is extensively used in research laboratories for separating out microscopic particles like viruses, hormones etc. from the solutions. This principle is also used in a cream separator.

The same principle is utilised in a **drying machine**. The wet clothes are placed inside a drum whose walls are perforated. When the drum is rotated rapidly round its axis, the clothes get pressed against the wall under the action of the centrifugal force but the water particles are flung out of the drum through the perforations. Consequently, the clothes dry up within a short time.

Example 5·9. *A body of mass 2·5 kg. is moving in circle of radius 20 cm. at angular speed 3 rps. Calculate (i) the centripetal acceleration and (ii) centripetal force acting on the body.*

Solution : Here, $\omega=3$ rps$=3\times 2\pi$ rad./sec., $r=20$ cm.

Centripetal acceleration, $f_c=\omega^2 r$

$$=(3\times 2\pi)^2\times 20 \text{ cm./sec.}^2$$
$$=720\pi^2 \text{ cm./sec}^2.$$

Centripetal force, $F_c=m\omega^2 r$

$$=2{\cdot}5\times 10^3\times 720\pi^2 \text{ dyne}$$
$$=17{\cdot}75\times 10^6 \text{ dyne}$$
$$=177{\cdot}5 \text{ newton}.$$

Example 5·10. *An artificial satellite revolves round the earth in a circular orbit of radius 7000 km. Its period of revolution is 2 hours. What is its centripetal acceleration* ?

Solution : The centripetal acceleration $f_c=\dfrac{v^2}{r}$

If T be the period of revolution, then $T=\dfrac{2\pi r}{v}$

$$\therefore\ f_c=\frac{4\pi^2 r^2}{rT^2}=\frac{4\pi^2 r}{T^2}=\frac{4\pi^2\times 7000\times 10^5}{(2\times 3600)^2}=532{\cdot}5 \text{ cm./sec}^2.$$

Example 5·11. *A stone of mass 100 gm. is swung in a horizontal circle at the end of a string 50 cm. long. Find the maximum angular speed at which it can swing if the cord can sustain a tension of 10 kg. wt. Find also the number of revolutions per minute executed by the stone.*

Solution : Let ω be the maximum angular speed in rad./sec.

Now, tension in the string$=$centripetal force

$$\therefore\ 10\times 10^3\times 980=100\times\omega^2\times 50$$

$$\text{or,}\quad \omega^2=\frac{10\times 10^3\times 980}{100\times 50}=2\times 980$$

$$\therefore\ \omega=44{\cdot}27 \text{ rad./sec.}$$

If N be number of revolutions per minute, then

$$\omega=2\pi\frac{N}{60}$$

$$\text{or,}\quad N=\frac{30\times 44{\cdot}27}{\pi}=423$$

Example 5·12. *A 2 kg. ball is swung in a vertical circle with a constant angular speed of 60 r. p. m. by means of a wire of length 50 cm. Find the tensions in the wire (a) at the top and (b) at the bottom of the circle.*

Solution : $m=2$ kg. ; $r=50$ cm.$=0{\cdot}5$ m. ; $\omega=\dfrac{60\times 2\pi}{60}=2\pi$ rad./sec.

(a) At the top, two radially inward forces are acting on the ball : (i) its weight and (ii) the tension T_1 in the wire. The resultant of these two forces provides the centripetal force required to keep the ball in the circular path.

$$\therefore\ T_1+mg=m\omega^2 r$$

$$\text{or,}\quad T_1=m\omega^2 r-mg=2(4\pi^2\times 0{\cdot}5-9{\cdot}8)=19{\cdot}84 \text{ newton}.$$

(b) At the bottom, the weight and the tension T_2 act in opposite direction the direction of the latter being towards the centre.

$$\therefore\ T_2-mg=m\omega^2r$$

or, $T_2=m\omega^2r+mg=2(4\pi^2\times0{\cdot}5+9{\cdot}8)=59{\cdot}04$ newton.

Example 5·13. *A ball of mass 1 kg. is whirled in a vertical circle by means of a rope of length 20 cm. (a) Calculate the minimum speed of the ball at the top of the circle in order that the rope does not slacken. (b) What would be its speed at the bottom of the circle if it has the above minimum speed at the uppermost point? Calculate also the tension in the rope at this stage.*

Solution : (a) The rope just slackens when $T_1=0$, i.e., when $mg=m\omega^2r=\frac{mv_1^2}{r}$ where v_1 is the minimum speed of the ball at the top of the circle so that the rope does not slacken.

$$\therefore\ v_1=\sqrt{gr}=\sqrt{980\times20}=140 \text{ cm./sec.}$$

(b) In moving from the bottom to the top of the circle, the ball rises through a height $2r$.

Now, K. E. at the bottom=K.E. at the top+gain in P. E. (vide Art. 7·9)

So if v_2 be the required velocity at the bottom of the circle, then

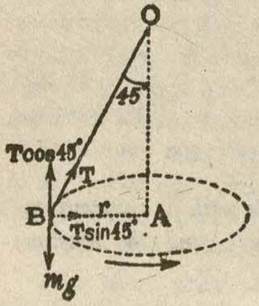
$$\tfrac{1}{2}mv_2^2=\tfrac{1}{2}mv_1^2+mg.2r=\tfrac{1}{2}m.gr+2mgr$$

$$\therefore\ v_2=\sqrt{5gr}=\sqrt{5}\ v_1=\sqrt{5}\times140=313 \text{ cm./sec.}$$

Tension in the rope is given by,

$$T_2=\frac{mv_2^2}{r}+mg=\frac{m.5gr}{r}+mg=6\ mg=6 \text{ kg. wt.}$$

Example 5·14. *A small bob of mass 1 gm. is attached to one end of a string 20 cm. long, the other end of the string being fixed to a point O. The bob is so projected that it describes a horizontal circle of radius AB=r about a point A vertically under O. Calculate the speed of the bob if the string makes an angle of 45° with the vertical.*



Solution : From the figure, the forces acting on the bob are (i) its weight mg and (ii) the tension T in the string. The vertical component of tension must balance its weight while the horizontal component of tension supplies the necessary centripetal force.

$$\therefore\ T\cos 45^\circ=mg\ ;\ T\sin 45^\circ=\frac{mv^2}{r}.$$

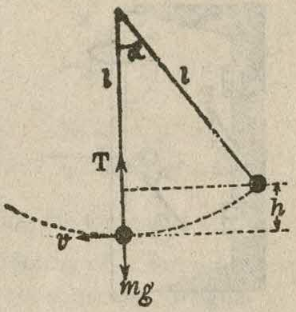
$$\therefore\ \frac{v^2}{rg}=\tan 45^\circ \quad \text{or,} \quad v=\sqrt{rg\tan 45^\circ}.$$

But $r=20\sin 45^\circ=10\sqrt{2}$ cm.

$$\therefore\ v=\sqrt{10\sqrt{2}.\ 980.1}=198 \text{ cm./sec. (approx.).}$$

Example 5·15. *The bob of a simple pendulum weighs m gm. It is deviated from its position of equilibrium through an angle α and then released. Find an expression for the tension of the thread at the moment the bob passes through the position of equilibrium.*

Solution : Let l be the length of the pendulum in cm. Also let h cm. be the height of the bob at the deviated position from its position of equilibrium as shown in the figure.



Let v be the velocity of the bob of mass m when it passes through the equilibrium position.

Since, K. E. of the bob at equilibrium position=its P. E. at the deflected position, we have,

$$\tfrac{1}{2}mv^2=mgh \qquad \therefore \quad v=\sqrt{2gh}$$

Again from figure, $h=l-l\cos\alpha=l\,(1-\cos\alpha)$.

At the equilibrium position, the tension of the thread is given by $T=mg+m\frac{v^2}{l}$

$$\text{or,} \quad T=mg+m.\frac{2gh}{l}$$

$$=mg+2mg\,(1-\cos\alpha)=mg\,(3-2\cos\alpha).$$

Example 5·16. *Find the maximum speed at which a car can round a curve of 25 m. radius on a level road. Given that the co-efficient of friction between the tyres and road is 0·3 and $g=980$ cm./sec.^{2}*

Solution : Let w be the weight of the car. Hence, the maximum centripetal force supplied by friction is $0{\cdot}3\,w$.

Force of friction=Centripetal force.

$$\therefore \quad 0{\cdot}3w=m(v^2/r)=\frac{w}{g}(\,v^2/r\,)$$

where mass of the car $m=\dfrac{w}{g}$

$$\therefore \quad v=\sqrt{0{\cdot}3gr}=\sqrt{0{\cdot}3\times980\times25\times10^2}$$

$$=857 \text{ cm./sec. (approx)}=8{\cdot}57 \text{ m./sec.}$$

Example 5·17. *In a road having a curve of radius 35 m. a car has taken a turn at the speed of 45 km./hr. without depending on friction. What must be the slope of the curve ?*

Solution : We have,

$$v=45 \text{ km./hr.}=\frac{45\times10^3}{60\times60}\text{m./sec} ; \quad g=9{\cdot}8 \text{ m./sec}^2 ; \; r=35 \text{ m.}$$

$\therefore$ from eqn. (5·22),

$$\tan\theta=\frac{v^2}{gr}=\frac{(\,45\times10^3/60\times60\,)^2}{9{\cdot}8\times35}=0{\cdot}455 \qquad \therefore \quad \theta=24{\cdot}5°.$$

5·14. Moment of a force about a point and about an axis ; Torque.

The effectiveness of a force in producing rotational motion depends not only upon its magnitude, but also on the position of the line of action of the force. This can be easily grasped if we consider a few familiar examples. A door rotates about the vertical axis passing through its hinges [Fig. 5·15]. It is a common experience that the door can be easily closed or opened if we apply the force, say F, normally at some point A near its outermost edge. Application of the same normal force F at some point B, nearer to the axis of rotation, does not produce the same effect. Thus, the effect of a given force upon the rotational motion of a body is greater, the farther the line of action of the force is from the axis of rotation. The distance of the line of action of the force, not its point of action, from the axis of rotation, is important ; this can be realised if we consider, in our previous example, the effect of a force F applied at C but in a direction towards the axis of rotation of the door. The line of action of the force then passes through the axis of rotation. No matter how much effort is given, the door will not rotate.

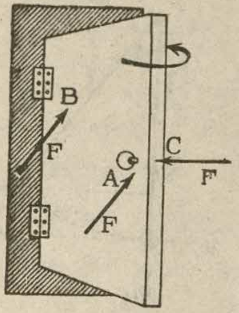


Fig. 5·15 : Illustration of the concept of moment of a force

A similar example is of turning a wheel, about an axis passing through its centre. A tangential force applied at a point A on the rim would easily produce rotation [Fig. 5·16] ; the same force applied at a point B near to the centre would produce smaller amount of motion. On applying the same force at the point C, no rotational motion can be produced ; the wheel would be simply lifted upward. The explanation of these observations, as in the previous example, leads to the same conclusion that in considering rotatory motion of a body, not only the magnitude of the force but also the distance of its line of action from the axis of rotation is important.

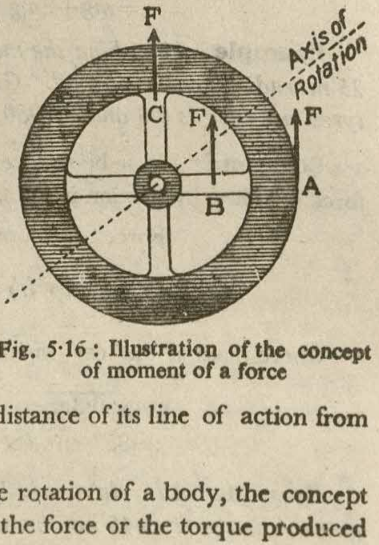


Fig. 5·16 : Illustration of the concept of moment of a force

To measure the tendency of a force to produce rotation of a body, the concept of a new physical quantity, called the moment of the force or the torque produced by the force about a given point or an axis, is introduced.

(*a*) The moment of a force or torque about a point :

The **moment of a force** *or the* **torque** *about a given point is defined as the product of the force and the perpendicular distance of the given point from the line of action of the force.* This distance is called the **force arm** or the **moment arm** of the force about the point. The Fig. 5·17 shows a body capable of rotating about an axis passing through O and perpendicular to the plane of the paper. A force F applied in the plane of the paper would cause it to turn about O in the direction shown. The moment of the force or torque produced is given by

$$L=F\times ON=F\times d \quad \ldots \quad \ldots \quad (5{\cdot}23)$$

where $d=ON$, ON being drawn from O at right angles to the line of action of F. Obviously, the moment would be zero, if either F or d is zero, i.e., if the force vanishes or if its line of action passes through the point of rotation. This line of action of the force is a mere geometrical construction. It may be extended indefinitely in either direction in order to meet the perpendicular ON [Fig. 5·18]. It has nothing to do with the length representing the force vector.

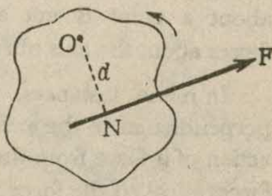


Fig. 5·17 : Moment of a force

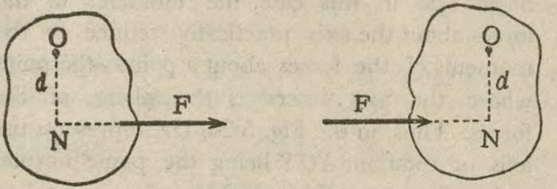


Fig. 5·18

Moment of a force (or torque) is usually designated as *positive* if it tends to produce counter clockwise rotation and as *negative,* if it tends to produce clockwise rotation.

The unit of moment (or torque) involves both the units of force and length. The unit is dyne-centimetre or gram-weight centimetre in the C. G. S. system ; poundal-feet or pound-weight feet in the F. P. S. system and newton-metre or kilogram-weight metre in M. K. S. system.

Representation of moment :

In the Fig. 5·19, let the force F be represented completely by the line PQ. O is joined to P and Q. Then moment of F about O

$$=F\times ON=PQ\times ON=2\triangle OPQ.$$

Thus, ***twice the area of a triangle, obtained by joining the two extreme points of the line representing a force to the point of rotation, gives the moment of the force about that point.***

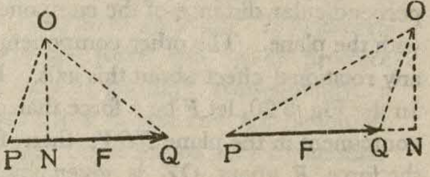


Fig. 5·19 : Representation of moment

The moment of any number of co-planar forces about a point :

It can be shown that *the moment of the resultant of a system of co-planer forces about any point is equal to the algebraic sum (i. e., with proper signs) of their individual moments about the same point.*

Thus, if a number of co-planar forces F_1, F_2, F_3......act on a body, d_1, d_2, d_3 ......being their respective moment arms about the point of rotation then

$$R\times s=F_1d_1+F_2d_2+F_3d_3+\ldots\ldots=\Sigma Fd \quad \ldots \quad \ldots \quad (5{\cdot}24)$$

where R is the resultant of the system of forces and s its moment arm about the same point.

(b) Moment of a force or torque about an axis.

In the case of a solid body rotating under the action of a number of forces acting at different points of the body, the conception of the moment of a force about a point is not applicable. We must then consider the moments of the forces about the axis of rotation.

In many instances, the system of forces acting on the body lies in a plane perpendicular to the axis of rotation. The perpendicular distance of the line of action of a force from the point of intersection of the axis with the plane of the forces is called the force arm or moment arm of the force about the axis. The moment of a force or torque about the axis is defined as the product of the magnitude of the force and its force arm. It should be noted that in this case, the moments of the forces about the axis practically reduce to the moments of the forces about a point—the point where the axis intersects the plane of the forces. Thus, in the Fig. 5·20, OZ represents the axis of rotation, XOY being the plane normal to it. Let us consider a force F_1, whose line of action lies in the plane XOY. From O, the point of intersection of OZ with the plane XOY, we draw the normal ON on the line of action of F_1. The moment or torque L of this force about OZ is then given by, $L=F_1 \times ON$.

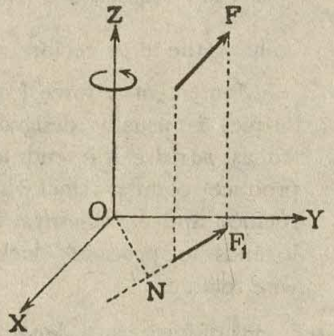


Fig. 5·20 : Moment of a force about an axis

In general, however, all the forces may not be confined in one plane. In that case, the moment of a force or torque about an axis is defined as the product of the magnitude of its component in a plane perpendicular to that axis and the perpendicular distance of the component from the point of intersection of the axis with the plane. The other component being parallel to the axis, does not produce any rotational effect about this axis. Its moment about the axis is obviously zero. In the Fig. 5·20, let F be a force that does not lie in the plane XOY. If F_1 be its component in the plane XOY, then from definition, the moment or torque L of the force F about OZ is given by $L=F_1 \times ON$. Obviously, if θ be the angle between the direction of the force F and that of the axis OZ, then $F_1=F \sin \theta$. Hence, moment or torque of the force F about OZ,

$$L=F \sin \theta \times ON \quad \ldots \quad \ldots \quad (5{\cdot}25)$$

It is now obvious that,

(*i*) The moment of a force about an axis OZ remains unchanged when the force is shifted along its line of action or when it is shifted in a direction parallel to OZ ;

(*ii*) the moment of a force about an axis OZ is zero if the line of action intersects OZ or is parallel to OZ.

Vector nature of the moment of a force or torque : So far we have not mentioned whether moment or torque is a vector or not. Close inspection, however, makes it clear that moment or torque is not a scalar because we must specify whether it is clockwise or anticlockwise. **Moment** *of a force* or **torque** *is thus a vector quantity*. The direction of the vector is along the axis of rotation in the sense shown in the Fig. 5·21. The sense is given by the direction of advancement of a right-handed screw if it is turned in the direction of rotation.

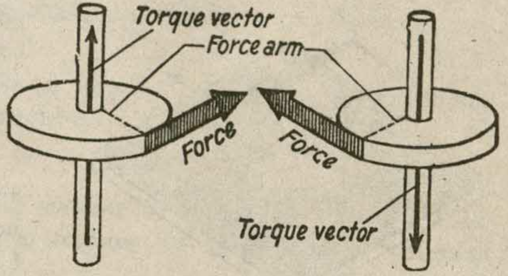


Fig. 5·21 : Vector representation of torque

Example 5·18. *A body of 5 kg. hangs at rest from a string wrapped around a wheel of diameter 10 cm. Calculate the torque about the axis of the wheel.*

Solution : Here $F=5\times10^3\times980$ dyne ; $r=5$ cm.

$$\therefore \text{ Torque } L=F\times r=5\times10^3\times980\times5$$
$$=245\times10^5 \text{ dyne-cm.}=2{\cdot}45 \text{ newton-m.}$$

5·15. Couple.

Two equal and opposite parallel forces, whose lines of action do not coincide, form a **couple**. The Fig. 5·22 shows a couple consisting of two forces, each of magnitude F.

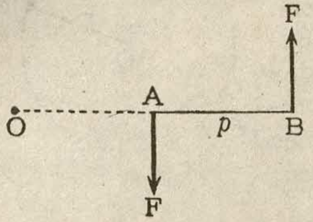


Fig. 5·22 : Couple

The resultant R of these two forces is $R=F-F=0$. This implies that a couple has no translatory effect on the body on which it acts. A couple, however, produces rotation. It is interesting to note that a single force can never produce the same effect as the couple ; consequently, a single force balancing a couple can not be found. Thus, a couple can be balanced only by another equal and opposite couple.

The effect of a couple is measured by the **moment** or **torque** *of the* **couple** *which is defined as the product of either of the forces forming the couple, and the perpendicular distance between their lines of action.*

This distance is called the *arm of the couple*. Thus the moment or torque of the couple, shown in Fig. 5·22 is,

$$L=F\times p \qquad \dots \qquad \dots \qquad (5{\cdot}26)$$

Let us consider any point O in the plane of the couple and draw OAB perpendicular to the lines of action of the forces. Then the algebraic sum (considering sign) of the moments of the forces about $O=F\times OB-F\times OA=F\times(OB-OA)$ $=F\times AB=F\times p=$moment of the couple. Since O is an arbitrary point, the *torque of the couple is given by the algebraic sum of the torques of the individual force about any point in the plane of the couple.*

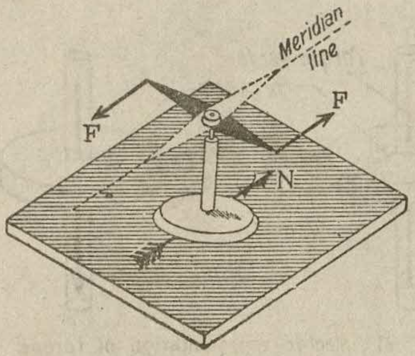


Fig. 5·23 : Couple acting on a deflected magnetic needle

Moment of the couple may be *positive* or *negative* according as the couple tends to rotate the body in anticlockwise or clockwise direction respectively.

Some common examples of couples are, the forces acting on a magnetic needle in the deflected position (Fig. 5·23), the forces acting on a rotating water sprinkler (Fig. 3·16) and the forces applied to the winding knob of a clock, etc.

5·16. Dynamics of Pure Rotation.

While discussing the angular velocity and the angular acceleration of a rotating particle, we did not take into consideration the torque producing the rotation or the mass of the particle. When these two quantities are introduced, the treatment is called *dynamics of pure rotation*.

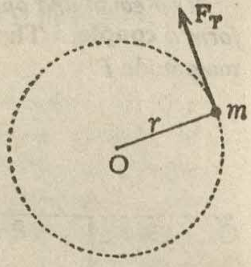


Fig. 5·24

Let us consider a particle of mass m, moving in a circular path of radius r about an axis which passes through O and is normal to the plane of the paper [Fig. 5·24]. Let a tangential force F_T be applied on the particle, so that its tangential acceleration f_T be given by Newton's second law of motion as,

$$F_T = mf_T$$

From equation (5·14), $f_T = r\alpha$, where α is the angular acceleration of the particle about the said axis. Then

$$F_T = mr\alpha \quad \ldots \quad \ldots \quad (5·27)$$

The torque L of this force about the axis through O is given by

$$L = F_T r = mr^2\alpha \qquad \text{[from equation (5·27)]}$$

The quantity mr^2 i. e., *the product of the mass of the particle and the square of its distance from the axis of rotation is called the* **moment of inertia** *of the particle about the said axis* and is usually designated by the letter I.

Thus $\quad I = mr^2 \quad \ldots \quad \ldots \quad (5·28)$

Hence $\quad L = I\alpha \quad \ldots \quad \ldots \quad (5·29)$

Comparison of this equation with the force equation $P = mf$, shows that L, α and I are analogous to P, f and m respectively. Due to this analogy of the moment of inertia with the mass of the particle, it is also called the *rotational inertia*.

For a rigid body, the same relation also holds ; the entire body can be imagined to be made up of a very large number of particles. For each particle, the quantity mr^2 is calculated and then summing over all of them the moment of inertia of the whole body about the axis under consideration can be found out. Thus

$$I=\Sigma mr^2 \qquad \ldots \qquad \ldots \qquad (5\cdot30)$$

Since each particle possesses the same angular acceleration, hence summing over all particles, we shall arrive at the relation,

$$L=\Sigma mr^2\alpha=\alpha\Sigma mr^2=I\alpha$$

Example 5·19. *A flywheel has a moment of inertia 0·05 kg. m². What constant unbalanced torque is required to increase its speed from 60 r. p. m. to 300 r. p. m. in 8 sec. ?*

Solution : Initial angular velocity, $\omega_o=60$ rpm$=2\pi$ rad./sec.

Final angular velocity, $\omega=300$ rpm$=10\pi$ rad./sec.

Moment of inertia $I=0\cdot05$ kg. m.$^2=0\cdot05\times10^7$ gm. cm.^{2}

$$\therefore \text{ Torque } \quad L=I\alpha=I\frac{\omega-\omega_o}{t}=0\cdot05\times10^7\times\frac{10\pi-2\pi}{8}$$

$$=15\cdot7\times10^5 \text{ dyne-cm.}=0\cdot157 \text{ newton-m.}$$

5·17. Angular Momentum or Moment of Momentum.

In rotational motion, the quantity which is analogous to momentum in translatory motion is known as angular momentum. Let us consider a particle of mass m moving in a circle of radius r with angular velocity ω [Fig. 5·25]. Its linear velocity v is then given by $v=r\omega$ and linear momentum by $p=mv=mr\omega$.

Its **angular momentum** about an axis through O perpendicular to the plane of the paper is then *defined as the product of its linear momentum and the perpendicular distance of its line of motion from that axis.* Hence,

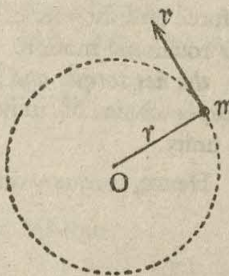


Fig. 5·25

$$\text{angular momentum, } J=pr=mvr$$

$$=mr^2\omega=I\omega \qquad \ldots \qquad \ldots \qquad (5\cdot31)$$

where $I=mr^2$ is the moment of inertia of the particle about the said axis through O.

Since the definition of angular momentum is analogous to the moment of a force, it is often called **moment of momentum.** The unit of angular momentum in C. G. S. system is gram centimetre² per second and in F.P.S. system is pound foot² per second. The corresponding unit in M.K.S. system is kilogram metre² per second.

For a rigid body the total angular momentum is found by summing over all the particles and since all the particles have the same angular velocity, we have,

total angular momentum of the body, $J=\Sigma mr^2\omega=\omega\Sigma mr^2=I\omega$.

Vector Representation : Angular momentum is a vector quantity and obviously this vector will be parallel to the angular velocity vector. Hence angular momentum vector is drawn along the axis of rotation in a proper scale so that the length represents the magnitude $I\omega$. The direction of the vector bears to the direction of rotation the same relation as that existing between the advancement and rotation of a right-handed screw [Fig. 5·26].

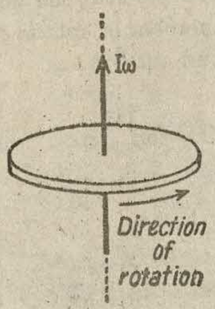


Fig. 5·26 : Vector representation of angular momentum

Example 5·20. *A boy whose mass is 40 kg. rides at the outer edge of a merry-go-round, 20 m. in diameter. Find his angular momentum if the merry-go-round is making 6 rpm.*

Solution :

Here, $\omega=\dfrac{6\times 2\pi}{60}=\frac{1}{5}\pi$ rad./sec., $m=40$ kg., $r=10$ m

Substituting for ω in eqn. (5·31),

$J=mr^2\omega=40\times 10^2\times \frac{1}{5}\pi$ kg. m.^{2}/sec.$=2{\cdot}512\times 10^3$ kg. m.^{2}/sec.

5·18. Relation between Angular Momentum and Torque.

Analogous to Newton's second law of motion which gives the relation between a force and the rate of change of momentum of a body, a similar relation exists for rotational motion. *The rate of change of angular momentum is proportional to the net torque and takes place in the direction of net torque.* As before, with proper choice of units, the value of the constant of proportionality is reduced to unity.

Hence, torque=rate of change of angular momentum

$$\text{i. e.,}\quad L=\frac{dJ}{dt} \qquad \cdots \qquad \cdots \qquad (5{\cdot}32)$$

This relation can be arrived at simply using calculus method. Since, $J=I\omega$,

$$\frac{dJ}{dt}=\frac{d}{dt}(I\omega)=I\frac{d\omega}{dt}=I\alpha=L.$$

5·19. Conservation of Angular Momentum.

This conservation principle states that if the resultant external torque on a system is zero, the angular momentum of the system remains constant.

From the above equation (5·32), when $L=0$,

$$\frac{dJ}{dt}=0 \qquad \text{or,}\quad J=\text{constant}.$$

The principle of conservation of angular momentum ranks with the principles of conservation of linear momentum and of conservation of energy as one of the most fundamental of physical laws.

Practical example.

A circus acrobat, a diver, or a skater whirling or turning about on the toe, all take advantage of the principle. Suppose a performer has just got detached from a swing with arms and legs extended and with a small clockwise angular momentum. As he pulls his arms and legs in, he is found to spin rapidly [Fig. 5·27]. This is because, in doing so his moment of inertia I becomes much smaller. Since his angular momentum $I\omega$ remains constant, as I decreases his angular velocity ω increases.

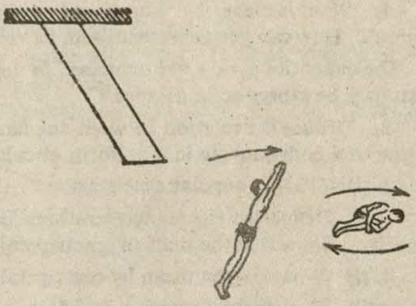


Fig. 5·27 : Illustration of the conservation of angular momentum

5·20. Kinetic Energy of a rotating body.

From the article 7·7, the kinetic energy of the rotating particle of mass m is

$$E_k=\tfrac{1}{2}mv^2.$$

Since $v=r\omega$,

$$E_k=\tfrac{1}{2}mr^2\omega^2=\tfrac{1}{2}I\omega^2 \quad \ldots \quad \ldots \quad (5\cdot33)$$

where $I=mr^2$, the moment of inertia of the particle about the axis of rotation.

For a rigid body, the total kinetic energy is found out by summing over all the particles, the angular velocities of which are the same. Hence

$$\text{Total K. E.}=\Sigma\tfrac{1}{2}mr^2\omega^2=\tfrac{1}{2}\omega^2\Sigma mr^2=\tfrac{1}{2}I\omega^2$$

I being the moment of inertia of the rigid body about the axis of rotation.

Example 5·21. *A disc has moment of inertia 0·04 kg. m.^{2} Find the work required to increase its speed from 60 r. p. m. to 150 r. p. m.*

Solution :

$$\omega_0=\frac{60\times 2\pi}{60}=2\pi \text{ rad./sec.}\ ; \quad \omega=\frac{150\times 2\pi}{60}=5\pi \text{ rad./sec.}$$

Work done=K. E. gained by the disc

$$=\tfrac{1}{2}I(\omega^2-\omega_0^2)=\tfrac{1}{2}\times 0\cdot04\times 21\pi^2=4\cdot15 \text{ Joule.}$$

5·21. Similarity between Rotational and Translational Motion.

The two types of motion are analogous is many respects. We have seen that the equations and formulae for rotational motion can be obtained simply from the corresponding equations for translational motion by mere substitutions of analogous terms. The analogy is brought out clearly in the following table.

Translational Motion	*Rotational Motion*
1. Linear displacement, s	1. Angular displacement, θ
2. Linear velocity, $v=\dfrac{ds}{dt}$	2. Angular velocity, $\omega=\dfrac{d\theta}{dt}$
3. Linear acceleration, $f=\dfrac{dv}{dt}=\dfrac{d^2s}{dt^2}$	3. Angular acceleration, $\alpha=\dfrac{d\omega}{dt}=\dfrac{d^2\theta}{dt^2}$
4. Mass, m	4. Moment of inertia, I
5. Force, $P=mf=m\dfrac{d^2s}{dt^2}$	5. Torque $L=I\alpha=I\dfrac{d^2\theta}{dt^2}$
6. Linear momentum, $p=mv$	6. Angular momentum, $J=I\omega$
7. Linear kinetic energy$=\tfrac{1}{2}mv^2$	7. Rotational kinetic energy$=\tfrac{1}{2}I\omega^2$

● EXERCISE ●

[A] Essay Type Questions

1. What is meant by 'Rotational motion' ? Define 'angular velocity' and 'angular acceleration'. How can you represent them as vector quantities ?

The quantities θ, ω, α are expressed in terms of radius instead of degrees. Why is it so ? Can they be expressed in degrees ?

2. Deduce the relation between angular velocity and linear velocity. Why is the acceleration of a body moving in a uniform circular motion always directed towards the centre ?

3. (i) What is angular acceleration ? Establish its relation with linear acceleration.

(ii) Define centripetal acceleration. Deduce its relations with angular velocity and linear velocity. Show that the units of centripetal acceleration and linear acceleration are the same.

4. (i) What do you mean by centripetal and centrifugal forces ? [*H. S.* '*83*]

(ii) Centripetal force is a real force and centrifugal force is a 'pseudo' force.—Justify the statement. [*H. S.* '*80*]

(iii) Find an expression for the centripetal force. [*H. S.* '*78*]

5. (i) Why is a centripetal force necessary to rotate a body in a uniform circular motion. [*H. S.* '*82*]

(ii) Does the centripetal force do any work ? Explain with reasons.

6. What is meant by banking of roads ? Why is it necessary ?

Deduce an expression for the angle of banking and hence show that a road of definite radius can be ideally banked for a particular value of highest speed. What would happen if a car rounds the curve with stlll higher speed ?

Does this value of the highest speed limit decrease when the road becomes wet in rain ?

7. Show that when a cyclist is going round a curved path of radius r with a velocity v, his body must become inclined to the vertical by an angle θ, where $\tan\theta = \frac{v^2}{rg}$, g being the acceleration due to gravity. [*H. S.* '*82*]

8. A constant force F is applied on a body of mass m, such that the direction of the force is always perpendicular to the direction of motion of the body. Describe the motion of the body, specifying its acceleration in direction and magnitude. Also find the rate at which work is done by F if the instantaneous speed has a value u. [*Jnt. Entrance '79*]

9. Define 'angular velocity', 'angular acceleration', 'angular momentum' and 'torque'. How is the torque related to the angular acceleration ? Explain the principle of conservation of angular momentum with suitable examples. [*H. S.* '*79*]

10. Define moment of a force about a point. Explain its difference with the moment of a force about an axis with a suitable example. State the units of the moment of a force in C. G. S., F. P. S. and M. K. S. systems. How do you represent the vector nature of the moment of a force ?

11. What is meant by 'couple' ? What do you understand by the moment of a couple ? Prove that the torque of the couple is given by the algebraic sum of the torques of the individual force about any point in the plane of the couple. Give a practical example of a couple.

12. What is moment of inertia ? Why is it called rotational inertia ? Deduce a relation of angular momentum with angular velocity and moment of inertia. Hence prove that the rate of change of angular momentum is proportional to the net torque.

13. Prove that—

(i) Mass in linear motion is analogous to moment of inertia in rotational motion.

(ii) Moment in linear motion is analogous to angular momentum in rotational motion.

(iii) Force in linear motion is analogous to torque in rotational motion.

14. (i) State and prove the principle of conservations of angular momentum.

(ii) Establish an expression for the kinetic energy of a rotating body.

[B] Short answer type Questions

1. If a heavy body on the end of a string is rotated rapidly enough, the string will break What force causes the string to break ?

2. Is the centripetal force ever greater than the centrifugal reaction ?

3. A stone is whirled in a vertical circle by means of a rope. In what position of the stone is the tension in the rope greatest. Justify your answer.

4. Name the force which provides the centripetal force in each of the following examples :—

(i) An earth satellite is kept moving in its orbit.

(ii) Railway tracks are banked on curves.

(iii) A stone tied at one end of a string is being whirled.

5. Explain the following statments with proper reasoning :

(i) Centripetal force is a 'no-work' force.

(ii) A cyclist rounding a curve leans inwards.

(iii) The earth is an oblate spheroid.

(iv) The force applied to a point near to the centre of a wheel produces smaller amount of motion.

(v) When milk is churned, the cream separates from it.

(vi) It is possible to rotate a bucket of water in a vertical plane without spilling the water.

(vii) In rectilinear motion, a body may move without any acceleration, but in circular motion it would always have an acceleration.

(viii) At a curve in a road, notice specifying the maximum speed limit is displayed.

(ix) When a car turns sharply towards right, the passengers lean towards left.

6. You want to remove a rusty bolt of your machine. Why do you prefer long arm of your wrench ?

7. If a wheel is at rest, at what point should a force be applied in order that it produces no rotational motion ?

8. How can a high diver turn his body in air so that he can spin rapidly ? What physical principle does he make use of ?

9. A small girl is standing on a rotating table at the centre with her arms closed together. What will happen when she extends her arms horizontally ?

10. 'It is easier to open or close a door by pushing or pulling near its outermost edge than by pushing or pulling near the hinges'.—Explain. [*Model Questions, H. S. Council*]

11. Explain why a motor-cyclist in a circus '*well of death*' can ride around the walls of the well without falling down. [*I. I. T. '67*]

12. The driver of a truck travelling with a velocity v suddenly notices a wall in front of him at a distance r. Is it better for him to apply brakes or to make a circular turn without applying brakes in order to just avoid crashing into the wall ? Why ? [*I. I. T. '77*]

13. Prove that if the earth suddenly contracts to half its radius, the length of a day will be shortened from 24 hours to 6 hours.

14. 'The earth moving about the sun in a circular orbit is always acted upon by a force and hence work must be done on the earth by this force'.—Do you agree with this statement ? [*I. I. T '73*]

[C] Simple Problems.

1. A railway line of length 31·4 m. is to curve through 10°. If the line forms an arc of a circle, what is the radius of curvature ? [*Ans.* 180 m.]

2. A flywheel rotates at 1200 rev./min. Find its (a) frequency, (b) angular speed, (c) time period and (d) time required to turn through 72°.

[*Ans.* (a) 20 r. p. s. (b) 125·6 rad./sec. (c) 0·05 sec. (d) 0·01 sec.]

3. An electric clock provides three hands. Find the angular speed of each of them.

[*Ans.* Hour., $1{\cdot}454 \times 10^{-4}$; Min., $1{\cdot}746 \times 10^{-3}$; Sec., $1{\cdot}047 \times 10^{-1}$ rad./sec.]

4. Find the angular velocity of the daily rotation of earth. [*Ans.* $7{\cdot}271 \times 10^{-5}$ rad./sec.]

5. To the passengers of an aeroplane, flying from east to west on the equator, the sun appears motionless. Find the velocity of the aeroplane. Neglect the height of the aeroplane and take equatorial radius of the earth=6000 km. [*Ans.* 1570 km./hr.]

6. A satellite revolves in a circular orbit around the earth 500 km. above its surface. It takes 100 min. for one complete rotation ; calculate its angular and linear speed assuming the earth's radius=6000 km. [*Ans.* $1{\cdot}047 \times 10^{-3}$ rad./sec., 6·806 km./sec.]

7. The linear velocity of a point on the rim of a rotating wheel is 3 times greater than that of a point 7 cm. closer to the wheel axle. Find the diameter of the wheel. [*Ans.* 21 cm.]

8. A fly-wheel of radius 50 cm. increases its speed from 720 r. p. m to 1440 r. p. m. in 1 min. Calculate (a) its angular acceleration in rad./sec.^{2} and (b) linear acceleration in cm./sec.^{2} of a point on its rim. [*Ans.* (a) 1·256 rad./sec.^{2} (b) 62·8 cm./sec.^{2}]

9. If a wheel revolving at 1800 r. p. m. slows down uniformly to 1200 r. p. m. in 2 sec., calculate (a) the angular acceleration of the wheel and (b) the number of revolutions it makes in this time. [*Ans.* 10π rad./sec.^{2}, 50 rev.]

10. A wheel starts from rest and accelerates uniformly to reach the angular speed 30 rad./sec. at the end of 15th revolution. Determine the angular acceleration of the wheel. [*Ans.* 4·77 rad./sec.^{2}]

11. A fan while rotating at 1500 r. p. m. is switched off. It then slows down uniformly and executes 150 revolutions before it comes to a stop. In how much time the fan stops after it has been switched off ? [*Ans.* 12 sec.]

12. A body revolving with a constant angular acceleration of 5 rad./sec.^{2} possesses an angular velocity of 300 rev./min. at the instant $t=0$. (a) Find the time during which it attains an angular velocity of 900 rev./min. (b) Also find the time required by the body to perform 120π revolutions after the instant $t=0$. [*Ans.* (a) 4π sec. (b) 8π sec.]

13. A wheel of radius 8 cm. starts from rest and accelerates uniformly. Find the linear acceleration of a point on its rim, if it attains a linear velocity of 120 cm./sec. at the end of 10th revolution of the wheel. [*Ans.* 14·33 cm./sec.^{2}]

14. A body of mass 50 gm. is moving in a circle of diameter 100 cm at an angular speed of 360 r. p. m Calculate (a) its centripetal acceleration and (b) the centripetal force acting on it. [*Ans.* (a) 710·8 m./sec.^{2} (b) 35·54 newton.]

15. To a first approximation the electron of a hydrogen atom moves in a circular orbit of diameter $1{\cdot}0 \times 10^{-8}$ cm. with linear velocity $2{\cdot}2 \times 10^8$ cm./sec. Find its centripetal acceleration. [*Ans.* $9{\cdot}68 \times 10^{24}$ cm./sec ^{2}]

16. Calculate the acceleration of a point on the equator assuming that the equatorial radius is 6000 km [*Ans.* 3·174 cm./sec ^{2}]

17. A point starts from rest and moves in a circle of radius 20 cm. with uniform angular acceleration. The linear velocity of the point is 20 cm /sec at the end of the tenth revolution. Find the centripetal acceleration of the point at the instant $t=20$ sec. after it starts moving. [*Ans* 0·51 cm /sec.^{2}]

18. Compute the force exerted inward on a ball weighing 6 kg. at any particular instant while it is describing a circle of radius 30 cm. and revolving at 100 r. p. m. [*Ans.* $19{\cdot}74 \times 10^6$ dyne]

19. A tram car weighing 6 metric ton travels round a curve of radius 120 m. with a speed of 15 km. p. h. Find the force with which the wheels press laterally against the rails. [*Ans.* 868 newton]

20. A body lies on a horizontal disc at a distance of 10 cm. from the centre. The disc rotates about a vertical axis passing through its centre at 60 r.p.m. What should be the co-efficient of friction between the body and the disc so that the body will not slide off the disc ? [*Ans.* $\mu \geqslant 0{\cdot}4024$]

21. Calculate the proper angle through which the rider must lean inward from the vertical in order to keep the balance if the mass of the cycle together with the rider is 180 lb. and the bicycle travels round a curve of 11 ft. radius at 10 mile/hr. [*Ans.* 31°26′]

22. What is the proper angle for banking a road around a curve of 60 m. radius to allow for speeds of 70 km./hr. ? [*Ans.* 32·73°]

23. A mountain road has a curve of radius 40 m. If at the curve the road is 4 m. wide and its outer edge is 80 cm. higher than the inner edge, find the maximum speed for which the curve is intended. [*Ans.* 31·88 km./hr.]

24. In a machine, a horizontal arm 2 ft. long is driven by a vertical shaft and makes 600 r. p. m. in horizontal plane. A 10 lb. block is attached to the free end of the arm. Find out the force on the block and the force on the vertical shaft due to rotation of the block.

[*Ans.* 78980 poundal ; 78980 poundal] (***Jt. Entrance*** '73)

25. A stone is swung uniformly in a horizontal circle at the end of a thread 30 cm. long. At what number of revolutions per sec., will the thread snap if it is known to do so under a road equal to ten-fold weight of the stone.

Repeat your calculation for the stone revolving in a vertical circle.

[*Ans.* 2·88 r.p.s ; 2·73 r.p.s.]

26. A bucket with water is swung uniformly in a vertical circle at the end of a rope 80 cm. long. Find (a) the minimum velocity of rotation of the bucket when the water will not spill out at the highest point, (b) the tension of the rope at this velocity at the highest and the lowest points of the circumference. The mass of the bucket with the water is 5 kg.

[*Ans.* (a) 3·5 rad./sec. (b) zero., 10 kg. wt.]

27. A stone tied to a rope rotates uniformly in a vertical plane. The difference between the maximum and the minimum tension of the rope is 500 gm. wt. Find the mass of the stone. [*Ans.* 250 gm]

28. A light horizontal bar is 10 metre long. A 3 kg. force acts vertically upward on it 25 cm. from the right hand end. Find the torque about each end.

[*Ans.* 7·35 newton-m. (clockwise) & 286·6 newton-m. (counter clockwise)]

29. A flywheel has moment of inertia 0·1 km m.^{2} What constant unbalanced torque is required to increase its speed from 3 r.p.s. to 9 r.p.s. in 18 revolutions ?

[*Ans.* 1·256 newton. m]

30. A boy weighs 50 kg. and stands on the edge of a large wheel of diameter 2 m. which makes 15 revolutions per minute. Calculate the angular, momentum of the boy.

[*Ans.* 78·54 kg. m.^{2}/sec.]

31. A flywheel has moment of inertia 0·02 kg. m^2. Find the work required to decrease its speed from 7 r.p s. to 3 r.p.s. [*Ans.* 15·8 Joule]

32. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Two objects each of man m, are attached gently on the opposite ends of a diameter of the ring. What will then be the angular velocity of the ring ?

[*Ans.* $\omega M/(M+2m)$]

[D] Harder Problems

1. An automobile having wheels of diameter 80 cm. changes its speed from 25 to 50 km./hr. in 5 sec. Calculate the angular acceleration of the wheel. [*Ans.* 3·47 rad./sec.^{2}]

2. In an electric clock, the extremity of the hour hand moves one twentieth as fast as that of the minute hand. What is the length of the hour hand if the minute hand is 10 cm. long ? [*Ans.* 6 cm.]

3. Determine the linear velocity of revolution of points on the earth's surface at a latitude of 60°. Given, earth's radius$=$6000 km. [*Ans.* 785 km./hr.]

4. Two disks are mounted 40 cm. apart on an axle. The axle with disks rotates with uniform angular velocity of 3000 r.p.m. A bullet flying parallel to the axle pierces both the disks. The hole in the second disk is displaced with respect to that in the first one by an angle of 18°. What was the velocity of the bullet ? [*Ans.* 400 m./sec.]

5. A particle starts from rest and moves along a circle of radius 40 cm. with a constant linear acceleration of 10 cm./sec^2. Find the time required after the motion begins for the centripetal acceleration of the particle to be (a) equal to the linear acceleration, (b) twice the linear acceleration. [*Ans.* (a) 2 sec (b) 2·828 sec]

6. What should be the minimum speed of an aeroplane at which it can execute a loop of radius 100 m. so that there will be no tendency for the pilot to fall out at the highest point ? [*Ans.* 31·3 m./sec]

7. An aeroplane flying at a speed of 600 km. p. h. loops a loop. What should be the radius of the loop so that the maximum force pressing the pilot against the seat is (a) 5 times his weight (b) 10 times his weight ? [*Ans.* (a) 708·6 m. (b) 314·9 m.]

8. A simple pendulum is suspended from the ceiling of a tram-car. The car passes a curve of radius 65 m. at a speed of 12 km. p.h. Through what angle will the bob deviate ? [*Ans.* 1°]

9. An object is being weighed on a spring balance in a railway wagon going round a curve of radius 625 m, at a speed of 90 km. p.h. The object has a weight of 10 kg. What is the reading on the spring balance ? [*Ans.* 10 05 kg]

10. A small body is tied to a point by an inextensible string of negligible mass and is rotated in a circle of radius 500 cm. in the vertical plane. What is the minimum speed that it must have at the uppermost point of the circle, so that the string does not slacken ? What would be its speed and angular velocity at the lowest point of the circle if it has the above minimum speed at the uppermost point. [*Ans.* 700 and 1565·2 cm /sec., 3·132 rad /sec.]
[*I. I. T.* '67]

11. A cylindrical steel drum, diameter 20 cm., rotates about its axis which is vertical. A small steel body in contact with the upper part of the inner wall is carried round with the drum if the rate of rotation exceeds 200 revolutions per minute, but falls at lower rates. Find the co-efficient of limiting friction between the steel surfaces. [*Ans.* 0·2235] (*London Univ.*)

12. A motor car travels over a convex bridge of radius 20 m. With what maximum speed the car travels with safety i. e., it does not move tangentially off the bridge. [*Ans.* 50·4 km./hr.]

13. A man whirls a stone round his head on the end of a string 1·5 m. long. Can the string be in a horizontal plane ? If the stone has a mass of 300 gm. and the string will snap if the tension in it exceeds 6N, what is the smallest angle the string can make with the horizontal ? What is then the speed of the stone ? (Take $g=10$ m./sec^2.) [*Ans.* No ; 30° ; 4·74 m./s.]

14. A rough horizontal circular disc is rotating with uniform angular velocity ω about a fixed vertical axis through its centre. Two bodies of masses M and m lie on the disc and are at rest relative to it, being on the same radius but at a distance a and $3a$ respectively from the centre. They are connected by a light inextensible string of length $2a$ and the coefficient of friction between either the bodies and disc is μ. Show that both the bodies are on the verge of slipping outwards if

$$\omega^2=\frac{\mu g\,(M+m)}{a\,(M+3m)}$$

Find also an expression for the corresponding tension in the string.

$$\left[\textit{Ans.}\quad T=\frac{2\mu gMm}{M+3m}\right]$$

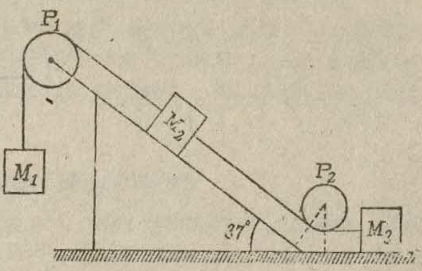
15. Two bobs of masses 6 m. and 10 m. are attached to the ends of a light inextensible string which passes over a frictionless ring at C. The lighter bob describes a horizontal circle about the heavier bob (which remans stationary a+c) as centre. Show that $OC=\frac{3a}{8}$ where a is the length of the string and the time period of revolution of the lighter bob is $\pi\sqrt{\frac{3a}{2g}}$ where g is the acceleration due to gravity.

16. A rubber ring is forced on the rim of a wheel of diameter 80 cm. The wheel is then rotated about its axis at an increasing rate. The ring becomes just slack on the wheel when it is rotating at 1200 r. p. m. What was the strain in the rubber before the wheel started rotation ? Given that, for rubber, Young's modulus $Y=5{\cdot}5\times10^9$ dyne/cm^2 and the density $\rho=0{\cdot}92$ gm./cc. [*Ans.* $4{\cdot}36\times10^{-3}$]

17. A cyclist negotiates a curve of radius 25 m. on a road, which is banked at an angle of 13° to the horizontal. If the co-efficient of friction between the tyres and the road is 0·6, determine the greatest speed at which the cyclist can negotiate the curve without skidding and his inclination to the vertical at this speed. [*Ans.* 55·33 km./hr. ; 44°]

[**Hints** : Three forces are acting on the system of the cycle and the cyclists, viz.
(1) normal reaction acting through the point of contact between the tyre and road,
(2) friction through the same point and (3) weight of the system through the c. g. of the system.]

18. The masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and frictionless pulleys P_1 and P_2 as shown in the fig. The masses move such that the portion of the string between P_1 and P_2 is parallel to the incline and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4·0 kg. each and the co-efficient of kinetic friction between the masses and the surfaces is 0·25. The inclined plane makes an angle of 37° with the horizontal. If the mass M_1 moves downwards with a uniform velocity, find (i) the mass M_1 (ii) the tension in the horizontal portion of the string. ($g=9{\cdot}8$ m./sec.^{2} ; $\sin 37°=\frac{3}{5}$) (*I. I. T. '81*)
[*Ans.* (i) 4·2 kg ; (ii) 9·8 N]



19. A body of mass m is whirled in a vertical circle by means of a rope. If the speeds of the body at the highest and the lowest points are u and $4u$ respectively, prove that the tension in the rope when its inclination to the downward vertical is θ, is approximately $mg\,(3\cos\theta+2{\cdot}27)$. If the length of the rope is 30 cm. and $\theta=45°$, find the speed of the body at that position. [*Ans.* 3·29 m./sec.]

20. A body of mass m hangs at one end of a string of length a, the other end of which is fixed. What velocity in the horizontal direction should be imparted to the body so that the string would become just slack, when it makes an angle of 60° with the upward vertical ? Find also the tension in the string when it makes an angle of 60° with the downward vertical.

$$\left[\textit{Ans.}\quad \sqrt{\frac{7ga}{2}}\ ;\ 3\,mg\right]$$

21. A smooth circular wire of radius r and centre O, is fixed in a vertical plane. A smooth small ring of mass m can slide on the wire. From a point P on the wire at a vertical distance $\frac{r}{2}$ above O, the ring is given a velocity v along the downward tangent to the wire. Show that the ring will just reach the highest point of the wire if $v=\sqrt{rg}$. Find also the reaction of the wire on the ring when the latter is at a vertical distance $\frac{r}{2}$ below O. [*Ans.* 3·5 mg]

22. A small body of mass m is placed on the top of a smooth sphere of radius r. If the body slides down the surface of the sphere, at what point does it flies off the surface ?

[*Ans.* At the point whose radius vector makes an angle $\cos^{-1}(\frac{2}{3})$ with the upward vertical through the centre of the sphere]

23. A particle slides from the top down the outside smooth surface of a fixed sphere of radius r. When the particle is at the highest position, what velocity in the horizontal direction should be imparted to it, so that it leaves the surface of the sphere at a point whose vertical height from the centre of the sphere is $4a/5$?

[*Ans.* $\sqrt{\frac{2gr}{5}}$]

24. A pipe of internal diameter 4 cm. delivers 600 kg. of water per minute. It bends to one side through a right angle and 70 cm. further on bends back to its original direction. Find the force exerted by the water at each bend. Also show that these forces form a couple and find its moment. [*Ans.* 112·54N ; 55·71 Nm]

25. A particle is projected at time $t=0$ from a point P on the ground with a speed v_0, at an angle 45° to the horizontal. Find the magnitude and direction of the angular momentum of the particle about P at time $t=v_0/g$. (*I. I. T, '84*)

[*Ans.* $\frac{mv_0^3}{2\sqrt{2}g}$ perpendicular to the plane of motion and directed from front to back when the P lies to the left of the figure]

6
CHAPTER

STATICS

6·1. Introduction.

The subject of **Statics** is concerned with systems of forces which acting on a body keep it in equilibrium. *If a body, acted on simultaneously by several forces, remains at rest or in a state of uniform motion in a straight line, it is said to be in* **equilibrium**. This means that a body in equilibrium possesses neither translational, nor rotational acceleration. It is to be noted that, a body in equilibrium need not necessarily be at rest—it may be in uniform motion in a straight line.

Since from Newton's second law, a body gets accelerated only under the action of unbalanced forces or couples, we conclude that forces or couples acting on a body in equilibrium must compensate one another. As we shall see, this leads us to two conditions of equilibrium.

6·2. Conditions of Equilibrium of a system of particles.

A body can be regarded as an aggregation of a large number of particles. The following discussions for the equilibrium of a body thus also holds true for a system of particles.

Let us consider a body on which the coplanar external forces F_1, F_2, F_3...... etc., are acting simultaneously [Fig. 6·1]. Let O be any arbitrary point in their plane. We first consider the force F_1, whose point of action is denoted by A. Let us introduce at O two equal and opposite forces, each of magnitude F_1, their lines of action being parallel to that of F_1. Since these two forces neutralise each other, introduction of them does not in any way alter the state of the body.

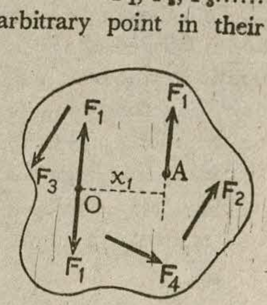


Fig. 6·1 : Equilibrium of a body under the action of several forces

The given force F_1 at A and the opposite parallel force F_1 at O constitute a couple of moment (or torque) $L_1 = F_1 \times x_1$, where x_1 is the moment arm of the given force F_1 about O. Thus, the original force F_1 at A is equivalent to an equal and parallel force F_1 at O and a couple of moment L_1.

In a similar way, each force of the system may be supposed to be equivalent to an equal force at O and a couple. The original system of forces is thus equivalent to the set of force F_1, F_2...acting at O, together with the same number of couples. The forces at O may be combined to give a single resultant force at O, while the couples are equivalent to a single couple whose moment is given by the algebraic sum of the moments of all the couples. Evidently, the moment of each couple is the moment of the corresponding force about the arbitrary point O. Thus, the algebraic sum of the moments of all the couples is simply the algebraic sum of the moments of all the forces about the arbitrary point O.

Now, as a force and a couple can not balance each other, hence for the body to be in equilibrium, both must separately vanish. We thus arrive at the following **two conditions of equilibrium** :

(*i*) *The resultant of all the forces acting on the body must be zero i. e., the algebraic sum of the resolved parts of all the forces along any two directions at right angles to each other must separately be zero*, and

(*ii*) *The algebraic sum of moments of all the forces about any arbitrary point is zero.*

The first condition ensures that the resultant force at O vanishes and the second condition ensures that the resultant couple is not in existence.

When the first condition is satisfied, there is no unbalanced force acting on the body and, therefore, the body will have no linear accleration. The condition is thus called **the condition of translational equilibrium.** Mathematically, if F_x and F_y be the resolved parts of any one of the forces along x and y axes respectively, then for all the forces,

$$\Sigma F_x=0 \text{ and } \Sigma F_y=0 \quad \dots \quad \dots \quad (6{\cdot}1)$$

Fulfilment of the second condition implies that the resultant torque acting upon the body is zero and so the body does not possess any angular acceleration. This condition is thus also known as **the condition of rotational equilibrium.** If the moments of the forces F_1, F_2...etc. about any point in their plane be L_1, L_2... etc., then this condition can be expressed analytically as

$$L_1+L_2+L_3+\dots\dots=0$$

$$\text{or,} \quad \Sigma L=0 \quad \dots \quad \dots \quad (6{\cdot}2)$$

Both the conditions must be simultaneously satisfied, for if only the first condition is satisfied, the couple is present and produces an angular acceleration of the body i. e., the body will not be in rotational equilibrium. On the other hand, if only the second condition is satisfied, the body will move under the action of the resultant force and hence will possess a linear acceleration. It cannot, therefore, remain in translational equilibrium.

The first condition of equilibrium can be put in another way. We know that the resultant of a number of forces can be found out by the polygon method for composition of vectors (vide Art. 1·3). If we draw a polygon whose sides, in succession, represent the different forces in direction and magnitude, then the resultant is represented by the closing side, drawn in reverse order, which completes the polygon. The resultant is zero means that this closing side is zero. Thus *the forces producing equilibrium can be represented in magnitude and direction by the sides of a closed polygon,*

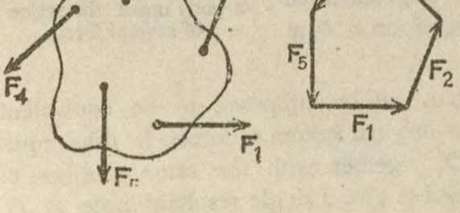


Fig. 6·2 : Polygon of forces

taken in order. This is shown in the Fig. (6·2), for the forces F_1, F_2, F_3 F_4, F_5 acting on a body.

Two simple cases may now be discussed to get a clear outlook of equilibrium problems :

(i) Equilibrium under only two forces : When a body is in equilibrium under the action of two forces F_1 and F_2, the two conditions of equilibrium give that (a) they must be equal in magnitude but opposite in direction, and (b) they must have the same line of action [Fig. 6·3]. Condition (a) means that the resultant of the two forces is zero and condition (b) ensures that there is no couple.

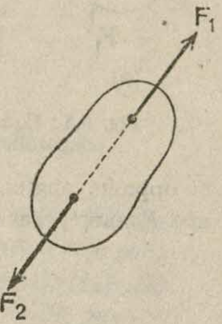


Fig. 6·3 : Equilibrium under two forces

(ii) Equilibrium under three forces : Let us consider a body in equilibrium under the action of three external forces F_1, F_2, F_3. We can replace any two of them, say F_1, and F_2, by their resultant R, so that the problem reduces to equilibrium under two forces, namely this resultant R and the third force F_3. Thus the conditions (a) and (b) mentioned above must be satisfied by R and F_3. It follows that the three forces must be *co-planar* ; otherwise the line of action of R can not be the same as that of F_3. Also, the force F_3, which is equal in magnitude but opposite in direction to the resultant R of F_1 and F_2, is called their **equilibrant**. *The equilibrant of a system of forces is, therefore, the force which would balance all the original forces taken together.*

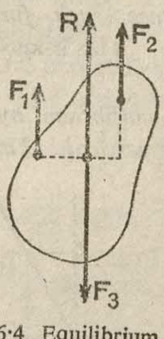


Fig. 6·4 Equilibrium under three parallel coplanar forces

The lines of action of the forces F_1 and F_2 may be either parallel or not. When F_1 and F_2 are parallel, their resultant R is parallel to them [vide Art. 6·3]. First condition of equilibrium thus requires that F_3 should be equal and opposite to R ; hence F_3 must be parallel to F_1 and F_2 [Fig. 6·4].

The second condition of equilibrium gives that the line of action of F_3 must be the same as that of R.

If the lines of action of F_1 and F_2 are not parallel, they must intersect at some point O [Fig. 6·5 (a)]. Their resultant R may then be obtained by the parallelogram method [Fig. 6·5 (b)]. For translational equilibrium. F_3 must be equal and opposite to R, while for rotational equilibrium, they must have the same line of action. Hence, the line of action of F_3 must pass through the point O. In other words, the three forces F_1, F_2, F_3 must be **concurrent**.

From [Fig. 6·5 (b)], we get that diagonal OB of the parallelogram $OABC$ represents R, the resultant of F_1 and F_2. Since F_3 is equal and opposite to R, it is obvious that BO completely represents F_3. Also, since CB is equal and parallel to OA, it completely represents F_2. Thus the sides OC, CB and BO of the triangle OCB represent the three forces completely [Fig. 6·5 (c)]. This is known as **Triangle of Forces Rule** which can be stated as :

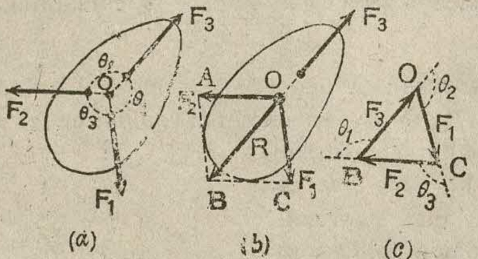


Fig. 6·5 : Equilibrium under three nonparallel coplanar forces

Three non-parallel forces in equilibrium can be represented in magnitude and direction by the three sides of a triangle taken in order.

The above rule can be put in an analytical form by using the property of triangles viz., the sides are proportional to the sines of opposite angles. If θ_1 be the angle between F_2 and F_3, θ_2 that between F_1 and F_3 and θ_3 that between F_1 and F_2, then from the force-triangle in Fig. 6·5 (c).

$$\frac{OC}{\sin\angle OBC}=\frac{CB}{\sin\angle COB}=\frac{BO}{\sin\angle OCB}$$

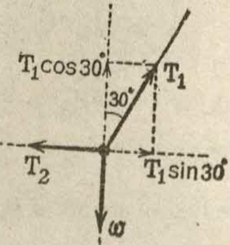
or, $$\frac{F_1}{\sin(180°-\theta_1)}=\frac{F_2}{\sin(180°-\theta_2)}=\frac{F_3}{\sin(180°-\theta_3)}$$

Since, $\sin(180°-\theta_1)=\sin\theta_1$, etc.

$$\therefore\quad \frac{F_1}{\sin\theta_1}=\frac{F_2}{\sin\theta_2}=\frac{F_3}{\sin\theta_3}\quad\cdots\quad\cdots\quad(6{\cdot}3)$$

Thus, *for equilibrium under three non-parallel coplanar forces, each force is proportional to the sine of the angle between the other two.* This is known as **Lami's Theorem.**

Example 6·1. *A load weighing 50 kg. is supported in equilibrium by two ropes, one horizontal and the other in a direction 30° with the vertical. Find the tension in each rope.*



Solution : Refer to the adjoining diagram. The load is in equilibrium under 3 forces :

(1) Tension $\mathbf{T_1}$ in the rope making 30° angle with vertical.

(2) Tension $\mathbf{T_2}$ in the horizontal rope.

(3) Weight $\mathbf{w}$ of the load acting vertically downwards.

Solution by Component method :

Sum of the components of the forces in horizontal direction $\Sigma F_x=0$

$$\text{or,}\quad T_1\sin 30°-T_2=0$$

$$\therefore\quad T_2=\frac{T_1}{2}\quad\cdots\quad\cdots\quad\text{(i)}$$

Sum of the components of the forces in vertical direction $\Sigma F_y=0$

$$\text{or,}\quad T_1\cos 30°-w=0$$

$$\therefore\quad T_1=\frac{w}{\cos 30°}=\frac{50\times 2}{\sqrt{3}}=57{\cdot}73\text{ kg.wt.}$$

From (i) $T_2=\frac{57{\cdot}73}{2}=28{\cdot}87$ kg. wt.

Solution by Lami's Theorem :

Angle between $\mathbf{T_1}$ and $\mathbf{T_2}=90°+30°$

Angle between $\mathbf{T_1}$ and $\mathbf{w}=180°-30°$

Angle between $\mathbf{T_2}$ and $\mathbf{w}=90°$

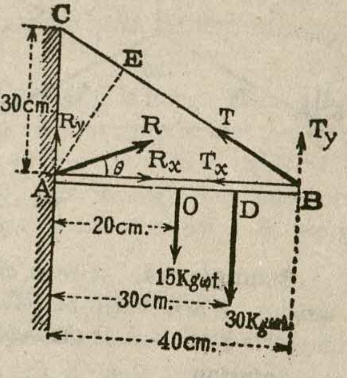
$$\therefore \quad \frac{T_1}{\sin 90°}=\frac{T_2}{\sin(180°-30°)}=\frac{w}{\sin(90°+30°)}$$

$$\therefore \quad T_1=\frac{50\times\sin 90°}{\cos 30°}=\frac{50\times 2}{\sqrt{3}}=57{\cdot}73 \text{ kg. wt.}$$

$$T_2=\frac{50\times\sin 30°}{\cos 30°}=\frac{50}{\sqrt{3}}=28{\cdot}87 \text{ kg. wt.}$$

Example 6·2. *A uniform horizontal beam AB 40 cm. long and weighing 15 kg. is hinged at A to a vertical wall and supported by a wire from B to a point C on the wall 30 cm. vertically above the hinge. A load of 30 kg. hangs from the beam at D 30 cm. from the wall. Find* (*a*) *the tension in the wire* (*b*) *the push of the beam and* (*c*) *the reaction at the hinge.*

Solution : The weight of the beam viz. 15 kg. wt. acts at the midpoint *O* of the beam. Let **T** be the tension in the wire and **R** be the reaction at the hinge. Now,



$$BC=\sqrt{AB^2+AC^2}=\sqrt{40^2+30^2}=50 \text{ cm.}$$

Drop *AE* perpendicular to *BC*. From the similar $\triangle$s *AEC* and *ABC*,

we have, $\frac{AE}{AB}=\frac{AC}{BC}$

$$\therefore \quad AE=\frac{AC}{BC}\times AB=\frac{30\times 40}{50}=24 \text{ cm.}$$

(a) Taking moments about *A*, we have,

$$\Sigma L=R\times 0+T\times 24-15\times 20-30\times 30=0$$

$$\therefore \quad T=\frac{30\times 30+15\times 20}{24}=50 \text{ kg. wt.}$$

(b) $T_x=T\cos\angle ABC=T\times\frac{AB}{BC}=T\times\frac{40}{50}=\frac{4T}{5}=\frac{4\times 50}{5}=40$ kg. wt.

$\therefore \quad T_x=$ push of the beam $=40$ kg. wt.

(c) $\Sigma F_x=R_x-T_x=0$

$\therefore \quad R_x=T_x=40$ kg. wt.

$\Sigma F_y=R_y+T_y-15-30=0 \quad \therefore \quad R_y=45-T_y$

But $T_y=T\sin\angle ABC=T\times\frac{AC}{BC}=T\times\frac{30}{50}=\frac{3T}{5}=\frac{3\times 50}{5}=30$ kg. wt.

$\therefore \quad R_y=45-30=15$ kg. wt.

$\therefore \quad R=\sqrt{R_x^2+R_y^2}=\sqrt{40^2+15^2}=42{\cdot}72$ kg. wt.

$$\text{and } \tan\theta=\frac{R_y}{R_x}=\frac{15}{40}=0{\cdot}375$$

$$\theta=20°33'$$

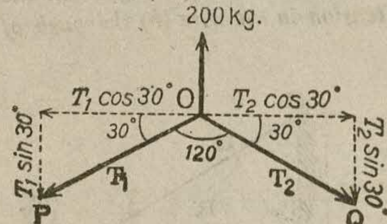
∴ The reaction at the hinge is 42·72 kg. wt. making an angle of 20°33′ above the horizontal.

Example 6·3. *To pull an automobile out of a ditch, one end of a rope POQ is wrapped around a piller at P and the other end is tied to the auto at Q. A pull of 200 kg. is applied to the centre O of the rope in a direction perpendicular to PQ. Find the tension in the rope when angle POQ is 120°.*

Solution : The point O is in equilibrium under the action of the forces T_1, T_2 and 200 kg. as shown in the adjacent figure. Hence by the component method, we have from eqn. (6·1),

$$\Sigma F_x=0 \text{ i. e., } T_2\cos 30°-T_1\cos 30°=0 \quad \ldots \quad \ldots \quad (1)$$

$$\text{and } \Sigma F_y=0 \text{ i. e., } 200-T_1\sin 30°-T_2\sin 30°=0 \quad \ldots \quad (2)$$



From equation (1) we get, $T_1=T_2=T$ (say).

Applying this result to eqn. (2), we find $T=200$ kg.

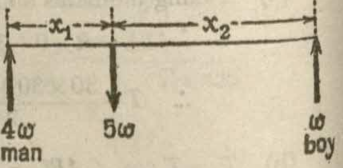
Alternative Proof. From Lami's Theorem, we get using eqn. (6·3),

$$\frac{T_1}{\sin 120°}=\frac{T_2}{\sin 120°}=\frac{200}{\sin 120°}$$

$$\therefore \quad T_1=T_2=200 \text{ kg.}$$

Example 6·4. *A man and a boy carry a load hung on a pole of negligible weight. Where must the load be hung so that the boy at one end of the pole supports ¼ as much as the man at the other end ?*

Solution : Refer to the adjoining fig. Let the load be hung at distances x_1 and x_2 respectively from the man and the boy at the end of the pole.



Also, let $5w$ be the weight of the load,

∴ w=weight supported by the boy

∴ $4w$=weight supported by the man

Since the pole is in equilibrium, taking movements of the forces about the point at which the load is hung, we have $\Sigma L=w\times x_2-4w\times x_1=0$.

∴ $x_1 : x_2=1 : 4$

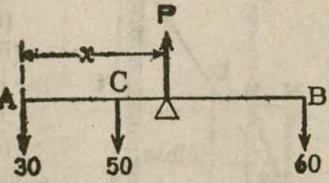
or, $x_1=\frac{1}{5}\times$length of the pole.

∴ The load must be hung at a distance equal to ⅕ the length of the pole from the man.

Example 6·5. *A non-uniform rod 15 ft. long and weighing 50 lb. is supported in a horizontal position on a fulcrum with weights of 30 lb. and 60 lb. suspen-*

ded from its ends. If the weight of the rod acts at a distance of 5 ft. from the end supporting the 30 lb. weight, compute the position of the fulcrum.

Solution : Let the weight of the rod act through the point C as shown in the adjacent figure. Let P be the upward force at the fulcrum.



Since the rod is considered to be in equilibrium under the action of the forces acting on it, the sum of these vertical forces must be zero. Thus, considering the upward forces as positive, we have from eqn. (6·1),

$$\Sigma F = -30 - 50 + P - 60 = 0, \quad \text{or,} \quad P = 140 \text{ lb.}$$

Again, the rod being in equilibrium, the sum of the torques about any point in the plane of the forces is zero. Taking moments about the point A (vide eqn. 6·2)

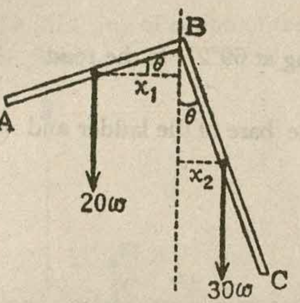
$$\Sigma L = -60 \times 15 + 140 \times x - 50 \times 5 + 30 \times 0 = 0.$$

$$\therefore \quad x = 8{\cdot}22 \text{ ft.}$$

Thus the fulcrum is at a distance of 8·22 ft. from the 30 lb. weight.

Example 6·6. *A thin heavy uniform iron rod 50 cm. long is bent at the 30 cm. mark forming a right-angled L shaped piece. The bent rod is hung on a peg at the point of bend. What angle does the 30 cm. side make with the vertical when the system is in equilibrium ?*

Solution : Let B be the point at which the rod ABC is hung. Then $AB = 20$ cm. and $BC = 30$ cm. Let us suppose that the weight per unit length of the rod is w. The weights of the portions AB and BC act through their mid points. Thus taking moments about the point B, we get for equilibrium,



$$\Sigma L = 20w(x_1) - 30w(x_2) = 0$$

$$\text{or,} \quad 20w(10 \cos \theta) - 30w\,(15 \sin \theta) = 0$$

$$\text{or,} \quad \tan \theta = \frac{4}{9} \quad \therefore \quad \theta = 23{\cdot}9°.$$

Example 6·7. *A uniform ladder 10 ft. long and weighing 40 lb. rests with its upper end against a point on a smooth vertical wall 8 ft. above the ground, and its lower end on a rough road. Find (a) the reaction of the wall, (b) frictional force due to the road, (c) the reaction of the road.*

If the ladder is at the point of sliding find also (d) the co-efficient of static friction between the base of the ladder and the road.

Solution : In the adjoining diagram, AB is the ladder.

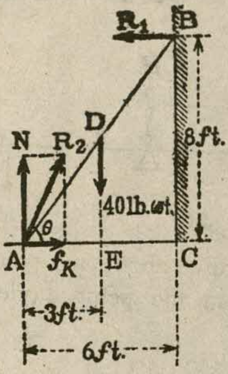
$\because \quad AB = 10$ ft. and $BC = 8$ ft.

$\therefore \quad AC = \sqrt{10^2 - 8^2} = 6$ ft.

The ladder is in equilibrium under the action of the three forces :

(1) The weight (40 lb. wt.) of the ladder acting vertically downwards through D, its mid-point,

(2) The reaction R_1 of the wall which is normal to the wall since it is smooth and hence has no friction.



(3) The reaction R_2 of the road at A. R_2 is composed of two forces :

(i) the normal reaction N perpendicular to the road,

(ii) the frictional force f_k, acting towards C to oppose the motion of the base of the ladder.

The ladder is in equilibrium. Hence,

(a) taking moments of the forces about A (This is advantageous, since the moments of the unknown forces N and f_k about A are zero).

$$\Sigma L = R_1 \times BC - 40 \times AE$$

$$= R_1 \times 8 - 40 \times 3 = 0 \left[\begin{array}{l} \because \ AD = \tfrac{1}{2} AB \\ \therefore \ AE = \tfrac{1}{2} AC \end{array} \right]$$

$$\therefore \quad R_1 = \frac{40 \times 3}{8} = 15 \text{ lb. wt.}$$

(b) $\Sigma F_x = f_k - R_1 = 0 \qquad \therefore \quad f_k = R_1 = 15$ lb. wt.

(c) $\Sigma F_y = N - 40$ lb. wt. $= 0 \quad \therefore \quad N = 40$ lb. wt.

$$\therefore \quad R_2 = \sqrt{N^2 + f_k^2} = \sqrt{40^2 + 15^2} = 42{\cdot}72 \text{ lb. wt.}$$

$$\text{and } \tan \theta = \frac{N}{f_k} = \frac{40}{15} = 2{\cdot}6667$$

$$\therefore \quad \theta = 69°27'$$

$\therefore$ The reaction of the road is 42·72 lb. wt. acting at 69°27′ to the road.

(d) At the point of sliding, $f_k = \mu N$

where μ = co-efficient of static friction between the base of the ladder and the road.

$$\therefore \quad \mu = \frac{f_k}{N} = \frac{15}{40} = 0{\cdot}375.$$

6·3. Resultant of Parallel Forces.

The resultant of a system of parallel forces cannot be found by using the parallelogram method, as such construction is not possible. The resultant can, however, be determined completely as follows* :

(i) Two like parallel forces : Let us consider two like parallel forces F_1 and F_2 [Fig. 6·6]. In order to find out their resultant, we remember that the sum of the components of the forces in any direction is equal to the component of the resultant in that direction. We take any arbitrary point O and draw a line Ox perpendicular to the lines of action of the forces. Let us also draw the line Oy normal to Ox. Oy is thus parallel to the direction of the forces. Since the two forces have no x-components their resultant R also must be so directed as to possess no component in x-direction and hence should be normal to Ox. The resultant of the two forces is, therefore, parallel to them. Taking component along the y-direction, the magnitude of the resultant is given by,

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*The method is not a rigorous one.

$$R=F_1+F_2 \qquad \dots \qquad \dots \qquad (6\cdot4)$$

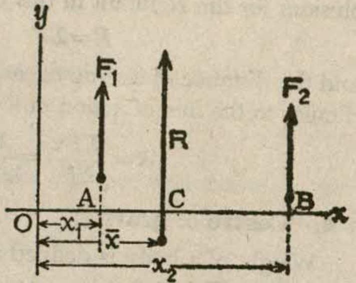


Fig. 6·6 : Resultant of two like parallel forces

If the perpendicular distances from O to the lines of action of F_1 and F_2 are given by x_1 and x_2 respectively, then the algebraic sum of their moments about O is given by $x_1F_1+x_2F_2$.

If $\bar{x}$ represents the perpendicular distance from O to the line of action of R, then the moment of R about O is $\bar{x}R=\bar{x}\ (F_1+F_2)$,

Since the moment of the resultant about any point equals the algebraic sum of the moments of the component forces, we have

$$\bar{x}(F_1+F_2)=x_1F_1+x_2F_2 \qquad \dots \qquad \dots \qquad (6\cdot5)$$

$$\text{or,} \quad \bar{x}=\frac{F_1x_1+F_2x_2}{F_1+F_2} \qquad \dots \qquad \dots \qquad (6\cdot6)$$

The resultant R is thus completely determined in magnitude, direction and line of action.

We can write equation (6·5) in a slightly different way.

Thus, $\quad F_1\ (\bar{x}-x_1)=F_2\ (x_2-\bar{x})$

or, $\quad F_1\times CA=F_2\times CB$

$$\text{or,} \quad \frac{F_2}{F_1}=\frac{CA}{CB} \qquad \dots \qquad \dots \qquad (6\cdot7)$$

i. e., the line of action of the resultant divides the line AB in the inverse ratio of the forces.

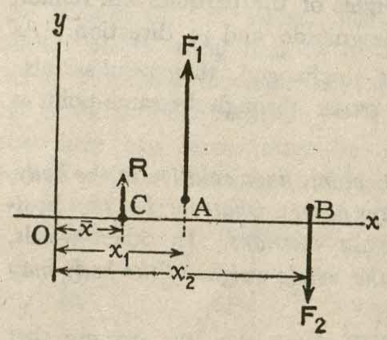


Fig. 6·7 : Resultant of two unlike parallel forces

(ii) Two unlike parallel forces : When the forces F_1 and $F_2 (F_1>F_2)$ are parallel and act in opposite sense [Fig. 6·7], we can proceed similarly and find that the resultant acts in the same sense as that of the large one and its magnitude is given by the difference of the magnitudes of the two forces. Thus

$$R=F_1-F_2$$

The line of action of R is given by the relation

$$\bar{x}=\frac{F_1x_1-F_2x_2}{F_1-F_2} \qquad \dots \qquad \dots \qquad (6\cdot8)$$

$$\text{or, by the relation,} \quad \frac{F_2}{F_1}=\frac{CA}{CB} \qquad \dots \qquad \dots \qquad (6\cdot9)$$

i. e., the line of action of the resultant divides the line AB *externally* in the inverse ratio of the forces.

(iii) **Any number of like parallel forces** : Arguments, similar to the finding of the resultant of two like parallel forces, lead us to the following conclusions for the resultant in this case. The magnitude of the resultant is

$$R=\Sigma F \qquad \ldots \qquad \ldots \qquad (6{\cdot}10)$$

and the distance of its line of action from a point O on a line Ox drawn perpendicular to the line of action of forces, is

$$\bar{x}=\frac{\Sigma Fx}{\Sigma F}=\frac{\Sigma Fx}{R} \qquad \ldots \qquad \ldots \qquad (6{\cdot}11)$$

6·4. Centre of gravity.

Weight of a body is defined as the force with which the earth attracts it (vide Art. 8·4). This definition, however, says nothing about the point of application of this force.

We can suppose a body to be built up of a large number of small particles, each having a definite weight w_1, w_2, w_3...etc. [Fig. 6·8]. Due to gravity each particle is attracted towards the centre of the earth with a force equal to the weight of the particle. The centre of the earth being at a very large distance, all the forces can be considered to be parallel to one another, i.e., they all act vertically downwards. The resultant of these parallel forces is the total weight W of the body, which acts vertically downwards at a definite point. This point is called the *centre of gravity* (written in abbreviated form as c. g.) of the body. This point is *fixed relative to the body* because if we turn the body to a new position, the weights of the particles will remain the same in magnitude and in direction. As the system of the parallel forces thus remains unchanged, their resultant also remains unchanged in every respect and so passes through the same point as before. We may therefore define that—

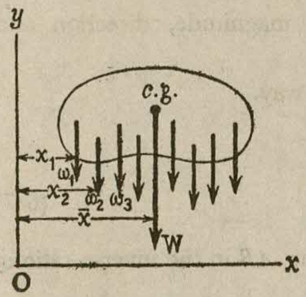


Fig. 6·8 : Centre of gravity of a body

The **centre of gravity** *of a body is that point, fixed relative to the body, through which resultant force due to gravity always passes, whatever be the position of the body, so long as its size and shape remain constant.* In other words, *the centre of gravity is the point at which the whole weight of the body may always be supposed to act.*

A body can not have more than one c. g. To prove this, we assume that if possible, let a body possess two centres of gravity, G and G'. From the definition of c.g. the line of action of the weight of the body should always pass through both G and G'. Since the weight always acts vertically downwards, this is not possible except for that particular position of the body when the line GG' is vertical. Hence the body can not have more than one centre of gravity.

How to find the position of the c. g. ? The position of the c. g. of a body can be determined analytically by adopting the procedure of finding the resultant of a system of parallel forces as discussed in the preceeding article. Let us consider the body in Fig. 6·8 as laminer and we imagine two mutually perpendicular axes Ox and Oy, the x-axis being horizontal. Then, if the coordinates of the particles of weights w_1, w_2......be (x_1, y_1), (x_2, y_2),......respectively, the total weight W of the body is given by,

$$W = w_1 + w_2 + \ldots\ldots = \Sigma w$$

and from eqn. (6·11) the x-coordinate of the c. g. is given by,

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + \cdots}{w_1 + w_2 + \cdots} = \frac{\Sigma wx}{\Sigma w} = \frac{\Sigma wx}{W} \quad \cdots \quad \cdots \quad (6{\cdot}12)$$

In a similar way by rotating the body so as to make the y-axis horizontal, we can find the y-coordinate of the c.g. as

$$\bar{y} = \frac{w_1y_1 + w_2y_2 + \cdots}{w_1 + w_2 + \cdots} = \frac{\Sigma wy}{\Sigma w} = \frac{\Sigma wy}{W} \quad \cdots \quad \cdots \quad (6{\cdot}13)$$

Centre of gravity of a regularly shaped body can, however, be located from symmetry considerations. For example, the centre of gravity of

(i) a uniform straight rod is at the middle point of its axis ;
(ii) a uniform circular lamina or sphere is at its geometrical centre ;
(iii) a uniform annular ring is at the centre of the ring ;
(iv) a uniform rectangular lamina is at the intersection of the diagonals ;
(v) a uniform triangular lamina is at the intersection of the medians [Fig. 6·9] etc.

It should be noted that the *c.g. of a body need not be in the body itself,* as is evident from the position of the c.g. of an annular ring.

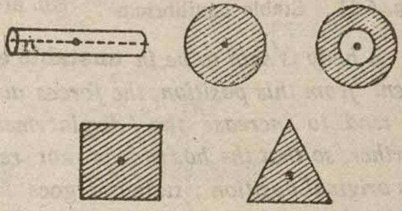
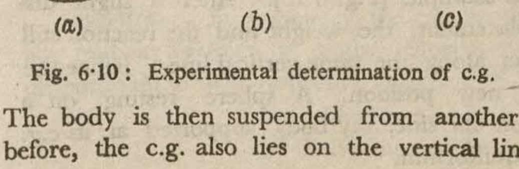


Fig. 6·9 : Centres of gravity of some regularly shaped bodies

If a body is suspended freely by a string, then it is in equilibrium under the action of two forces, namely, the tension in the string and its own weight acting vertically through the c.g. From the condition of equilibrium, it is obvious that these two forces must be equal and opposite and must have the same line of action. Thus for equilibrium, the point of suspension must lie on the vertical line through the c.g.



(a) (b) (c)

Fig. 6·10 : Experimental determination of c.g.

The c.g. of a thin sheet or a laminar body can be found experimentally as follows. The body is first suspended by a string from any point A [Fig. 6·10(a)]. When the body is in equilibrium, the vertical line through A is drawn. Obviously the c.g. must lie on this line. The body is then suspended from another point B [Fig. 6·10(b)]. Since as before, the c.g. also lies on the vertical line through B, it is, therefore, located at the point of intersection of this line and the previous one. If we now hang the body from a third point *C* it will be found that the vertical line through *C* passes through the same point [Fig. 6·10(*c*)], thus verifying that it is the c.g. of the body.

6·5. States of Equilibrium of a body.

As explained earlier, a body can remain in a state of equilibrium under the action of gravity, when the external force of reaction acts upward through its c.g. This state of static equilibrium (i.e., of rest) depends, among other conditions, upon the location of the centre of gravity and may be classified into three types, viz. *stable, unstable* and *neutral.*

A body is said to be in a **stable equilibrium** *position, when after it is slightly displaced from this position, the forces acting on it are such that the body tends to return to its original position.* A cone resting on its base provides an example of such equilibrium [Fig. 6·11]. When displaced slightly, the couple acting on the cone tries to restore it back to its original position. A block resting on its side, a weight hung by a string, a ball inside a bowl, the beam of a balance etc. are all examples of stable equilibrium.

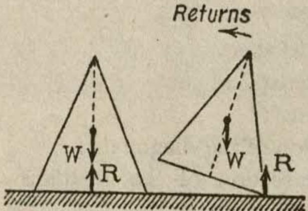


Fig. 6·11 : Stable equilibrium

A body is said to be in **unstable equilibrium**, *when after a slight displacement from this position, the forces acting on it tend to increase the displacement still further, so that the body does not return to its original position* ; rather it goes far away from the original position. Thus, if a cone resting on its apex is slightly disturbed [Fig. 6·12], the couple acting on it tends to increase the displacement ; the cone upsets and the equilibrium is said to be unstable. An egg resting on its end, a stick balanced on the finger, a ball resting on the top of an inverted bowl etc. provide some examples of unstable equilibrium.

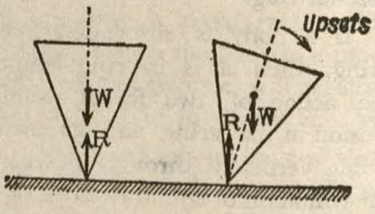


Fig. 6·12 : Unstable equilibrium

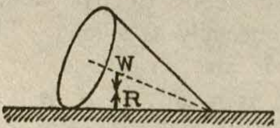


Fig. 6·13 : Neutral equilibrium

A body is said to be in **neutral equilibrium**, *when on being slightly displaced, the body remains in equilibrium also in the new position.* A cone resting on its side provides an example [Fig. 6·13]. After a slight displacement, the weight and the reaction still act along the same vertical line ; hence the cone remains in equilibrium in the new position. A sphere resting on a horizontal surface, a cylinder resting on its side, any body supported at its c.g. etc., offer some examples of neutral equilibrium.

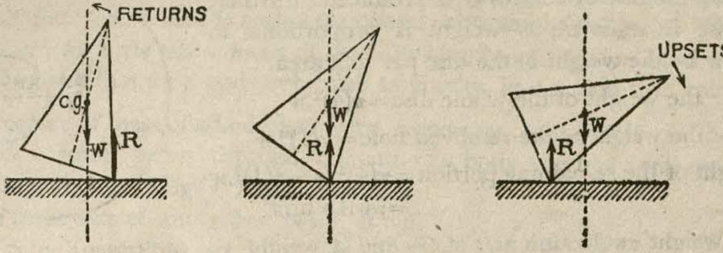
6·6. Static equilibrium and potential energy.

A careful examination of a body in stable equilibrium will show that in the displaced position of the body, its c.g. rises above the original position. Thus, displacement from the equilibrium position raises the gravitational potential energy (vide Art. 7·8) of the body. As the body returns to its original position, its c.g. comes down and the potential energy decreases. So, *when the c.g. occupies the lowest position or when the potential energy is minimum, the body is in stable equilibrium*. In the case of unstable equilibrium, a slight displacement lowers the c.g. and thereby lowers the potential energy of the body. Hence, *for a body in unstable equilibrium, its c.g. is at the highest possible position and the potential energy of the body is maximum*. The body, therefore, always tends to move to a more stable position. For a body in neutral equilibrium, its c.g. is neither raised nor lowered by a small displacement from this position i.e., the potential energy remains the same. The body thus remains stable in any position.

6·7. Limit of stability.

Let us consider the case of a cone resting on its base. When tilted slightly, the vertical through its c.g. passes through its base [Fig. 6·14(*a*)]. Its weight W and the reaction force R form a restoring couple and the equilibrium is stable.

If the cone is gradually tilted further on, a time will come when the vertical through its c.g. will pass through the supporting point of its base [Fig. 6·14(*b*)].



(*a*) (*b*) (*c*)

Fig. 6·14 : Limit of stability

Its weight W and the reaction force R then act along the same vertical line opposite to each other. This is called the state of *limiting equilibrium*.

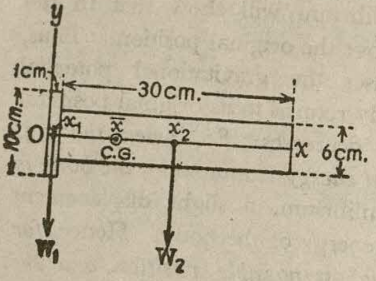
Any further tilt will cause the cone to fall over on its side. This is so because the vertical through the c.g. falls in this case outside its base [Fig. 6·14(*c*)]. The weight W and the reaction R then form a couple which upsets the cone.

Example 6·8. *A machine part constructed of a homogeneous material has two parts, one being a disc of 10 cm. diameter and 1 cm. thickness and another a rod 6 cm. in diameter and 30 cm. long. Find the centre of gravity of the machine part.*

Solution : By symmetry, the c.g. lies on the axis and the c.g. of each part lies midway between its ends. Moreover, the weights of both the parts are proportional to their respective volumes.

Now, the volume of the disc $=\pi(5)^2 . \ 1$ c.c.
and the volume of the rod $=\pi(3)^2 . \ 30$ c.c.

If the weight of the disc and the rod are W_1 and W_2 respectively, we have,



$$\frac{W_1}{W_2}=\frac{25\pi}{270\pi}=\frac{5}{54}.$$

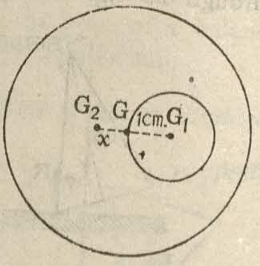
We consider a system of co-ordinates with origin at the point O on the left face of the disc and situated on its axis, which is taken as the x-axis. Then $x_1=0{\cdot}5$ cm. and $x_2=16$ cm. From equation (6·12) we get,

$$\bar{x}=\frac{W_1\times 0{\cdot}5+\frac{54}{5}W_1\times 16}{W_1+\frac{54}{5}W_1}=14{\cdot}7 \text{ cm.}$$

Thus, the c.g. of the machine part is on the axis at a distance of 14·7 cm to the right of O.

Example 6·9. *Calculate the position of the c.g. of a disc of uniform thickness and 8 cm. diameter in which a circular hole of 2 cm. diameter has been drilled, the circumference of the hole passing through the centre of the disc.*

Solution : In the figure, the c.g. of the whole disc is at G, its centre and the c.g. of the hole lies at its centre, G_1. Let G_2 be the c.g. of the disc after the drilling of the hole. Symmetry consideration shows that it must lie on the line of centres G_1G produced. Further, since the disc is uniform, its weight is proportional to area. Let w be the weight of the disc per unit area.



$\therefore$ the weight of the whole disc $=\pi(4)^2.w$

the weight of the removed hole $=\pi(1)^2.w$

$\therefore$ weight of the remaining portion $=\pi(4)^2.w-\pi(1)^2.w$

$=\pi w.15$ unit.

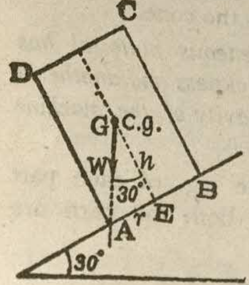
Hence, weight $\pi w.15$ unit acts at G_2 and a weight πw unit acts at G_1. But their resultant acts at G. $\therefore$ We have

$$\pi w.15\times GG_2=\pi w\times GG_1$$

But $GG_2=x$; $GG_1=1$ cm., $\therefore$ $x=\frac{1}{15}$ cm.

Thus the required c.g. is at G_2. $\frac{1}{15}$ cm. from the centre of the disc along the line of centres, in a direction away from the removed portion.

Example 6·10. *Calculate the maximum height of a uniform cylinder of diameter 8 cm. that can be placed on its base on a rough inclined plane of angle 30° without the cylinder toppling over.*



Solution : The cylinder $ABCD$ is placed on its base AB on the inclined plane. When its height is maximum, say $2h$, so that the cylinder just does not topple, the vertical through its c.g. G passes through the extreme point A of its base.

Then, $\angle AGE$ = angle of the plane $=30°$. From $\triangle$ AGE

$\frac{AE}{EG}=\tan 30°$; or, $\frac{r}{h}=\frac{1}{\sqrt{3}}$ $\therefore$ $h=\sqrt{3}\,r=4\sqrt{3}$

$\therefore$ the maximum height$=2h=8\sqrt{3}=13{\cdot}856$ cm.

6·8. Centre of Mass.

If a body be taken to a place where gravity does not exist, the concept of centre of gravity would have no meaning. Though the body would have no weight then, its mass would still exist. Since weight=mass × acceleration due to gravity (g), we note that the equations (6·12) and (6·13), can be written as follows in terms of the masses of the constituent particles,

$$\bar{x}=\frac{\Sigma mx}{M},\quad \bar{y}=\frac{\Sigma my}{M}\qquad \ldots \qquad \ldots \qquad (6{\cdot}14)$$

where $M=m_1+m_2+\ldots\ldots=\Sigma m=$Total mass of the body, m_1, m_2......being the masses of the constituent particles.

These equations always define a certain point, irrespective of the presence of the gravitational forces, and this point is called the *centre of mass* (abbreviated as c.m.) of the body.

Thus, *if we imagine a system of parallel forces acting on the particles in which a body may be supposed to be divided, the magnitude of the force on each particle being proportional to its mass, then the point at which the resultant of these parallel forces acts is called the* **centre of mass** *of the body. Hence, it is the point at which the whole mass of the body may be assumed to be concentrated.* It is to be noted that for a body subjected to gravity, its c.g. and c.m. coincide.

The centre of mass of a body has very important *properties*. If the line of action of a force passes through the c.m. of a body, the body will execute pure translational motion [Fig 6·15(a)]. If, however, the force acts at any other point, it will produce both translational and rotational motion of the body [Fig. 6·15(b)]. Hence in considering pure translational motion of a body, we can assume that (i) its whole mass is concentrated at its centre of mass and (ii) the resultant of all the external forces is being applied there. This explains why in our earlier discussions on motion of a body, we tacitly imagined it to be as a point mass, and the forces were taken to be acting at that point. Obviously this point is the c.m. of the body.

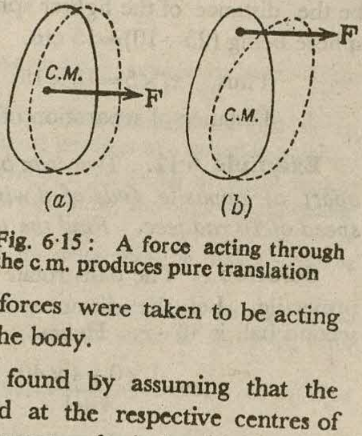


Fig. 6·15 : A force acting through the c.m. produces pure translation

The centre of mass of a system of bodies may be found by assuming that the masses of the constituent bodies are concentrated at the respective centres of masses and then, by applying relation (6·14). If the system be isolated i.e., if the net external force acting on the system be zero, the centre of mass of the system either remains at rest or continues to move with uniform velocity, though there may be internal forces acting on different bodies of the system due to internal causes. Thus, let us imagine an artillery shell moving along the path OA with uniform velocity v [Fig. 6·16]. At the point A, it explodes, the fragments flying off in all directions (only two are shown in the figure). However, the centre of mass of all these fragments would continue to move along the same path with the same velocity v as if nothing had happened.

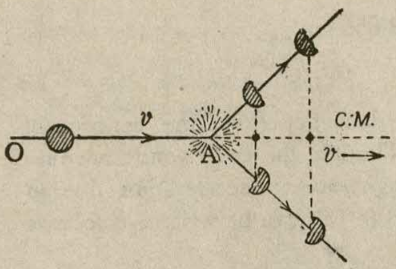
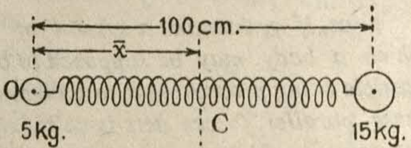


Fig. 6·16 : Motion of the centre of mass

Example 6·11. *Two spheres of mass 5 kg. and 15 kg. respectively are connected by a light spring. The spring is stretched until the particles are 100 cm. apart. (i) Find the position of the centre of mass of the system as referred to the heavier sphere ; (ii) When the spheres are released, find the separation of the pair after the heavier sphere has moved a distance of 10 cm. from its rest position.*

Solution : (i) Let the c.m. of the system consisting of the two spheres be located at C, at a distance $\bar{x}$ from O, the position of the lighter sphere.



Then from eqn. (6·14), the position of the c.m. is given by

$$\bar{x}=\frac{5\times 0+15\times 100}{15+5}=\frac{1500}{20}=75 \text{ cm.}$$

Hence, the c.m. is situated at a distance of $(100-75)=25$ cm. from the heavier sphere.

(ii) On getting released, the two spheres move towards each other under the action of the spring. Since no external force acts on the system, the c.m. of the system will remain at the same position. Measuring distance from the cm., let x_1 be the distance of the lighter sphere from the c.m., the distance of the heavier sphere being $(25-10)=15$ cm.

Thus, $x_1\times 5=15\times 15$ or, $x_1=45$ cm.

$\therefore$ Distance of separation of the pair$=45+15=60$ cm.

Example 6·12. *Two iron balls of masses 1 kg. and 4 kg. are tied 40 cm. apart at opposite ends of a wire and thrown whirling into the air with an angular speed of 10 rad./sec. Find the tension in the wire.*

Solution : The balls rotate about their centre of mass which travels like a projectile. Let the distance of the first ball from the c.m. be r ; so that of the second ball is $40-r$. Hence

$$r=\frac{1\times 0+4\times 40}{4+1} \quad \text{or,} \quad 5r=160 \quad \therefore \quad r=32 \text{ cm.}$$

Hence, tension $T=m\omega^2 r=1\times 10^2\times \frac{32}{100}$ newton$=32$ newton.

● EXERCISE ●

[A] Essay type questions

1. Define equilibrium of a body. State and explain the conditions under which forces acting on a body maintain it in equilibrium. Why is one called the conditions of translational equilibrium and the other the condition of rotational equilibrium ?

2. What is polygon of forces ? Show that this expresses the first condition of equilibrium. A ball thrown vertically upwards is momentarily at rest when at the top of its path. Is it in equilibrium at that instant ?

3. What is the condition for equilibrium of a body under three forces ? State and prove the triangle of forces rule.

4. State and prove Lami's theorem. Three equal forces acting at a point are in equilibrium. Show from Lami's theorem that they are equally inclined to one another.

5. What do you mean by like and unlike parallel forces ? Find the resultant of two like parallel forces and its line of action.

Can there be a single force whose effect on a body is equivalent to the joint effect of two unlike parallel forces ? Explain your answer.

6. Explain the meaning of the term 'Centre of gravity.' Can a body possess more than one centre of gravity ? Give reasons.

7. What are meant by stable, unstable and neutral equilibrium ? Explain with examples. What part does the centre of gravity play in defining the nature of equilibrium ? Explain your answer with suitable illustrations.

8. How is the static equilibrium of a body related to its potential energy ? Give illustrations.

9. A body is held on an inclined plane. What factors would determine whether it would remain at rest or get upset ? Explain your answer.

10. Show that a body freely suspended from a fixed point rests with its centre of gravity vertically below the point of suspension.

How is the centre of gravity of a laminar body determined ?

11. What is 'centre of mass' ? What is its difference with centre of gravity ? When do centre of gravity and centre of mass of a body coincide ? State the importance of centre of mass.

[B] Short answer type questions

1. 'It is possible that the centre of gravity of a body may lie outside the body'—Is the statement correct ? If so explain how it is possible.

2. Show that for a body to be in equilibrium under the action of three non parallel forces, (i) these forces must be coplanar, (ii) their lines of action must intersect at a common point.

3. Three forces are acting on a body simultaneously, two of them being of equal magnitude. If these two forces act in opposite directions at different points, the body cannot remain in equilibrium. Why ?

4. 'For a body executing a pure translational motion its mass may be assumed to be concentrated at its centre of mass'. Explain.

5. If a force acting on a body be so directed that its line of action passes through (i) the centre of mass of the body, (ii) any point other than the centre of mass of the body, what would be the nature of motion of the body in each case ?

6. If the two arms of a common balance are unequal, how can you determine the true mass of a body with it ?

7. A tradesman uses a false balance. The lengths of the two arms of the balance are l_1 and l_2 respectively. He weighs out two customers W lb. of tea as indicated by his balance. But in serving one of the customers he puts the weights in one pan while in serving the other he puts them on the other pan. How much does he gain or lose by this ?

[*Model Questions, H. S. Council*]

8. Would the centre of gravity and the centre of mass of the Kutub Minar be identical ? Justify your answer.

9. A ping-pong ball is floating on the top of a vertical water jet. Is it in stable, unstable or in neutral equilibrium in the vertical direction ? [*I. I. T.* '73]

10. A man is standing against a wall with his right leg and right shoulder in contact with the wall. If he raises his left leg, then in doing so can he remain in equilibrium ? If not, in which direction he would lean ?

11. Explain the following phenomena :—

(i) It is dangerous to stand in the upper deck of a double decker bus.
(ii) It is preferable to stand on two legs than merely on one leg.
(iii) There is a type of doll that immediately returns to the sitting position if left in a lying position.
(iv) It is better to carry load in two hands, than in one hand.
(v) While an acrobat walks on a rope, he carries a long rod in his hand.
(vi) A plumb line is used for determining the vertical.
(vii) When a rod is suspended from its centre of gravity it remains in neutral equilibrium.
(viii) A stick balanced on the finger remains in unstable equilibrium.
(ix) The centre of gravity is fixed whatever may be the position of the body.

[C] Simple Problems.

1. A 60 kg. load is hung from a steel wire. (a) Determine the tension in the rope if the load is held to one side with a horizontal force of 30 kg. wt. (b) If the wire is 130 cm. long what horizontal force is required to hold the load at a distance of 50 cm. from the vertical line through the point of support. [*Ans.* (a) 67·08 kg. wt. (b) 25 kg. wt.]

2. Two ropes *X* and *Y* suspended from the ceiling of a room holds a 50 kg. body. Rope *X* makes an angle of 30° with the horizontal and the rope *Y* makes an angle of 60° with the horizontal. Find the tension in each rope. [*Ans.* 43·3 kg. wt ; 25 kg. wt.]

3. An electric light fitting is supported in a room by means of wires respectively 4 ft. and 7 ft. long attached to nails in the same horizontal plane in two opposite walls, the walls being 9 ft. apart. If the fitting weighs 10 lb., find the tension in each wire. [*Ans.* 6·96 lb. and 9·44 lb.]

4. A light rod *AB* of length 12 ft. is hinged at *A* to a wall and carries a load 100 lb. at the other end *B*. The rod is kept horizontal by means of a rope one end of which is attached to the end *B* and the other end attached to a point on wall 9 ft. vertically above the hinge. Find the tension in the rope and the push of the rod. Neglect the weight of the rod.
[*Ans.* 166·67 lb. wt. and 133·33 lb. wt.]

5. A uniform steel bar 5 ft. long weighing 60 lb. is hinged at one end to a vertical wall and supported by a wire from the other end to a point on the wall 12 ft. vertically above the hinge. Find the tension in the wire and the reaction of the hinge.
[*Ans.* 32·5 lb. wt. ; 32·5 lb. wt. directed 67°23′ with the horizontal]

6. An advertising sign weighing 20 lb is suspended at the end of a horizontal bar weighing 5 lb. and 6 ft. long. The bar is hinged to a vertical wall at the other end and is kept horizontal by a wire fastened to its mid-point and to a point on the wall 4 ft. above the hinge. Calculate the tension in the wire. [*Ans.* 56·25 lb. wt.]

7. A uniform plank 4 m. long and weighing 40 kg. is supported horizontally by two vertical ropes hung from its ends. Find the tension in each rope when a 60 kg. painter stands 1 m. from one end. [*Ans.* 65 kg. wt. ; 35 kg. wt.]

8. A metre scale balances about its mid-point. A weight of 60 gm. is hung at 25 cm. mark. What weight must be hung from 65 cm. mark for the scale to be in equilibrium ?
[*Ans.* 100 gm.]

9. Two men carry a block of stone of weight 150 kg. on a light plank. How must the block be placed so that one of the men should bear a weight 100 kg. more than the other ?
[*Ans.* The stone must be placed on the plank at a point dividing it in the ratio 1 : 5]

10. A light horizontal plank of length 3 m. on which is placed a load of 15 kg. at a point 1 m. from one end, rests on supports at its ends. If the load be removed from its position and placed at the middle of the plank, find by how much the pressure on each support is altered.
[*Ans.* 2·5 kg.]

11. If three men support a heavy triangular board of weight W at its three corners, how much weight is supported by each man ? [*Ans.* Each man supports a weight $\frac{1}{3}W$]

12. A V-shaped trough whose sides form an angle of 60° contains a sphere weighing 200 gm. What is the normal force exerted by the sphere on each side of the trough ?

[*Ans.* 200 gm. wt.]

13. A ladder AB 13 ft. long and weighing 90 lb. with its centre of gravity G $\frac{2}{5}$ up, rests with one end A on the ground and the other end B 12 ft. above the ground and against a smooth vertical wall. Find (a) the reaction of the wall (b) the frictional force due to the ground (c) the reaction of the ground.

If the ladder is at the point of sliding, find also (d) the co-efficient of static friction between the base of the ladder and the ground.

[*Ans.* (a) 15 lb. wt. (b) 15 lb. wt. (c) 91·24 lb. wt. at 80°36′ with the ground (d) 0·167]

14. A uniform ladder weighing 50 lbs. and of length 24 ft. rests on a rough ground against a smooth vertical wall and makes an angle of 60° with the horizontal. A man weighing 120 lb. is on the ladder at a point 16 ft. from its base when the ladder begins to slip. Find the co-efficient of friction between the base of the ladder and the ground. [*Ans.* 0·3566]

15. A force P acting parallel to the length and a force Q acting parallel to the base of an inclined plane are able while acting singly to support a weight W on the plane. R and S are respectively the forces of reaction acting on the weight. Prove that

$$\frac{1}{P^2}-\frac{1}{Q^2}=\frac{1}{W^2} \text{ and } RS=W^2$$

16. An object is weighed on a balance whose pans are not equal in weight. When placed in the left hand pan, the object appears to weight 10·2 gm. But when placed in the right hand pan it appears to weight 10·42 gm. Find : (a) the correct weight of the object ; (b) the difference in the weights of the pans. (*Cambridge Univ.*) [*Ans.* (*a*) 10·31 gm. (*b*) 0·11 gm.]

17. A radio antenna 1 m. long consists of 4 uniform tubes joined together. The rods are each 25 cm. long and weigh respectively 30 gm., 40 gm., 50 gm. and 60 gm. Find the distance of its centre of gravity from one end. [*Ans.* 56·945 cm. from 30 gm. end]

18. A disc of uniform density and radius 20 cm. has a circular hole of diameter 20 cm. whose two extremities touch the circumference and the centre of the disc. Find the centre of mass. [*Ans.* 3·33 cm. from the centre of the disc away from the centre of the hole]

19. A non-uniform rod remains horizontal when a load of 10 kg. is applied at its one end and the fulcrum is placed 2 m. away from that end. The rod is also found to remain horizontal when the load at the same end is 4 kg. and the fulcrum is 3 m. away. Find the weight of the rod and the position of its centre of gravity. [*Ans.* 8 kg. ; 4·5 m. from the loaded end]

20. Prove that the c. g. of three particles of equal masses placed at the vertices of a triangle is the same as that of the triangle.

21. Two cylinders of the same material and of the same length 10 cm. are joined together to have a common axis. Their diameters are 6 cm. and 3 cm. respectively. Find the c. g. of the combination. [*Ans.* On the axis, 2 cm. from the c. g. of the first]

22. A cylinder of diameter 4 cm. and height 16 cm. is placed on its base on a rough horizontal board. The board is then slowly titled. Find the inclination of the board to the horizontal when the cylinder just gets upset. [*Ans.* 14°2′]

23. Two bodies of mass 2 gm. and 8 gm. respectively are placed 15 cm. apart. Find their centre of mass. [*Ans.* 3 cm. from the second body]

24. Two skaters of masses 60 and 80 kg. respectively, stand facing each other 7 m. apart. They then pull, hand over hand, on a light rope stretched between them. How far has each moved when they meet ? Neglect friction. [*Ans.* 4 m. and 3 m.]

25. Show that the centre of mass of two particles is on the line joining them at a point whose distance from each particle is inversely proportional to the mass of the particle.

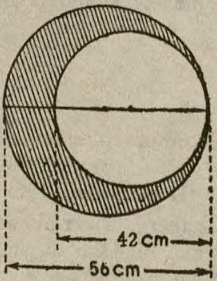
26. The distances between the centres of the earth and the moon is about 60 times the radius of the earth. The earth is about 76·923 times massive than the moon. Find the distances of the centre of mass of the earth-moon system from the centre of the earth. Given, radius of the earth $=6{\cdot}4\times10^6$ m. [*Ans.* $4{\cdot}93\times10^6$ m]

27. Two bodies initially at rest, move towards each other under a mutual attraction. Find the speed of the centre of mass of the system at the instant when the speed of A is V and the speed of B is $2V$. [*Ans.* Zero]

28. A dishonest grocer uses a false balance having ratio of the arms 6 : 7. He weighs out to his customer 7 kg. of sugar by the balance to his own advantage. But at this stage the customer detects the fraud and the grocer now agrees to weigh out another lot of 7 kg. with the same balance using into the customer's advantage. Is the compensation exact ? If not, who gains and by how much ? [*Ans.* No ; Customer gains by 167 gm.]

29. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. Find the minimum value of F for which the cube begins to tip about an edge. Assume that the cube does not slide. [*Ans.* $\frac{2}{3}mg$]

30. A car weighing 4000 kg. has a wheel base (distance between front and back axles) of 3·6 m. Its c. g. is located 2 m. behind the front axle. Determine the force exerted on each of the front wheels (assuming equal) and the force exerted on each of the back wheels (also assuming equal). [*Ans.* 1111 kg. wt. ; 889 kg. wt.]



31. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of 42 cm. is removed from one edge of the plate as shown in the figure.

Find the position of the centre of mass of the remaining portion. (*I. I. T. '80*)

[*Ans.* 9 cm. from the centre of the plate on the line joining the centre of the plate and the centre removed portion on the side opposite to the centre of the removed portion]

32. A passenger of weight w moves a distance a from the front to the rear of a bus. Show that a weight wa/b is transferred from the front springs to the back springs where b is the distance between the axles (i. e., the springs).

[D] Harder Problems

1. A uniform timber is carried by three men, one is holding the timber at one end and the other two support it by means of a cross-bar placed underneath. At what point of timber must the bar be placed so that each may carry one-third of the weight of the timber ? [*Ans.* $\frac{1}{4}$ length from free end]

2. A body weighing 10 kg. is supported by two strings attached to it. If the direction of one string be at 30° to the vertical, find the direction of the other in order that its tension may be as small as possible. Find also the magnitude of the tensions in the two strings in this case. [*Ans.* 60° to the vertical ; $5\sqrt{3}$ kg. wt. ; 5 kg. wt.]

3. A thin heavy uniform rod 1 m. long has a right-angle bend at a point 20 cm. from one end and can turn freely in a vertical plane about a pivot at the bend. When a load of 3 kg. is attached to the rod at the end of the shorter arm, the bend rod comes to rest with the shorter arm making 45° with the horizontal. Determine the weight of the bar. [*Ans.* 2 kg.]

4. The lengths of the left and right arms of the beam of a balance are 10 cm. and 10·5 cm. respectively and the beam balances when no scale pans are attached. The beam also balances when two scale pans of unequal weights are attached. Now, an object is put on the right hand pan and the beam is horizontal with a 100 gm. weight on the left hand pan. What is the true weight of the object. [*Ans.* 95·23 gm.]

5. A uniform plank AB, 5 m. long and weighing 50 kg. rests on two supports, one 1·5 m. from A and the other 2 m. from B. A man weighing 60 kg. is moving about on the plank. How near he could approach to A and B respectively without upsetting the plank ? [*Ans.* $\frac{2}{3}$ m. from A ; $1\frac{7}{12}$ m. from B]

6. A balloon just remains on the ground when carrying a load of 100 kg. After being unloaded it rises being tied to the end of a cable and finally remains steady with the cable at an angle 30° to the vertical. If the balloon is acted upon by a horizontal wind, find the tension in the cable and the force of the wind on the balloon. [*Ans.* 115·4 kg. wt. ; 57·7 kg. wt.]

7. A uniform plank 6 metre long and weighing 60 kg. leans against a smooth vertical wall and stands on the level concrete floor having a co-efficient of sliding friction equal to 0·3. Find the distance of the plank from the wall at which it will just start to slip.

(*Jt Entrance '79*) [*Ans.* 3·09 m.]

8. A uniform ladder 5 m. long and weighing 20 kg. rests with one end on the rough ground and the other end against a smooth wall. The ladder makes an angle of 60° with the ground, and the co-efficient of static friction between the base and the ladder is 0·25. How far can a 50 kg. boy go up the ladder before it begins to slip ? [*Ans.* 2·03 m.]

9. A paper covered book, 10 cm. broad when closed has 200 pages, that is 100 leaves, including the cover. Show that as each page is turned by a reader, the centre of gravity of the book shifts one millimetre to the left. [*Oxford Univ.*]

10. A metal square of uniform thickness and side 18 cm. is divided into four equal triangles by drawing the diagonals and one of the triangles is then cut out. Find the position of the c. g. of the remainder. [*Ans.* 2 cm. away from the vertex of the removed triangle]

11. A square plate of uniform thickness and side 12 cm. has one corner removed along the line joining the middle points of two adjacent sides. Find the c. g. of the remainder.

[*Ans.* On the diagonal and 0·808 cm. from the centre of the original square]

12. Three masses 2 gm., 3 gm., and 5 gm. are situated at the corners of an equilateral triangle with sides 5 cm. long. Find the centre of mass.

[*Ans.* 3·5 cm. and 2·18 cm. from 2 gm. and 5 gm. masses respectively]

13. A string 120 cm. long passes through a smooth ring of mass 100 gm. The ends of the string are attached to two fixed points *A* and *B* 60 cm. apart at the same level. The ring is now acted upon by a horizontal force *F* which holds it in equilibrium vertically below *B*. Find the tension in the string and the magnitude of *F*. [*Ans.* 62·5 gm. wt. ; 50 gm. wt.]

14. A table has a heavy circular top of radius 1 m. and mass 20 kg. It has four legs of length 1 m. fixed symmetrically on its circumference.

(i) What is the maximum mass that may be placed *anywhere* on the table without toppling the table ?

(ii) What is the area of the table-top over which any weight may be placed without toppling it ? (*I. I. T. '74*) [*Ans.* 48·31 kg ; 2 m^2]

15. A length 3 m. of a uniform plank 10 m. long and weighing 70 kg. projects over the side of a quay. What minimum weight must be placed on the end of the plank so that a man weighing 70 kg. may be able to walk on the other end without the plank tipping over ?

[*Ans.* 10 kg.]

16. Two uniform beams *AB* and *BC* each of length *L* and mass *M* gm. are hinged smoothly together at *B*, and a light inextensible string of length $L/\sqrt{2}$ is tied between their midpoints *D* and *F*. The beams stand in a vertical plane with points *A* and *C* resting on a smooth horizontal plane.

(a) Show all the forces acting on both rods.

(b) Calculate the tension in the string. (*I. I. T. '71*) [*Ans.* (b) *Mg*]

17. A smooth fixed hemispherical bowl with smooth horizontal rim contains a particle of mass 90 gm. which is attached to another particle of mass 15 gm. by means of a light inextensible string passing over the rim of the bowl and hanging vertically. Prove that in the position of equilibrium the radius through the first particle makes an angle $\sin^{-1}(\frac{1}{8})$ with the vertical.

7

CHAPTER

WORK, ENERGY AND POWER

7·1. The concept of work.

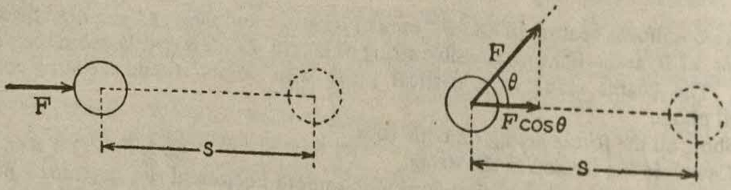
The term *work*, commonly used in connection with widely different activities, is restricted in physics to cases in which a force acting on a body causes displacement to that body.

Let us consider the following cases. Suppose a man lifted a suitcase and held it on his shoulders for a long time. The physicist will argue that he did work in lifting the suitcase, for he applied force against gravity causing the displacement of the suitcase from the ground upto the height of his shoulder. But though he might become very tired holding it for a long time on his shoulders, in the scientific sense, however, he did not perform any work, since there was no displacement of the suitcase. Imagine the amazement on the tired face of the greek God Atlas, who spent his time supporting the world on his shoulders, when a scientist tells him that he is not at all doing any work, because he is exerting only an upward force on the world without moving it.

Work *done is measured by the product of the force and the corresponding displacement along the direction of the force.* Thus, if a force F acts on a body and if the body undergoes a displacement s in the direction of the force [Fig. 7·1 (*a*)], the work done

$$W=Fs \qquad \ldots \qquad \ldots \qquad (7{\cdot}1)$$

In general, however, the line of action of the force, and the direction of displacement do not coincide. In that case, the **work** *done is defined as the product of the magnitude of the displacement by the component of the force in the direction of the displacement.* If under the action of a force F, a body undergoes a displacement s at an angle θ with the line of action of the force [Fig. 7·1 (*b*)], the work done is



(*a*) (*b*)

Fig. 7·1 : Work done by a force

$$W=(F\cos\theta)s \qquad \ldots \qquad \ldots \qquad (7{\cdot}2)$$

Obviously, when the force and the corresponding displacement are in the same direction, i.e., when $\theta=0$, this expression reduces to the previous one.

On the other hand, if $\theta=90°$, $\cos\theta=0$ and W becomes equal to the zero. No work is, therefore, done when the displacement occurs at right angles to the applied force.

Although work is the product of two vector quantities, force and displacement, it is itself a scalar quantity. This provides an example of the product of two vectors being a scalar.

Work done by a system of forces : If a body is acted upon by several forces, then the work done by the system of forces is the sum of the individual work contributions by the various forces acting on that body. This can be shown to be equal to the work done by the resultant of this system of forces.

"No-Work" forces : It frequently happens that one or more of the system of forces is perpendicular to the displacement of the body. For example, the force of gravity mg and the normal reaction N on the block B moving in a horizontal plane [Fig. 7·2] do not influence its motion. Hence they do not contribute anything to the work done. The reason is very simple. As the block moves horizontally, the components of the gravitational force and of the reaction along the direction of displacement become zero because for both of them $\theta=90°$. These forces are, therefore, said to be 'no-work' forces. "**No-work**" *forces, then, are those which are perpendicular to the direction of motion of a body and so do not contribute anything to the work done in such motion.*

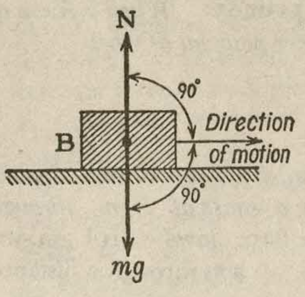


Fig. 7·2 : No-work forces

Work done along a curved path : In the most general case of a body moving in a curved path under the action of a variable force, we should divide the whole path into a large number of very small intervals $\Delta s_1, \Delta s_2 \ldots \Delta s_n$, which can be regarded practically as straight lines [Fig. 7·3]. Since the intervals are very small, the force along each of them may be taken as constant in direction and magnitude. Thus if $F_1, F_2 \ldots F_n$ give the magnitudes of the forces for the corresponding intervals, and $\theta_1, \theta_2 \ldots \theta_n$ be the angles made by the respective forces with the path intervals, the total work done is given by,

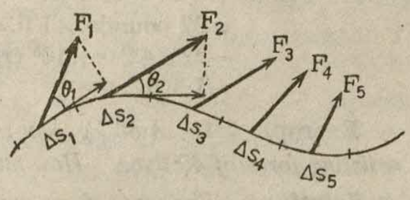


Fig. 7.3 Work done along a curved path

$$W=F_1\Delta s_1 \cos\theta_1+F_2\Delta s_2 \cos\theta_2+ \quad \ldots \quad +F_n\Delta s_n \cos\theta_n$$

$$=\sum_{k=1}^{n} F_k\Delta s_k \cos\theta \quad \ldots \quad \ldots \quad (7{\cdot}3)$$

If the number of intervals is increased indefinitely, each interval being made infinitesimally small, this process of summation becomes equivalent to the process of integration and the total work done can be expressed as,

$$W=\int F\cos\theta\, ds$$

7·2. Units of work.

(a) The Absolute Units : Unit work is done when unit force produces unit displacement along its line of action. The C.G.S. absolute unit of work is the **erg** and is defined as *the work done when a force of 1 dyne acts through a distance of 1 cm.*

Since erg is a very small unit, the practical unit is taken to be **Joule**, where 1 Joule$=10^7$ erg.

In M.K.S. system, newton-metre (n.m) is the absolute unit and is defined as the amount of work done when a force of 1 newton acts through a distance of 1 metre.

$\therefore$ 1 newton-metre$=$1 newton $\times$ 1 metre
$=10^5$ dyne $\times\, 10^2$ cm.$=10^7$ erg$=$1 Joule.

In F.P.S. system, the absolute unit of work is **foot poundal**. *When a force of 1 poundal acts through a distance of 1 foot, it does 1 foot poundal of work.*

$\therefore$ 1 foot poundal$=$1 poundal $\times$ 1 ft.
$=13825$ dyne $\times\, 30{\cdot}48$ cm.
$=4{\cdot}214\times10^5$ erg (approx.)

(b) The Gravitational Unit : In C.G.S. system, gravitational unit of work is **gm. cm.** and is defined as *the work done in lifting a mass of 1 gm. through a height of 1 cm. on the earth's surface.* Since the force *involved* is 1 gm.-wt. 1 gm. cm. of work is done when a force of 1 gm. wt. acts through a distance of 1 cm.

In F.P.S. system, the corresponding unit is **foot pound**, and it is *the amount of work done in raising a mass of 1 lb. through a height of 1 ft. on the earth's surface* i.e., it is equal to the work done by a force of 1 lb. wt. acting through a distance of 1 ft.

$\therefore$ 1 ft. lb.$=$1 lb. wt. $\times$ 1 ft.
$=32$ poundal $\times$ 1 ft.$=32$ ft. poundal
$=32\times4{\cdot}214\times10^5$ erg$=1{\cdot}356\times10^7$ erg
$=1{\cdot}356$ Joule

Example 7·1. *A 60 kg. box is pushed 200 cm. across a floor against a resisting force of 40 dyne. How much work is done ?*

Solution : The work done against the force of friction of magnitude 40 dyne in producing a displacement of 200 cm. is given by eqn. (7·1) as,

$$W=F.s=40\times200 \text{ dyne}\times\text{cm.}=8\times10^3 \text{ erg.}$$

The force due to the weight of the body, being perpendicular to the direction of the displacement of the box, is a 'no-work' force. Hence this force does not contribute anything to the work done.

Example 7·2. *A block of ice is drawn through a distance 500 cm. along a smooth horizontal surface. The pull in the rope is 100 dyne and the angle between the rope and ground is 30°. Find the work done.*

Solution : Here $F=100$ dyne, $\theta=30°$ and $s=500$ cm.

Then applying eqn. (7·2), we get,

$W=$ Component of the force parallel to the displacement × displacement
$=(F\ \cos\theta)\times s=(100\ \cos\ 30°)\times 500=43300$ erg.

7·3. Work done by and against force.

Work is said to be done *by a force* when its point of application moves in the direction of the force. When a body falls from the top of a tower, work is done by the force of gravity since the direction of the force and that of motion are in the same sense. This work is also termed as ***'negative-work.'***

On the contrary, work is said to be done ***against a force,*** when an external agent causes a body to move against the direction of the force acting on that body. For example, when a man raises a suitcase against its weight he does work against that force. Dragging a body on a rough horizontal surface also provides another example of work done against a force, since frictional force always acts opposite to the direction of motion. This type of work is also called ***'positive-work'***.

7·4. Graphical representation of work.

If the component of the force in the direction of displacement, is plotted against displacement, the area under the curve will represent the work done by the force. Thus, if under the action of a constant force, a body gets displaced from x_1 to x_2, the curve will be a straight line parallel to the x axis (shown in the Fig. 7·4 by a heavy line). The area of the rectangle formed by the line and the x-axis and the two ordinates at x_1 and x_2, evidently gives the work done in this process.

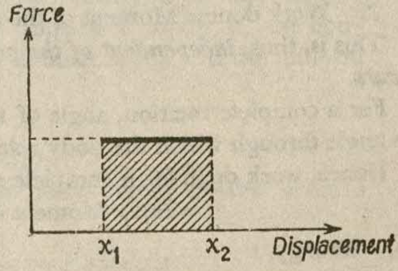


Fig. 7·4 : **Diagram of work done by a constant force**

If, however, the force varies continuously, the curve may be of any shape [Fig. 7·5]. The above assertion is still true. We consider a very small displacement Δx of the body, such that the variable force F remains practically constant during such displacement. The product $F\Delta x$ which gives the work done during the displacement Δx, represents the area of the horizontally shaded strip. We can imagine the displacement from x_1 to x_2 as made up of a very large number of such small displacements Δx, for each of which the product $F\Delta x$ can be computed. The total work done can be obtained by summing over all such elements and is, thus, given by the area included between the curve and x axis, and the two ordinates at x_1 and x_2.

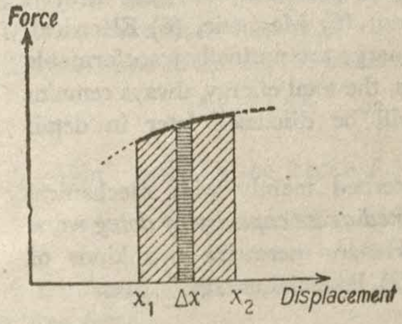


Fig. 7·5 : Diagram of work done by a varying force

7·5. Work done by a couple.

Let a couple P, P act on a body as shown in the Fig. 7·6. If AB be the perpendicular distance between the lines of action of the forces, the moment of the couple is

$$P \times AB = P \times a, \qquad \text{where } AB = a.$$

Let us suppose that this couple causes the body to rotate through a small angle $\Delta\theta$ about an axis through O, perpendicular to the plane of the paper. The points A and B describe the arcs AC and BD in this process. Evidently, $AC = OA.\Delta\theta$ and $BD = OB.\Delta\theta$.

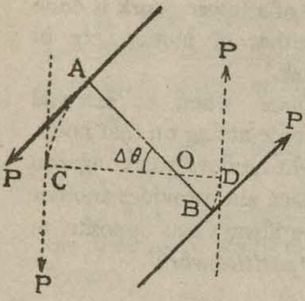


Fig. 7·6 : Couple performing work

Since $\Delta\theta$ is small, we can regard the arcs described, as practically straight. Hence, work done by the force P acting at A=force × displacement$=P.AC=P.OA.\Delta\theta$.

Similarly, the work done by the force P acting at $B=P.OB.\Delta\theta$.

∴ The total work done by the couple as the body rotates under its action through an angle $\Delta\theta = P.OA.\Delta\theta + P.OB.\Delta\theta = P(OA+OB).\Delta\theta = P.AB.\Delta\theta = P.a.\Delta\theta$ = Moment of the couple × angle of rotation.

∴ Work done = Moment of the couple × angle of rotation.

This is, thus, *independent of the position of the point O about which rotation occurs.*

For a complete rotation, angle of rotation$=2\pi$; so for n complete rotations, the angle through which the body rotates$=2\pi n$.

Hence, work done for n complete rotations

$$= 2\pi n \times \text{Moment of the couple} \qquad \cdots \qquad \cdots \qquad (7{\cdot}4)$$

7·6. Energy.

Energy is stored up work. A body is said to possess energy when it is capable of doing work. Thus, *energy is measured by the total quantity of work that a body or a system of bodies can do.*

Different types of energy can be broadly classified under the following heads : (1) Mechanical, (2) Heat, (3) Light, (4) Sound, (5) Magnetic, (6) Electrical, (7) Chemical, (8) Nuclear. All these forms of energy are mutually transformable from one to other and in such transformations, the total energy always remains constant. This law of conservation of energy will be discussed later in detail [vide Art. 7·13].

In the following sections, we shall be concerned mainly with mechanical energy. **Mechanical energy** *of a body is defined as its capacity of doing work by dint of its motion, configuration or both.* Hence, there are two kinds of mechanical energy, viz. (1) Kinetic energy and (2) Potential energy.

7·7. Kinetic energy.

Kinetic energy of a moving body is defined as the energy it possesses by virture of its motion. The work done by a moving body against external impressed *forces, before coming to rest gives the measure of its kinetic energy.* Hence, kinetic energy is expressed in the same units as those of work. The placid mass of water in the reservoir of a hydel power project is itself unable to drive a dynamo ; but water gushing out of the bottom of the reservoir with a high velocity easily drives a dynamo. Due to the high velocity, the outflowing water has high kinetic energy, which is then used up in driving the dynamo.

A hammer driving a nail into some substance (e.g. wood) provides a good example of kinetic energy possessed by a body. It is a common experience that in order to drive the nail, the hammer must be struck on it with a high velocity ; the same hammer left simply in touch with the nail for eternity would not produce any effect. The kinetic energy of the hammer is thus used up in over-coming the resistance of the material to the nail.

As the motion of a body may be of two types viz., translatory and rotatory, so also its kinetic energy may be either translational or rotational or both. Example of translational kinetic energy is that of a freely falling body, while the energy of a rotating fly-wheel is purely rotational. The kinetic energy of the wheels of a moving car, or of a moving football, is both translational and rotational.

Calculation of kinetic energy of translation.

Let us consider a body of mass m moving with a velocity u at any instant. If a constant force P opposing the motion be impressed upon it, the body decelerates and comes to rest after traversing a distance say, s. If the uniform retardation produced be f, then,

$$P=mf$$

$$\text{Also,}\quad 0=u^2-2fs \qquad \therefore\ u^2=2fs.$$

Since the work done against the force as the body comes to rest gives a measure of the kinetic energy, we have,

Kinetic energy of the body $=E_k=$ force × distance $=P.s=mf.s$

$$=m\frac{u^2}{2s}.s=\frac{1}{2}\,mu^2 \text{ (in absolute units)} \qquad \ldots \qquad (7{\cdot}5)$$

Thus, *the kinetic energy of a moving body at any instant is given by half the product of its mass and the square of its velocity at that particular instant.*

If under the action of a force P the velocity of the body increases from u to v, the distance covered during the interval being s, then,

$$v^2=u^2+2fs \quad \text{or,} \quad s=\frac{v^2-u^2}{2f}$$

$$\therefore\ \text{Work done by the force}=P.s=mf.s=mf.\frac{v^2-u^2}{2f}=\tfrac{1}{2}mv^2-\tfrac{1}{2}mu^2$$

$$=\text{Final K.E}-\text{Initial K.E.}$$

$$=\text{Increase of kinetic energy of the body} \qquad \ldots \qquad (7{\cdot}6)$$

So the *increase of kinetic energy of a body is equal to the work done by the impressed force.*

Sometimes the K.E of a body is conveniently expressed in terms of its momentum. As we know, momentum p of the moving body is given by,

$$p=mu$$

$$\therefore \text{ K.E. of the body} = E_k = \tfrac{1}{2}mu^2 = \frac{m^2u^2}{2m} = \frac{p^2}{2m} \quad \cdots \quad (7\cdot7)$$

7·8. Potential Energy.

Potential energy of a body is defined as the energy possessed by it by virtue of its position or configuration. Potential energy is measured by the amount of work that is available when the body returns from its present position or configuration to some standard position or configuration, usually called the zero position or configuration. It is, therefore, expressed in the same units as those of work.

If a weight be raised through a certain finite height, work is done against the force of gravity. This work is stored up in the body in its new elevated position as potential energy. If it is now allowed to descend again to its former level, it can do work ; it can raise another weight or it can compress a spring or if allowed to fall freely, it can drive a pile through some distance into the ground or can demolish something it hits.

When a watch is wound, work is done ; its spring then possesses potential energy and while returning to its natural shape, it does work in moving the balance wheel of the watch. Similarly, any stretched or compressed spring, the stretched string of a bow, compressed steam in the boiler of a locomotive etc. are said to possess potential energy. In returning to their normal configuration, each one of them can do work. It should be noted, from the previous example, that the body acquires this capacity by virtue of work done on it previously.

Another important thing should be mentioned in this connection. A body or more correctly, a system of bodies, remains in a state of stable equilibrium only when the potential energy is minimum for that state or configuration. If not, the state is unstable. When allowed to move freely, the system always returns to the position of minimum potential energy. Thus a raised weight when allowed to move freely, always goes downward ; a stretched spring always shrinks ; compressed gas always expands.

Gravitational Potential Energy : *The particular kind of potential energy of a raised object is called gravitational potential energy.* The zero configuration is usually taken as the surface of the earth, which is also called the reference level.

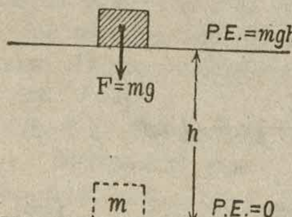


Fig. 7·7 : Gravitational potential energy

The gravitational potential energy of a body of mass m raised to a height h above the surface of the earth can be easily calculated. The force F acting on the body is its weight mg ; hence as the body is raised against this force through the height h, its point of application also gets displaced by the same amount. Hence,

the work done=force × displacement$=mg.h$

This work is stored up as potential energy of the body and can be regained by dropping the body back to the ground. Hence by definition,

Potential energy$=mgh$ (in absolute units) ... ... (7·8)

It should be noted this equation has been deduced assuming h to be small compared to the radius of the earth so that g may be taken as constant over the distance h.

The above equation is valid even if the body is not raised straight upward to the height h, but if carried there by any other path. We shall prove this statement only for the simple case when the body is pulled up a frictionless inclined plane to the height h [Fig. 7·8]. As the body of mass m is pulled upwards along the plane, work must be done against the downward component of its weight mg along the plane viz., $mg \sin \theta$. The other component $mg \cos \theta$ being normal to the direction of displacement, does not contribute anything to the total work done. If the body is raised to the point B which is at a height $BC=h$ above the ground,

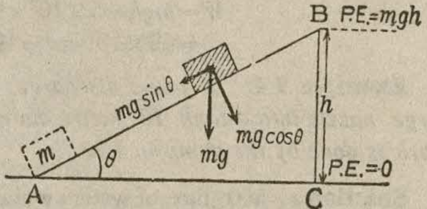


Fig. 7·8 : P.E. is independent of the path by which the body is raised

the total work done=force × displacement

$$=mg \sin \theta \times AB = mg \times AB \sin \theta = mg \times BC = mgh.$$

By definition, this is the potential energy of the body, which is the same as that obtained before, when the body is pulled straight up through a distance h.

As mentioned earlier, gravitational potential energy must always be specified with respect to some reference level. The expression, mgh, for gravitational potential energy, implies that its value is zero when $h=0$, that is when the body is at the reference level. However, the choosing of the reference level is purely arbitrary, since in any practical case, one is concerned only with difference in potential energy, which is independent of the reference level. The floor of the laboratory, or the top of the table may also be taken as reference level. It is also evident that the elevation of earth's surface, which is usually taken as reference level, varies from place to place.

Potential energy of a body may also be negative [Fig. 7·9]. Located at any point above the reference level, a body has positive potential energy ; at points below that level, it has negative potential energy. Suppose position B is on the reference plane. In lifting the mass m from B to C, work is done. In returning from C to B the body releases energy. Similarly, in going from B to A, the body releases energy and in the end, it would evidently possess smaller amount of energy at A than that at B. The energy at the point B on the reference plane is regarded as zero. The energy at A is therefore negative. To raise it again to B, work must be done to

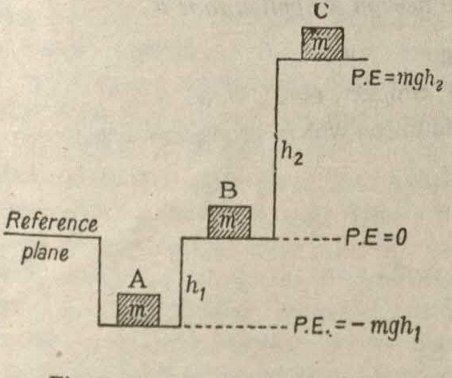


Fig. 7·9 : P.E. may be negative

supply the necessary energy. Thus, with earth's surface as reference level, an object lying inside a mine has negative potential energy while an object on the top of a nearby building has positive potential energy.

Example 7·3. *A body of mass 5 kg. is raised 100 cm. Find the work done.*

Solution : From eqn. (7·8), we have

$$W = mgh = 5 \times 10^3 \times 980 \times 10^2 \text{ erg}$$
$$= 49 \times 10^7 \text{ erg} = 49 \text{ Joule.}$$

Example 7·4. *A pump discharges 500 litre of water per minute through a large nozzle into a tank 10 metre above the ground. Calculate how much useful work is done by the pump in 1 hr.* ?

Solution : 500 litre of water weighs 500 kg. Amount of water lifted to the tank in 1 hr. $= 500 \times 60 = 3 \times 10^4$ kg.

From eqn. (7·8), we have for work done in 1 hr.,

$W = 3 \times 10^4 \times 9{\cdot}8 \times 10$ Joule, where $g = 9{\cdot}8$ m/sec.^{2}
$= 2{\cdot}94 \times 10^6$ Joule.

Example 7·5. *A well is 50 feet deep and 8 feet in diameter, and is full of water. Calculate the work done in pumping the whole of the water upto the ground level.*

Solution : The volume of water $= \pi \dfrac{64}{4} \times 50$

Since the density of water is 62·5 lb./cu. ft., hence the total mass of water pumped out $= 8\pi \times 62{\cdot}5 \times 10^2$ lb. The c.g. of the water column lies $50/2 = 25$ feet below the ground.

$\therefore$ the work done in pumping = P. E. gained $= 8\pi \times 62{\cdot}5 \times 10^2 \times 25$ ft. lb.
$= 3{\cdot}924 \times 10^6$ ft. lb.

Example 7·6. *A train of mass 1000 kg. is running at 60 km. per hour. Calculate* (*a*) *the work done to bring it to rest, and* (*b*) *what distance it will move before coming to rest if a braking force of 1000 newton is applied to it* ?

Solution : (a) Here $m = 1000$ kg. $= 10^6$ gm.
$u = 60$ km./hr. $= 1{\cdot}67 \times 10^3$ cm./sec., $v = 0$

If F be the force applied to bring the train to rest which produces a negative acceleration f, then,

$$F = -mf \quad \therefore \quad f = -\frac{F}{m}$$

We have from the eqn., $v^2 = u^2 + 2fs$, since $v = 0$,

$$0 = u^2 - 2.\frac{F}{m}s \quad \therefore \quad F.s = \tfrac{1}{2}mu^2$$

But $F.s = W$. $\therefore$ We have, $W = \frac{1}{2}mu^2$
$= \frac{1}{2} \times 10^6 \times 2{\cdot}78 \times 10^6$
$= 1{\cdot}39 \times 10^{12}$ erg $= 1{\cdot}39 \times 10^5$ Joule.

(b) $s=\frac{W}{F}=\frac{1\cdot39\times10^{12}}{10^8}$ [$\because$ 1 newton$=10^5$ dyne]

$=1\cdot39\times10^4$ cm.$=139$ m.

Example 7·7. *A bullet of 50 gm. is fired with a speed of 100 cm./sec. from a gun weighing 10 kg. Calculate (a) the kinetic energy of the bullet and the gun ; (b) the average resistance that the bullet will experience when it penetrates a distance 50 cm. into a sand bank.*

Solution : (a) From eqn. (7·5), the kinetic energy of the bullet

$$=\frac{1}{2}\times50\times(1\times10^2)^2=2\cdot5\times10^5 \text{ erg.}$$

Momentum of the bullet$=50\times1\times10^2$ gm. cm./sec.

From the principle of conservation of momentum,

momentum of the gun$=$momentum of the bullet

But the momentum of the gun$=10\times10^3$ gm.$\times v$, where v is the velocity of recoil of the gun.

$$\text{or,}\quad 10^4\times v=50\times10^2$$

$$\therefore\quad v=0\cdot5 \text{ cm/sec.}$$

$\therefore$ kinetic energy of the gun$=\frac{1}{2}\times10\times10^3\times(0\cdot5)^2=1\cdot25\times10^3$ erg.

(b) Let W represent the work done by the bullet in penetrating through the distance s against the resisting force F. Obviously, W is the initial kinetic energy of the bullet just before penetration. Then we have,

$$W=F.\,s$$

$$\therefore\quad F=\frac{W}{s}=\frac{2\cdot5\times10^5}{50}=5000 \text{ dyne.}$$

7·9. Transformation and conservation of Mechanical Energy.

In the course of its motion, a body might possess both kinetic energy by virtue of the motion and, potential energy by virtue of its position. For example, a ball thrown upwards or a flying aeroplane possesses both kinetic and potential energies. The total mechanical energy of such a body is the sum of both these energies.

$$\text{Total energy, } E_T=E_K+E_P \qquad \ldots \qquad \ldots \qquad (7\cdot9)$$

In many mechanical processes, kinetic energy of the system is converted into its potential energy or vice versa. Examples of such transformations of mechanical energy from one form to another are frequent. If a ball of mass m is thrown vertically upwards [Fig. 7·10] with a velocity v_0, from a horizontal plane taken as the reference, then initially it starts with a kinetic energy $\frac{1}{2}mv_0^2$, the potential energy being zero. As it goes upwards doing work against the force of gravity, its gravitational potential energy continually increases ; however at the same time its velocity and hence, its kinetic energy decreases continually. So during this upward journey of the ball its kinetic energy is being continually transformed to its potential energy. In the end of its motion, the ball is momentarily at rest at the

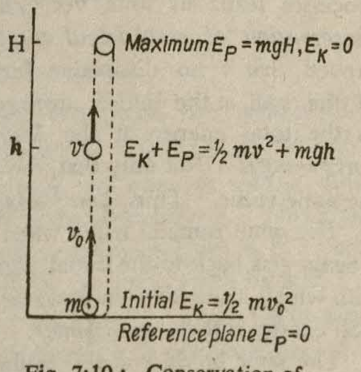

H
Maximum $E_P=mgH, E_K=0$
h
v
$E_K+E_P=\frac{1}{2}mv^2+mgh$
v_0
m
Initial $E_K=\frac{1}{2}mv_0^2$
Reference plane $E_P=0$

Fig. 7·10 : Conservation of mechanical energy topmost point ; its kinetic energy is then nil, while the potential energy is maximum, given by mgH, where H is the maximum height reached. At any intermediate point, its total energy is partly kinetic and partly potential. Thus when the ball is passing with an upward velocity v through a point at a height h, its total energy is $\frac{1}{2}mv^2+mgh$.

The reverse process occurs when the ball descends ; its potential energy continually decreases, while its kinetic energy increases. The ball descends with ever increasing speed until it again comes back to the reference horizontal plane from where it started ; its energy is then again wholly kinetic, its potential energy being zero.

Another interesting example is provided by the collision of two cars moving towards each other [Fig 7·11 (*a*)]. The energy of the system is then wholly kinetic. When they collide, they are momentarily at rest [Fig. 7·11 (*b*)] ; the kinetic energy of the system evidently becomes zero, being converted into its potential energy. If the cars interact with each other through a spring, work will be done on the spring when it is compressed. The changed configuration of the spring accounts for the potential energy stored. The spring then expands doing work and in this process releases the stored potential energy. The cars separate in opposite direction [Fig. 7·11 (*c*)], the energy of the system being then again wholly kinetic.

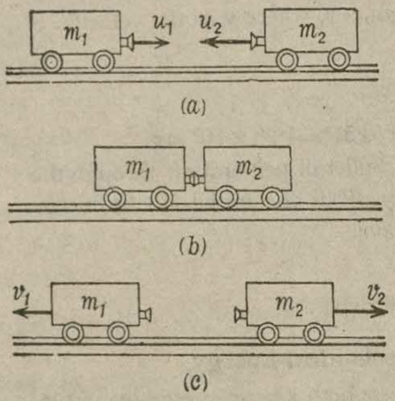


Fig. 7·11 : Conservation of mechanical energy

More such examples can be cited, two important ones being the oscillation of a simple pendulum and a body moving along an inclined plane. A careful and closer examination of these transformation processes leads us to a very important conclusion known as the ***principle of conservation of mechanical energy***. Thus, in our first example, it can be proved that if no dissipative forces, like friction are present, the total energy of the ball at the initial starting point, when it is wholly kinetic, is the same as the total energy at the highest point when it is wholly potential, i.e., $\frac{1}{2}mv_0^2=mgH$. Not only that, the total energy at any intermediate point also have the same value. Thus, $\frac{1}{2}mv_0^2=mgH=\frac{1}{2}mv^2+mgh$.

The same remains true, when the ball falls back freely downwards. When it again gets back to the initial starting point, it returns with the same velocity v_0 with which it was thrown upwards. Since its energy is then wholly kinetic, the total energy is then again $\frac{1}{2}mv_0^2$.

The same holds true also in the second example. As above, if no dissipative forces are present and if the collision is perfectly elastic, the total energy remains the same. Thus

Initial K. E.=P. E. stored=Final K. E.

or, $\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2$=P.E. stored$=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$.

When a quantity remains constant in a process, it is said to be conserved, and we may, therefore, say that the total energy is conserved in these processes. Hence we may state—

Dissipative forces like friction being absent, the total mechanical energy i.e., the sum of the P.E. and K.E. of a system of bodies, remains constant.

This statement is known as **the principle of conservation of mechanical energy.** This statement is equivalent to saying that any increase or decrease in the P.E. of a system, leads to a corresponding decrease or increase in its K.E. and vice versa. The principle is a very important one and much useful in solving mechanical problems without requiring any knowledge of intermediate states of the system ; only the knowledge about initial and final energies are required. Let us now demonstrate its validity with some simple examples.

Total energy of a falling body is constant : Let a body of mass m be raised to a height h above the ground level which is taken as the reference plane [Fig. 7·12]. Hence, its gravitational potential energy is mgh and since the body is at rest, total energy is also the same.

If the body now descends freely through a height x, its potential energy reduces to $mg(h-x)$. If its velocity at that point be v, then its kinetic energy is $\frac{1}{2}mv^2$, so that its total energy is given by $mg(h-x)+\frac{1}{2}mv^2$.

But, since the body starting from rest, moved with the downward acceleration g, then $v^2=2gx$.

$\therefore$ Total energy$=mg(h-x)+mgx=mgh$, which is the same as the initial value.

When the body reaches the reference plane, its potential energy is zero and its energy is wholly kinetic. If v_0 be its velocity, then $v_0^2=2gh$.

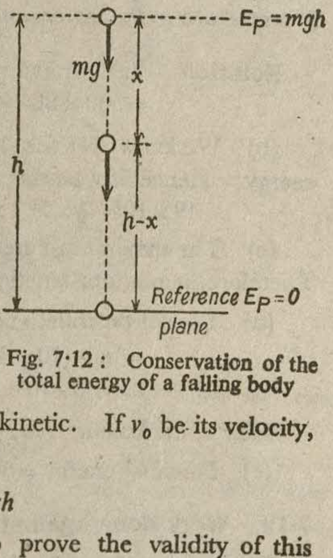


Fig. 7·12 : Conservation of the total energy of a falling body

$\therefore$ Total energy$=$its kinetic energy$=\frac{1}{2}mv_0^2=mgh$

Thus total energy is conserved in this case. To prove the validity of this principle when a body is thrown upwards, is left for the students as an exercise.

The inclined plane : Let us consider a body of mass m sliding down a smooth, frictionless plane inclined at an angle θ to the ground level. This is taken as the reference plane. If the body starts from rest from a point A, at a vertical height h, then at that point its total energy is wholly potential, given by mgh.

The body slides down under the force $mg \sin\theta$ and at a point B distance x from A, let its velocity be v. Since its acceleration is $g \sin\theta$, we have $v^2=2g \sin\theta.x$, so that its kinetic energy at that instant is $\frac{1}{2}mv^2=mg\sin\theta.x$.

But, as shown in the Fig. 7·13, the body has descended through the vertical height $x \sin\theta$, so that its potential energy at that position is $mg(h-x\sin\theta)$. Hence its total energy at that instant

$$=mg\sin\theta.x+mg(h-x.\sin\theta)=mgh$$

which is the same as the initial energy.

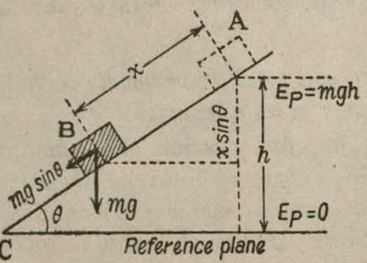


Fig. 7·13 : Total energy of a body sliding down an inclined plane

At the lowest point *C*, when the particle is on the reference plane, its potential energy is zero, and the total energy is wholly kinetic. If its velocity at that point be v_0, then $v_0^2=2g\sin\theta.\ AC$.

$\therefore$ Total energy=its kinetic energy $=\frac{1}{2}mv_0^2=mg.\ AC\sin\theta$

$=mgh$ [$\because$ from the Fig. 7·13, $AC\sin\theta=h$]

Total energy is thus conserved.

Example 7·8. *A 500 gm. object falls from the top of a ship and lands in the water 10 metre below. Find (a) its initial potential energy, (b) its velocity of impact on the water, (c) the maximum kinetic energy, (d) the kinetic energy 3 metre above the water, and (e) the potential energy 3 metre above the water.*

Solution : (a) Initial potential energy$=mgh$

$=500\times980\times(10\times100)=49\times10^7$ erg$=49$ Joule.

(b) We know that the loss of potential energy equals the gain of kinetic energy. Hence, if v be the velocity of impact of the water,

$49\times10^7=\frac{1}{2}\times500\times v^2 \quad \therefore\ v^2=49\times4\times10^4;\ v=1400$ cm./sec,

(c) The maximum kinetic energy is equal to the initial potential energy. Therefore, maximum kinetic energy$=49$ Joule.

(d) Loss of potential energy in this case

$=m.g.(10-3)\times10^2=500\times980\times7\times10^2$ erg

$=34{\cdot}3\times10^7$ erg$=34{\cdot}3$ Joule.

Hence, the kinetic energy$=34{\cdot}3$ Joule.

(e) Potential energy$=500\times980\times3\times10^2$ erg$=14{\cdot}7$ Joule.

7·10. Work done against friction.

Let us now consider how frictional forces, which have been neglected in previous discussions, affect the mechanical energy of a moving body. In practice, it is impossible to consider a mechanical process where frictional or other resistive forces are absent. These forces always act in a direction opposite to that of the motion. Thus, if a ball is thrown upwards in air, a downwards resistive force will act on it ; in its downward return journey, the resistive force will act upwards. Hence work must be done against these forces and mechanical energy is not conserved. Total mechanical energy of the system always decreases. We say that mechanical energy is dissipated. So the ball returns to the hand of the thrower with less energy (smaller speed) than it had, when thrown. The swing of a pendulum continually gets shorter and then stops altogether. On imparting a definite kinetic energy, if a body is allowed to move on a surface, it decelerates (loses energy) and then stops.

As an illustration of the way in which mechanical energy is dissipated, let us consider a body sliding down an inclined plane. The body starts from rest from the point A [Fig. 7·14], where its total energy is all potential given by mgh.

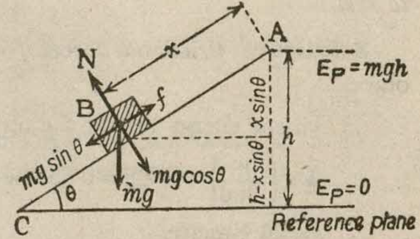


Fig. 7·14 : Energy lost due to friction

As the body slides down under the force $mg \sin\theta$, force f due to friction, acts upwards along the plane. Since the normal force

$N=mg\cos\theta$, the force of friction has the magnitude,

$$f=\mu N=\mu\, mg\cos\theta$$

Where μ is the co-efficient of sliding friction.

The body, therefore, moves under the action of the resultant force F down the plane, given by

$$F=mg\sin\theta-f.$$

The downward acceleration along the plane produced by this force is $a=F/m$. Thus, the speed v of the body at a point B [Fig. 7·14] at a distance x from A is given by

$$v^2=2ax=2\frac{F}{m}x$$

so that the kinetic energy of the body

$$=\tfrac{1}{2}mv^2=xF=mgx\sin\theta-fx \quad \dots \quad (7{\cdot}10)$$

The potential energy of the body at B is evidently $mg\,(h-x\sin\theta)$,

$\therefore$ Total energy of the body at that instant

$$=mg\,x\sin\theta-fx+mg\,(h-x\sin\theta)$$
$$=mgh-fx \quad \dots \quad \dots \quad \dots \quad (7{\cdot}11)$$

The first term, on the right hand side of this equation, represents the total energy at the starting point A, while the second term gives the work done against the frictional force f for a displacement x. So, the total mechanical energy of the body has decreased by the amount fx and is said to be ***dissipated in friction.*** Thus, the kinetic energy at the bottom point C on the reference plane is not as great as the potential energy mgh at the top ; it is smaller by the amount $f.AC$, the work done against friction.

7·11. Work-Energy Principle.

Principle of conservation of mechanical energy does not hold, when forces of friction or other resistive forces are present. However, the principle can be generalised, so that it is valid even when dissipative forces are present, provided that such forces or the work done against them is known. The eqn. (7·11) is

Total energy at B=Total energy at $A-fx$.

Rewritting, Total energy at A=Total energy at $B+fx$.

i.e. Total mechanical energy of a system at any instant=Total mechanical energy at a later instant+work done against dissipative forces in that interval.

This general statement is known as the **work-energy principle.**

Example 7·9. *An object is thrown along the ground with a speed of 10 m./sec. The co-efficient of sliding friction between the object and the ground is 0·25. Calculate the time and the distance the object will travel before coming to rest.*

Solution : Frictional force, $f = \mu . w = 0{\cdot}25 \times w$, where w is the weight of the object.

$\because$ kinetic energy lost by the object=work done against friction,

$\therefore$ $\frac{1}{2} \times \left(\frac{w}{980} \right) \times (10^3)^2 = 0{\cdot}25 \times w \times s$, where s is the distance by the object.

$\therefore$ $s = 20{\cdot}4$ metre.

Now, time required$= \dfrac{\text{distance traversed}}{\text{average speed}}$

But the average speed$= \frac{1}{2}$ (10+0) m./sec.=5 m./sec.

Hence, required time$= \dfrac{20{\cdot}4}{2}$ sec.=4·08 sec.

Example 7·10. *A sphere of mass 10 kg. is projected vertically upward with a velocity of 250 m./sec. It rises to a height of 3000 m. How much work is done against friction? With what velocity will it strike the earth after it comes down?*

Solution : Mass of the body$= m = 10^4$ gm.

Initial velocity$= 25 \times 10^3$ cm./sec.

$\therefore$ Initial K. E. of the body$= \frac{1}{2} \times 10^4 \times (25 \times 10^3)^2$

$= 312{\cdot}5 \times 10^{10}$ erg.

The P. E. at this position is nil.

$\because$ the maximum height reached by the body$= 3 \times 10^5$ cm.,

its P. E. at this position$= 10^4 \times 980 \times 3 \times 10^5 = 294 \times 10^{10}$ erg.

Its K. E. at this position is zero.

$\therefore$ Work done against friction=total initial mechanical energy—total final mechanical energy$= 312{\cdot}5 \times 10^{10} - 294 \times 10^{10}$

$= 18{\cdot}5 \times 10^{10}$ erg$= 18{\cdot}5 \times 10^3$ Joule.

As the sphere comes back to the earth, it will lose the same amount of energy due to friction. Since, its energy is then wholly kinetic, therefore if the velocity with which it strikes the earth be v,

$$\frac{1}{2} \times 10^4 \times v^2 = 294 \times 10^{10} - 18{\cdot}5 \times 10^{10}$$

$$\therefore \quad v = 2{\cdot}35 \times 10^4 \text{ cm./sec.} = 235 \text{ m./sec.}$$

Example 7·11. *A block of mass 4 kg. slides down an inclined plane 6 m. high and 10 m. long. Find (a) the K. E. of the block at the base of the plane (b) the distance travelled by the block over the horizontal part of the route until it stops. Assume that the co-efficient of friction over the entire route is constant and equal to 0·2.*

Solution : (a) The base of the inclined plane is of length

$\sqrt{10^2 - 6^2} = 8$ m.

If θ be the inclination of the plane, then $\cos\theta = \frac{8}{10}$

Frictional force $= \mu\, mg \cos\theta = 0{\cdot}2 \times 4 \times 9{\cdot}8 \times \frac{8}{10}$ newton

Energy dissipated = work done against friction

$$= 0{\cdot}2 \times 4 \times 9{\cdot}8 \times \tfrac{8}{10} \times 10 = 6{\cdot}4 \times 9{\cdot}8 \text{ Joule}$$

P. E. of the block at the top $= 4 \times 9{\cdot}8 \times 6$ Joule

$\therefore$ K.E. of the block at the base of the plane = P.E. = Energy dissipated

$$= 24 \times 9{\cdot}8 - 6{\cdot}4 \times 9{\cdot}8 = 172{\cdot}48 \text{ Joule}$$

(b) Frictional force $= \mu\, mg = 0{\cdot}2 \times 4 \times 9{\cdot}8$ Newton.

Now, Energy dissipated against friction = Initial K.E. of the block.

If x be the distance travelled by the block before coming to rest, then

$0{\cdot}2 \times 4 \times 9{\cdot}8 \times x = 172{\cdot}48$ or, $x = 22$ m.

7·12. Conservative and Dissipative forces.

A system in which mechanical energy is conserved, is called a *conservative system*. The forces acting in such a system are *conservative force*. Thus gravitational force is a conservative one.

The presence of the frictional force or of other resistive forces leads to the non-conservation of mechanical energy ; mechanical energy is dissipated during motion. Forces like those of friction are *non-conservative* or *dissipative forces.*

When a body is lifted vertically, work is done against the force of gravity ; the gravitational potential energy increases. When the body falls back downwards, its potential energy decreases and the descending body can, therefore, be made to do work. In other words, the work is *recoverable*, or in still other words, the net work done in a round trip is zero.

However, when a body is made to slide along a rough horizontal surface, work must be done against the force of friction. The potential energy of the body does not change, because the body does not gain any capacity of doing work in this process. When the body is slided back to its original position, the force due to friction reverses and instead of recovering the work done in the first displacement, work must again be done against the frictional forces in the return journey. The net work done in a round trip is not zero.

This is the criterion which determines the difference between a conservative and a non-conservative force. *Forces where the work is recoverable, are called* **conservative forces** ; *forces where this is not possible, are called* **non-conservative forces**.

7·13. Conservation of Energy.

The answer to the question—"what happens to the mechanical energy that disappears in a non conservative system", leads us to a principle of great importance, the principle of conservation of energy. Whenever work is done against dissipative forces, the mechanical energy that disappears is not destroyed, but is converted into thermal energy. Recognition of this fact, leads us to the formulation of a more general conservation principle called the **Universal Principle of Conservation of Energy**. This can be stated as follows :

Energy can not be created or destroyed, but may be transformed from one form to another. The sum total of all energies in the universe remains constant.

Thus when we rub our hands they get warmed ; mechanical energy of rubbing is converted into heat energy. When a falling body stops on striking the ground, its machanical energy is transformed mostly into heat and partly into sound energies. Machines get warmed due to friction between their different parts ; hence the necessity to cool them with a continuous flow of oil. Shooting stars are nothing but fast moving meteors heated to incandescence by friction with earth's atmosphere.

So, we find that whether the forces be conservative or non-conservative, the total energy of the system remains constant but we must keep a careful track of all the different forms in which it may be transformed.

7·14. Power.

In the foregoing discussions of work and energy, the time spent in doing the work was never mentioned. To raise a given weight through a given height the work done is always the same whether it is done in one second, or one minute, or one month. However, in many instances, the rate of doing work is of utmost concern to us. For example, in choosing a machine, the time in which a given work is done by the machine is of great importance to the buyer ; a machine doing a given work in a shorter time is more useful than another, doing the same work in a longer time.

The time rate of doing work by an agent is defined as the **Power** *of the agent*. Thus

$$\text{Power}=\frac{\text{Work done}}{\text{Time interval}}$$

If a constant force F acts on a body for a time t, while the body undergoes a displacement s in the direction of the force, with a uniform velocity v, the power developed is,

$$P=\frac{F.s}{t}=F.v=\text{force}\times\text{velocity} \qquad \ldots \qquad \ldots \qquad (7\cdot12)$$

If the displacement does not take place in the direction of the force, the expression becomes,

$$P=Fv\cos\theta \qquad \ldots \qquad \ldots \qquad (7\cdot13)$$

where θ is the angle between the force and the displacement.

Units of Power : The C.G.S. absolute unit of power is one erg per second, while the gravitational unit is one gm. cm. per second.

In M.K.S. system, unit of power is one Joule per second.

This is given a special name, the **watt** (W).

Thus, 1 watt=1 Joule/sec.$=10^7$ erg/sec.

Two multiples of the watt are commonly used : the kilowatt (1 K.W.=1000W.) and the mega watt (1 M. W.$=10^6$ W).

In F.P.S. system absolute unit of power is one foot poundal per sec. The gravitational unit in the same system is one foot-pound per sec.

In this system, one commonly used unit of power is the horse power (H.P), which was introduced by James Watt.

1 H.P.=550 ft. lb/sec.

Relation between Horse Power and Watt.

Since 1 foot-pound$=1 \cdot 356$ Joule (vide Art. 7·2)

1 H.P.$=550 \times 1 \cdot 356$ Joule/sec.$=746$ watt.

$\therefore$ 1 Kilowatt$=\frac{1000}{746}=1 \cdot 34$ H.P.

Sometimes, a conveniently large unit of energy is used. The unit known as kilowatt-hour (k. W. h) is obtained from previous discussions. Thus, one kilowatt-hour is the work done in one hour by an agent of power one kilowatt. So

$$1 \text{ k.W.h.}=1 \text{ k.W.} \times 1 \text{ h.}=1000 \text{ Joule/sec.} \times 3600 \text{ sec.}=3 \cdot 6 \times 10^{6} \text{ Joule.}$$

Example 7·12. *A boy who weighs 50 kg. finds that he can reach upstairs in 25 sec., there being 60 stairs each of 15 cm. height. Find the average horse power of the boy.*

Solution : Total work done=weight × vertical height raised

$$=5 \times 10^{4} \times 15 \times 60 \text{ gm. cm.}=4 \cdot 5 \times 10^{7} \text{ gm. cm.}$$

$$\therefore \text{ Average power}=\frac{\text{Total work done}}{\text{Time taken}}=\frac{4 \cdot 5 \times 10^{7}}{25} \text{ gm. cm./sec.}$$

$$=0 \cdot 18 \times 10^{7} \text{ gm. cm./sec.}=0 \cdot 18 \times 980 \text{ Joule/sec.}$$

But 1 H.P.$=746$ Joule/sec.

$$\therefore \text{ Average power of the boy}=\frac{0 \cdot 18 \times 980}{746} \text{ H.P.}=0 \cdot 24 \text{ H.P. (nearly)}$$

Example 7·13. *On a level road a horse is pulling a car with a force of 50 lb. wt. If the horse is walking with a uniform velocity of 6 mph., calculate the horse power it is developing.*

Solution : The velocity of the horse$=v=6$ mph.$=8 \cdot 8$ ft./sec.

From eqn. (7·12), the power developed by the horse

$$=P=Fv=50 \times 8 \cdot 8 \text{ ft. lb./sec.}=\frac{50 \times 8 \cdot 8}{550} \text{ H.P.}=0 \cdot 8 \text{ H.P.}$$

Example 7·14. *A pump raises 1500 gallon of water per minute through a height of 8 feet, and discharges through a nozzle with a velocity of 48 feet per second. Find the horse-power of the pump, given that one gallon of water weighs 10 lb. Find also the force of reaction on the nozzle.*

Solution : In one sec. the pump discharges $(1500 \times 10)/60=250$ lb. of water.

Work done to lift 250 lb. of water through 8 ft.$=250 \times 8=2000$ ft. lb.

$$\text{K.E. of 250 lb. water at 48 ft./sec.}=\tfrac{1}{2} \times \frac{250 \times 48^{2}}{32} \text{ ft. lb.}=9000 \text{ ft. lb.}$$

$$\therefore \text{ Power of the pump}=(2000+9000) \text{ ft. lb./sec.}=\frac{11000}{550} \text{ H.P.}=20 \text{ H.P.}$$

The reaction on the nozzle is equal to the force required to set the water in motion. We know that

$$F=\text{Rate of change of momentum}=\frac{m \times v}{t}$$

Here, $m=250$ lb., $v=48$ ft./sec., $t=1$ sec.

$\therefore$ Reaction on nozzle$=F=250 \times 48$ poundal$=375$ lb. wt.

Example 7·15. *Find the power developed by the engine of a vehicle of mass 500 kg. if it moves at a constant speed of 72 km./hr. (a) over a horizontal road (b) up a grade 1 in 25 (c) down the same grade. The co-efficient of friction is 0·1.*

Solution : 72 km./hr.=20 m./sec.

(a) Opposing force of friction$=F=\mu\, mg=0{\cdot}1\times500\times9{\cdot}8$ Newton.

Since the vehicle travels at a constant speed, hence its engine develops just this amount of force.

$\therefore$ Power$=Fv=0{\cdot}1\times500\times9{\cdot}8\times20$ watt$=9{\cdot}8$ kw.

(b) If θ be the inclination of the plane, then $\sin\theta=1/25$, $\cos\theta\simeq1$.

$\therefore$ Downward force along the plane=component of weight+force of friction

$$=mg.\sin\theta+\mu\, mg.\cos\theta=500\times9{\cdot}8\times\tfrac{1}{25}+0{\cdot}1\times500\times9{\cdot}8\times1$$
$$=196+490=686 \text{ Newton}$$

$\therefore$ Power$=686\times20$ watt$=13{\cdot}72$ kw.

(c) Upward force along the plane=force of friction$-$component of weight

$$=\mu\, mg.\cos\theta-mg.\sin\theta=490-196=294 \text{ Newton}$$

$\therefore$ Power$=294\times20$ watt$=5{\cdot}88$ kw.

● EXERCISE ●

[A] Essay type Questions.

1. Define work. Name and define its absolute and practical units in different systems. Define gm-centimeter, and foot-pound. Why are these called gravitational unit ? Obtain the relations between (i) erg and Joule, (ii) foot-poundal and erg, (iii) foot-pound and Joule. Is work done whenever a force is exerted ? Explain with a suitable example.

2. (a) What is the difference between—'work done by a force' and 'work done against a force ?

(b) Obtain an expression for the kinetic energy of a particle of mass *m* moving with a velocity *v*.

(c) Prove that the sum of kinetic energy and potential energy is everywhere constant for a freely falling body. [*H. S.* '*83*]

What happens to its energy when it touches the ground ?

3. (i) Explain what are meant by kinetic energy and potential energy. State the principle of conservation of energy and establish it for a body falling freely under gravity.

(ii) Define Watt and Horse-Power. Show that 1 H. P.=746·4 watts. [*H. S.* '*82*]

(iii) "The power of an engine is 10 H. P."—What is the meaning of the statement ? [*H. S.* '*80*]

4. Under what condition the work done by a force is (i) positive, (ii) negative, and (iii) Zero ? What is meant by 'No-work force'. Explain with examples. If the force acting on a body be (i) constant and (ii) varying, then draw curves for both the cases plotting force against displacement. How do you obtain the work done from these curves ?

5. What do you mean by 'work done by a couple ? Find an expression for it. Indicate which pair of physical quantities have the same unit : Force and energy, work and power, energy and work, Acceleration and power.

6. Define gravitational potential energy and deduce an expression for it.

Give an example of a force acting on a moving body, yet doing no work.

7. State the work energy principle. When a watch is wound, what type of energy is stored in it ?

A force acts on a body whose velocity changes thereby. Prove that the change of kinetic energy is equal to the work done by the impressed force.

8. A body of mass *m* is pulled upwards along an inclined plane through a vertical height *h* ; find the expression for the work done.

Is it possible for a body to possess negative value of the potential energy ? Explain your answer with a suitable example.

9. What is mechanical energy ? State the principle of conservation of mechanical energy. Justify the principle with reference to a body sliding down an inclined plane.

When do we call a system to be conservative ? Is it ever possible that such system can exist in nature ?

In the motion of a simple pendulum, show that the work done by the tension in the string is zero.

10. Deduce an expression for the total energy at any instant of a body sliding down a rough inclined plane. Is it same throughout its journey ? If not, how do you explain ?

11. State the universal principle of conservation of energy. Develop the idea of the principle with some suitable examples.

Name the different kinds of energy to which the heat energy obtained during the explosion of a bomb, gets converted .

12. (i) What are meant by a conservative and a non-conservative force ? Prove that the gravitational force is conservative but the frictional force is non-conservative.

(ii) A car is moving with a uniform velocity. Is the engine of the car doing any work under this condition ? [*Model Questions, H. S. Council*]

13. (i) What is power ? Name and define its absolute units in C. G. S. and F. P. S. systems. What is the gravitational unit of power in F. P. S. system ? What is its relation with the practical unit in the same system ? Deduce the relation between Kilowatt and Horse-Power.

(ii) What unit does kilowatt-hour represent ? Define it. What is its relations with Joule ?

Does the work done in raising a body to a certain height depend on how fast it is being raised ?

[B] Short answer type questions

1. A man rises in a lift carrying a bucket of water. Explain (a) if any work is done by the man on the bucket of water, and (b) if the energy of the bucket of water would remain unaltered. [*H. S. '78*]

2. (i) A man is swimming against the current in a river. If he remains stationary relative to the ground, does he perform any work ?

(ii) Two boys are playing a ball within a railway compartment, should the kinetic energy of the ball depend on the velocity of the train ? [*Model Questions, H. S Council*]

3. In a tug of war, team *A* is slowly giving ground to team *B*. Is any work being done ? If so, by what force ?

4. If a body of mass m possesses kinetic energy E, show that its momentum is $\sqrt{2mE}$.

5. 'In many mechanical processes kinetic energy of the system is converted into its potential energy'—Justify the statement with reference to the following examples :

(a) A ball of mass m thrown vertically upwards.

(b) The collision of two cars moving towards each other.

(c) The oscillation of a simple pendulum.

Is it possible that a body has an energy without momentum ? Explain with an example.

7. A light and a heavy body possess equal momentum. Which one will have a larger kinetic energy ? [*Model Questions, H. S. Council*]

If their kinetic energies are equal, then which one will have greater momentum ?

8. A lorry and a car moving with the same kinetic energy are brought to rest by the application of brakes which provide equal retarding forces. Which of them will come to rest in a shorter distance. [*I. I. T. '73*]

9. Dyne-second is the unit of (i) force, (ii) momentum, (iii) energy (iv) power, which is correct ?

10. Explain why a falling body becomes hotter when it strikes the ground.

11. A boy tries to raise a bucketful of water but fails to do so. What is the amount of work done ? [*Model Questions, H. S. Council*]

12. An ascending gas balloon gains both K. E. and P. E. How does the principle of conservation of energy hold in this case ?

13. **Answer** the following questions with proper reasonings :—

(i) Work and torque are expressed in the same unit. Are they identical physical quantities ?

(ii) Does the centripetal force acting on a body moving in a circular path perform any work ?

(iii) Work is always done against friction. Why work is not done by friction ?

(iv) A meteorite burned completely in the atmosphere before it reached the earth's surface. What happened to its energy ?

(v) Is the resistive force of air a conservative one ?

[C] Simple Problems.

1. A force 50 dyne acts constantly on a body and moves it through 1 m. (along the direction of the force). How much work is done ? [*Ans.* 5000 erg]

2. A body of mass 5 kg. is raised from ground to a height 20 m. How much work is done ? What is the potential energy of the body ? [*Ans.* 980 Joule, 980 Joule]

3. A box is drawn through a distance of 10 m. along the horizontal floor by means of a rope. The pull in the rope is 4 newton and the angle between the rope and the ground is 30°. Calculate the work done. [*Ans.* 34·64 Joule]

4. A uniform iron chain 10 m. long and weighing 6 kg. per metre hangs vertically. Calculate the work required to wind it up. [*Ans.* $2·94 \times 10^3$ Joule]

5. Find the work done by a man weighing 144 lb. in carrying 56 lb. of brick up a ladder 20 ft. long which makes an angle of 30° with horizontal. (*H. S.* '69 *Comp.*) [*Ans.* 2000 ft. lb.]

6. A body has 1 joule of kinetic energy. It is opposed by a force of 1 megadyne. How far will it move before it comes to rest ? (*H. S. 1964*) [*Ans.* 10 cm.]

7. A non-uniform rod weighing 3 kg. and 4 m. long has its centre of gravity at 1·5 m. from one end. A load of 8 kg. is tied with its other end. Calculate the amount of work required to raise the rod from a horizontal position on the ground to vertical position with the loaded end at the top [*Ans.* 357·7 Joule]

8. How much work is done to build a column 16 ft. high with 4 marble blocks, each of height 4 ft. and mass 200 lb. ? [*Ans.* 4800 ft. lb.]

9. A uniform brick of mass 2·5 kg. and dimensions 6 by 8 by 10 cm. rests with its largest face on a horizontal plane. Find how much work must be done to stand it on end. [*Ans.* 0·49 Joule]

10. A cylindrical tank of diameter 2 m. and height 4 m. is full of water. Calculate the work done in pumping the water to a height 6 m. above the top of the tank. [*Ans.* $9·856 \times 10^5$ Joule]

11. An automobile weighing 1000 kg. changes its speed from 30 km./hr. to 60 km./hr. How much work is necessary to produce this acceleration of the automobile ? [*Ans.* $1·04 \times 10^5$ Joule]

12. A ball of mass 50 gm. is held 10 m. above the ground and then released, so that it rebounds to a height of 6 m. Calculate (i) its potential energy and (ii) kinetic energy, (a) when it is released, (b) just before it hits the ground for the first time, (c) immediately after it first hits the ground, (d) at the end of the first rebound, (e) immediately before it hits the ground for the second time. Take $g = 1000$ cm./sec.^{2}

[*Ans.* (i) (a) 5 Joule, (b) 0, (c) 0, (d) 3 Joule, (e) 0,
(ii) (a) 0, (b) 5 Joule, (c) 3 Joule, (d) 0, (e) 3 Joule]

13. A truck weighing 8000 kg. runs at the speed of 60 km./hr. and then it is stopped in 30 sec. with the application of brakes. How much heat is developed ? [*Ans.* $2·645 \times 10^6$ cal.]

14. An aeroplane weighing 5000 kg. ascends and reaches a velocity of 360 km./hr. at an altitude of 5 km. Find the work done. Neglect friction of air. [*Ans.* $2·7 \times 10^8$ Joule]

15. The work 98 joule was performed to lift a load of 2 kg. vertically to a height 1 m. above ground by a constant force. What acceleration was the load raised with ? Neglect friction of air. [*Ans.* 29·4 m./sec.]

16. The effective length of a pendulum is 50 cm. and the mass of its bob is 4 gm. The bob is drawn to a side till the string is horizontal and then released. Find its K. E. when the string makes an angle of 60° with the vertical. [*Ans.* $9·8 \times 10^4$ erg.]

17. If a train moving at 60 km. per hour can be pulled up by its brake in ½ km. From what speed could it be pulled up in 125 metre ? What is the time taken in coming to a stand still in each case ? [*Ans.* 30 km./hr., 1 min., 30 sec.]

18. A body of mass *m* accelerates uniformly from rest to a speed *V* in time *T*. Show that the work done on the body as a function of time *t* in terms of *V* and *T* is

$$\tfrac{1}{2}m\ \frac{V^2}{T^2}\,t^2$$

19. A fielder catches a cricket ball of mass 100 gm. moving horizontally at a speed of 40 m./sec. If his glove moves back a distance of 20 cm. while bringing the ball to rest, what average force was exerted on his hand ? [*Ans.* 400 newton]

20. A cloud 5 km. above the ground condenses into rain which collects 1 cm. deep over an area of 10^4 m^2. What is the loss of P. E. of the cloud ? [*Ans.* $4{\cdot}9\times10^9$ Joule]

21. A lift weighing 2000 kg. rises from rest in the ground floor to the fifth floor, a height of 20 m. As it passes the fifth floor, it has a speed of 4 m./sec. There is a constant frictional force of 10 kg. wt. Calculate the work done by the lifting mechanism. [*Ans.* $4{\cdot}1\times10^5$ Joule]

22. A heavy body weighing 600 kg. is pulled up an incline 5 m. long and 3 m. high. Calculate the work done in moving the body if the coefficient of kinetic friction is 0·50.
[*Ans.* $29{\cdot}4\times10^3$ Joule]

23. A 10 kg. gun fires a bullet of mass 20 gm. The kinetic energy of the bullet at the end of the barrel is 900 Joule. What kinetic energy is imparted to the gun by the recoil ?
[*Ans.* 1·8 Joule]

24. A bullet of mass 2 gm. and moving at 500 m. per sec. pierces a plate and moves on with a velocity of 100 m. per sec. With what velocity would it emerge if the plate had only half its thickness ? It may be assumed that the work done in piercing a plate is proportional to its thickness. [*Ans.* 360·6 m./sec.]

25. A block of mass 2 kg. slides down an inclined plane 1 m. high and 10 m. long. Find (a) the kinetic energy of the block at the base of the plane, (b) the velocity of the block at the base of the plane, (c) the distance travelled by the block over the horizontal part of the route until it stops. Assume that the co-efficient of friction over the entire route is constant and equal to 0·05. [*Ans.* (a) 9·85 Joule (b) 3·14 m./sec. (c) 10 m.]

26. A tone of mass 50 gm. is projected vertically upwards with a velocity of 100 m./sec. It rises to a height of 400 m. (a) Find the resistance of air. (b) With what velocity will it strike the earth after it comes down ? [*Ans.* (a) $1{\cdot}35\times10^4$ dyne (b) 75·36 m./sec.]

27. A crane lifts a 1500 lb. steel beam to a height of 44 ft. in 10 sec. Find the horse-power developed. (*H. S.* '67) [*Ans.* 12 H. P.]

28. What should be the H. P. of an engine which is intended to pump 250 gallons of water per minute to a height of 40 yards ? (1 gallon of water weighs 10 pounds). (*H. S. '61)*
[*Ans.* $9\frac{1}{11}$ H. P.]

29. 400 kg. of air moving at 20 m./sec., impinge on the vanes of a windmill every second. At what rate in kilowatts is the energy arriving at the windmill ? What is the maximum mass of water that could be pumped each second through a vertical height of 5 m. ? (Assume $g=$ 10 m./sec^2.) (*Oxford Univ.*) [*Ans.* 80 kw., 1600 kg./sec.]

30. A boy weighing 100 lb. ascends a flight of 20 steps, each 9 inches high, in 5 second. What horse-power does he employ ? (*H. S.* '60) [*Ans.* 6/11 H. P.]

31. An engine expends 4 H. P. in pulling a car on a horizontal road at 30 mile/hr. If the co-efficient of friction between the tyres of the car and the road is 0·02, calculate the weight of the car. [*Ans.* 2500 lb. wt.]

32. A 1000 lb. car travels up a grade of 1 in 25 at 45 mile/hr. Find the horse power required. Neglect friction. [*Ans.* 4·8 H. P.]

33. Find the power developed by the engine of a vehicle of mass 1000 kg. if it moves at a constant speed of 36 km/hr. (a) over a horizontal road (b) up a grade 1 in 20 (c) down the same grade. The co-efficient of friction is 0·07.

[*Ans.* (a) 6·86 kw. (b) 11·76 kw. (c) 1·96 kw.]

34. A uniform rectangular parallelopiped of sides l, $2l$ and $4l$ lies on a horizontal plane on each of its three different faces, in turn. What is the potential energy of parallelopiped in each of these positions ? Which position is the most stable ?

[*Ans.* 2 mgl, mgl and $\frac{1}{2}$ mgl respectively on the small, medium and large face ; when lies on the large face]

35. A particle is falling down a rough inclined plane. Assuming the frictional resistance to be 0·2 times the normal reaction and the angle of the inclined plane to be 30°, calculate the acceleration of the particle. Calculate also the change in the sum of the kinetic and potential energies as the particle falls through a length of 1 metre along the inclined plane. The mass of the particle is 1 gm. (*Jt. Entrance '82*) [*Ans.* 0·33g cm./sec.^{2} ; 17g erg]

36. A body is moved along a straight line by a machine delivering constant power. Show that the distance moved by the body in time t is proportional to $t^{3/2}$.

37. An object of mass 2 kg falls from rest through a vertical height of 30 m. and reaches the ground with a velocity of 20 m./sec. How much work is done by the push of the air on the object ? ($g=10$ m./sec.^{2}) [*Ans.* −200 J]

38. A ball flying at a velocity $v_1=15$ m./sec. is thrown back by a racket in the opposite direction with a velocity $v_2=20$ m./sec. Find the change in momentum of the ball if the kinetic energy changes by $\triangle E=8{\cdot}75$ Joules. (*Jt. Entrance '84*) [*Ans.* 3·5 kg. m./sec.]

39. A vertical shield is made of two plates of wood and iron respectively, the iron plate being 3 cm. and the wooden plate 6 cm. thick. A bullet fired horizontally goes through the iron plate first and then penetrates 3 cm. into the wood. If the shield is reversed the same bullet goes through the wood first and then penetrates 2 cm. into the iron. Find the ratio of the average resistances offered by the iron and the wood. [*Ans.* 3 : 1]

40. A train of mass 200 tonnes (including the engine) can run at a maximum speed of 108 km./hr. on a straight level track, the resistance of motion being 10 kg.wt per tonne. With what constant speed it can move up a similar track of inclination 1 in 100. [*Ans.* 54 km./hr.]

41. An engine working at a constant rate draws a train of total mass 500 tonnes up a plane whose inclination to the horizontal is $\sin^{-1}(\frac{1}{100})$. If the frictional resistance to the motion is 5 kg. wt. per tonne and the greatest speed up this incline is 10 metre/sec, calculate the power of the engine in kilowatts. (*Jt. Entrance '81*) [*Ans.* 735 kW.]

42. A uniform flexible chain of length 4 m. is held on a smooth horizontal table so that 1 m overhangs one edge, the chain being perpendicular to the edge. If the chain be released from rest, find the velocity with which it leaves the table.] *Ans.* 6·06 m./sec.]

[D] Harder Problems

1. A horse pulls a cart with a force of 50 lb. at an angle of 30° with the horizontal and moving along with a speed of 6 miles per hr. How much work does the horse perform in 10 minutes ? What is the power output of the horse in practical units ?

(*Jt. Entrance, '62*) [*Ans.* $2{\cdot}287\times10^6$ ft. lb., 0·693 H. P.]

2. A water-fall discharges 10^3 cu-ft./sec. from a height of 200 ft. If 75% of the available energy at the bottom of the water-fall is converted into power, calculate the horse-power developed. Take 1 cu-ft. of water=62·5 lb.

If this power is utilised to raise water from a tube-well of depth 600 ft., find how much water can be raised per sec. (*Jt. Entrance '69*) [*Ans.* $17{\cdot}046\times10^3$ H. P., 250 cu. ft.]

3. A man in a boat A, mass of which including the man is 400 kg. pulls a rope with a force of 10 kg. wt. In one case, the other end of the rope is tied to a tree on the shore, and in another, to a boat B having a mass of 300 kg. (a) Find in both cases the velocity of boat A at the end of the fourth second. (b) Also find the work done in this time and (c) the power developed by the man in both cases at the end of the fourth second.

[*Ans.* Case I : (a) 0·98 m./sec. (b) 192 Joule (c) 96 watt.
Case II : (a) 0·98 m./sec. (b) 448·1 Joule (c) 224 watt.]

4. Calculate the power required to raise 300 lt. of water per minute through a vertical height of 6 m. and then to deliver it through a nozzle of diameter 2·4 cm. (*London Univ.*)

[*Ans.* 600 watt.

5. An engine pumps water from the surface of a river 15 ft. below its own level and discharges it through a nozzle of diameter 2 in. with a speed of 100 ft./sec. Find the horse power of the engine assuming its efficiency to be (a) 100% and (b) 70%. Given, density of water= 62·5 lb./cu. ft. [*Ans.* (a) 42·47 H. P. (b) 60·67 H. P.]

6. A body first slides along an inclined plane forming an angle of 30° with the horizontal and then over a horizontal plane. If the body covers the same distance of the horizontal plane as on the inclined one, calculate the co-efficient of sliding friction. [*Ans.* 0·268]

7. The maximum speed of a motor van weighing 3 tons is 12 miles an hour on a level road, but drops to 5 miles an hour up an incline of 1 in 10. Assuming resistance per ton to vary as the square of the velocity, find the horse power of the engine. (1 ton—2240 lb.)

(*London Univ.*) [*Ans.* 9·66 H. P.]

8. Prove that the kinetic energy of two particles of masses m_1 and m_2 moving in any manner in a plane is equal to

$$\tfrac{1}{2}(m_1+m_2)\,v^2+\tfrac{1}{2}m_1\,m_2\,u^2/(m_1+m_2)$$

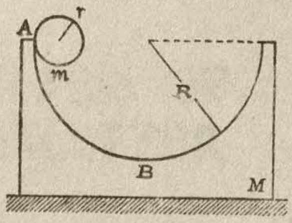
where v is the velocity of the centre of mass of the two particles and u is the relative velocity of one particle w. r. t. the other.

9. The human heart forces 60 c. c. of blood at each beat against an average pressure of 12 cm. of mercury. If the pulse frequency is 72 per minute, calculate the power of the heat in watt ? (Density of mercury—13·6 gm./c. c.) [*Ans.* 1·15 W]

10. It requires 400 H. P. to drive a ship through water at a speed of 20 miles per hour. If the resistance of the water is proportional to the second power of the speed, what horse power is acquired to drive the ship at a speed of 30 miles per hour ? [*Ans.* 1350 H. P.]

11. A block of mass M with a semicircular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see Fig.). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track ?

(*I. I. T. '83*) $\left[\textit{Ans.}\ \dfrac{m\,(R-r)}{(m+M)};\quad \dfrac{m}{M}\sqrt{2g\,(R-r)} \right]$



12. A shell of mass 10 kg. moving vertically upwards explodes into two pieces when its velocity is 22·5 m./sec. and is at a height of 23·6 m. above the ground. The lower piece of mass 2·5 kg. returns to the ground in 1·5 sec after the explosion. Find how much higher the upper piece of mass 7·5 kg. will rise after the explosion. Find also the energy of explosion.

[*Ans.* 54·86 m ; 1588·47 J]

13. A vehicle weighing 7 metric tons with its engine shut-off moves down an incline 1 in 35 at a constant speed of 20 km./hour. What amount of engine power in H. P. would be required to drive the vehicle up the same incline at the same speed, assuming frictional resistance to be the same in each case ? (*Jt. Entrance '80*) [*Ans.* 29·2 H. P.]

14. A motorcycle whose mass including rider is 400 kg. can go at 5 m./sec. up a plane of inclination 1 in 14 and 30 m./sec. down the same plane. If the resistance varies as the square of the speed and the power developed by the machine is constant, find the power developed.

[*Ans.* 1445 W.]

15. In the figure (a) and (b) AC, DG and GF are fixed inclined planes. $BC = EF = x$ and $AB = DE = y$. A small block of mass M is released from rest from the point A. It slides down AC and reaches C with a speed V_C. The same block is released from rest from the point D. It slides down DGF and reaches the point F with speed V_F. The co-efficients of kinetic friction between the block and both the surfaces AC and DGF are μ.

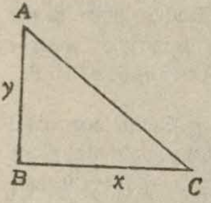


Fig. (a)

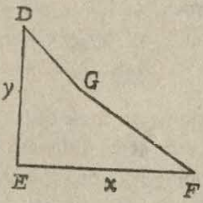


Fig. (b)

Calculate V_C and V_F.

(*I. I. T.* '80) [*Ans.* $V_C = V_F = \sqrt{2g(y - \mu x)}$]

16. A horse pulls a wagon of 400 kg. from rest against a constant resistance of 100N. The pull exerted initially is 800N and it decreases uniformly with the distance covered, to 400 N at a distance of 50 m. from the start. Find the velocity of the wagon at this point.

[*Ans.* 12·74 km./hr.]

2

GENERAL PROPERTIES OF MATTER

8
CHAPTER

GRAVITATION

8·1. Newton's Law of Universal Gravitation.

Gravitation *is the name given to the universal attraction between massive bodies,* as a consequence of which the members of the solar system, for example, maintain their stable orbits. The idea of such a general influence was held somewhat vaguely by Greek metaphysicians and also by Indian philosophers. But its emergence as clear and quantitative concept required the exact principles of motion introduced by Galileo, the observational generalizations of Kelper and the conception of force given, by Newton. From a close study of the experimental observations upon the motion of the planets, Kepler formulated three famous laws of planetary motion. In order to explain these laws, Newton synthesized the abovementioned concepts and put forward the famous **law of universal gravitation** which states :

Every particle of matter in the universe attracts every other particle with a force, which is directly proportional to the product of the masses of the two particles and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

If m and M be the masses of two particles, separated by a distance d, and F be the gravitational force of attraction which each particle exerts on the other [Fig. 8·1], then from this law,

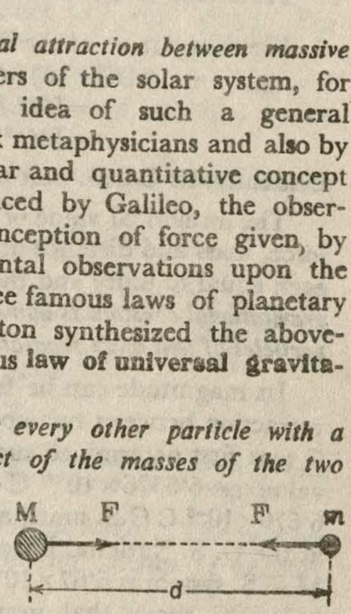


Fig. 8·1 : Gravitational attraction between two particles

$$F \propto mM \quad \text{and} \quad F \propto \frac{1}{d^2}$$

Combining, we get $$F \propto \frac{Mm}{d^2}$$

$$\text{or,} \quad F = G\frac{Mm}{d^2} \qquad \ldots \qquad \ldots \qquad (8·1)$$

where G is a constant of proportionality, known as the **universal gravitational constant.**

The *law is universal* in the sense that it is applicable to all bodies ; whether the bodies are big or small, whether they are celestial or terrestrial, the value of G remains always the same. Numerous experiments have been designed to determine the possible influence of various factors on the gravitational attraction, but all such experiments so far have been negative. The only factors of influence are the masses of the attracting bodies and their distance apart. The gravitational attraction does not depend upon the nature of the attracting bodies, nor also upon the nature of the medium between them. It is also independent of the state (i.e. whether solid, liquid or gaseous) of the attracting bodies. It does not depend upon temperature or any other physical factor.

8·2. Constant of gravitation.

From eqn. (8·1), we can write $G=\dfrac{Fd^2}{mM}$ ... (8·2)

We thus, arrive at the dimensional formula for G as

$$\begin{aligned}[G]&=[F]\,[d^2]/[mM]\\&=[MLT^{-2}]\,[L^2]/[M^2]\\&=[M^{-1}L^3T^{-2}]\end{aligned}$$

Evidently, its C.G.S. unit is *cm.^{3} gm.$^{-1}$ sec.$^{-2}$*.

The numerical value of the constant G depends on the units in which force, mass and distance are expressed. If m and M in eqn. (8·2) be put each equal to unity and the distance d be also unity, then $F=G$. Thus, *gravitational constant is numerically equal to the force of attraction between two particles of unit mass separated by unit distance.*

Its magnitude can be found experimentally by measuring the force of attraction between two bodies of known masses at a known separation. G was first determined successfully by Cavendish in 1798 and he found the value as $6{\cdot}6576\times10^{-8}$ C.G.S. units. The best present value of G is $6{\cdot}670\times10^{-8}$ C.G.S. units, as determined by P.R. Heyl and P. Chrzanowski at the U. S. National Bureau of Standard in 1942. The value of G in M.K.S. system is $6{\cdot}67\times10^{-11}$ m.^{3}kg.$^{-1}$sec^{-2}. The extremely small magnitude of G shows that gravitational forces between terrestrial bodies are extremely small and quite negligible for all ordinary purposes.

Example 8·1. *Two lead spheres of 20 cm. and 2 cm. diameters are placed with their centres 100 cm. apart. Calculate the force of attraction between them. Given that, density of lead = 11·5 gm./c.c. and* $G=6{\cdot}67\times10^{-8}\text{cm}^3.\text{gm.}^{-1}\text{sec.}^{-2}$

Solution : $M=\frac{4}{3}\times\pi\times(10)^3\times11{\cdot}5$ gm.

$m=\frac{4}{3}\times\pi\times(1)^3\times11{\cdot}5$ gm.

$$F=G\frac{Mm}{d^2}=6{\cdot}67\times10^{-8}\times\frac{(\frac{4}{3})^2\times(\pi)^2\times(11{\cdot}5)^2\times10^3\times1^3}{(100)^2}$$

$$=15{\cdot}46\times10^{-6}\text{ dyne.}$$

Example 8·2. *Two equal masses, which may be considered as particles, attract each other with a force of 4×10^{-6} dyne when they are 10 cm. apart. What is the distance between the two masses when the attraction is 8×10^{-6}* dyne ?

Solution : If the mass of each particle be m, then from eqn. (8·1),

$$4\times10^{-6}=G\frac{m^2}{10^2}\quad\text{or,}\quad Gm^2=4\times10^{-4}$$

If the distance between the particles be d, when the force of attraction is 8×10^{-6} dyne, then,

$$8\times10^{-6}=\frac{Gm^2}{d^2}=\frac{4\times10^{-4}}{d^2}$$

$$\text{or, } d^2=\frac{10^2}{2} \quad \therefore \quad d=7{\cdot}1 \text{ cm.}$$

8·3. Gravitational attraction for extended bodies.

The law of gravitation, in the form given by equation (8·1), applies only when the dimensions of the attracting masses are very small. For extended bodies separated by a very large distance, we can neglect their dimensions in comparison with the distance separating them and can treat them as point masses. The masses of the bodies are then assumed to be concentrated at their respective centres of masses.

If the above condition is not fulfilled, the attraction for extended bodies can not be obtained easily. We can regard any body of finite size as made up of a very large number of small point masses m_1, m_2, m_3 etc. To get the force of attraction between an extended body and a particle, we first find out the force of attraction between the particle and each of these point masses m_1, m_2 etc. [Fig. 8·2]. The total force of attraction is then given in magnitude and direction by the vector sum of all these forces. The process is rather complicated and can be solved readily only for bodies of regular shape. Some of these results are very useful.

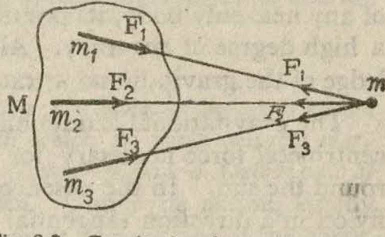


Fig. 8·2 : Gravitational attraction between an extended body and a point mass

Thus, *for a uniform solid sphere or for a uniform hollow spherical shell*, the force of attraction at points outside the sphere, or shell, is found to be the same as that due to a particle situated at the centre of the sphere or shell, the particle having the same mass as the sphere or the shell, as the case may be.

For a uniform hollow spherical shell, the force of attraction on particles at points within the shell is zero. We shall make use of these results in subsequent discussions.

To find the force of attraction between two extended bodies is still more complex. Firstly, we should divide the two bodies into point masses and then find out the forces of attraction between every pair of such point masses. The resultant attraction is obtained by summing up all these forces vectorially.

8.4. Gravitational attraction between heavenly bodies.

Our universe consists of millions and millions of heavenly bodies. Of these, those which emit light and radiant heat of themselves are called the *stars*. *Planets* are those celestial bodies which revolve round the sun in almost circular orbits. The sun has nine planets. Unlike the stars, the planets do not radiate light or heat ; they shine with the light they receive from the sun and reflect. The planet nearest to the sun is the Mercury and the furthest one is the Pluto. The other planets arranged according to their distances from the sun are the Venus, the Earth, the Mars, the Jupiter, the Saturn, the Uranus and the Neptune. Jupiter is the biggest planet ; the Mercury is the smallest one.

The heavenly bodies, called *satellites*, revolve round the planets in definite orbits. Each planet except the Mercury, the Venus and the Pluto has its own satellite. The Moon is the only satellite of the Earth. The Jupiter has the largest number of satellites, viz. eleven, of different sizes.

The motions of the stars, planets and satellites are highly regular. For example, it is possible to predict the time at which the sun rises on any day of any year accurately upto the fraction of a second. We can locate the position of any planet on any day very accurately. In fact, the orbit of any heavenly body, its position and velocity etc. can be determined with a high degree of accuracy. All these calculations are based on the knowledge of the gravitational attraction among heavenly bodies.

The gravitational attraction between the sun and its planet provides the centripetal force necessary for the planets to describe the circular motion round the sun. In the absence of this gravitational force, the planet would fly off in a direction tangential to the orbit. Exactly in a similar fashion, for the rotational motion of a satellite round a planet, the gravitational attraction between the planet and the satellite provides the necessary centripetal force. We shall now calculate the gravitational attraction between the sun and the earth and between the earth and the moon.

(i) Gravitational attraction between the sun and the earth.

In the previous article, we have seen that for a uniform solid sphere, the force of attraction at points outside it may be obtained by considering its total mass to be concentrated at its centre. So in order to determine the gravitational attraction between two heavenly bodies, considered as solid spheres, we can regard each of them as a point of equal mass situated at its centre. We know that

$$\text{the mass of the sun, } M_1 = 1{\cdot}99 \times 10^{33} \text{ gm}$$

$$\text{the mass of the earth, } M_2 = 5{\cdot}96 \times 10^{27} \text{ gm}$$

The orbit of the earth is almost circular, the average distance of the earth from the sun being $d = 1{\cdot}497 \times 10^{13}$ cm. Hence, the gravitational attraction between the earth and the sun is

$$F = G\frac{M_1 M_2}{d^2} = 6{\cdot}67 \times 10^{-8} \times \frac{1{\cdot}99 \times 10^{33} \times 5{\cdot}96 \times 10^{27}}{(1{\cdot}497 \times 10^{13})^2}$$

$$= 3{\cdot}53 \times 10^{27} \text{ dyne} = 3{\cdot}53 \times 10^{22} \text{ newton.}$$

(ii) Gravitational attraction between the earth and the moon.

The radius of the moon's circular orbit round the earth is $d=3{\cdot}84\times10^{10}$ cm. Now,

the mass of the earth, $M_1=5{\cdot}96\times10^{27}$ gm
the mass of the moon, $M_2=7{\cdot}33\times10^{25}$ gm

Hence, the gravitational attraction between the earth and the moon is

$$F=G\frac{M_1M_2}{d^2}=6{\cdot}67\times10^{-8}\times\frac{5{\cdot}96\times10^{27}\times7{\cdot}33\times10^{25}}{(3{\cdot}84\times10^{10})^2}$$

$$=1{\cdot}976\times10^{25}\text{ dyne}=1{\cdot}976\times10^{20}\text{ newton.}$$

The sun is about 30 million times heavier than the moon ; yet the above calculations show that the gravitational attraction between the sun and the earth is only about 125 times larger than that between the earth and the moon. Clearly, this occurs due to the relative proximity of the moon to the earth.

Tides are created under the action of the gravitational attraction exerted by the sun and the moon on the earth.

8·5. Gravitational attraction of the earth.

In general, *the force of attraction between any two material bodies is known as* **gravitation.** In particular, *the force with which the earth attracts every body on or near its surface is called* **gravity.** *Gravity is thus a special case of gravitation and may be called, earth's gravitation.* Under its action, a body when released, falls vertically downward towards the surface of the earth. The attraction is mutual ; the body also attracts the earth towards itself with an equal and opposite force. But as the mass of the earth is very large in comparison with that of terrestrial bodies, any motion of the earth under the action of this force is quite imperceptible.

Assuming the earth to be a homogeneous sphere of radius R, we can regard its total mass M to be concentrated at its centre. The force of gravity acting on a body of mass m on its surface is then given by

$$F=G\frac{Mm}{R^2}\qquad\cdots\qquad\cdots\qquad(8{\cdot}3)$$

and is directed towards the centre of the earth. Thus in its vertical fall under gravity, a body actually moves towards the centre of the earth. A plumb line, when suspended, points towards the centre of the earth and thus hangs in a perfectly vertical line.

Acceleration due to gravity : As a body falls downwards, it is continuously acted upon by the force of gravity. The body thus possesses an acceleration, called the *acceleration due to gravity.* It is usually denoted by the letter 'g'. From Newton's second law,

$$F=mg$$

$$\therefore\quad mg=G\frac{Mm}{R^2}$$

$$\text{or,}\quad g=G\frac{M}{R^2}\qquad\cdots\qquad\cdots\qquad(8{\cdot}4)$$

If the mean density of earth be ρ, then, $M=\frac{4}{3}\pi R^3\rho$

$$\therefore\ g=G\frac{M}{R^2}=\frac{4}{3}\pi GR\rho \qquad \cdots \qquad \cdots \qquad (8{\cdot}5)$$

The eqns. (8·4) and (8·5) give the value of g on the earth's surface.

If a body is thrown vertically upwards, the force of gravity opposes its motion ; the body therefore gets decelerated, the magnitude of deceleration being the same i.e., g.

The value of g at a particular place remains constant. However, due to the shape of the earth not being perfectly spherical and due to the diurnal motion of the earth, the value of g varies from place to place on the earth's surface (vide Art. 8·6). The minimum value is obtained at the equator being about 978 cm./sec.^{2} and the maximum value is obtained at the poles, viz., 983 cm./sec.^{2} The variation is thus very small. The *standard value* is taken as 980·6 cm./sec.^{2} or 32·2 ft./sec.^{2} which is obtained at the sea-level at latitude 45°.

Weight of a body : The force with which the earth attracts a body is also called its **weight** (W). Weight of a body is thus synonymous with the force of gravity acting on it. Hence,

$$W=mg \qquad \cdots \qquad \cdots \qquad (8{\cdot}6)$$

The weight acts at its centre of gravity vertically downward i.e., towards the centre of the earth. Variation in the value of g produces a proportional change in the weight of a body. Weight of a body is therefore, not an invariant quantity.

Mass and Weight : In our everyday utterances, we do not make any distinction between the mass and the weight of a body. A body is usually said to weigh 5 kg., its mass being also said to be equal to 5 kg. However, they are distinctly different quantities, the differences being :

(1) The mass of a body is the quantity of matter contained in it. Mass is a measure of the inertia of the body. It is thus a scalar quantity.

Weight of a body is a force viz., the gravitational force exerted by the earth on it. It is thus a vector quantity.

(2) The weight of a body at a place is given by the product of its mass and the acceleration due to gravity at that place. Hence, the mass and the weight of a body can never be equal.

(3) The mass of a body remains constant at all places. But the weight of a body varies from place to place due to variation in the value of g. At the centre of the earth, $g=0$ and so the weight of a body vanishes there ; but its mass remains the same. Similarly, if the body is taken to the moon, its weight decreases due to the fact that the gravitational attraction of the moon is about one-sixth times of that due to the earth. Due to similar reasons, a body will weigh twenty-seven times more when taken to the sun. However, its mass on the moon or on the sun is the same as that on the earth.

Being an invariant quantity, the mass of a body is considered as one of its intrinsic property. Such consideration is inapplicable in the case of its weight, because it varies from place to place.

8·6. Variation of acceleration due to gravity.

As mentioned in the previous article, the value of g varies from place to place due to different causes. These are now discussed below fully :

(i) Variation with altitude : Let us consider a body of mass m situated at a height h above the surface of the earth. We assume the earth to be a perfect sphere of radius R and mass M. From the law of gravitation, the force of attraction of the earth on the body is given by

$$F=G\frac{Mm}{(R+h)^2}$$

If g' be the acceleration due to gravity at that place, then $F=mg'$.

Therefore,
$$g'=G\frac{M}{(R+h)^2} \quad \cdots \quad \cdots \quad (8·7)$$

But from eqn. (8·4), the value of acceleration due to gravity on the surface of the earth is given by

$$g=G\frac{M}{R^2}$$

$$\therefore \quad \frac{g'}{g}=\frac{R^2}{(R+h)^2}=\frac{1}{\left(1+\frac{h}{R}\right)^2} \quad \cdots \quad (8·8)$$

When h is very small compared to R, then expanding binomially and neglecting the higher order terms, we have from eqn. (8·8)

$$g'=g\left(1-\frac{2h}{R}\right) \quad \cdots \quad (8·9)$$

As h increases, g' decreases. Hence, *the acceleration due to gravity decreases with altitude.*

(ii) Variation with depth : Let us consider now a body of mass m situated at A at a depth h below the surface of the earth [Fig. 8·3], say at the bottom of a mine. The distance of the body from the centre of the earth is then $(R-h)$. We draw a sphere concentric with the earth and of radius $(R-h)$. The body lies on the surface of this inner sphere of radius $(R-h)$ and just inside the hollow spherical shell of thickness h. As has been stated earlier (Art. 8·3), the gravitational attraction inside a hollow shell is zero. Hence, this portion of the earth does not exert any gravitational force on the body. Thus the force of attraction on the body is only due to the inner sphere of radius $(R-h)$ and is directed towards the centre.

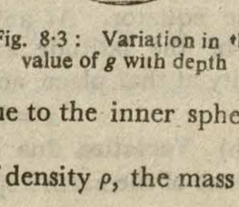


Fig. 8·3 : Variation in the value of g with depth

Assuming the earth to be a homogeneous sphere of density ρ, the mass of the inner sphere = volume × density $=\frac{4}{3}\pi(R-h)^3\rho$.

From the law of gravitation, the force of attraction on the body of mass m is

$$F = G.\frac{\frac{4}{3}\pi(R-h)^3\rho.m}{(R-h)^2} = \frac{4}{3}\pi G(R-h)\rho m$$

If g' be the acceleration due to gravity at that place at a depth of h, then, $F=mg'$.

$$\therefore \quad g' = \tfrac{4}{3}\pi G(R-h)\rho \qquad \cdots \qquad \cdots \qquad (8{\cdot}10)$$

g' is, therefore, directly proportional to $(R-h)$. *The acceleratian due to gravity inside the earth, then varies directly as the distance from the centre of the earth.* As we go deeper inside the earth, the acceleration due to gravity proportionally diminishes and ultimately *at the centre of the earth, it compltely vanishes.* The weight of the body is thus zero at the centre of the earth.

Now, from eqn. (8·5) we have $g=\frac{4}{3}\pi GR\rho$

$$\therefore \quad \frac{g'}{g} = \frac{R-h}{R} = 1 - \frac{h}{R}$$

$$\text{or,} \quad g' = g\left(1 - \frac{h}{R}\right) \qquad \cdots \qquad \cdots \qquad (8{\cdot}11)$$

From the above discussions, it is evident that the value of acceleration due to gravity is maximum on the surface of the earth. For points outside the earth, the value of g is inversely proportional to the square of the distance from the centre of the earth [vide eqn. 8·7] and for points inside the earth, g varies directly as the aforesaid distance. This variation of g is shown by plotting its values against the corresponding distances [Fig. 8·4], R being the earth's radius.

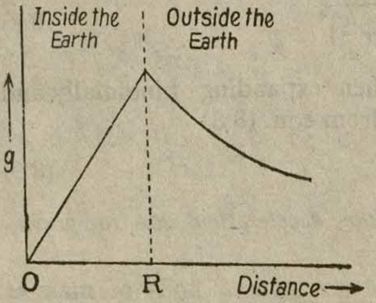


Fig. 8·4 : Variation of g with distance from the centre of the earth

(iii) Variation on the earth's surface : On the surface of the earth also, the value of g varies from place to place. This is due to the following two causes :

(a) **Variation due to the peculiar shape of the earth :** The earth is an oblate spheroid, being slightly flattened at the poles and bulging at the equator ; the equatorial radius is greater than the polar radius by about 21·72 km. Since the value of g on the earth's surface varies inversely as the square of the radius, its value is thus greatest at the poles and least at the equator. At any other place, g will have an intermediate value. If λ be the latitude at any place, then the value of acceleration due to gravity at that place, according to **Helmert**, is given by

$$g' = 978{\cdot}00(1 + 0{\cdot}00531 \sin^2\lambda) \text{ cm./sec.}^2$$

(b) **Variation due to the rotation of the earth about its axis :** On account of the earth's spinning motion about its axis, all the bodies on the surface of the earth, except those lying on the poles, execute circular motion, the centres of the circles being situated on the axis of the earth. Consequently, a body is subjected to a centrifugal force due to which it tends to fly off. This force is largest at the equator and vanishes at the poles. Since this force acts in a directicn opposite to the force of gravity, the weight of the body appears less than its true value.

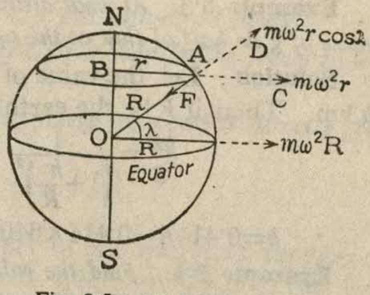


Fig. 8·5 : Variation of g due to the earth's diurnal motion

Let us consider a body of mass m situated on the earth's surface at a point A of latitude λ [Fig. 8·5]. The angular velocity of rotation of the earth around the polar axis NS is given by ω. The body describes a circle of radius $AB=r$ where $r=R\cos\lambda$, R being the earth's radius. The centrifugal force acting on the body along AC is thus given by $m\omega^2 r$. Due to gravity, a force $F=mg$ acts on the body towards the centre of the earth i.e., along AO. The component of the centrifugal force on the body along AD i.e., opposing the force of gravity is $m\omega^2 r\cos\lambda$. Hence, the apparent weight of the body at A

$$=mg-m\omega^2 r\cos\lambda=mg-m\omega^2 R\cos^2\lambda$$

If g' be the acceleration due to gravity at A, then the apparent weight is also given by mg'.

$$\therefore\quad g'=g\left(1-\frac{\omega^2 R\cos^2\lambda}{g}\right)\qquad\cdots\qquad\cdots\qquad(8\cdot12)$$

$$=g\left(1-\frac{\cos^2\lambda}{288}\right)\quad\left[\begin{array}{l}\text{Taking } g=981\text{ cm./sec.}^2\text{ ; } R=6\cdot367\times10^8\text{ cm.}\\ \text{and }\ \omega=2\pi/24\times60\times60\text{ rad./sec.}\end{array}\right]$$

At the equator, $\lambda=0°$; so, $\cos\lambda=1$

$$\therefore\quad g'=g\left(1-\frac{\omega^2 R}{g}\right)=g-\omega^2 R$$

At the poles, $\lambda=90°$; so, $\cos\lambda=0\quad\therefore\quad g'=g$.

Hence due to the diurnal motion of the earth, the variation in the acceleration due to gravity is such that it is minimum at the equator and maximum at the poles. At intermediate places, its value lies between these two extremes. It should be noted that the variation in the value of g due to the causes (*a*) and (*b*) are of similar nature.

8·7. Mass and Mean Density of the Earth.

The mass of the earth (M) can be calculated from the relation $g=G\dfrac{M}{R^2}$ if the values of g, R and G are known. Taking $g=981$ cm./sec.², $R=6\cdot367\times10^8$ cm., and $G=6\cdot67\times10^{-8}$ c.g.s. units, the mass of the earth is given by

$$M=\frac{R^2 g}{G}=5\cdot96\times10^{27}\text{ gm.}$$

The mean density ρ of the earth can similarly be calculated from eqn. (8·5) as

$$\rho=\frac{3g}{4\pi GR}=5{\cdot}46 \text{ gm./c.c.}$$

Example 8·3. *At what altitude above the earth's surface would the numerical value of g be half of that at the surface? Given the radius of the earth=6400 km.*

Solution : Let the value of g be half of that at the surface at an altitude h km. Then if R be the earth's radius, we have from eqn. (8·8),

$$\frac{\frac{1}{2}g}{g}=\frac{1}{\left(1+\frac{h}{R}\right)^2} \quad \text{or,} \quad 1+\frac{h}{R}=\sqrt{2}$$

$$\therefore \quad h=0{\cdot}414R=0{\cdot}414\times 6400=2650 \text{ km.}$$

Example 8·4. *Find the value of g in a mine four kilometres deep if g at the surface be 980 cm./sec². and earth's radius be 6400 km.*

Solution : We have from eqn. (8·11), the value of acceleration due to gravity inside the mine as,

$$g'=g\left(1-\frac{h}{R}\right)=980(1-\tfrac{4}{6400})=979{\cdot}5 \text{ cm./sec.}^2$$

Example 8·5. *The planets Earth and Mars have diameters 8000 mi. and 4000 mi. respectively and the mass of the Mars is 0·12 that of the Earth. If a body weighs 100 lb. on the surface of the Earth, what would be its weight on the Mars? What is the acceleration due to gravity on the Mars?*

Solution : Let the weight of the body on the Earth be w_1 and the mass and radius of the Earth be M_1 and R_1 respectively. If the mass of the body be m and the acceleration due to gravity on the Earth's surface be g_1, then,

$$w_1=mg_1=G\frac{mM_1}{R_1^2}$$

Similarly, if the weight of the body on the Mars be w_2, where the acceleration due to gravity is g_2 and if M_2 and R_2 be the mass and the radius of the Mars, then

$$w_2=mg_2=G\frac{mM_2}{R_2^2} \qquad \therefore \quad \frac{w_2}{w_1}=\frac{g_2}{g_1}=\frac{M_2}{M_1}\times\frac{R_1^2}{R_2^2}$$

But, $w_1=100$ lb., $M_2=0{\cdot}12\,M_1$, $R_1=4000$ mi. and $R_2=2000$ mi.

$$\therefore \quad w_2=0{\cdot}12\times\left(\frac{4000}{2000}\right)^2\times 100 \quad 48 \text{ lb.}$$

$$\because \quad g_1=32 \text{ ft./sec}^2.$$

$$\therefore \quad g_2=\frac{w_2}{w_1}g_1=\frac{48}{100}\times 32=15{\cdot}4 \text{ ft./sec.}^2$$

8·8. Laws of falling bodies.

It is a common experience that a body when released falls downwards and as we now know, this happens due to the action of gravity. For all practical purposes the height from which the body is released being very small compared to the radius of the earth, the acceleration due to gravity may be assumed to remain constant throughout the downward motion. When such downward motion takes place only under the action of gravity, the resistive forces due to air being absent, the following simple laws known as the **laws of falling bodies**, are found to be applicable. These laws were first formulated by Galileo by dropping objects from the top of of the leaning tower of Pisa.

(1) *In vacuum, all bodies starting from rest fall with equal rapidity.*

(2) *In a given time, the velocity acquired by a body falling freely from rest is directly proportional to the time.*

(3) *The space traversed by a body falling freely from rest is directly proportional to square of the time.*

Discussion of the laws.

First Law—The first law seems contrary to our everyday experience as a heavy body appears to fall to the ground more quickly than the lighter one. But by experiments with bodies of different sizes and materials. Galileo established that all bodies fall with almost equal rapidity. He pointed out that the observed slight differences between the time of fall of heavier and lighter bodies are due to the resistance of air. After the invention of exhaust pump, Newton conclusively proved the validity of the law by his well-known guinea and feather experiment.

Guinea and feather experiment : A hollow glass tube, about 5 cm. in diameter and one metre long with a cap screwed at one end and a stop-cock at the other end is taken [Fig. 8·6]. A guinea coin and a feather are introduced by opening the screwed cap. On inverting the tube, the coin will be found to fall more quickly than the feather. Air is then exhausted from the tube by connecting it with an exhaust pump. The coin and feather are then found to fall together.

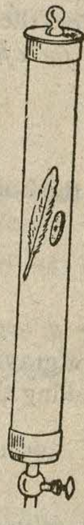
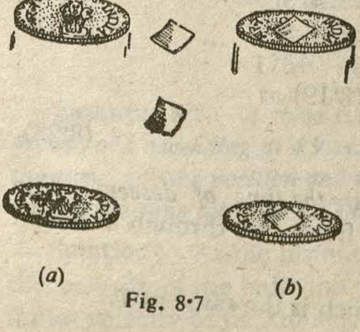


Fig. 8·6 : Guinea and feather experiment

This experiment thus confirms that the force of gravity causes an equal acceleration in all bodies. Inequality in the time of fall is attributable to air resistance.



(*a*) (*b*)

Fig. 8·7

A simple demonstration experiment : A coin and a small piece of paper are dropped from the same height simultaneously. The coin will reach the ground faster [Fig. 8·7 (*a*)]. But if the paper is put on the coin, then it will be found that both reach the ground at the same time [Fig. 8·7(*b*)].

Second Law—Starting from rest, if a

body falls for a time t and thus acquires a velocity v, then according to the second law

$$v \propto t, \qquad \therefore \frac{v}{t}=\text{constant.}$$

Since acceleration is the rate of change of velocity, this law implies that the acceleration of a freely falling body is constant.

Third Law—If a body starting from rest, falls through a height h in a time t, then according to the third law

$$h \propto t^2 \qquad \text{i.e. } \frac{h}{t^2}=\text{constant.}$$

8·9. Equations of Vertical Motion.

Since the downward motion of a body under the action of gravity is a rectilinear motion with constant acceleration, the equations of article (2·10) hold true. Replacing s by h for the height of descent, and f by g for the acceleration, we get the following equations of motion under gravity *for a falling body* :

$$v=u+gt \quad \text{.... ...} \quad (8\cdot13)$$

$$h=ut+\tfrac{1}{2}gt^2 \quad \text{... ...} \quad (8\cdot14)$$

$$v^2=u^2+2gh \quad \text{... ...} \quad (8\cdot15)$$

When the body falls from rest, $u=0$, and the above equations reduce to

$$v=gt \quad \text{... ...} \quad (8\cdot16)$$

$$h=\tfrac{1}{2}gt^2 \quad \text{... ...} \quad (8\cdot17)$$

$$v^2=2gh \quad \text{... ...} \quad (8\cdot18)$$

For a body projected vertically upwards with an initial velocity u, the force of gravity opposes its motion. The body thus rises with a retardation g. Taking the upward direction to be positive, the equations now become

$$v=u-gt \quad \text{... ...} \quad (8\cdot19)$$

$$h=ut-\tfrac{1}{2}gt^2 \quad \text{... ...} \quad (8\cdot20)$$

$$v^2=u^2-2gh \quad \text{... ...} \quad (8\cdot21)$$

The maximum height attained by the body can be found by remembering that at the highest point, the velocity of the body is zero. The body stops there momentarily and then again falls downwards. Putting $v=0$, in equation (8·21), the *maximum height of ascent* H is given by,

$$H=\frac{u^2}{2g} \quad \text{... ...} \quad (8\cdot22)$$

The *time of ascent* T is given from equation (8·19) as

$$T=\frac{u}{g} \quad \text{... ...} \quad (8\cdot23)$$

We can easily prove that *the time of ascent=the time of descent.* The time of descent T_1 is the time necessary to fall from rest through a height of $u^2/2g$. Hence from equation (8·17),

$$\frac{u^2}{2g}=\tfrac{1}{2}gT_1^2 \qquad \text{or, } T_1=u/g, \text{ which is the same as } T.$$

So, the *whole time of flight* $=\frac{2u}{g}$ ... ... (8·24)

Example 8·6. *An anti-aircraft shell is fired vertically upwards with a muzzle velocity of 294 m./sec. Calculate (a) the maximum height reached by it, (b) the time taken to reach this height, (c) the velocities at the ends of 20th and 40th sec. (d) When will its height be 2450 m.? Given $g=980$ cm./sec^2.*

Solution : (a) Here the initial velocity $u=294$ m./sec. and $g=9{\cdot}8$ m./sec.^{2}

∴ From eqn. (8·22), the maximum height reached by the shell is,

$$H=\frac{u^2}{2g}=\frac{294^2}{2\times 9{\cdot}8}=4410 \text{ m.}=4{\cdot}41 \text{ km.}$$

(b) From eqn. (8·23), the time taken to reach the height is,

$$T=\frac{u}{g}=\frac{294}{9{\cdot}8}=30 \text{ sec.}$$

(c) From eqn. (8·19), the velocity at the end of 20th sec. is given by,

$$v=u-gt=294-9{\cdot}8\times 20=98 \text{ m./sec. upward,}$$

and, the velocity at the end of 40th sec. is given by,

$$v=294-9{\cdot}8\times 40=-98 \text{ m./sec.}$$

The negative sign implies that the shell is falling downward.

(d) From eqn. (8·20), $h=ut-\frac{1}{2}gt^2$ or, $2450=294t-\frac{1}{2}\times 9{\cdot}8t^2$
or, $t^2-60t+500=0$ ∴ $t=10$ and 50 sec.

At $t=10$ sec. the shell is at a height of 2450 m. and is ascending, and at the end of 50 sec. it is at the same height, but is falling.

Example 8·7. *A ball dropped from a tower strikes the ground in 6 sec. Find the velocity with which the ball strikes the ground and the height of the tower. Find the distance travelled in the last 3 sec.*

Solution : From eqn. (8·16), the velocity of the ball is given by,

$$v=gt=980\times 6=5880 \text{ cm./sec.}=58{\cdot}8 \text{ m./sec.}$$

The height of the tower can be obtained from eqn. (8·17). Thus,

$$h=\tfrac{1}{2}gt^2=\tfrac{1}{2}\times 980\times 6^2=17640 \text{ cm.}=176{\cdot}4 \text{ m.}$$

The distance travelled in the last 3 sec.

=the distance fallen in 6 sec.—the distance fallen in the first 3 sec.

$$=17640-\tfrac{1}{2}\times 980\times 3^2=13230 \text{ cm.}=132{\cdot}3 \text{ m.}$$

Example 8·8. *A stone is dropped from a balloon which is 441 m. above the ground and ascending at 4·9 m./sec. Find (a) the maximum height reached by the stone, (b) the position and velocity of the stone 5 sec. after being dropped, and (c) the time taken to hit the ground.*

Solution : At the instant the stone is dropped, its velocity is equal to the velocity of the balloon. Taking the upwards direction as positive, the initial velocity of the stone is $u=490$ cm./sec.

(a) From eqn. (8·22), the maximum height reached by the stone above the point at which it is dropped is given by,

$$H=\frac{u^2}{2g}=\frac{490^2}{2\times980}=122{\cdot}5 \text{ cm.}=1{\cdot}225 \text{ m.}$$

Hence, the maximum height above the ground reached by the stone $=(441+1{\cdot}225)=442{\cdot}225$ m.

(b) From eqn. (8·20), the height of the stone after 5 sec. is given by,

$$h=ut-\tfrac{1}{2}gt^2=490\times5-\tfrac{1}{2}\times980\times25=-9800 \text{ cm.}=-98 \text{ m.}$$

The negative sign indicates that the stone is at that moment 98 m. below the point of release. Hence it is at a height of

$$(441-98)=343 \text{ m. above the ground.}$$

From eqn. (8·19), the velocity of stone after 5 sec. is given by,

$$v=u-gt=490-980\times5=-4410 \text{ cm./sec.}=-44{\cdot}1 \text{ m./sec.}$$

The negative sign implies that the stone is falling downwards.

(c) When the stone hits the ground, its displacement $h=-44100$ cm. below the point of release.

$\therefore$ From eqn. (8·20), $h=ut-\tfrac{1}{2}gt^2$

or, $-44100=490t-\tfrac{1}{2}\times980\times t^2$ or, $490t^2-490t-44100=0$

or, $(t-10)(t+9)=0$ $\therefore$ $t=10$ sec. or -9 sec.

As the negative answer is absurd, the required time is 10 sec.

8·10. Simple pendulum.

A heavy point mass suspended by means of a weightless, inextensible and perfectly flexible string and executing to and fro oscillation without friction, constitutes a **simple pendulum.** Obviously this can not be realized in practice. The nearest approach to such a simple pendulum is achieved by suspending a small heavy body from a fixed support by a very fine long thread. The weight of the thread can be neglected in comparison to that of the body.

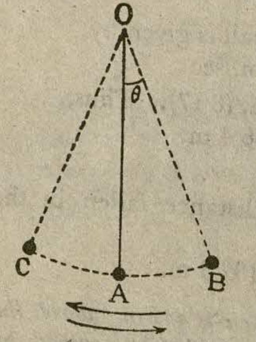


Fig. 8·8 : Simple pendulum

A small heavy spherical ball suspended by a string from the point *O* [Fig. 8·8] constitutes in reality a simple pendulum. The body is called the *bob* of the pendulum and *O* is known as the *point of suspension.* The **effective length** of the pendulum is the distance between the point of suspension and the centre of gravity of the bob. For a spherical bob, the effective length *l* is the distance from the point *O* to the centre of the bob and is thus equal to $(L+r)$, where L is the length of the string and r the radius of the bob.

At the position *A*, when the centre of gravity of the bob is vertically below the point of suspension, the bob is at rest. If it is slightly displaced to a position *B* and then released, it moves back towards *A* with increasing speed. Due to the inertia of motion, the bob crosses *A* and then moves with decreasing speed until it reaches the point *C*, where it stops momentarily. For negligible air resistance, it is found that $AC=AB$. The bob then swings back and the motion is continued as described above, but in the opposite direction. This to and fro motion of the bob continues until the resistive force offered by air gradually brings it to rest. The pendulum is said to be oscillating about the point *A* within the limits *B* and *C*. It can be proved that this motion is *simple harmonic* (vide Art. 22·7, Vibrations and Waves). The maximum displacement of the bob (*AB* or *AC*) on either side from its mean position (*A*) is called its **amplitude.** It can also be measured by the angle between its mean position and the extreme position on either side and is called the *angular amplitude*. Thus in the figure, $\theta=\angle AOB$, or $\theta=\angle AOC$ gives the angular amplitude of the pendulum. When the pendulum starting from any point of its path again comes back to the same point moving in the same direction, it is said to have described one **complete oscillation**. Thus during its oscillatory motion, as the bob passes through *A* towards *B* and on reaching *B* comes back to *A*, moves towards *C* and then again returns to *A*, it executes one complete oscillation. The time required to complete one full oscillation is called the **time period** or simply the **period** of oscillation. The number of full oscillations executed in one second is called the **frequency**. If the period be *T* and *n* the frequency, then

$$nT=1\text{ ; or, } n=\frac{1}{T}$$

8 11. Laws of simple pendulum.

The motion of a simple pendulum obeys the following laws, known as '*Laws of simple pendulum*'. These laws explain how its motion depends on the angular amplitude, the effective length of the pendulum, the acceleration due to gravity at a place and the mass of the bob.

I. **Law of isochronism** : *Angular amplitude remaining small (not exceeding 4°), the period of a pendulum of given length at a given place, remains the same being independent of the amplitude.*

II. **Law of length** : *For oscillations of small amplitude, the period of a simple pendulum at a given place varies directly as the square root of its effective length.*

If T be the period of a simple pendulum of effective length l at a given place, then $T\propto\sqrt{l}$.

III **Law of gravity** : *For a simple pendulum of given length, if the amplitude be small, the period varies inversely as the square root of the acceleration due to gravity.*

If g represents the acceleration due to gravity, then $T\propto\frac{1}{\sqrt{g}}$, the effective length l being constant.

IV. Law of mass : *The period of a simple pendulum at a given place is independent of the mass or material of the bob, provided the effective length of the pendulum remains the same.*

The laws can be summarized as

(i) if l and g are constant, T is constant (**Laws I & IV**)

(ii) if g is constant, $T \propto \sqrt{l}$ (Law II)

(iii) if l is constant, $T \propto \frac{1}{\sqrt{g}}$ (Law III)

Combining, we get $T \propto \sqrt{\frac{l}{g}}$

$$\text{or,}\quad T = \text{constant} \times \sqrt{\frac{l}{g}}$$

The value of this constant may by shown to be equal to 2π.

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}} \qquad \cdots \qquad \cdots \qquad (8{\cdot}25)$$

Rearranging the equation (8·25), we get

$$g = 4\pi^2 \frac{l}{T^2} \qquad \cdots \qquad \cdots \qquad (8{\cdot}26)$$

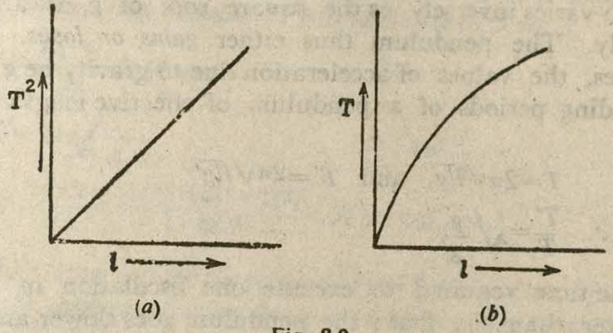
This shows that when the period and the effective length of a simple pendulum are known, the acceleration due to gravity at a place can be easily determined.

8·12. Verification of the Laws of Pendulum.

The laws of pendulum may be experimentally verified as follows :

Law I : A simple pendulum with a given effective length is made to oscillate with a small angular amplitude, not exceeding 4°. The period of oscillation is measured by noting the time necessary to complete a given number of oscillations, say 20. On changing the amplitude of oscillations, taking care that the angular amplitude does not exceed the above-mentioned limit of 4° and repeating the measurement, it will be found that the period of oscillation always remains the same. Thus the oscillations of a simple pendulum are isochronous.

Law II : The effective length l, of the pendulum is measured. For a spherical bob, this is obtained by adding the radius r of the bob to the length L of the string. The radius of the bob is found out with the help of a slide callipers. The period of oscillation is obtained by observing the time necessary to execute a given number of oscillations, say 20. On changing the length of the string, the effective length is altered. The experiment is thus repeated with different lengths of the string and the corresponding periods are measured. The value of $\frac{l}{T^2}$ will be found to be always the same, provided the amplitude of oscillation is small. Thus, $\frac{\sqrt{l}}{T}$=constant ; or, $T \propto \sqrt{l}$.



(a) (b)

Fig. 8·9

A plot of T^2 against l will be found to be a straight line [Fig. 8·9(a)]. The curve obtained by plotting T against l will be parabolic [Fig. 8·9(b)].

Law III : A simple pendulum with a given effective length is taken to different places and the periods of oscillation are measured. Since the value of g varies from one place to another on the surface of the earth, the periods will be found to be different. However, the variation is such that the product $T^2 \times g$ comes out to be always the same, i.e.

$$T \times \sqrt{g} = \text{constant, whence } T \propto \frac{1}{\sqrt{g}}$$

Law IV : Spherical bobs of different materials and different sizes are taken. Altering the lengths of the string, the effective lengths in each case are kept the same. On measuring the periods of these pendulums in the usual way, it will be found that the period always remains same for oscillations of small angular amplitude.

8·13. Seconds Pendulum.

If for a pendulum, the time required to swing from one extremity to the other is one second, it is called a **seconds pendulum**. Since this means that the time necessary for half an oscillation is one second, *the period of a seconds pendulum is obviously two seconds.*

Since, $T=2\pi\sqrt{l/g}$, for a seconds pendulum, $2=2\pi\sqrt{l/g}$

$$\therefore\ l=\frac{g}{\pi^2} \quad \cdots \quad \cdots \qquad (8·27)$$

In C.G.S. system, taking $g=981$ cm./sec.²

$$l=\frac{981}{\pi^2}=\frac{981}{9·87}=99·39 \text{ cm.}$$

In F.P.S. system, $g=32·2$ ft./sec.²

$$l=\frac{32·2}{9·87}=3·26 \text{ ft.}$$

8·14. Gain or loss of time of a pendulum due to change of place.

As discussed in article (8·5), the value of acceleration due to gravity, g, varies from place to place. Since the period of a pendulum of given effective length varies inversely as the square root of g, it also changes correspondingly. The pendulum thus either *gains or loses.* If at two different places, the values of acceleration due to gravity be g and g' and the corresponding periods of a pendulum of effective length l be T and T', then

$$T=2\pi\sqrt{l/g}, \text{ and } T'=2\pi\sqrt{l/g'}$$

$$\therefore \frac{T'}{T}=\sqrt{\frac{g}{g'}} \qquad \ldots \qquad \ldots \qquad (8{\cdot}28)$$

If $T'>T$, the time required to execute one oscillation in the second place is greater than the first ; the pendulum goes slower and loses. If $T'<T$, the pendulum runs faster and gains.

(i) **Effect of height** : With increasing altitude above the earth's surface, the value of acceleration due to gravity diminishes and thus the period of a pendulum increases. From eqn. (8·8), we get,

$$\frac{g'}{g}=\frac{R^2}{(R+h)^2}$$

$$\therefore \text{ from eqn. (8·28), } \frac{T'}{T}=\frac{R+h}{R}=1+\frac{h}{R}$$

$$\text{or, } T'=T\left(1+\frac{h}{R}\right) \qquad \ldots \qquad \ldots \qquad (8{\cdot}29)$$

where T' and T are the periods at a height h above the earth's surface and on the earth's surface respectively, R being the radius of the earth. Since, $T'>T$ a pendulum loses when taken upwards. Measurement of this loss provides a method for the determination of the height of a place (*c.f.* Ex. 8·12).

(ii) **Effect of depth** : Since the acceleration due to gravity diminishes as we go deeper inside the earth, the period of a pendulum increases. From equation (8·11),

$$\frac{g'}{g}=\frac{R-h}{R}$$

$$\therefore \text{ from eqn. (8·28), } \frac{T'}{T}=\sqrt{\frac{R}{R-h}}$$

$$\text{or, } T'=T\left(1+\frac{h}{2R}\right) \qquad \ldots \qquad \ldots \qquad (8{\cdot}30)$$

where T' is the period at a depth h below the earth's surface. Since, $T'>T$, in this case also, the pendulum loses.

(iii) **Effect of latitude** : As explained in article (8·5), the value of acceleration due to gravity increases with latitude, being minimum at the equator and maximum at the poles. Hence, the period of a pendulum is largest at the equator and smallest at the poles. A pendulum thus gains i.e., runs faster as it is taken from the equator to the poles.

Example 8·9. *Calculate the acceleration due to gravity at a place where a simple pendulum 150 cm. long executes 25 oscillations in 61·5 sec.*

Solution : $T=\frac{61{\cdot}5}{25}=2{\cdot}46$ sec.

$$T=2\pi\sqrt{\frac{l}{g}} \qquad \therefore \quad T^2=4\pi^2\frac{l}{g}$$

Hence, $g=\frac{4\pi^2 l}{T^2}=\frac{4\times(3{\cdot}14)^2\times 150}{(2{\cdot}46)^2}=977{\cdot}9$ cm./sec.^{2}

Example 8·10. *How will the time period of oscillations of a simple pendulum transferred from the earth to the moon change ? Given mass of earth$=5{\cdot}96\times10^{27}$ gm., of moon$=7{\cdot}3\times10^{25}$ gm. ; radius of earth$=6{\cdot}37\times10^{8}$ cm., of moon $=1{\cdot}74\times10^{8}$ cm.*

Solution : Let T_e =Time period on earth

T_m =Time period on moon

R_e =Radius of earth

R_m =Radius of moon

M_e =Mass of earth

M_m =Mass of moon

g_e =Acceleration due to gravity on earth

g_m =Acceleration due to gravity on moon

We have from eqn. (8·28),

$$\frac{T_m}{T_e}=\sqrt{\frac{g_e}{g_m}}$$

Again from eqn. (8·4) we have,

$$g_e=G\frac{M_e}{R_e^2} \text{ and } g_m=G\frac{M_m}{R_m^2}$$

$$\therefore \quad \frac{T_m}{T_e}=\frac{R_m}{R_e}\sqrt{\frac{M_e}{M_m}}=\frac{1{\cdot}74\times10^8}{6{\cdot}37\times10^8}\sqrt{\frac{5{\cdot}96\times10^{27}}{7{\cdot}3\times10^{25}}}=2{\cdot}47$$

Example 8·11. *A pendulum which beats seconds at the place where $g=981$ cm./sec^2 is taken to a place where $g=980{\cdot}32$ cm./sec^2. How many seconds does it lose or gain in a day ? By how much its length be altered to keep correct time ?*

Solution : Let T_1 and T_2 be the original and new period of the pendulum respectively.

We have $T_1=2\pi\sqrt{\frac{l}{981}}$ and $T_2=2\pi\sqrt{\frac{l}{980{\cdot}32}}$

Hence, $\frac{T_1}{T_2}=\sqrt{\frac{980{\cdot}32}{981}}=\sqrt{\frac{981-0{\cdot}68}{981}}=\sqrt{1-\frac{0{\cdot}68}{981}}$

It is obvious that $T_2 > T_1$ and so the pendulum will lose time. Let $n=$ no. of sec. lost per day. Since the number of sec. in a day is $(24 \times 60 \times 60)$ or 86400.

$$\therefore \quad (86400 - n)T_2 = 86400 T_1$$

$$\text{or,} \quad 86400 - n = 86400 \frac{T_1}{T_2} = 86400 \left(1 - \frac{0{\cdot}68}{981}\right)^{\frac{1}{2}}$$

$$= 86400 \left(1 - \frac{1}{2} \times \frac{0{\cdot}68}{981}\right) \text{ (approx.)} = 86400 - 30$$

$\therefore \quad n = 30$; hence the pendulum loses 30 sec. per day.

As the pendulum loses 30 sec. a day, it beats $(86400 - 30)$ or 86370 times in a day i.e., in 86400 sec.

$\therefore$ Time of one half-oscillation $= \frac{86400}{86370}$ sec.

Since the time of one half-oscillation is $\pi\sqrt{l/g}$, we have

$$\pi\sqrt{l/g} = \frac{86400}{86370} \quad \therefore \quad \pi^2 \frac{l}{g} = \left(\frac{86400}{86370}\right)^2 \quad \cdots \quad (1)$$

Let the length of the pendulum be shortened by x, so that it keeps correct time. It then becomes a true seconds pendulum and time for one half-oscillation is, therefore, equal to 1 second.

$$\therefore \quad \pi\sqrt{\frac{l-x}{g}} = 1, \quad \text{or,} \quad \pi^2 \frac{l-x}{g} = 1 \quad \cdots \quad \cdots \quad (2)$$

$$\text{From (1) and (2)} \quad \frac{\pi^2 x}{g} = \frac{\pi^2 l}{g} - 1 = \left(\frac{86400}{86370}\right)^2 - 1 = \left(1 + \frac{30}{86370}\right)^2 - 1$$

$$= \left(1 + \frac{2 \times 30}{86370}\right) - 1 \text{ (approx.)} = \frac{60}{86370}$$

$$\therefore \quad x = \frac{g}{\pi^2} \times \frac{60}{86370}$$

Since, $g = 980{\cdot}32$ cm./sec.^{2}, $\quad \therefore \quad x = \dfrac{980{\cdot}32 \times 60}{\pi^2 \times 86370} = 0{\cdot}07$ cm.

Hence, the length must be shortend by 0·07 cm. to keep correct time.

Example 8·12. *A pendulum beats seconds in a certain place on the surface of the earth but when it is taken to the peak of a mountain, it loses 20 sec. per day. Find the height of the peak. [Earth's radius is 6400 km.]*

Solution : The period T on the earth's surface $= 2$ sec. Let T' be the period of the pendulum on the peak of the mountain of height h km. If R be the radius of the earth, then from eqn. (8·29),

$$T' = 2\left(1 + \frac{h}{R}\right) \quad \cdots \quad \cdots \quad (1)$$

Since at the peak, pendulum loses 20 sec. per day, it therefore completes $(86400 - 20) = 86380$ half oscillations per day.

$\therefore$ Time of half an oscillation $= \frac{86400}{86380}$ sec.

$$\text{i.e.} \quad \frac{T'}{2} = \frac{86400}{86380} \quad \cdots \quad \cdots \quad (2)$$

From (1) and (2), $1+\frac{h}{R}=\frac{86400}{86380}$

$$\therefore \quad h=\frac{20\times 6400}{86380}=1\cdot 48 \text{ km.}$$

8·15. Motion of planets and satellites (Essential points).

We all know that the planets revolve round the sun. The heavenly bodies, called satellites, revolve round the planets in the same way as the planets keep going round the sun. In 1957, man was first able to put an artificial satellite in orbit around the earth. The theoretical possibility of this was, however, discussed by Newton himself. There is no essential difference in the dynamical behaviour of an artificial satellite or a natural one like the moon and a planet.

The shape of the orbits of the planets round the sun is elliptic. The sun lies at one of the foci of the ellipse. Fig 8·10 shows such an elliptic orbit of a planet with the sun situated at one of the foci, viz. F_1. The point F_2 represents another focus of the ellipse. The line *AB* and *CD* are called the major axis and minor axis of the ellipse respectively. The point *A*, where the planet is nearest to the sun is called *perihelion* and the point *B*, where it is furthest from the sun is known as *aphelion*. From the geometry of ellipse, it can be inferred that the sum of the distances of the planet from the two foci F_1 and F_2 remains always the same. It should also be noted that if the foci F_1 and F_2 come more and more closer together, the major axis *AB* and the minor axis *CD* become more and more nearly equal; when the foci coincide, the axes are equal and the ellipse becomes a circle. For the planets of our solar system, the orbits are very nearly circular, the maximum variation in distances from the sun being just less than 3 percent. While moving in elliptic orbit, the speed of a planet increases when it comes nearer to the sun. Thus the speed of a planet is greatest at the perihelion and least at the aphelion. For the earth, the maximum and minimum values of the speed are 18·8 mile/sec and 18·2 mile/sec. The near equality of those two values shows that the earth's orbit is very nearly circular.

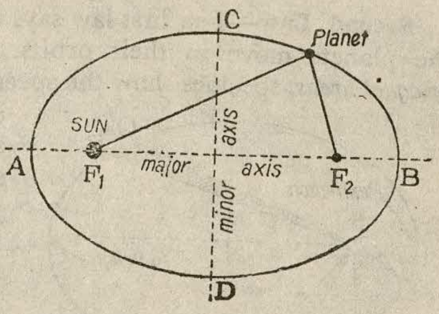


Fig. 8·10 : The orbit of a planet

As mentioned earlier, the above discussion also holds true for the motion of a satellite, whether natural or artificial, around a planet.

8·16. Motion of the Planets : Kepler's laws of planetary motion.

Kepler after a thorough and careful study of the vast accumulation of data collected by Tycho Brahe, a Danish astronomer, concerning the motions of the planets round the sun, formulated three famous laws which are now known as *Kepler's laws of planetary motion.* The laws may be stated as follows :

I. *The planets revolve round the sun in elliptical orbits with the sun at one of the foci.*

II. *The straight line joining the sun and any planet sweeps out equal areas in equal intervals of time.*

III. *The squares of the periodic times of orbital motion of different planets are proportional to the cubes of the mean distances of the planets from the sun.*

Experimental observations show that except for some minor deviations which can be properly accounted for, Kepler's laws give an accurate description of the motion of the planets round the sun. We shall now briefly present a closer examination of the contents of these laws :

First Law—This law simply specifies the shapes of the orbits described by the planets revolving round the sun. A detailed discussion has been made in the previous article.

Second Law—The first law says nothing about the speed with which the planets move in their orbits. The second law, also known as *law of equal areas*, specifies how the speed of a planet varies with its position on the orbit. In Fig. 8·11, the shaded regions represent the areas sweept out by the line joining the planet to the sun in equal intervals of time. From second law, all these areas are equal. It is obvious from the figure that in order to maintain this equality, the *planet must move with a larger speed when it is nearer to sun.*

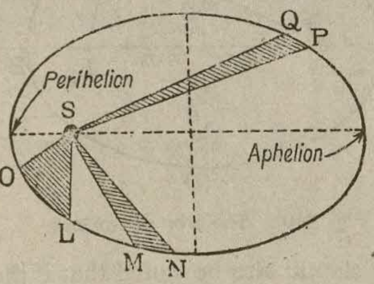


Fig. 8·11 : Illustration of Kepler's second law

Third Law—The period of a planet is defined as the time spent in making a complete revolution round the sun. The mean distance of the planet from the sun equals the arithmetical average of its largest and smallest distances. It is obvious from Fig. 8·10 that this equals the semi-major axis of the ellipse. Thus according to the third law, *square of the period of a planet moving in an elliptic orbit is proportional to the cube of the semi major axis of the ellipse.*

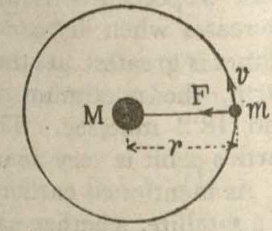


Fig. 8·12

This law can be simply proved assuming the orbit to be circular. In Fig. 8·12, *M* is the mass of the sun, *m* is the mass of the planet moving with a speed *v* and *r* is the distance between their centres. The gravitational force of attraction on the planet towards the sun is given by, $F=G\,\frac{Mm}{r^2}$ which provides the necessary centripetal force $\frac{mv^2}{r}$, so that the plant moves in a circle. Hence, $G\frac{Mm}{r^2}=\frac{mv^2}{r}$

$$\text{or,}\quad \frac{GM}{r}=v^2 \quad \cdots \quad \cdots \quad (8{\cdot}31)$$

If the period of the planet be T, then from the relation $s=vt$,

$$v=\frac{2\pi r}{T} \quad \cdots \quad \cdots \quad (8{\cdot}32)$$

Combining these two equations, we get

$$\frac{GM}{r}=\frac{4\pi^2 r^2}{T^2}$$

$$\text{or,}\quad T^2=\left(\frac{4\pi^2}{GM}\right)r^3 \quad \cdots \quad \cdots \quad (8{\cdot}33)$$

Since all the quantities inside the bracket are constants, $T^2 \propto r^3$, which is Kepler's third Law.

It follows that for two planets,

$$\frac{T_1^2}{T_2^2}=\frac{r_1^3}{r_2^3} \quad \cdots \quad \cdots \quad (8{\cdot}34)$$

Thus if the period and the mean distance from the sun be known for one planet, then the mean distance of another planet can be deduced from its observed period. We also conclude that the period i.e., length of the year is shorter for planets nearer to the sun.

Mass of the sun : From a knowledge of the period and the distance of a planet, say earth, from the sun, the mass of the sun can be calculated from equation (8·33). Thus considering the planetary motion of the earth,

Period $T=365\frac{1}{4}$ days $=365\frac{1}{4}\times 24\times 60\times 60$ sec.

Mean distance $r=1{\cdot}497\times 10^{13}$ cm. from the sun. Since, $G=6{\cdot}67\times 10^{-8}$ c.g.s. unit, we get

$$M=\frac{4\pi^2 r^3}{GT^2}=1{\cdot}99\times 10^{33}\text{ gm.}$$

Considering the data for any other planet, the mass of the sun can also be calculated.

Comparison of masses of heavenly bodies : Making use of the third law, masses of heavenly bodies which have satellites at known distances and of known periodic times can be compared. From equation (8·33), we have

$$\frac{T^2 M}{r^3}=\frac{4\pi^2}{G}=\text{a constant}$$

Considering two heavenly bodies of masses M_1 and M_2, having two satellites at distances r_1 and r_2 from their respective parent bodies, and of periods T_1 and T_2, we get

$$\frac{T_1^2 M_1}{r_1^3}=\frac{T_2^2 M_2}{r_2^3}$$

$$\text{or,}\quad \frac{M_1}{M_2}=\frac{T_2^2}{r_2^3}\times\frac{r_1^3}{T_1^2} \quad \cdots \quad (8{\cdot}35)$$

Example 8·13. *Calculate the average rate at which the line joining the sun to the earth is sweeping out area, in km.^{2}/sec. The average linear speed of the earth in the orbit is 29·5 km./sec., and the average radius of the orbit is* 16×17^7 *km.*

Solution : From eqn. (8·32), we have

$$T=\frac{2\pi r}{v}=\frac{2\pi\times16\times10^7}{29{\cdot}5}$$

The area of orbit$=\pi r^2$

$\therefore$ the average rate of sweeping out area by the line joining the sun to the earth $=\dfrac{\pi r^2}{T}=\dfrac{\pi\times(16\times10^7)^2\times29{\cdot}5}{2\pi\times16\times10^7}=23{\cdot}6\times10^8$ km^2/sec.

Example 8·14. *The mean angular velocity of the earth around the sun is 1° per day. The distance from the sun to the earth is* $1{\cdot}5\times10^8$ *km. Determine the mass of the sun. Given that* $G=6{\cdot}67\times10^{-8}$ *C.G.S. unit.*

Solution : Period $T=\dfrac{360^\circ}{1^\circ}=360$ days$=360\times24\times60\times60$ sec.

From eqn. (8·31),

$$M=\frac{4\pi^2r^3}{GT^2}=\frac{4\times(3{\cdot}14)^2\times(1{\cdot}5\times10^8\times10^5)^3}{6{\cdot}67\times10^{-8}\times(360\times24\times60\times60)^2}$$
$$\simeq2\times10^{33}\text{ gm.}$$

Example 8·15. *The planet Pluto is, on an average, 40 times as far from the sun as the earth is. What is Pluto's orbital period, in years ?*

Solution : Let T_1 be the period of the Pluto in years and r_1 its distance from the sun. If T_2 be the period of the earth and r_2 its distance from the sun, then from eqn. (8·34),

$$\frac{T_1^2}{T_2^2}=\frac{r_1^3}{r_2^3}$$

Now, $T_2=1$ year, $r_1=40r_2$

$\therefore$ $T_1^2=40^3$ or, $T_1=253$ years.

Example 8·16. *A satellite of Jupiter completes one revolution in 17 days, the radius of the orbit being* $1{\cdot}86\times10^6$ *km. The moon completely rounds the earth once in 27 days, distance of the moon from the earth being* $3{\cdot}6\times10^5$ *km. Compare the masses of Jupiter and earth.*

Solution : Let M_1 be the mass of Jupiter and r_1 be the distance of the satellite of period T_1 from Jupiter. Similarly if M_2 be the mass of earth and T_2 the period of moon which is at a distance of r_2 from the earth, then $r_1=1{\cdot}86\times10^6$ km. ; $T_1=17$ days ; $r_2=3{\cdot}6\times10^5$ km. ; $T_2=27$ days.

$\therefore$ From eqn. (8·35),

$$\frac{M_1}{M_2}=\frac{27^2}{(3{\cdot}6\times10^5)^3}\times\frac{(1{\cdot}86\times10^6)^3}{17^2}=348$$

8·17. Motion of the satellites.

All the laws of Kepler are, quite valid for describing the motion of satellites. Let us consider a satellite of mass m moving round a planet of mass M, the distances of their centres being r [Fig. 8·13]. r is also called the *orbital radius.* From equation (8·31), the velocity v of the satellite, called the *orbiting velocity*, is given by

$$v=\sqrt{\frac{GM}{r}} \quad \dots \quad \dots \quad (8{\cdot}36)$$

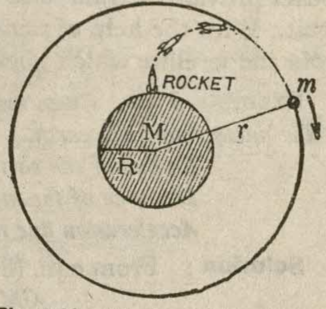


Fig. 8·13 : Motion of the satellite

This equation applies universally to any satellite moving in a circular orbit around any heavenly body.

In the case of an artificial satellite circling the earth at a height h above the earth's surface, we have

$$r=R+h \text{ and } v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{GM}{R+h}}$$

where R and M are the radius and mass of the earth respectively.

Now, if g be the acceleration due to gravity at earth's surface, then for any body of mass m, we have

$$mg=G\frac{Mm}{R^2} \quad \text{or,} \quad GM=gR^2$$

Therefore, the orbiting velocity of an artificial satellite is given by

$$v=\sqrt{\frac{gR^2}{r}}=R\sqrt{\frac{g}{R+h}} \quad \dots \quad \dots \quad (8{\cdot}37)$$

The period of revolution of the satellite about the earth is given by

$$T=\frac{2\pi r}{v}=\frac{2\pi(R+h)}{v} \quad \dots \quad \dots \quad (8{\cdot}38)$$

When h is very small compared to R i.e., when the satellite is circling the earth very close to the surface, $r\simeq R$ and we get from eqns. (8·37) and (8·38)

$$v=\sqrt{gR} \text{ and } T=\frac{2\pi R}{v}=2\pi\sqrt{\frac{R}{g}} \quad \dots \quad (8{\cdot}39)$$

Putting $g=981$ cm./sec.^{2} and $R=6{\cdot}367\times10^8$ cm., we get $v=$about 7850 m./sec. or about 17500 mi/hr. The value of T comes out to be 1 hr. 23 min. Satellites orbiting the earth at larger distances will have longer periods ; obviously the orbiting velocity in larger orbits is smaller. If the velocity imparted to a body be less than this particular velocity for rotating in an orbit it will drop down to the surface of the earth.

Launching of artificial satellites : In practice this high velocity is given to an artificial satellite with the help of multistage rockets. To avoid air resistances, the satellite is carried to a height of several miles by very powerful multistage rockets. To penetrate the dense lower part of the atmosphere by the shortest possible route, initially the rocket rises vertically. Afterwards, it is gradually tilted by a guidance system. Ultimately when the requisite speed is reached, the rocket provides a transverse motion to the satellite, thereby placing it in orbit. With the help of remote control devices, the rocket is then separated from the satellite which goes on revolving round the earth.

Example 8·17. *Given the following data, calculate the period of revolution of the moon round the earth.*

Radius of the earth=6400 km.
Distance of the moon from the earth=$3{\cdot}84\times10^5$ km.
Acceleration due to gravity on earth's surface=980 cm./sec.^{2}

Solution : From eqn. (8·4) we get

$$GM=gR^2$$

Combining with eqn. (8·33) we get

$$T^2=\frac{4\pi^2r^3}{gR^2}$$

where M=mass of the earth
R=radius of the earth
r=distance of the moon from the earth

$$\therefore\quad T=\frac{2\pi r}{R}\sqrt{\frac{r}{g}}=\frac{2\times3{\cdot}14\times3{\cdot}84\times10^5\times10^5}{6400\times10^5}\sqrt{\frac{3{\cdot}84\times10^5\times10^5}{980}}\text{ sec.}$$
$$=27{\cdot}3\text{ days.}$$

Example 8·18. *What should be the orbital radius of an artificial satellite that will hover over a given place on the equator i.e., will turn in the same rate as the earth ?*

Solution : Let T be the period of revolution of the satellite about the earth, which must be the same as the period of revolution of the earth about its own axis. Therefore, T=1 day=86400 sec. If r be the radius of the orbit of the satellite, then from eqn. 8·33 we have,

$$r^3=\frac{T^2GM}{4\pi^2}$$

$\because\ G=6{\cdot}67\times10^{-8}$ C.G.S. unit, $M=5{\cdot}96\times10^{27}$ gm.
$\therefore\ r=4{\cdot}22\times10^4$ km.

Example 8·19. *An artificial satellite was projected horizontally from a rocket when it is at a height of 800 km. above the surface of the earth. Calculate the velocity of projection necessary so that it can circle round the earth and also its period of revolution. Given $G=6{\cdot}66\times10^{-8}$ C.G.S. unit, $M=6\times10^{27}$ gm. and R=6400 km.*

Solution : The radius of the orbit of the satellite r=6400+800=7200 km.=$7{\cdot}2\times10^8$ cm. From eqn. (8·36), the orbiting velocity i.e., the velocity of projection is given by,

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{6{\cdot}66\times10^{-8}\times6\times10^{27}}{7{\cdot}2\times10^8}}=7{\cdot}45\text{ km/sec.}$$

For the period of revolution we get,

$$T=\frac{2\pi r}{v}=\frac{2\pi\times 7200}{7{\cdot}45}\text{ sec.}=1{\cdot}7\text{ hr.}=1\text{ hr. }42\text{ min.}$$

Example 8·20. *An artificial satellite of the Earth moves in a circular orbit at a distance of 600 km. from the earth's surface. Find the linear velocity of the satellite and its period of revolution around the earth. Given, radius of the earth 6400 km. and acceleration due to gravity at earth's surface=980 cm./sec.*^{2}

Solution : From eqn. (8·37),

$$v=R\sqrt{\frac{g}{R+h}}=6400\times 10^5\sqrt{\frac{980}{(6400+600)\times 10^5}}\text{ cm./sec.}$$
$$=7{\cdot}57\text{ km./sec.}$$

From eqn. (8·38),

$$T=\frac{2\pi(R+h)}{v}=\frac{2\times 3{\cdot}14\times 7000}{7{\cdot}57}\text{ sec.}=96{\cdot}8\text{ min.}=1\text{ hr. }36{\cdot}8\text{ min.}$$

8·18. Escape Velocity.

Escape velocity is defined as the velocity a body must have in order to 'break the bonds of gravity' or 'to escape the earth's gravitational pull'. Since the earth's gravitational pull extends to infinity (however weak it may be at great distances), escape velocity is apparently the velocity that must be possessed by a body in order that it can move infinite distance from the earth into space. It is obvious that the orbiting velocity of a satellite must be less than the escape velocity ; otherwise it would fly into outer space.

Using a method involving calculus, it can be proved that the escape velocity for a body projected from the earth's surface is given by,

$$v_e=\sqrt{\frac{2GM}{R}}$$

where M is the mass of the earth and R its radius. Since $gR^2=GM$, we get

$$v_e=\sqrt{2gR} \qquad \cdots \qquad \cdots \qquad \cdots \qquad (8{\cdot}40)$$

Comparison with equation (8·39) gives,

$$v_{\text{escape}}=\sqrt{2}\,v_{\text{orbit}}=1{\cdot}41\,v_{\text{orbit}} \qquad \cdots \qquad \cdots \qquad (8{\cdot}41)$$

Putting the values of g and R, we get $v_e=11200$ m./sec., or about 25,000 mi/hr. Hence if a body is projected with a velocity exceeding 25,000 mph, it will never return to the earth. The velocity for launching artificial satellites must, therefore, lie within the range of 17,500 mph to 25,000 mph.

8·19. Weightlessness in orbiting satellites.

If a body rests on a surface, its weight acts downwards i.e., the body presses the surface. According to the Newton's third law, the surface exerts an equal and opposite reaction on the body. The sensation of weight is felt due to this vertical reaction acting on us applied by the surface on which we stand.

Let us now consider an artificial satellite rotating round the earth in an orbit very close to its surface. From equation (8·39), the orbiting velocity is given by $v=\sqrt{gR}$. Considering an astronaut inside the satellite, the forces acting on him are the weight mg towards the centre of the earth and the oppositely directed reaction force N applied by the surface on which he stands [Fig. 8·14]. The resultant of these forces must provide the centripetal force F_c which acts on him for moving in the circular orbit. Thus, $F_c=mg-N$

or, $\frac{mv^2}{R}=mg-N$; putting $v=\sqrt{gR}$, we get, $N=0$

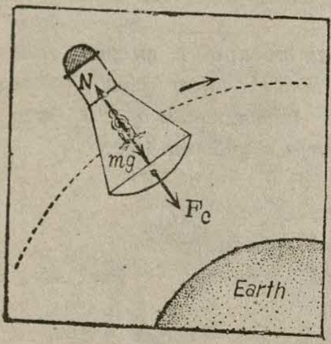


Fig. 8·14

The vanishing of the reaction force means that there is no action i.e., the force of gravity on the astronaut has apparently vanished. The astronaut will thus have the feeling of having no weight at all. This is said to be the condition of "weightlessness". This is a very peculiar sensation and the astronaut will simply float inside the satellite. Every object inside it will also become weightless.

This condition can be simulated on the earth's surface for a very short interval of time by flying an aeroplane in a loop with a requisite velocity, so that the centripetal acceleration v^2/r equals g. The astronauts are trained beforehand so as to get accustomed with this state of weightlessness.

An important branch of space travel is concerned with the study of the effects of prolonged weightlessness on human beings. To avoid this, the orbiting satellites are nowadays made to spin around their axes; the resulting centripetal force provides the sensation of weight.

● EXERCISE ●

[A] Essay type questions.

1. (a) State Newton's Law of Gravitation. What do you mean by 'acceleration due to gravity'? Which one is greater—the attraction of the earth on 1 lb. of lead or the attraction of 1 lb. of lead on the earth ?

(b) The point of suspension of a simple pendulum moves with a uniform acceleration in the horizontal direction. How is the period of the pendulum affected ?

(c) A person will feel weightless on an artificial satellite of the earth—explain. [*H. S.* '83]

2. (a) What is 'acceleration due to gravity'? How is it related to the universal gravitational constant ?

(b) Explain with diagram why and how the acceleration due to gravity varies from place to place on the earth's surface. [*H. S.* '82]

3. (a) What do you mean by a 'seconds pendulum'? Calculate its length in cm. at a place where $g=980$ cm./sec^2.

(b) State and explain whether the time period of the pendulum will change in the following cases :—

(i) if a hollow bob is taken instead of a solid bob.

(ii) if the hollow bob is partially filled with water.

(iii) if the pendulum is taken to the top of a mountain. [*H. S. 78*]

4. (a) State and explain Newton's Law of Gravitation. What is meant by acceleration due to gravity ? How does it vary with the altitude above sea-level and latitude of a place ? [*H. S.* '*79*]

(b) What is the unit of universal gravitational constant in C. G. S. system ? [*H. S.* '*81*]

5. State and explain Newton's law of universal gravitation. Why the law is termed 'universal' ? How do you find the gravitational attraction for extended bodies ?

6. (a) Distinguish between 'gravity' and 'gravitation'. How do you determine the mass and the mean density of the earth from the given values of gravitational constant, acceleration due to gravity, and the radius of the earth ?

(b) Find the variation in the value of the acceleration due to gravity with the distance from the centre of the earth, both inside and outside it. Describe the said variation graphically.

(c) Prove that—"The value of the acceleration due to gravity at the centre of the earth is zero".

7. Define 'mass' and 'weight' of a body. What are the differences between them ? Will the weight of a body be greater at the poles or at the equator ? Explain with reasons.

While buying commodities at Darjeeling, which method is more profitable to weigh the commodities in a spring balance or in a common balance ?

8. (a) State and explain the laws of falling bodies.

(b) Two bodies of different masses are dropped from two different heights. Will the ratio of time taken by the two bodies to drop through these distances be same as the ratio of the square root of the distances ? Explain your answer.

(c) Write down the equations of vertical motion. Prove that the time of ascent is equal to the time of descent.

(d) Obtain the expression for the whole time of flight of a body projected vertically upwards:

9. Two reasons for which the value of the acceleration due to gravity varies from place to place on the earth's surface are :—(i) the oblate spheroid shape of the earth and (ii) the diurnal motion of the earth. Explain.

10. (a) What is simple pendulum ? 'A simple pendulum can not be realised in practice'. Explain the statement.

(b) State the laws of a simple pendulum. How can you measure acceleration due to gravity with its help ?

11. (a) How do you measure the altitude of a mountain and the depth of a mine with the help of a simple pendulum ?

(b) A hollow sphere is filled with water through a small hole in it. It is hung by a long thread and as the water flows slowly out of the hole at the bottom, one finds that the period of oscillation first increases and then decreases. Explain. [***Model Questions, H. S. Council***]

(c) Will a pendulum clock gain or lose when it is taken to the pole from the equator ?

12. (a) State Kepler's laws of planetary motion. How can you establish Kepler's third law from the law of gravitation for a planet moving in a circular orbit.

(b) How can you compare the masses of different planets having satellites from Kepler's third law ?

(c) Show that the smallest period of revolution of an artificial satellite in a circular orbit around a planet (assumed spherical) is determined only by the average density of matter of the planet. [*Jt. Entrance* '*75*]

13. (a) Obtain from Newton's Law of Gravitation expressions for the orbiting velocity and period of revolution of a planet moving round the sun in a circular orbit.

(b) Explain how the mass of the sun can be calculated from the above expressions.

(c) Prove that the length of a year is shorter for a planet nearer to the sun.

14. What do you mean by the orbiting velocity of a satellite ? Calculate the orbiting velocity and the period of revolution of an artificial satellite assuming its orbital radius to be equal to the radius of the earth.

15. (a) What is 'escape velocity' ? Write down its magnitude. Find its relation to the orbiting velocity of an artificial satellite circling the earth very close to the surface.

(b) Is it possible to keep an artificial satellite stationary at a particular place of the sky with respect to an observer on the earth's surface ? If this is possible, find its height and the direction of its motion.

[B] Short answer type questions

1. Justify with reasons whether the period of a pendulum increases, decreases or remains the same and whether a clock gains, loses or maintains correct time in the following cases when :—

(i) The diameter of the bob is increased (ii) The mass of the bob is increased, its diameter remaining the same. (iii) The clock is taken to Darjeeling from Calcutta. (iv) The clock is taken to the moon from the earth. (v) It is taken to the bottom of a mine. (vi) It is taken upwards in a rocket with an acceleration g. (vii) It is placed in a lift moving up with uniform velocity or with uniform acceleration. (viii) It is placed in a lift moving downward with uniform velocity or with uniform acceleration. (ix) It is kept inside an artificial satellite. (x) It is allowed to fall freely. (xi) The bob is hollow and filled completely with a liquid. (xii) The clock is taken to different places on the earth's surface. (xiii) It is taken to different places on the Tropic of cancer.

2. Explain with reasons the following statements :—

(i) A freely falling body has no weight. (ii) In different places of the earth, same amount of matter is not contained in 1 kg. wt. of a substance if the weight is measured by a spring balance. (iii) A road in a mountainous region does not rise straight but spirals upwards. (iv) The time periods of a simple pendulum at the pole and at the equator would not be same even if the earth is considered to be a perfect sphere. (v) The time periods of a simple pendulum at the pole and at the equator would not be same even if the diurnal motion of the earth ceases. (vi) If the earth is considered as a perfect sphere, yet the velocities of projection necessary for a body to attain a particular height at different places on the earth will be different. (vii) The weight of a body is not its intrinsic property. (viii) The escape velocity for a rocket projected from the earth's surface does not depend on the mass of the rocket. (ix) The orbiting velocity of an artificial satellite does not depend on its mass. (x) The value of the escape velocity is less at the moon's surface than at the earth's surface. (xi) The moon has no atmosphere. (xii) In the earth's atmosphere light gases like hydrogen, helium etc are scarce. (xiii) A plumb-line does not indicate the vertical exactly due to the earth's rotation except on the equator and at the poles.

3. Write down the correct alternative with justification : A satellite is moving in a circular orbit round the earth. The satellite moves with (i) constant velocity, (ii) constant acceleration, (iii) changing acceleration.

[*I. I. T.* '75]

4. "The force of sun's attraction acts on all bodies on the surface of the earth. By day (when the sun is 'above our heads') it acts against the earth's attraction. At night (when the sun is beneath our feet) this force gets combined with the force of earth's attraction. Therefore, at night all objects should be heavier than they are by day."—Is this true ?

[*Model Questions, H. S. Council*]

5. Write down the correct alternative with justification. If mass remaining same, the diameter of the earth gets reduced by 1%, then the value of the acceleration due to gravity at the earth's surface (i) remains the same, (ii) decreases, (iii) increases.

[*I. I. T.* '81]

6. A body is thrown vertically upwards. It rises to a certain height and then falls down to the earth. Answer the following questions :—

(i) As it ascends or descends does any other force other than gravity affect its acceleration ?
(ii) Can you support the statement that the net time of ascent is equal to the time of descent,

if no force except gravity acts on the body ? (iii) Does the body always possess acceleration during its upward and downward journey ? (iv) Does the body possess a negative acceleration at any instant ?

7. A ball is dropped from the top of a building while another is thrown horizontally at the same instant. Which ball will strike the ground first ? [*I. I. T. '74*]

8. "A body possessing larger mass has a larger weight and smaller mass has a smaller weight. The mass of the earth is very large. Hence the weight of the earth in enormous." Justify the statement.

9. A block slides down a smooth inclined plane when released from the top, while another falls freely from the same point. Which one of them will strike the ground (i) earlier (ii) with greater velocity ? [*I. I. T. '74*]

10. An instrument package is released from an orbiting artificial satellite by simply detaching it from the outer wall of the satellite. What happens to the package ?

11. When a ball is thrown up the magnitude of its momentum decreases and then increases. Does this violate the conservation of momentum principle? [*I. I. T. '79*]

12. Will the length of the year increase or decrease, if the distance between the sun and the earth decreases due to any reason ? Justify your answer.

13. An astronaut on the surface of the moon finds that the period of a simple pendulum there is much larger than on the earth and that the pendulum continues to oscillate for a much longer time than on the earth. What informations regarding the moon could be obtained from these observations ? [*I. I. T '71*]

[C] Simple Problems

1. Find the force of attraction between neutrons which are at a distance of $d=10^{-8}$ cm. from each other. The mass of a neutron $m=1{\cdot}675\times10^{-24}$ gm. and $G=6{\cdot}67\times10^{-8}$ C. G. S. unit. [*Ans.* $1{\cdot}87\times10^{-39}$ dyne]

2. Given that, mass of earth $m_e=5{\cdot}98\times10^{24}$ kg. ; mass of moon $m_m=0{\cdot}0123\ m_e$, and the mean distance between them $d=3{\cdot}84\times10^{5}$ km. ; Calculate (a) the gravitational force of attraction for each other, (b) the acceleration of each that is produced by this force.
[*Ans.* (a) $1{\cdot}99\times10^{20}$ newton, (b) $2{\cdot}7\times10^{-3}$ m./sec.^{2}, $3{\cdot}33\times10^{-5}$ m./sec^2.]

3. A rocket flies to the moon. At what point of the straight line connecting the centres of the moon and earth will the rocket be attracted by the earth and the moon with the same force ? Given that, mass of earth$=5{\cdot}96\times10^{24}$ kg., mass of moon$=7{\cdot}3\times10^{22}$ kg., mean distance between them$=3{\cdot}84\times10^{8}$ m. [*Ans.* $3{\cdot}46\times10^{8}$ m. from earth]

4. Calculate the mass of earth from the following data :
$g=980$ cm./sec.^{2}, $G=6{\cdot}67\times10^{-8}$ c. g. s. units and $R=6\times10^{8}$ cm. [*Ans.* $5{\cdot}29\times10^{27}$ gm.]

5. (a) If the mass of the moon is $\frac{1}{78}$th times the mass of the earth and the diameter is $\frac{4}{15}$th of that of the earth, what is the acceleration due to gravity on the moon ? (b) How far will a 2 gm. mass fall from rest in 2 sec. on the moon ? [*Ans.* (a) $0{\cdot}19\,g$, (b) 372·4 cm.]

6. Two spheres each of mass 1 kg. are placed with their centre 20 cm. apart. Find the gravitational force of attraction between them. Given $g=980$ cm./sec.^{2}, mass of the earth$=4\times10^{27}$ gm. and radius of the earth$=6\times10^{8}$ cm. [*Ans.* $2{\cdot}2\times10^{4}$ dyne]

7. If the earth were a solid sphere of iron of radius $6{\cdot}37\times10^{6}$ m. and of density 7·86 gm./c.c. what would be the value of acceleration due to gravity at its surface, taking the gravitational constant to be $6{\cdot}58\times10^{-8}$ C. G. S. unit. [*Ans.* 1380 cm./sec.^{2}]

8. Taking the acceleration due to gravity at the earth's surface equal to $g=980$ cm./sec.^{2} and $G=6{\cdot}67\times10^{-8}$ c. g. s. unit, calculate the average density of the planet Mars, given that its diameter$=6780$ km. and the acceleration due to its gravity on its surface$=0{\cdot}38\,g$.
[*Ans.* 3·93 gm./c. c.]

9. Show that if the earth were not rotating about its axis, the acceleration due to gravity at the equator would exceed its present value by 3·41 cm./sec.^{2} Given that the radius of the earth$=6{\cdot}4\times10^{8}$ cm. and the angular velocity of the earth$=7{\cdot}3\times10^{-5}$ rad/sec.

10. At what altitude will the acceleration due to gravity be 25 percent of that at earth's surface ? [*Ans.* At an altitude equal to earth's radius]

11. At what depth will the acceleration due to gravity be 25 per cent of that at earth's surface ? Consider the earth's density to be constant.
[*Ans.* 0·75 *R*, where *R* is the earth's radius]

12. The radii of earth and mercury are $6{\cdot}371\times10^6$ m. and $2{\cdot}42\times10^6$ m. respectively. The mass of the mercury is 0·054 that of the earth. If a body weighs 10 kg. on the surface of the earth, what would it weigh on the mercury ? What is the acceleration due to gravity on the mercury ? Take g on earth=980 cm./sec.^{2} [*Ans.* 366·7 cm./sec.^{2} ; 3·74 kg:]

13. What would be the length of a day if the earth rotated so fast that objects at the equator had no weight ; Take the diameter of the earth as 6400 km. [*Ans.* 1·41 hr.]

14. If a body falls freely from rest, calculate (a) its acceleration, (b) distance it falls in 4 sec., (c) its velocity after falling 90 metre, (d) time required for its velocity to become 98 m./sec. (e) time of fall through 44·1 metre.
[*Ans.* (a) 9·8 m./sec^2. (b) 78·4 m. (c) 42 m./sec. (d) 10 sec. (e) 3 sec.]

15. From what height must a stone fall to acquire a velocity of (a) 140 cm./sec. (b) 4·9 m./sec. [*Ans.* (a) 10 cm., (b) 122·5 cm.]

16. A boy can throw a stone to a height of 10 m. With what velocity must the stone leave his hand ? Calculate also the velocity of the stone when it returns to the boy's hand.
[*Ans.* 14 m./sec. ; 14 m./sec.]

17. A ball thrown vertically upwards from ground returns to the ground in 8 sec. Calculate its initial speed. [*Ans.* 39·2 m./sec.]

18. A stone is thrown vertically upwards with a speed of 29·4 m./sec. from ground. Find (a) time of ascent, (b) the maximum height attained, (c) time of return to the ground and (d) the time when its speed equals 24·5 m./sec.
[*Ans.* (a) 3 sec. (b) 44·1 m. (c) 6 sec. (d) 0·5 and 5·5 sec.]

19. A stone is projected vertically upwards with a speed of 30 m./sec. from a height of 40 m. Find (a) the maximum height attained (b) the time required to strike the ground and (c) the speed with which it strikes the ground. [*Ans.* (a) 85·9 m. (b) 7·24 sec. (c) 41 m./sec.]

20. The cage of a coal pit is descending at a speed of 15 m./sec. When it is 75 m. from the bottom of the shaft a stone is released. Find the velocity of the stone when it reaches the bottom of the shaft and also the time it takes. [*Ans.* 41·17 m./sec. ; 2·67 sec.]

21. A stone is dropped from a balloon at an altitude of 300 m. How much time is required for the stone to reach the surface of the earth if the balloon is (a) stationary (b) ascending at 5 m./sec. (c) descending at 5 m./sec. [*Ans.* (a) 7·8 sec. (b) 8·4 sec. (c) 7·3 sec.]

22. A stone dropped from a balloon strikes the ground in 30 sec. Find the height of the balloon (a) if it is at rest and (b) if it is ascending with constant speed of 100 cm./sec., when the stone was released. [*Ans.* (a) 4410 m. (b) 4380 m.]

23. A stone is dropped from the height 19·6 m. What distances it travels during (a) the first 0·1 sec. (b) the last 0·1 sec. of its motion ? [*Ans.* (a) 4·9 cm. (b) 1·9 m.]

24. A stone is dropped from the height 19·6 m. How long the stone takes to travel (a) the first metre (b) the last metre ? [*Ans.* (a) 0·45 sec. (b) 0·05 sec.]

25. Two bodies fall freely from different heights and arrive at the earth simultaneously. The time of fall of the first body is 2 sec. and of the other 1 sec. At what height was the first body when the second body began to fall ? [*Ans.* 14·7 m.]

26. A body *A* is thrown vertically upward with the initial velocity u_1, and a body *B* falls from a height *h* with the initial velocity $u_2=0$. Find how the distance *x* between the bodies *A* and *B* depends on the time *t*, if the bodies began moving simultaneously. [*Ans.* $x=h-u_1t$]

27. A stone dropped from the top of a cliff traverses 53·9 m. during the last second of its fall. Find the height of the cliff and also the time the stone took to fall.
[*Ans.* 176·4 m. ; 6 sec.]

28. Two bodies begin a free fall from the same height at an interval of 2 sec. apart. How long after the first body begins its fall will the two bodies be separated by a distance of 40 m. ?
[*Ans.* 3·04 sec.]

29. A stone is dropped from the top of a tower 90 m. high and at the same time another stone is projected vertically upwards from the foot of the tower with a speed of 30 m./sec. When and where will the stone pass each other ? [*Ans.* 3 sec. ; 45·9 m. from the ground]

30. A simple pendulum of length 98·45 cm. has time period of 1·99 sec. Find the value of *g*. [*Ans.* 980·2 cm./sec.^{2}]

31. Calculate the length of a second's pendulum (to 3 significant digits) at a place where $g=980$ cm./sec.^{2} What will be the periodic time if the length is increased 2·25 times ? (*H. S. '64*) [*Ans.* 3 sec.]

32. If the length of a second's pendulum be 100 cm., find the length of a pendulum which makes 25 oscillations per minute at that place. [(*H. S. '61* (*Comp.*)] [*Ans.* 144 cm.]

33. The period of a simple pendulum is 2 sec. Find the change in its period if its length is shortened by 10 cm. Assume $g=981$ cm./sec.^{2} [*Ans.* 0·103 sec.]

34. A pendulum beats seconds at the basement of a tall building. How much time will this pendulum lose in one day if it is moved to the top floor of this building ? The top floor is 200 m. above the basement. The radius of the earth is 6400 km. [*Ans.* 2·7 sec.]

35. A second's pendulum loses 5 second a day. In what way and by how much the length is to be changed in order that the clock may keep correct time ? ($g=981$ cm./sec.^{2})
[*Ans.* Shortened by 0·0115 cm.] [*H. S.* '72]

36. Two pendulums begin to swing simultaneously. The first one executes 15 full oscillations in the time the second makes 10. Find the ratio of their lengths. [*Ans.* 4 : 9]

37. What is the ratio between the height H of a mountain and the depth h of a mine if a simple pendulum swings with the same period at the top of the mountain and at the bottom of the mine ? [*Ans.* $h=2H$]

38. A pendulum beats seconds at the surface of the earth. But when it is taken at the bottom of a mine, it loses 5 sec. in a day. Find the depth of the mine. The radius of the earth is 6400 km. [*Ans.* 740·9 m.]

39. Assuming that the earth's orbit round the sun be circular, find the linear velocity of its motion and the period of revolution about the sun. Given, $G=6{\cdot}67\times10^{-8}$ C. G. S. units, mass of the sun$=1{\cdot}99\times10^{33}$ gm. mean distance between the sun and the earth$=1{\cdot}497\times10^{13}$ cm.
[*Ans.* 29·78 km./sec. ; 365 days 9·6 hr.]

40. The distances of Earth and Neptune from sun are $1{\cdot}495\times10^{8}$ km. and $44{\cdot}956\times10^{8}$ km. respectively. Find in years the orbital period of Neptune. [*Ans.* 164·9 yr.]

41. The periods of revolution of the planets Mercury and Saturn are respectively 0·24 and 29·46 years. Find their mean distances from the sun in astronomical units (1 astronomical unit=mean distance from sun to earth). [*Ans.* 0·39 and 9·54 astronomical unit]

42. The period of revolution of a satellite of Jupiter is 16·7 days. Its orbital radius is $1{\cdot}87\times10^{6}$ km. Calculate the mass of Jupiter. Given, $G=6{\cdot}67\times10^{-8}$ C. G. S.
[*Ans.* $1{\cdot}857\times10^{27}$ kg.]

43. How does the period of a satellite change if its mass be doubled keeping its orbit same ? [*Ans.* No change]

44. Find the linear velocity of an artificial satellite of the earth which moves in a circular orbit :

(a) near the earth's surface, (b) at an altitude of 7000 km.

Find also the periods of revolution of the satellite in these orbits. Assume $g=980{\cdot}7$ cm./sec.^{2} and radius of the earth$=6{\cdot}37\times10^{6}$ m.

[*Ans.* Velocity : (a) 7·903 km./sec. (b) 5·454 km./sec.
Period : (a) 1 hr. 24 min. (b) 4 hr. 17 min.]

45. A shell was projected horizontally from a rocket when it is at a height of 600 km. above the earth's surface. Calculate the velocity of projection necessary for the shell to circle round the earth as satellite. Given, $G=6{\cdot}66\times10^{-8}$ C. G. S. unit ; mass of earth$=6\times10^{27}$ gm. and radius of earth=6400 km. [*Ans.* 7·556 km./sec.]

46. In a balloon expedition, a seconds pedulum carried on the balloon, on reaching the topmost position is found to loose 400 seconds a day. Estimate the height of ascent, assuming Earth's radius to be 6450 km (*Jt. Entrance '78*) [*Ans.* 30 km.]

47. The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is a seconds pendulum on earth ? [*I. I. T* '73]
[*Ans.* 2·83 sec.]

48. Water drips from the nozzle of a tap onto the floor 3·24 m. below. The drops fall at regular intervals of time, the first drop striking the floor at the instant the fourth drop begins to fall. Find the location of the drops when a drop strikes the floor.
[*Ans.* 3·24, 1·44, 0·36, 0 m. below the nozzle of the tap]

49. Two particles are let fall from the top of a tower at an interval of 2 seconds. Find the relative velocity and relative acceleration of the two particles at any instant when both are still falling. (*Jt. Entrance '82*) [*Ans.* $v_{12}=2g$ directed vertically downwards ; zero]

50. If the radius of the earth was to shrink by one percent, its mass remaining the same, how and by how much the value of g would change ? [*Ans.* Increases by about 2%]

51. A 5 lb stone is thrown vertically up and at the end of the first second is moving upwards at the rate 64 ft. per. sec. Calculate the maximum height attained by it, the time for attaining the maximum height, kinetic energy at the maximum height. Kinetic energy at the moment when it reaches the ground and potential energy at the ground ? [Assume $g=$ 32 ft./sec.^{2}] (*Jt. Entrance '83*) [*Ans.* 144 ft. ; 3 sec ; zero ; 720 ft. lb ; zero]

52. A rocket is fired vertically from the ground with a resultant vertical acceleration of 10 m/sec.^{2} The fuel is finished in 1 minute and it continues to move up. What is the maximum height reached ? (*I. I. T. '75*) [*Ans.* 36·367 km.]

53. A conical pendulum of length 1 m. and mass of the bob 100 gm. has time period $\frac{\pi}{2}$ sec. Calculate (*a*) the angle of inclination of the string with the vertical, (*b*) the tension in the string, and (*c*) linear velocity of the bob. [*Ans.* (*a*) $\cos^{-1}\frac{49}{80}$, (*b*) 1·6 N, (*c*) 3·16 m./sec.]

[D] Harder Problems

1. A spherical mass of 20 kg. is attracted by another spherical mass of 160 kg. with a force equal to the weight of 0·15 mg ; when the centres of masses are 40 cm. apart. Calculate the mass of the earth, given radius of the earth$=6\times10^8$ cm. [*Ans.* $4\cdot8\times10^{27}$ gm.]

2. At what angular velocity would the earth have to rotate for the weight of a body at the equator to be zero. Given, the density of the earth is 5·6 gm./c.c. ; its radius is 6400 km. and $G=6\cdot67\times10^{-8}$ C. G. S. unit. [*Ans.* $1\cdot25\times10^{-3}$ rad./sec.]

3. During the last second of its free fall a body covers half the total distance covered. Find (a) the height h from which the body falls. (b) the duration of falling.
[*Ans.* (a) 56·65 m. (b) 3·4 sec.]

4. Two bodies are thrown up vertically from the same point on earth 1 sec. apart with the same initial velocity of 30 m./sec. When and where do they meet ?
[*Ans.* 3·56 sec. after the projection of the first body ; 44·7 m. above ground.]

5. A body A is projected vertically upwards from the top of a tower 528 ft. high with a velocity of 128 ft./sec. After t sec. a body B is dropped from the same point and both the bodies reach the ground at the same instant. Calculate t and the velocity of A and B on reaching the ground. [*Ans.* 5·26 sec. ; 224 ft./sec. ; 183 83 ft./sec.]

6. A stone falling from the top of a vertical tower has descended x ft. when another is let fall from a point y ft. below the top. If they fall from rest and reach the ground together, show that the height of the tower is $(x+y)/4x$ ft.

7. A, B, C, D are four points on the same vertical line and are such that $AB=BC=CD$. If a particle falls freely from A, show that the time taken by it to describe AB, BC, CD are respectively as $1:(\sqrt{2}-1):(\sqrt{3}-\sqrt{2})$.

8. The acceleration due to gravity on a planet is 196 cm./sec.^{2} If it is safe to jump from a height of two metre on the earth, what will be the corresponding safe height on the planet ?
(*I. I. T. '72*) [*Ans.* 10 m.]

9. (*a*) Two bodies are thrown up vertically with the same initial velocities at an interval of t sec. apart. (*i*) What is the velocity of the second body relative to the first ? (*ii*) Find an expression for the distance between the two bodies.

(*b*) Solve the problem for the case in which the initial velocity of the second body u_0 is half the initial velocity of the first body.

[*Ans.* (*a*) (*i*) $v_{21}=gs$, directed upwards both during the upward and downward motions of both bodies, (ii) distance at any instant t, $x=us-gts+\frac{1}{2}gs^2$ where u is the common initial velocity.

(*b*) (*i*) $v_{21}=-u_0+gs$ directed upwards and $v_{21}=0$ for downward motion if $s=u_0/g$. (*ii*) $x=u_0t+u_0s-gts+\frac{1}{2}gs^2$; $x=3u_0^2/2g$ for $s=u_0/g$.]

10. A juggler is maintaining four balls in motion, making each in turn rise to a height of 90 cm. from his hand. With what velocity does he project them and where will the other three balls be at the instant when one is just leaving his hand ?

[*Ans* : 420 cm./sec. ; 67·5 cm., 90 cm., 67·5 cm.]

11. How would the greatest height you could throw a ball on the moon compare with the greatest height you would throw it on earth ? Given, mass of moon$=\frac{1}{76}$ mass of earth and diameter of the moon$=\frac{5}{19}$ diameter of earth.

[*Ans.* 5·26 times that on earth]

12. A stone is allowed to fall from rest from the top of a cliff and takes t sec. to fall the last h ft. Show that the total time of fall is $\left(\frac{t}{2}+\frac{h}{gt}\right)$ sec.

13. A lift starts ascending from ground level with uniform acceleration of f cm./sec.^{2} A stone is dropped from it after t second ; show that it reaches the ground after $\frac{t\sqrt{f}}{2}(\sqrt{f}+\sqrt{f+g})$ second.

14. The length of a second's pendulum is increased by 1%. How much time will it gain or lose per day ?

[*Ans.* loses 7 min. 12 sec.]

15. A simple pendulum is placed inside a lift. Compare its period when the lift is (a) ascending with uniform velocity (b) ascending with uniform acceleration of 8 ft./sec.^{2} (c) stationary (d) descending with uniform velocity and (e) descending with uniform acceleration of 8 ft./sec.^{2}

[*Ans.* $1 : \frac{2}{\sqrt{5}} : 1 : 1 : \frac{2}{\sqrt{3}}$]

16. The radius of the earth is 6400 km., the distance from the earth to the sun is $1{\cdot}5\times10^8$ km. ; the density of the earth is 5·6 gm./cm.^{3} ; the period of revolution of the earth around the sun is 365 days. Using these data determine the gravitational force of attraction between the sun and the earth.

[*Ans.* $3{\cdot}656\times10^{27}$ dyne]

17. Find the relationship between the period of revolution T of an artificial satellite moving in a circular orbit at the surface of a planet and the density ρ of the planet.

[*Ans.* $T=\sqrt{\frac{3\pi}{G\rho}}$]

18. A small satellite revolves round a planet of mean density 10 gm./c.c., the radius of the orbit of the satellite being slightly greater than the radius of the planet. If the constant of gravitation is $6{\cdot}6\times10^{-8}$ c. g. s. units, calculate approximately the time of revolution of the satellite.

(*Jt. Entrance, '71*) [*Ans.* 1 hr. 3 min. (approx.)]

19. In the case (b) of Prob. 44, find (i) the orbital kinetic energy and (ii) the total energy expended in putting the satellite into orbit, if its mass be 2000 kg.

[*Ans.* (i) $2{\cdot}97\times10^{10}$ J ; (ii) $9{\cdot}507\times10^{10}$ J]

20. An upward force $F=190$ Newtons is applied upon a body of mass 10 kg. till it is raised vertically upwards by a distance of 10 metres.

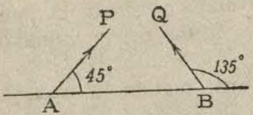
Calculate the work done by F and work done against gravity. Here the work done by F is much greater than the gain in P. E. Show using clear calculations that the law of conservation of energy is quantitatively satisfied here.

(*Jt. Entrance '78*) [*Ans.* 1900 J : 980 J]

21. A train is moving at a constant speed of 45 km. p.h. A passenger throws a stone through the window from a height 3 m. above the ground horizontally and normally to the direction of motion of the train with a velocity 5 m./sec. 'Neglecting air resistance, determine the velocity with which the stone strikes the ground.

[*Ans.* 15·5 m./sec. at an angle $\tan^{-1}(0{\cdot}57)$ with the horizontal]

22. Particles P and Q of mass 20 gm and 40 gm. respectively are simultaneously projected from points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles with the horizontal AB as shown in the figure. Each particle has an initial speed of 49 m./sec. The separation AB is 245 m. Both particles travel in the same vertical plane and undergo a collision. After the collision, P retraces its path. Determine the position of Q when it hits the ground. How much time after the collision does the particle Q take to reach the ground? Take $g=9{\cdot}8$ m./sec.^{2} (*I. I. T.* '82) [*Ans.* At the midpoint of AB ; 3·53 sec]



23. Two particles are projected at the same instant from two points A and B on the same horizontal level where $AB=42$ m, the motion taking place in a vertical plane through AB. The particle from A has an initial velocity of 58·5 m./sec. at an angle $\sin^{-1}\frac{5}{13}$ with AB and the particle from B has an initial velocity of 37·5 m./sec. at an angle $\sin^{-1}\frac{3}{5}$ with BA. Show that the particles will collide in mid-air and find when and where the collision occurs.

[*Ans.* 0·5 sec. after projection ; at height 10·025 m. above a point on the line AB, the point being at a distance 27 m. from A]

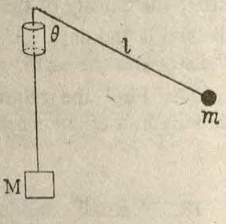
24. If g is the acceleration due to gravity on earth's surface, calculate the gain in potential energy of an object of mass m raised from the surface of earth to a height equal to the radius R of the earth [*Ans.* $\frac{1}{2}mgR$]

25. A pilot jumps out of an aeroplane with a parachute when the plane is just above the point A on the surface of the earth and moving horizontally with a velocity 600 km./hr. After 10 sec. the parachute unfolds when the man descends with a retardation of 2 m./sec^2. When he is about to touch the ground, his velocity is 8 m./sec. Find the height of the plane from the surface of the earth and the distance of the spot where the man drops from the point A. ($g=9{\cdot}8$ m/sec.^{2}) (*Jt. Entrance* '84) [*Ans.* 2875 m ; 9166·7 m.]

26. A man and his parachute together have a mass of 96 kg. If the air has a density of 1·2 kg /m^3, what must be the effective area of the parachute, if he is to have a terminal velocity of 8 m./sec^2 ? (Given $g=9{\cdot}8$ m /sec.^{2}) [*Ans.* 12·25 m^2]

27. A large mass M and a small mass m hang at the two ends of a string that passes over a smooth tube as shown in the figure. The mass m moves around a circular path which lies in a horizontal plane. The length of the string from the mass m to the top of the tube is l and θ is the angle this length makes with the vertical. What should be the frequency of rotation of the mass m so that the mass M remains stationary? (*I. I. T.* '78)

$$\left[\textit{Ans.}\ \frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}\right]$$



9
CHAPTER

ELASTICITY

9·1. Elasticity.

In the chapter on Newton's Laws of motion, we saw that when a system of unbalanced external forces acts on a body at rest, the body begins to move. If, however, a system of balanced external forces acts on the body, the body does not move as a whole but its different portions generally undergo a relative displacement. The original size or shape of the body changes, which is then said to be *deformed.* As a simple example, let us consider a spiral spring with one of its ends fixed rigidly to a support [Fig. 9 1]. If we now pull the other end with a force *F*, this will be communicated by the spring to the support. Then from Newton's third law, the support will in turn exert an equal and opposite reaction *F* on the spring. These two forces balance each other so that the spring does not move as a whole but gets elongated. This implies that its different portions have undergone a relative displacement. Generally, we regard this as the effect of the external deforming force ; the resulting balancing force which comes into play is not overtly mentioned.

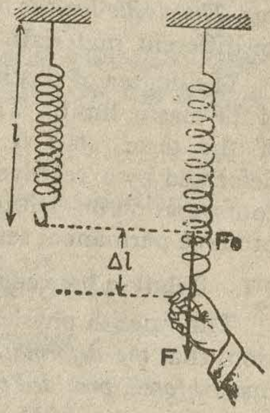


Fig. 9 1 : Deformation of a spiral spring

Now it is a common experience that as the spring is pulled and gets stretched, it seems that the spring is also exerting a pull in the opposite direction on our hand. Existence of this reaction force follows from Newton's third law. The spring is said to be resisting the deformation and as soon as we release the spring, it regains its original length. These phenomena are common to every object and are attributed to a general property of matter called **elasticity.**

Elasticity *is defined as the property by which a body is able to resist deformation, either in shape or in volume or both, and recovers its original configuration when the deforming force is removed.*

A body is said to be **perfectly rigid**, *when no relative displacement between its pairs occurs under the action of a deforming force, however large it might be.* However, a perfectly rigid body cannot be found in practice ; glass provides a very near approximation.

On the other hand, *a body is said to be* **perfectly elastic** *when it recovers its original configuration after the removal of the deforming force, however large it might be.* No body is perfectly elastic.

A body is said to be **perfectly inelastic** *or* **plastic** *when it maintains its changed configuration even after the removal of the deforming force.* This is also an ideal case and does not exist in practice.

A body is said to be **partly elastic** *when after the removal of the deforming force, it regains only partly its original configuration.* This is the case encountered in practice.

In practice, the recovery is found to be practically complete for many kinds of materials, provided that the deforming force is not too large. *Whenever the magnitude of the deforming force crosses a certain maximum limit, called* **elastic limit**, *the body does not completely regain its original shape or size after the force is removed, but acquires a permanent set or permament deformation. The elastic limit is the property of the material of the body* i.e. it is different for different materials.

The degree of elasticity of a substance is determined by the magnitude of the elastic limit. Thus contrary to common belief, in the scientific sense of the term, *steel is more elastic than rubber,* since even after getting deformed by a relatively large force, the former returns to its original configuration on withdrawal of the force, whereas the same force can produce permanent set in the latter (rubber).

9·2. Relation between Force and Deformation.

The English physicist Robert Hooke found from experimental observations that *the deformation of an elastic body is directly proportional to the applied force, provided the elastic limit is not exceeded.* This statement which bears the name of Hooke, was later modified by Young. Nowadays, this generalised version is usually known as Hooke's Law, which we shall discuss in the next article.

In the example of spring discussed above, let $\triangle l$ be the elongation of the spring when a pull F is applied. By Newton's third law an equal and opposite force F_e, called elastic force is exerted by the spring. It will be found that $\triangle l \propto F$, which proves the validity of Hooke's law.

$$\therefore \quad F = k\triangle l, \text{ and } F_e = -k\triangle l \qquad \ldots \qquad (9{\cdot}1)$$

where the proportionality constant k is called the *force constant* of the spring and is defined as force per unit elongation. It is a measure of the stiffness of the spring and is constant for that particular spring only. The negative sign in eqn. (9·1) only indicates that the elastic force F_e is directed opposite to the elongation $\triangle l$.

9·3. Elastic potential energy.

When a body is deformed, the force acting on the body operates through a certain distance and work is done. This work is stored in the body as potential energy.

The change in the elastic potential energy of a body is measured by the work done by the applied force F during slow elongation or by the negative of the work done by the elastic force F_e.

To calculale its magnitude let us consider the case of the spring cited above. During the elongation of the spring, the elastic force changes linearly from 0 to $-k\triangle l$. Its mean value, average w.r.t. distance is $-\frac{1}{2}k\triangle l$. Hence, for the displacement $\triangle l$, it does work equal to $-\frac{1}{2}k\triangle l\times\triangle l=-\frac{1}{2}k\triangle l^2$. So from the definition, the elastic potential energy (E_p) is given by,

$$\begin{aligned} E_p &= -(-\tfrac{1}{2}k\triangle l^2) \\ &= \tfrac{1}{2}k\triangle l^2 \\ &= \tfrac{1}{2}F\times\triangle l \end{aligned}$$

∴ Elastic $P.E.=\frac{1}{2}$ Applied force × elongation

This energy may be imagined to be distributed throughout the material of the spring.

Example 9·1. *Find the force which must be applied to a string in order to compress it by 5 cm. Given that, the spring obeys Hooke's law and a force of 400 gm. wt. compresses the spring by 1 cm.*

Also find the work done during the compression.

Solution : By Hooke's law, the compression is proportional to the force applied.

Now, 1 cm. compression is produced by 400 gm. wt.

∴ 5 cm. compression is produced by $400\times5=2000$ gm. wt.

Again, the average force applied during compression $=\dfrac{0+2000}{2}$

$=1000$ gm. wt.

∴ Work done during compression = average force × the distance compressed

$$\begin{aligned} &= 1000\times980\times5 \text{ erg.} \\ &= 4{\cdot}9\times10^6 \text{ erg.} \end{aligned}$$

9 4. Stress and strain : Generalised Hooke's law.

The relation described in the previous article, is only applicable to particular bodies ; value of the constant k remains same only for a particular body and changes from one body to another, even though they may be made of the same material. However, it is possible to generalise it in terms of elastic constants for materials. The elastic constants of bodies can then be calculated from the elastic constants of their materials. Introduction of the concepts of *stress* and *strain* enables us to state Hooke's law in such a general form.

Stress is related to the force producing deformation. *Whenever a body is subjected to a deforming force, internal force of reaction comes into play within it, which tends to resist the applied force and maintains the original configuration of the body. This reaction force generated per unit area of the body is called* **stress.** By Newton's third law, the reaction force is equal and opposite to the applied force. Hence, *the stress is measured by the applied force per unit area of the body.*

Obviously stress has units same as those of pressure in all systems i.e., dyne/cm.^{2}, newton/metre^2, and poundal/ft^2.

Strain is a measure of the amount of deformation produced. The different parts of a body under stress suffer relative displacements and a consequent change in its size or shape occurs ; the body is then said to be strained. *The fractional change in its length, volume or shape relative to that in its original configuration is a measure of the* **strain.**

The strain, being a ratio of two similar quantities, is a pure number without unit.

Thomas Young modified Hooke's law in the light of the above conceptions of stress and strain. In this modified form the law can be stated as :

Within elastic limit, stress is proportional to strain, the proportionality constant being a characteristic of the material of the boby under stress. This is called the **generalised Hooke's Law.**

Thus according to this law, within elastic limit.

$$\text{Stress} \propto \text{Strain or, } \frac{\text{Stress}}{\text{Strain}} = \text{constant} \quad \ldots \quad (9{\cdot}2)$$

This proportionality constant is called the **elastic modulus** *of the material of a body and may be defined as stress developed per unit strain.*

Strain being a pure number, elastic modulus has the same units as those of stress i.e. dyne/cm.^{2}, newton/meter.^{2}. poundal/ft.^{2}, etc.

9·5. Longitudinal Stress and Strain ; Young's modulus.

If a body whose other dimensions are small in comparison to its length (e.g. a bar or a wire etc.) be subjected at both ends to a pair of equal and opposite forces F, F acting along its length, the body gets deformed in length ; the strain in this case is called the *longitudinal strain.* The corresponding stress is called the *longitudinal stress.*

Fig. 9 2(a) shows a long elastic bar AB of initial length l and uniform cross-sectional area a. Fig. 9·2 (b) shows the stretched condition of the bar under the action of two equal and opposite pulls F,F at its ends. The bar is said to be in tension. If a bar is subjected to pushes at its ends by applying the forces F,F in reversed direction [Fig. 9·3], it is said to be in compression. *The longitudinal strain of the bar under tension (or compression) is defined as the change in length per unit length.* Thus, if $BC = \Delta l$, be the elongation or compression of the bar, then

$$\text{longitudinal strain} = \frac{\Delta l}{l} \quad \ldots \quad (9{\cdot}3)$$

Let us imagine a section of the bar at right-angles to its length through any point E on the bar as indicated by the dotted line. The equilibrium of the portions of the bar to the right, and to the left of the section through the point E, gives us an idea about the forces acting in the interior of the rod. We shall focus our attention on a bar under tension (Fig. 9·2).

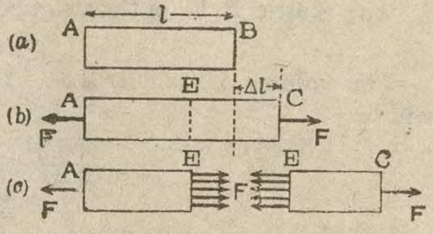


Fig. 9·2 : A bar in tension

Considering the equilibrium of the portion AE of the bar, since external force F acts at A towards left, this must be balanced by an equal internal force towards right, acting across the section through E and exerted by the section CE of the bar [Fig. 9·2 (c)]. Since CE is also in equilibrium, the rightward force F at C must be similarly balanced by a force F exerted by the portion AE, acting leftwards through the section at E. These forces are uniformly distributed over the section at E as shown in Fig. 9·2(c). Similar considerations apply to a bar under compression ; the directions of all the forces are just reversed as shown in the Fig. 9·3(c). These internal forces resist the effort of the externally applied forces to deform the body and on their removal, tend to bring the body back to its original length. A body under the action of such forces in its interior is said to be in a state of longitudinal stress. Since the point E is arbitrarily chosen along the length of rod, the entire bar is under such stress.

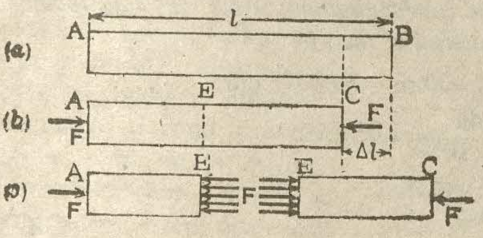


Fig. 9·3 : A bar in compression

Longitudinal stress *of the bar under tension (or compression) is defined as the internal forces developed per unit cross-sectional area.* Since the magnitude of the internal forces equals that of the external deforming forces, we have

$$\text{longitudinal stress} = \frac{F}{a} \qquad \cdots \qquad (9{\cdot}4)$$

Experiments on bars of same material but of different cross-sections show, in accordance with Hooke's law, that *within elastic limit, longitudinal (i.e. tensile or compressive) stress is proportional to longitudinal strain. The proportionality constant, called Young's modulus, is independent of the sizes of the bars.* Young's modulus is different for different material ; it is the property of the material itself.

Young's modulus *of the material of a bar or wire under tension (or compression) is the ratio of the longitudinal stress to the longitudinal strain.* It is generally denoted by Y.

Thus, for example, if F be the force which acting on a wire of length l and cross-section a produces an elongation $\triangle l$, then from equations (9·3) and (9·4), we have

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F/a}{\Delta l/l} \qquad \cdots \qquad (9{\cdot}5)$$

As fluids have no definite length, Young's modulus is a special property of solids.

The elastic P. E. of the strained wire is given by

$$E_p = \tfrac{1}{2} F \triangle l$$

The volume of the wire is *al*. Therefore, elastic P. E. per unit volume will be

$$\frac{1}{2}\frac{F\triangle l}{al} = \frac{1}{2}\,\frac{F}{a} \times \frac{\triangle l}{l}$$

$$= \tfrac{1}{2}\ \text{stress} \times \text{strain}.$$

So, *the elastic potential energy per unit volume of a strained body is equal to one-half the stress multiplied by the strain.*

Example 9·2. *A wire, 30 m. long and 2 sq. mm. cross-section, is stretched with the weight of 5 kg. by 0·49 cm. Find (a) longitudinal stress, (b) longitudinal strain and (c) Young's modulus of the material of the wire.*

Solution : Here, $\triangle l = 0{\cdot}49$ cm. ; $l = 30$ m. $= 30 \times 10^2$ cm

$\therefore$ Longitudinal strain $= \dfrac{\triangle l}{l} = \dfrac{0{\cdot}49}{30 \times 10^2} = 1{\cdot}63 \times 10^{-4}$

Also $F = 5$ kg. wt. $= 5 \times 10^3 \times 980$ dyne.

$a = 2$ sq. mm. $= 2 \times 10^{-2}$ sq. cm.

$\therefore$ Longitudinal stress $= \dfrac{F}{a} = \dfrac{5 \times 10^3 \times 980}{2 \times 10^{-2}} = 2{\cdot}45 \times 10^8$ dyne/cm.^{2}

and Young's Modulus $= \left(\dfrac{F}{a}\right) \Big/ \left(\dfrac{\triangle l}{l}\right) = \dfrac{2{\cdot}45 \times 10^8}{1{\cdot}63 \times 10^{-4}} = 1{\cdot}5 \times 10^{12}$ dyne/cm.^{2}

Example 9·3. *The diameter of a brass wire is 1 mm. What force in dyne is necessary to stretch it by 0·1 percent of its original length ? Given Y for brass* $= 9 \times 10^{11}$ *dyne/cm.*^{2}

Solution : Here, $\dfrac{\triangle l}{l} = \dfrac{0{\cdot}1}{100} = 1 \times 10^{-3}$

If F be the required force, then from eqn. (9·5), $Y = \left(\dfrac{F}{a}\right) \Big/ \left(\dfrac{\triangle l}{l}\right)$

$$\text{or, } 9 \times 10^{11} = \frac{F}{\pi\left(\frac{0{\cdot}1}{2}\right)^2} \Big/ 1 \times 10^{-3}$$

$$\therefore\ F = 7{\cdot}07 \times 10^6 \text{ dyne.}$$

Example 9·4. *A vertical steel wire of length 3m. and diameter 0·38 mm. is fixed at its upper end and stretched by a load of 4 kg. attached to its lower end. Calculate the increase in length and the potential energy stored in the stretched wire. The value of Young's modulus for steel may be taken as* $2{\cdot}0 \times 10^{12}$ *c.g.s. units.*

Solution : Here, $l = 3$ m. $= 300$ cm. ; $F = 4$ kg. wt. $= 4 \times 10^3 \times 980$ dyne.

cross-sectional area $a = \pi \times \left(\dfrac{0{\cdot}038}{2}\right)^2$ cm.^{2}

Now, from eqn. (9·5), $Y=\left(\frac{F}{a}\right)\Big/\left(\frac{\triangle l}{l}\right)$

or, $2{\cdot}0\times10^{12}=\frac{4\times10^{3}\times980}{\pi\times(\frac{0{\cdot}038}{2})^{2}}\Big/\frac{\triangle l}{300}$

or, $\triangle l=0{\cdot}5186$ cm.

Again, during the extension $\triangle l$, the average load is $\frac{0+4}{2}=2$ kg.

$\therefore$ Work done on the wire = average force × the distance extended

$=2\times10^{3}\times980\times0{\cdot}5186$ erg.

$=1{\cdot}016\times10^{6}$ erg.

Potential energy stored in the wire $=1{\cdot}016\times10^{6}$ erg.

9·6. Volume stress and strain ; Bulk modulus.

Under the action of external deforming forces, if only a change in the volume of a body takes place without any change in its shape, the body is said to have suffered a volume strain. This occurs when the body is subjected to uniform normal pressure at every point on its surface. By pressure we mean normal compressive force per unit area of its surface. The Fig. 9·4 illustrates a cube and a sphere subjected to such pressure ; under its action the cube or the sphere will be in equilibrium but their volumes will get reduced.

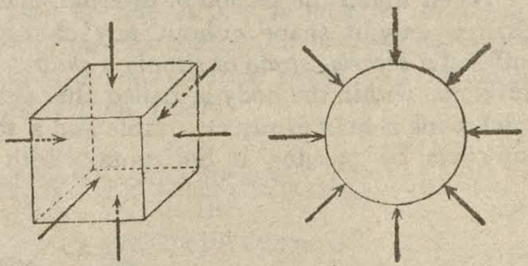


Fig. 9·4 : Volume stress

Let us consider a body of initial volume v which is under a uniform normal pressure P at each point of its surface. Let $\triangle v$ be the corresponing decrease in volume. We then define **volume strain** *as the change in volume per unit volume and* **volume stress** *as the normal force per unit area i.e. pressure on the body*. Hence, volume stress $=P$

$$\text{and volume strain}=\frac{\triangle v}{v} \qquad \cdots \qquad \cdots \qquad (9{\cdot}6)$$

Again, it may be found experimentally that within the elastic limit, volume stress is proportional to volume strain, the proportionality constant, called *bulk modulus*, being a characteristic of the material of the body.

Thus **bulk modulus** *is defined as the ratio of volume stress to volume strain* and is usually denoted by K.

$$\therefore \quad K=\frac{P}{\Delta v/v} \qquad \cdots \qquad \cdots \qquad \cdots \qquad (9{\cdot}7)$$

The **compressibility** *of a material is the reciprocal of its bulk modulus.*

Matter, in all three states, possesses certain volume. Hence bulk modulus, which involves a volume change, is a general property of matter.

It is very high for solids and liquids, because large forces are necessary to produce even a small change in volume. Gases are more easy to compress ; hence they have much smaller bulk modulii.

Example 9·5. *A litre of mercury contracts 0·2 c.c., when subjected to a pressure of 50 kg. per sq. cm. Calculate its bulk modulus.*

Solution : Hence, volume stress$=50\times10^3\times980$ dyne/cm^2.

$$\text{Volume Strain}=\frac{0{\cdot}2}{10^3}$$

$$\therefore\ \text{Bulk modulus}=\frac{\text{Volume Stress}}{\text{Volume Strain}}=\frac{50\times10^3\times980\times10^3}{0{\cdot}2}$$

$$=2{\cdot}45\times10^{11}\ \text{dyne/cm.}^2$$

9·7. Shearing stress and strain ; Modulus of rigidity.

When under the action of external deforming forces, a body suffers change only in shape without any change in volume, it is said to have suffered a *shearing strain* or simply a *shear*. The corresponding stress which develops within the body is called the *shearing stress*. For example, if a thick book is held firmly on a table and a tangential force is applied on the top cover by pushing it horizontally with a hand, the shape of the book will be found to have changed [Fig. 9·5]. The force causes the consecutive pages to be slightly displaced, or sheared, relative to one another. If we draw a small rectangle on the end side face of the book in front of the observer, it becomes a parallelogram after shear.

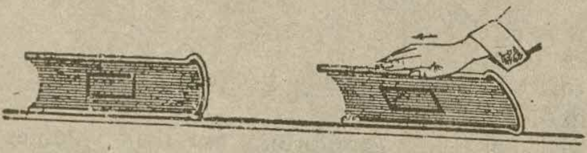


Fig. 9·5 : Shear of a book

Let us now consider, in general, a rectangular block *ABCD* with its lower face *DG* being fixed on a horizontal surface [Fig. 9·6]. Let a force *F* act tangentially over the face *AK*. This will produce a reaction force *F* also acting tangentially over the face *DG*. As a result, the shape of the block changes, the rectangular face *ABCD* assumes the shape of a parallelogram *A′B′CD*. The rectangular block assumes the shape of a parallelopiped. The body is then said to undergo a shear. The **sharing strain** *is defined as the angle* ϕ *(expressed in radians) by which a line BC or AD normal to the applied force gets displaced due to shear*. The angle ϕ is called the *angle of shear*. Usually this angle is very small. Then if $AA'=\Delta x$, and $AD=l$, we have from the Fig. 9·6,

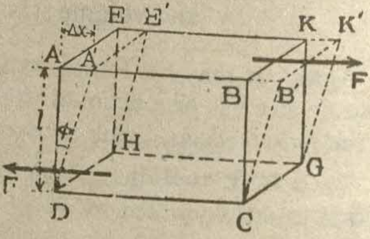


Fig. 9·6 : Shearing stress

shearing strain$=\phi=\tan\phi$ (as ϕ is small)$=\frac{\Delta x}{l}$

$$=\frac{\text{Relative displacement of two planes viz., } DG \text{ and } AK \text{ of the body}}{\text{Distance between the two planes}}$$

=Relative displacement for planes at unit distance apart.

Shearing stress *is the tangential force per unit area.* Thus, if the area of either of the face AK or DG be a, then

the shearing stress$=F/a$.

Modulus of rigidity or **shear modulus** *of a material is defined as the ratio of the shearing stress to the shearing strain* and is usually denoted by n.

$$\therefore \quad n=\frac{F/a}{\phi}=\frac{F/a}{\Delta x/l} \qquad \dots \qquad \dots \qquad (9{\cdot}8)$$

As solids only possess definite shape, modulus of rigidity is a special property of solids.

Example 9·6. *A metallic cube of side 10 cm. is subjected to a shearing force of 900 kg. wt. The top face of the cube is displaced through 0·03 cm. with respect to the bottom. Calculate (a) the shearing stress. (b) the shearing strain and (c) the modulus of rigidity of the metal.*

Solution :

$$\text{Shearing stress}=\frac{\text{tangential force}}{\text{area of a face}}=\frac{900\times 10^3\times 980}{10^2}\ \text{dyne/cm.}^2$$

$$=8{\cdot}82\times 10^6\ \text{dyne/cm.}^2$$

$$\text{Shearing strain}=\frac{\text{displacement}}{\text{height}}=\frac{0{\cdot}03}{10}=3\times 10^{-3}$$

$$\text{Modulus of rigidity}=\frac{\text{shearing stress}}{\text{shearing strain}}=\frac{8{\cdot}82\times 10^6}{3\times 10^{-3}}=2{\cdot}94\times 10^9\ \text{dyne/cm.}^2$$

9·8. Torsional deformation.

The shape of a solid may be changed in a number of ways ; we shall briefly consider the case in which a cylinder is twisted about its axis. This occurs when one end of the cylinder is fixed rigidly and a torque is applied at the other end. This produces a uniform shear. This can be clearly seen if we consider a long rectangular element (shown by the dotted lines) on the cylindrical surface before deformation. When the cylinder is twisted, the 'rectangle', becomes a parallelogram' [Fig. 9·7]. Therefore, the material experinces a pure shear. The angular displacement θ (in radian) suffered by each point on the free end of the cylinder is called the, *angle of twist* and may be taken as the measure of the deformation produced. If L be the applied torque, then Hooke's law gives,

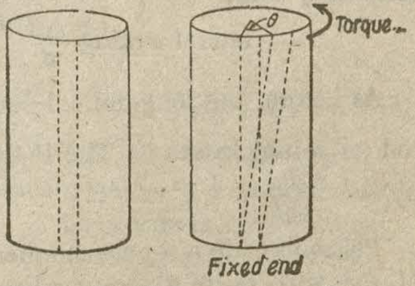


Fig. 9·7 : Torsional deformation

$$L=c\theta \qquad \dots \qquad \dots \qquad (9{\cdot}9)$$

where the proportionality constant c is called the *torsion constant* of the cylinder. It can be proved that the torsion constant is related to the modulus of rigidity of the material of the cylinder by the relation $c=\frac{\pi n r^4}{2l}$ where r is the radius and l is the length of the cylinder.

Example 9·7. *A wire fixed at one end is twisted through 45° by applying a couple of moment 1500 dyne-cm. What should be the moment of the couple required to produce a twist of 0·75 radian ?*

Solution : Since $180^\circ=\pi$ rad., $\therefore\ 45^\circ=\frac{\pi}{4}$ rad.

From equation (9·9), $1500=c\frac{\pi}{4}$ ... ... (i)

If L be the required moment of the couple, $L=c\times 0{\cdot}75$ ... (ii)

Dividing (ii) by (i) we get, $\frac{L}{1500}=\frac{0{\cdot}75\times 4}{\pi}$ or, $L=1432$ dyne-cm.

9·9. Poisson's ratio.

When a wire or a rod is subjected to longitudinal stress, its elongation is always accompanied by lateral contraction. Similarly, when a compressive force is applied, a lateral expansion takes place along with longitudinal compression. *We define the* **lateral strain** *as the lateral change in length per unit length. The ratio of the lateral strain and the longitudinal strain is called* **Poisson's ratio.** The Fig. 9·8 illustrates the lateral contraction suffered by an elongated rod in exaggerated form. If d be the initial diameter of the rod and Δd be the change in the diameter in the elongated condition, then,

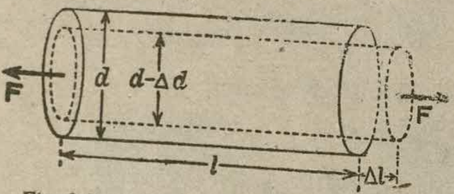


Fig. 9·8 : Lateral contraction accompanies longitudinal elongation

$$\text{lateral strain}=\frac{\Delta d}{d} \quad \cdots \quad \cdots \quad (9{\cdot}10)$$

As before, the longitudinal strain$=\frac{\Delta l}{l}$, Δl being the elongation of the rod of initial length l. The Poisson's ratio, usually denoted by σ is given by, $\sigma=\frac{\Delta d/d}{\Delta l/l}$ ... ... (9·11)

Poisson's ratio is a pure number and is the characteristic of the material of the wire or of the rod. For most metals it has a value nearly 0·3. Obviously, it is a special property of solids.

Example 9·8. *A metal wire of length 150 cm. is loaded and an elongation of 2 mm. is produced. If the diameter of the wire is 1 mm., find the change in the diameter of the wire when elongated. Given Poisson's ratio of the wire as 0·24.*

Solution : Length of the wire $l=150$ cm., elongation $\triangle l=0{\cdot}2$ cm., and diameter of the wire $d=0{\cdot}1$ cm.

$\therefore$ From eqn. (9·11), change in the diameter

$$\triangle d=\frac{\sigma\times\triangle l\times d}{l}=\frac{0{\cdot}24\times0{\cdot}2\times0{\cdot}1}{150}=3{\cdot}2\times10^{-5}\text{ cm.}$$

9·10. Relations among elastic modulii.

Out of the four constants described, only Young's modulus and modulus of rigidity are comparatively easier to determine experimentally. These four constant are, however, connected by the four equations given below :

$$\sigma=\frac{3K-2n}{6K+2n} \qquad \frac{Y}{3K}=1-2\sigma$$

$$Y=\frac{9Kn}{3K+n} \qquad \frac{Y}{2n}=1+\sigma$$

Obviously, if two of these four elastic constants are determined experimentally, the other two can be calculated from these relations.

Now, Y, K and n are always positive ; so from the above equations, it can be proved that the maximum value of the Poisson's ratio σ is $+\frac{1}{2}$ and its minimum value is -1 i.e., $0{\cdot}5 \geqslant \sigma \geqslant -1$ for all solids.

The derivation of these relations needs advanced treatment ot mechanics and is beyond the scope of this book.

9·11. Stress-strain relation ; Elastic limit.

When a wire of elastic material is subjected to an increasing tensile stress, the strain changes as shown in the Fig. 9·9. The figure shows that up to the point A the relation between stress and strain is linear i.e., the material obeys Hooke's law. The wire returns to its original length when the stress is removed. In other words, OA is the region of *perfect elasticity*. The point A represents the *elastic limit* of the material.

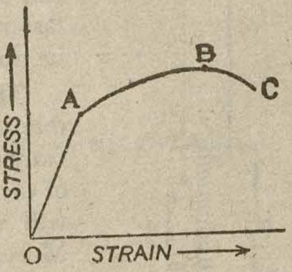


Fig. 9·9 : Stress-strain curve

If the stress on the wire is increased beyond the value corresponding to the elastic limit, **Hooke's** law is unable to account for the elastic behaviour ; the wire acquires **permanent set** i.e., *if the stress is now removed, the strain does not vanish completely.* If the process is carried on for some time more, a stage is reached when the wire is found to undergo a relatively large strain with practically no increase of stress. The stress at which this large increase in strain occurs is called the *yield point* (point B in the figure).

Still further increase in stress ultimately results in the breaking of the wire. The corresponding point C on the stress-strain curve is called the *breaking point. The maximum stress that can be applied to a material before it* *ruptures is called its* **breaking stress.** For a wire fixed at one end, the **breaking-load** *is obtained by multiplying breaking stress of its material with its area of cross-section.*

9·12. Elastic fatigue.

Machine parts e.g., locomotive wheels, parts of aeroplanes and rockets etc. are sometimes subjected to repeated stress over long periods of time. Though the stress may be within elastic limit, each time the stress is removed, the body does not readily return to its original configuration. This failure of a body to regain its original size and shape as soon as the deforming force is removed is called *elastic lag* or *hysteresis.* If sufficient time is not allowed between two successive stresses, the body retains some permanent set at each stress. As this process continues, certain regions of the body are weakened, particularly in the vicinity of microscopic cracks that may be present within the body at the time of casting. Ultimately, the body ruptures in these areas. *This loss of strength due to repeated stress is called elastic fatigue or simply fatigue.* Hence it is very important to detect flaws in such parts of machine. Generally X-rays are used for such purpose.

9·13. Determination of Young's modulus for a wire.

Two wires of the same material are suspended side by side from chucks attached to a rigid support in the ceiling of the laboratory [Fig. 9·10]. One of them *A* is the experimental wire and the other *B* is the reference wire. The wire *B* carries a main scale *S* and is held straight by a fixed load *L*. The experimental wire *A* is attached to a vernier *V* which can slide along the main scale *S*. The end of *A* is fastened to a carrier *C* on which slotted weights *W* can be put in order to alter the load on *A*.

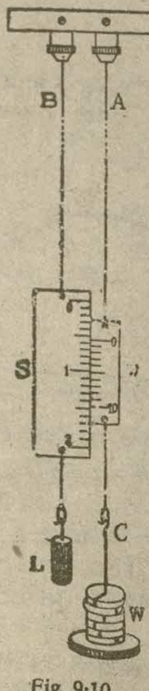


Fig. 9·10

The arrangement of two wires prevents any error due to yielding of the support being measured as an extension of the wire. This is so because any yielding of the support will affect both the wires equally and will not produce any relative motion of *V* over *S*. Moreover as the wires are made of the same material, any change in room temperature produces equal changes in their lengths and the readings remain unaffected.

At first, the wire *A* is made free from kinks by placing a suitable load on the carrier *C*. This is called the *dead load* and is not included in the stretching load. The length *L* of *A* from the point of suspension to the point at which the vernier is attached is determined. Its diameter *d* is measured with a screw gauge in several places and the mean value is taken.

The breaking load and the limiting load of the wire are then calculated from the following formulae :

$$\text{Breaking Load} = \text{Breaking stress} \times \text{area of cross-section}$$

$$= \text{Breaking stress} \times \frac{\pi d^2}{4}$$

$$\text{Limiting load} = \tfrac{1}{2} \times \text{Breaking load.}$$

During experiment, the total load applied to the experimental wire is kept with this calculated limiting load. This is necessary to ensure that the elastic limit of wire is not exceeded.

The load on the carrier is then increased by equal steps until the total load almost reaches the limiting value and at each step the readings of the main scale and vernier are noted. The observations are repeated by decreasing the load by similar steps till the dead load is reached. The two sets of readings should agree to within the limits of accuracy of observation. The mean of the two readings for each load is calculated.

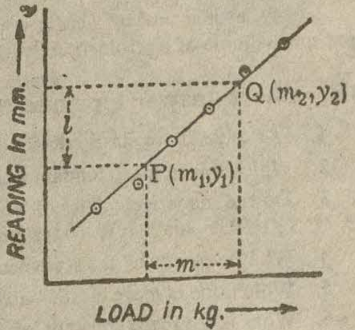


Fig. 9·11

A graph is drawn with load m along x-axis and the corresponding reading y along y-axis [Fig. 9·11]. It will be found to be a straight line. Taking any two points $P(m_1, y_1)$ and Q (m_2, y_2) on the graph except the experimentally obtained points, the corresponding loads (m_1 and m_2) and the readings (y_1 and y_2) are ascertained. Then

Elongation $= l = y_2 - y_1$

and the corresponding load,

$$m = m_2 - m_1$$

$$\therefore \quad \text{strain} = l/L$$

$$\text{stress} = mg \Big/ \frac{\pi d^2}{4} \text{ dyne/cm.}^2$$

Hence Young's modulus Y for the material of the wire is given by

$$Y = \frac{mg \Big/ \frac{\pi d^2}{4}}{l/L} = \frac{4mgL}{\pi d^2 l} \text{ dyne/cm.}^2$$

● EXERCISE ●

[A] Essay-type questions

1. Define—'longitudinal stress,' 'longitudinal strain', 'Young's modulus' and 'Poisson's ratio'. Explain with suitable diagram how longitudinal stress develops when a bar is strained longitudinally.

Derive the unit in which the Young's modulus should be expressed in C. G. S. system.

2. Define the terms 'stress' and 'strain'. What are their units in C. G. S. and M. K. S. system ? What is elastic limit ? What do you mean by 'perfectly elastic', 'partly elastic' and 'perfectly inelastic' bodies ? Explain with examples that provide very near approximations.

In what way do the numerical magnitude of (a) stress, (b) strain and (c) modulus of elasticity depend on the units of force and length ?

3. What is 'elasticity' ? State and explain Hooke's Law. Describe a laboratory method for determination of Young's modulus of the material of a wire. What is the reason for using two wires in this experiment ? Why are they suspended from the same support ? Is it possible to provide verification of Hooke's law from this experiment ?

4. Define—'Volume strain', 'Volume stress' and 'Bulk modulus'. What are their units in C. G. S. system ? Explain with diagram the nature of the load-extension curve for a solid.

5. Define 'shearing stress', 'shearing strain', and 'modulus of rigidity.' With the help of stress-strain curve, explain the following terms : (a) perfect elasticity, (b) yield point, (c) elastic limit, (d) breaking point, (e) breaking stress, and (f) breaking load.

6. Write short notes on :

Angle of twist, Poisson's ratio, Elastic fatigue, Compressibility.

7. What is meant by 'elastic potential energy' ? Prove that the elastic potential energy per unit volume of a strained body$=\frac{1}{2}\times$stress$\times$strain.

[B] Short answer type questions

1. Is rubber more elastic than steel ? Explain with reasons.

2. Which one is more fundamental—stress or strain ?

3. Do the numerical values of Young's modulus of a substance differ if it is calculated in C. G. S., F. P. S. and M. K. S. systems ? Explain your answer.

4. What is meant by the statement that Young's modulus of steel is 19×10^{11} dyne/cm.^{2} ?

5. Which one is more elastic—steel or diamond ? Explain with reason.

6. What is the force constant of a spring ? What property of the spring is expressed by it ?

7. When an elastic wire is stretched on application of tension, potential energy is stored in it. (a) What will happen to this potential energy when the wire regains its original length after the tension is removed ? (b) Will the temperature of the wire increase or decrease if it snaps under tension ?

8. Two springs have their force constants as k_1 and k_2 ($k_1>k_2$). On which spring is more work done (i) when their lengths are increased by the same amount, (ii) when they are stretched by the same force ?

[*I. I. T. '76*]

9. A certain force is required to break a piece of cord. What force is required to break a cord made of the same material which is (*a*) twice as long, (*b*) twice as large in diameter and is of the same length ?

10. Explain the following statements with proper reasoning :—
   (i) Fluids do not possess Young's modulus.
   (ii) Modulus of rigidity is a special property of solids.
   (iii) Poisson's ratio is a pure number and its value lies between 0·5 and −1.
   (iv) A perfectly rigid body cannot be found in nature.

[C] Simple Problems

1. A boy stretches the rubber cord of his catapult so that is becomes 10 cm. longer. Find the average force he applied to the rubber cord if stretching of the cord by 1 cm. requires a force of 100 gm. wt.

With what velocity will a stone of mass 10 gm. be projected under such condition ?

[*Ans.* 0·5 kg. wt. ; 9·898 m./sec.]

2. A brass wire 1 m. long and 1 mm. diameter is stretched with the weight of 2 kg. Find the elongation of the wire. Given, *Y* for brass$=1\times10^{12}$ dyne/cm.^{2} [*Ans.* 0·025 cm.]

3. Find the load in kilogram, required to stretch a vertical steel wire 628 cm. long and 2 mm. in diameter by one more millimetre in length. Y for steel$=2\times10^{12}$ C. G. S. unit and $g=980$ C. G. S. unit. (*H. S.* '60) [*Ans.* 10·2 kg.]

4. A copper wire 2 metre long and 0·5 mm. in diameter supports a mass of 10 kg. It is stretched by 2·38 mm. Calculate the Young's modulus of the wire.
[*Ans.* $4{\cdot}197\times10^{12}$ dyne/cm.^{2}]

5. What will be the elongation of a wire 10 metre long when the strain is 0·001% ? If the wire has a cross-section of 2 sq. mm. and is stretched by a load of 1 kg., what is the stress in dyne/cm.^{2} ? (*H. S.* '66) [*Ans.* 0·1 mm. ; 49×10^6 dyne/cm ^{2}]

6. Find the force necessary to stretch by 2 mm. a steel wire 1 m. long and 1 mm. diameter. Also calculate the energy stored in the stretched wire. Given Y for steel$=2\times10^{12}$ dyne/cm.^{2} [*Ans.* $3{\cdot}14\times10^7$ dyne ; $3{\cdot}14\times10^6$ erg]

7. A scale pan is suspended from the ceiling by 3 steel wires each of length 3 m. The diameter of each wire is 0·8 mm. By how much the pan will be depressed if a load of 7·5 kg. is placed on the pan ? Given, Y for steel$=2\times10^{12}$ dyne/cm^2. [*Ans.* 0·073 cm.]

8. A steel wire 1 m. long and a brass wire 70 cm. long each 0·01 cm.^{2} in cross-section are fastened together end to end and are then subjected to a tension of 10 kg. wt. Calculate the elongation of each wire. Given Y for steel$=2\times10^{12}$ dyne/cm.^{2} and that for brass$=1\times10^2$ dyne/cm.^{2} [*Ans.* $4{\cdot}9\times10^{-2}$ cm. ; $6{\cdot}86\times10^{-2}$ cm.]

9. Could a steel wire 2 m. long be stretched 10·0 mm. without exceeding its elastic limit, which is about $8{\cdot}26\times10^9$ dyne/cm.^{2} ? Given Y for steel$=2\times10^{12}$ dyne/cm.^{2}
[*Ans.* No, elastic limit would be reached for $\triangle l=8{\cdot}26$ mm.]

10. Elastic limit is exceeded when the strain of a wire becomes just greater than 1/2000. Its cross-section is 0·02 cm.^{2} and Y for its material is 14×10^{11} dyne/cm.^{2}. Find the value of maximum load that can be hanged from the wire without causing permanent set.
[*Ans.* 14·3 kg.]

11. A vessel keeps 300 c. c. of air at a pressure of 760 mm. of mercury. If the pressure be altered by 2 mm. of mercury, the volume is observed to decrease by 0·3 c.c. without change of temperature. Calculate the coefficient of the volume elasticity of the gas.
[*Ans.* $2{\cdot}665\times10^6$ dyne/cm.^{2}]

12. Calculate the contraction in volume of a solid copper cube, having side 10 cm. when subjected to a pressure of 70 kg. wt./cm.^{2} normally on all its sides. Given, Bulk modulus for copper$=1{\cdot}4\times10^{12}$ dyne/cm.^{2} [*Ans.* $4{\cdot}9\times10^{-2}$ c. c]

13. A nickel sphere is immersed in a liquid and the normal pressure on its surface is increased by 8×10^{10} dyne/cm.^{2} The volume is thus reduced by 5 percent. What is the Bulk modulus of nickel ? [*Ans.* $1{\cdot}6\times10^{12}$ dyne/cm.^{2}]

14. The compressibility of water is 44×10^{-6} per atmosphere pressure. Find the decrease in volume of 200 c. c. of water when subjected to 125 atmosphere pressure. [*Ans.* 1·1 c. c.]

15. A metallic cube of side 5 cm. has its upper surface displaced through 0·03 mm. by a tangential force of 10,000 kg. wt. Calculate (a) the shearing stress (b) the shearing strain and (c) the modulus of rigidity of the metal.
[*Ans.* (a) $3{\cdot}92\times10^8$ dyne/cm.^{2} (b) 6×10^{-4} (c) $6{\cdot}53\times10^{11}$ dyne/cm.^{2}]

16. Two parallel and opposite forces each of 1000 kg. wt. are applied tangentially to opposite faces of a cube having side 25 cm. Calculate the angle of shear and the relative displacement. Given that the shear modulus of steel is $2{\cdot}95\times10^9$ dyne./cm.^{2}
[*Ans.* $5{\cdot}315\times10^{-4}$ radian, $1{\cdot}329\times10^{-2}$ cm.]

17. A metal wire of length 200 cm. is stretched and an elongation of 1 mm. is produced. If the diameter of the wire is 2 mm. find its change in diameter when elongated. Given, Poisson's ratio of the material of the wire is 0·25. [*Ans.* $2{\cdot}5\times10^{-5}$ cm.]

18. The diameter of a brass rod is 6 mm. It is subjected to a tension of 5×10^8 dyne. If its diameter changes by $3{\cdot}6\times10^{-4}$ cm., calculate (a) the longitudinal strain and (b) the Poisson's ratio of brass. Given Y for brass$=9\times10^{11}$ dyne/cm.^{2}.
[*Ans.* (a) $1{\cdot}966\times10^{-3}$ (b) 0·305]

19. A light rod of length 200 cm. is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wire is made of steel and is of cross-section 0·1 sq. cm. and the other is of brass of cross-section 0·2 sq. cm. Find the position along the rod at which a weight may be hung to produce (i) equal stresses in both wires (ii) equal strains in both wires. (Y for brass $=10\times10^{11}$ dynes/sq. cm., Y for steel $=20\times10^{11}$ dynes/sq. cm.) (*I. I. T.* '74)

[*Ans.* (i) At a distance 66·67 cm. from brass wire, (ii) At middle of the rod.]

20. Two weightless springs X and Y of equal length L, have force constants k_1 and k_2 respectively. Those are connected as shown in the following figures? Find the force constant k of spring system in each case.

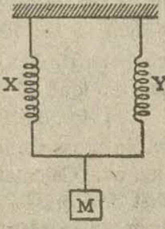


Fig. (a)

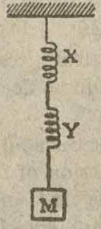


Fig. (b)

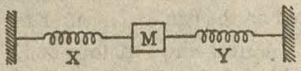


Fig. (c)

[*Ans.* (a) $k=k_1+k_2$; (b) $k=\frac{k_1k_2}{k_1+k_2}$; (c) $k=k_1+k_2$]

21. When the load on a wire is increased slowly from 4 to 6 kg. wt. the elongation increases from 0·65 to 1·13 mm. How much work is done during the extension of the wire?

[*Ans.* 0·0205 J]

22. A wire of length 1 m. when stretched by a load of 10 kg. increases in length by 0·1 cm. The cross-section of the wire is 0·1 cm.^{2} Calculate the Young's modulus of the material. If the Poisson's ratio be $\frac{1}{3}$, calculate also the change in volume of the wire.

(*Jt. Entrance* '82) [*Ans.* $9{\cdot}8\times10^{10}$ dyne/cm.^{2}; 0·0033 c.c.]

23. A vertical steel column 10 metres high supports a load of 80 metric tons. Taking Young's modulus for steel to be 2×10^6 kg. wt./cm.^{2}, area of cross-section to be 100 sq. cm. and Poisson's ratio to be 0·2, find the decrease in length and the decrease in volume of the column when the load is applied. (*Jt. Entrance* '79) [*Ans.* 0·4 cm.; 56 c.c.]

24. A mass of 3 kg. is attached to one end of a wire of length 130 cm. The other end of the wire is fixed. The mass is kept moving with uniform speed in a horizontal circle of radius 50 cm. Find the increase in length of the wire if its radius is 0·40 mm. and Young's modulus of its material is $1{\cdot}8\times10^{11}$ dyne/cm.^{2}

[*Ans.* 0·46 cm.]

[D] Harder Problems

1. A railway engine of mass 20 ton struck a wall and each of its two buffers was found to be compressed by 10 cm. If the spring of each buffer is compressed by 1 cm under the action of a force of 1 ton. wt. find the velocity with which the engine struck the wall. (1 ton = 1000 kg.)

[*Ans.* 3·56 km./hr.]

2. A wire AB, of diameter 0·6 mm. and length 300 cm. is attached to a fixed support at A and hangs vertically. The wire carries a load of 4 kg. wt. fixed to its mid-point, and also a further load of 4 kg. wt. at its lower end B. Taking the value of Young's modulus for the material to be 2×10^{12} dyne/cm.^{2}, find the depression of the end B.

(*Oxford Univ.*) [*Ans.* 0·312 cm.]

3. A copper write 8 m. long hangs vertically. Find the elongation due to its own weight. Given, Y for copper $=1{\cdot}3\times10^{12}$ dyne/cm.^{2} and density of copper $=8{\cdot}9$ gm./c.c.

[*Ans.* $2{\cdot}147\times10^{-3}$ cm.]

4. A steel ball of diameter 20 cm. and mass 1 kg. is suspended from a point 311 cm. above the floor by a steel wire of unstretched length 290 cm. The diameter of the wire is

0.4 mm. If the ball is set swinging so that its centre passes through the lowest point at 6m./sec. by how much does it clear the floor ? Given, Y for steel$=2\times10^{12}$ dyne/cm.^{2}

[*Ans.* 0·7483 cm.]

5. A vertical steel wire and a parallel brass wire, each 4 m. long and 0·0040 cm.^{2} cross-section, hang from a ceiling and are 30 cm. apart. The lower ends of the wires are attached to a light horizontal bar. Find the mass of the load which must be hung from the bar to cause each wire to extend 0·10 cm. At what distance from the brass wire must the mass be suspended ? Given Y for steel$=2\times10^{12}$ dyne/cm.^{2} and Y for brass$=1\times10^{12}$ dyne/cm.^{2} (*London Univ.*)

[*Ans.* 3061 gm ; 20 cm.]

6. A certain type of wire is found to cross its elastic limit when the strain exceeds $\frac{1}{1000}$. It has a diameter of 0·04 inch and Young's modulus for its substance is 10×10^6 pound per sq. inch. What is the greatest weight that can be hung from a length of it without causing permanent stretching ? (*Jt. Entrance '71*) [*Ans.* 12·57 lb.]

7. A light bar of length 50 cm. is suspended horizontally by means of two wires of equal lengths connected to its two ends. One wire is of steel having cross-section 0·05 cm.^{2}, the other being of brass having cross-section 0·025 cm^2. Find the point on the bar where a weight must be suspended in order that both the wires experience (a) the same strain (b) the same stress. Given Y for steel$=20\times10^{11}$ dyne/cm.^{2}, Y for brass$=10\times10^{11}$ dyne/cm.^{2}

[*Ans.* (a) 10 cm. (b) 16·67 cm. from the end suspended by steel wire]

8. A body of mass 2 kg. and density 2·7 gm /c. c. is hung from a steel wire of length 1 m. and diameter 1 mm. If the body is now completely immersed in water, find the change in length of the wire. Assume Y for steel$=2\times10^{12}$ dyne/cm.^{2} and $g=1000$ cm./sec.^{2}.

[*Ans.* $4{\cdot}72\times10^{-3}$ cm.]

9. A steel wire of diameter 2 mm. is clamped horizontally between two rigid clamps A and B which are 200 cm. apart. Find the load which, when attached to the mid-point of the wire, will pull this down to a distance of 5 cm. below AB. Given, Y for steel$=2\times10^{12}$ c. g. s. units.

[*Ans.* 7·68 kg.]

10. To the mid-point C of an unstretched elastic string of length $2a$, secured at its ends by two pegs A and B (also $2a$ apart), is attached another elastic string of length a. The other end D of this string is pulled at right angles to AB until C is displaced through a distance equal to $a/10$. Calculate the shift of D assuming the strings to have the same modulii of elasticity and same cross-section.

[*Ans.* 0·101 a]

11. Show that if $\triangle\rho$, the change in density, be very small compared to the original density ρ, then the relation $P=K.\frac{\Delta v}{v}$ can be replaced by the relation $P=K.\frac{\Delta\rho}{\rho}$ where K is the Bulk modulus.

Hence find the pressure that would be required to change (a) the density of steel by 0·1 percent, (b) the density of glycerine by 0·1 percent. Given K for steel$=1{\cdot}7\times10^{12}$ dyne/cm.^{2}, for glycerine$=4\times10^{10}$ dyne/cm.^{2}. [*Ans.* (a) $1{\cdot}7\times10^{9}$ dyne/cm.^{2} ; (b) 4×10^{7} dyne/cm.^{2}]

12. When a rubber cord is stretched, the change in volume due to change in its linear dimensions is negligible. Show that the Poissons' ratio of rubber is 0·5.

13. A steel wire of length 2 m. and diameter 0·6 mm. is stretched horizontally between rigid supports attached at its ends. When a load is hung from the midpoint of the wire, it is found that a depression of 1 cm. is produced. If the Young's modulus of steel be 2×10^{12} c. g. s. units, calculate the load.

[*Ans.* 5·77 gm. wt.]

14. Two blocks A and B are connected to each other by a string and a spring ; the string passes over a frictionless pulley as shown in the figure. Block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C, both with the same uniform speed. The coefficient of friction between the surfaces of blocks is 0·2. Force constant of the spring is 1960 newtons/m. If the mass of block A is 2 kg, calculate the mass of block B and the energy, stored in the spring.

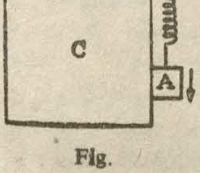
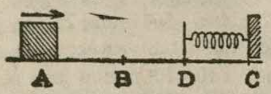


Fig.

(*I. I. T. '82*) [*Ans.* 10 kg ; 0·098 J]

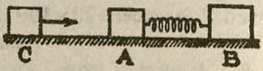
15. A 1 kg. block moving on a horizontal surface along the axis of a spiral spring collides with one end of the spring, the other end of which is fixed to a wall. The block thus compresses the spring 4·0 m. from the rest position. Assuming the force constant of the spring to be 2·0 N/m. and the co-efficient of kinetic friction between the block and the horizontal surface be 0·25, find the speed of the block at the instant of collision. [*Ans.* 7·2 m./sec.]

16. A 0·5 kg. block slides from a point *A* (see Fig.) on a horizontal track with an initial speed of 3 m./s. towards a weightless horizontal spring of length 1 m. and force constant 2 newton/m. The part *AB* of the track is frictionless and the part *BC* has the coefficients of static and kinetic friction as 0·22 and 0·2 respectively. If the distances *AB* and *BD* are 2 m. and 2·14 m. respectively, find the total distance through which the block moves before it completely. (Take $g=10$ m /s.^{2})



(*I. I. T.* '83) [*Ans.* 4·24 m.]

17. Two bodies *A* and *B* of masses m and $2m$ respectively are put on a smooth floor. They are connected by a spring. *A* third body *C* of mass m moves with a velocity v_0 along the line joining *A* and *B* and collides elastically with *A* as shown in the Fig. At a certain instant of time t_0 after collision, it is found that the instantaneous velocities of *A* and *B* are the same. Further, at this instant the compression of the spring is found to be x_0. Determine (i) the common velocity of *A* and *B* at time t_0 ; and (ii) the spring constant.



(*I. I. T.* '84) $\left[\textit{Ans.} \text{ (i) } \frac{v_0}{3} \text{ ; (ii) } \frac{2}{3}\frac{mv_0^2}{x_0^2} \right]$

18. One end of a wire 1·5 m. long (without load) is fixed at a point *A* and the other end is attached to a load of 2 kg. wt. The length of the wire is now found to be 1·53 m. The load is then held at *A* and released.

Find the maximum distance below *A* it will descend. [*Ans.* 1·83 m.]

10
CHAPTER

HYDROSTATICS

10·1. Introduction.

The branch of physics which is concerned with the study of fluids at rest is known as **hydrostatics.**

A fluid is a substance which flows or is capable of flowing. Fluids are of two types—liquids and gases. A liquid has a definite volume but no definite shape ; it always adapts its shape to that of the container. On the other hand, a gas possesses neither a definite volume nor a definite shape ; a gas always completely fills up the container whatever its size may be. There is another marked difference in the properties of a liquid and a gas. A liquid is practically incompressible while a gas can be easily compressed.

Since maintenance of a definite shape is due to rigidity, absence of any definite shape of a fluid implies that a fluid has no rigidity i.e., a fluid cannot exert or sustain shearing stresses. Under the action of such stresses, it always gives in and begins to flow. Thus for fluids at rest, shearing stresses are always absent.

10·2. Density.

We often feel that a small piece of one material may be heavier than a much larger piece of another material. The explanation is provided by the concept of density. The heaviness or lightness of a substance is attributed to its larger or smaller density.

The **density** *of a substance is defined as its mass per unit volume.* Hence,

$$\text{Density } \rho=\frac{\text{Mass}}{\text{Volume}}=\frac{M}{V} \qquad \cdots \qquad (10{\cdot}1)$$

In C.G.S. system, unit of density is gm. per c.c. ; in M.K.S. system kg. per cubic metre, while in F.P.S. system, the unit is pound per cubic foot.

Solids and liquids are compressed only very slightly even under very large stresses ; on the other hand gases are readily compressed. Moreover, with rise in temperature all substances expand i.e., their densities decrease. This change is also very marked for gases. For these reasons, it is necessary to state the conditions of temperature and pressure while referring to the density of a substance, particularly a gas. However, for solids and liquids, it is not necessary to mention the pressure because its effect on their densities is negligible as had been mentioned earlier.

10·3. Specific Gravity.

If we want to find out how the density of one material compares with that of another, we can always do it by simple division. For example, the density of gold (19·3 gm./c.c.) is 1·84 times the density of silver (10·5 gm./c.c.) and the density of silver is 3·9 times that of aluminium (2·7 gm./c.c.). To show all possible comparisons between a very large number of materials would require an impossibly large table, and it is, therefore, easier to choose one substance as a standard of density, and to compare the densities of all other substances with it. This leads us to the concept of *specific gravity*. The standard chosen for solids and liquids is water at the temperature of its maximum density, 4°C. (39·2°F.)

The **specific gravity** *of a solid or liquid is defined (1) as the ratio of the density of the substance to the density of water at 4°C.* ; or put in another way, *it is defined (2) as the ratio of the weight of a given volume of that substance to the weight of an equal volume of water at 4°C.*

We shall now show that these two definitions are completely equivalent. To do so, let us start with the first definition in which specific gravity is defined as the relative density. Thus,

Specific gravity of a substance

$$=\frac{\text{density of the substance}}{\text{density of water at } 4^\circ\text{C.}}$$

$$=\frac{\text{mass of unit volume of the substance}}{\text{mass of unit volume of water at } 4^\circ\text{C.}}$$

$$=\frac{\text{mass of any volume of the substance}}{\text{mass of same volume of water at } 4^\circ\text{C.}}$$

Since, the mass of the body at a given place is proportional to its weight, we have

$$\text{Specific gravity}=\frac{\text{weight of any volume of the substance}}{\text{weight of an equal volume of water at } 4^\circ\text{C.}}$$

As each of the two densities has the same unit, their quotient is dimensionless and has, therefore, no unit ; thus *specific gravity is a pure number.* The value of the specific gravity, therefore, remains the same in all systems of unit ; thus the concept of specific gravity provides us a very important advantage over the density.

In the case of gases, the standard chosen for defining specific gravity is hydrogen at N. T. P. (Normal Temperature and Pressure).

10·4. Difference between density and specific gravity.

Density of a substance has a definite unit. But the specific gravity being a ratio of two densities, is a pure number.

If ρ be the density of a substance and ρ_w the density of water at 4°C., then the sp. gr. S of the substance is given by,

$$S=\frac{\rho}{\rho_w}, \quad \text{or} \quad \rho=\rho_w\times S \qquad \cdots \qquad \cdots \qquad (10{\cdot}2)$$

In C.G.S. system, $\rho_w=1$ gm./c.c., so, $\rho=S$ gm./c.c.

Hence, *the density of a substance in C.G.S. system of units is numerically equal to its specific gravity.*

In M. K. S. system, $\rho_w = 1000$ kg./m³. $\therefore \quad \rho = 1000\,S$ kg./m³.

The density of a susbtance in M. K. S. system of unit is numerically equal to 1000 times the specific gravity.

In F. P. S. system, $\rho_w = 62{\cdot}5$ lb/cu. ft., at 4°C., and thus,

$$\rho = 62{\cdot}5 \times S \text{ lb./cu. ft.}$$

Hence, *the density of a substance in F. P. S. system of unit is numerically equal to 62·5 times the specific gravity.*

For example, in C.G.S. units, the density of lead$=11{\cdot}3$ gm./c.c. and also the sp. gr. of lead$=11{\cdot}3/1=11{\cdot}3$. Moreover, in F. P. S. system, the sp. gr. of lead has the same value i.e., 11·3, but its density$=11{\cdot}3 \times 62{\cdot}5 = 706$ lb./cu. ft. (Approx.)

Example 10·1. *How many gm. of glycerine oil can be poured into a bottle which holds 247·2 gm. of milk? (Densities given : milk 1·03 gm./c.c., glycerine 1·26 gm./c.c.)*

Solution : If V be the volume of liquid the bottle can hold, then

$$V = \frac{\text{mass of milk}}{\text{density of milk}} = \frac{247{\cdot}2}{1{\cdot}03} = 240 \text{ c.c.}$$

$\therefore$ required mass of glycerine$=1{\cdot}26 \times 240 = 302{\cdot}4$ gm.

Example 10·2. *A can weighs 15 gm. when empty, 75 gm. when filled with water, and 69 gm. when filled with olive oil. Find the specific gravity of the olive oil.*

Solution : Specific gravity of olive oil

$$= \frac{\text{Weight of olive oil}}{\text{Weight of equal vol. of water}} = \frac{69-15}{75-15} = 0{\cdot}9$$

Example 10·3. *When equal volumes of alcohol and water are mixed in a jar, the volume of the mixture is found to be 0·96 of the original volume. Calculate the specific gravity of the mixture, given that specific gravity of alcohol=0·76.*

Solution : Let V c.c. of water be mixed with an equal volume of alcohol. Taking the density of water as 1 gm./c.c., the mass of water$=V$ gm.

Now, mass of alcohol$=0{\cdot}76 \times V$ gm.

$\therefore$ Mass of mixture$=(0{\cdot}76+1)V = 1{\cdot}76\,V$ gm.

Since the volume of the mixture is $(0{\cdot}96 \times 2\,V) = 1{\cdot}92\,V$ c.c.

$$\therefore \text{ Sp. gr. of the mixture (or density)} = \frac{1{\cdot}76\,V}{1{\cdot}92V} = 0{\cdot}92$$

Example 10·4. *An alloy contains 64 c.c of iron to 32 c.c. of nickel. The volume of a bar of this alloy is 1·05 cu. metre. Calculate the mass of the bar given sp. gr. of iron 7·5 ; and of nickel 8·9).*

Solution : Since 96 c.c. of the alloy contains 32 c.c. of nickel and 64 c.c. of iron, we have for the bar of this alloy,

Volume of nickel $=\frac{32}{96}\times 1{\cdot}05\times 10^6=0{\cdot}35\times 10^6$ c.c.

Volume of iron $=\frac{64}{96}\times 1{\cdot}05\times 10^6=0{\cdot}7\times 10^6$ c.c.

Hence, mass of nickel $=0{\cdot}35\times 8{\cdot}9\times 10^6=3{\cdot}115\times 10^6$ gm.

and, mass of iron $=0{\cdot}7\times 7{\cdot}5\times 10^6=5{\cdot}25\times 10^6$ gm.

$\therefore$ The total mass of the bar $=8{\cdot}365\times 10^6$ gm. $=8365$ kg.

10·5. Pressure of a fluid ; Thrust.

When a fluid is contained in a vessel, it exerts a force on every part of the surface of the vessel in contact with the fluid. This force exerted by the fluid always acts perpendicular to any surface in contact with the fluid [Fig. 10·1]. We define *the* **pressure** *at a point due to a fluid as the normal force exerted by the fluid on unit area containing the point.* Thus, if F be the normal force exerted by the fluid on a very small area A surrounding a point, the pressure p at the point is given by

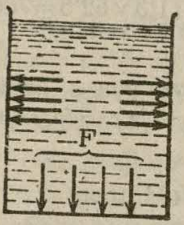


Fig. 10·1 : Pressure of a fluid

$$p=\frac{F}{A} \quad \ldots \quad \ldots \quad (10{\cdot}3)$$

$$\text{or,}\quad F=p\times A$$

This normal force exerted by a fluid on any surface in contact with it is called **thrust**.

$\therefore$ Thrust = pressure × area

It is obvious that the unit of pressure is the unit of force divided by the unit of area. Thus in C.G.S. system, the unit is dyne per square centimeter, in M.K.S system newton per square metre, while in F.P.S. system the unit is poundal per square foot. Similar to force, pressure is also a vector quantity.

The magnitude of pressure at a point in a liquid :

Let us imagine a horizontal area A containing the point P which is at a depth h below the surface of the liquid [Fig. 10·2]. We consider the liquid inside the vertical column BH having the base A.

If ρ be the density of the liquid, the mass of the liquid column = volume × density $=Ah\times\rho$.

Hence the weight of the liquid column $BH=h\rho g\times A$ where, g is the acceleration due to gravity.

$$\text{Thus,}\quad F_A=h\rho gA$$

Now from the eqn. (10·3), $p=\frac{F_A}{A}=h\rho g \quad (10{\cdot}4)$

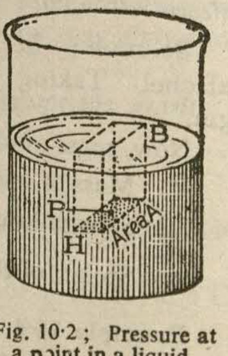


Fig. 10·2 ; Pressure at a point in a liquid

Thus, *the pressure p due to liquid at a point is equal to hρg i.e., to the product* *of depth, density and acceleration due to gravity. It follows that at a particular place* $p \propto h$ *when* ρ *is constant and* $p \propto \rho$ *when* h *is constant.*

Liquid pressure has also the following characteristics :

(i) Pressure at a point in a liquid at rest is the same in all directions.

Within a liquid contained in a vessel, let us consider the equilibrium of a small volume of liquid bounded by imaginary surfaces *KL*, *LM* and *MK*. Let p, p_x, p_y be the pressures on these surfaces [Fig. 10·3]. The volume is supposed to be so small that the pressure may be assumed uniform over each face and the weight of the liquid contained in that volume may be neglected.

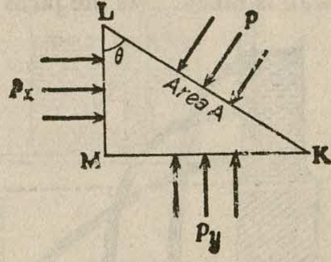


Fig. 10·3 ; Pressure in a liquid at rest is the same in all direction

Let A be the area of the surface *LK* and let the surfaces *LM* and *MK* be perpendicular to each other. The force on each area can be obtained by multiplying each pressure by the corresponding area. Resolving forces along and perpendicular to the surface *MK* we get,

$$pA \times \cos\theta = p_x \times A \cos\theta$$

$$\text{and } pA \times \sin\theta = p_y \times A \sin\theta$$

$$\therefore \quad p_x = p_y = p$$

If the volume is supposed to be extremely small, it practically reduces to a point. And since the surfaces *LK*, *LM*, *MK* may be taken in any direction, it follows that the *pressure at a point in a liquid at rest has the same value in all directions* and is, therefore, given by $h\rho g$ at any depth h in a liquid of density ρ.

(ii) Pressure at all points at the same horizontal level in a liquid at rest is the same.

To prove this, let us consider two points C and D [Fig. 10·4] at the same horizontal level in a liquid at rest. Let us imagine within the liquid a horizontal cylinder having axis CD ; the end surfaces of the cylinder pass through the points C and D. The cross-section A of the cylinder is assumed to be small.

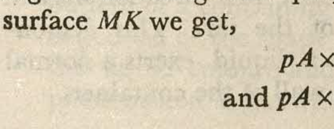


Fig. 10·4

Let the pressures at C and D be P and P', assumed to be different. These must act normal to the end surfaces of the cylinder, i.e., along the line CD. The thrust on the end surfaces through C and D are therefore $P.A$ in the direction CD, and $P'A$ in the direction DC respectively. The thrust on the curved surface of the cylinder has no component along CD, since it acts normal to the curved surface and hence to the line CD. As the liquid in the cylinder is in equilibrium, we must have

$$P.A - P'.A = 0, \text{ or, } P = P'$$

Hence, for a liquid at rest pressure is the same at the same horizontal level.

Demonstration Experiment.

(1) A tall glass jar having three openings at different heights in the wall is taken. As the jar is filled with water, small jets of water at once spurt out through the openings. It is found that the jet issues out of the lowest opening with the greatest speed and hence travels furthest [Fig. 10·5]. This shows that the pressure at a point in a liquid increases with depth.

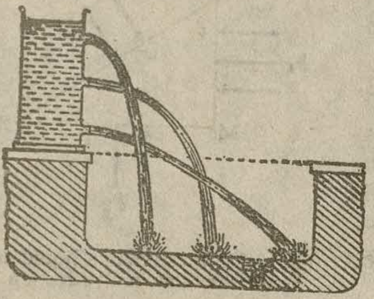


Fig. 10·5 : Pressure increases with depth

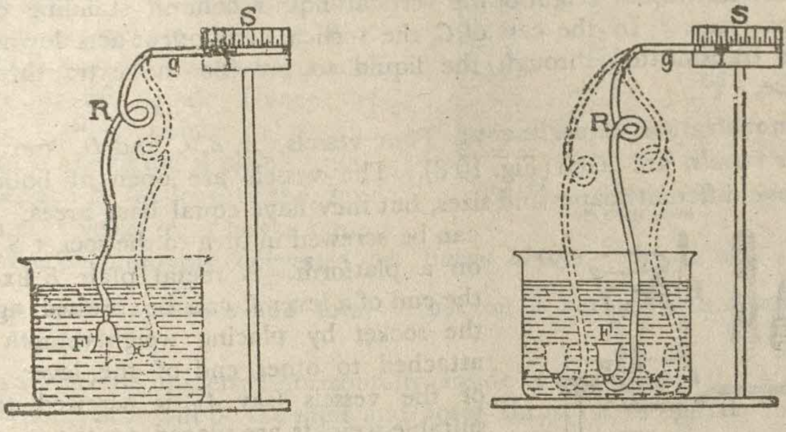
It is also observed that the jets of water spurt out in a direction normal to the wall of the jar. This demonstrates that a liquid exerts a normal force on the wall of the container.

(2) A funnel F whose mouth has been closed with a stretched rubber membrane is connected by means of a rubber tube R to a glass tube g of small bore. The glass tube is kept horizontal with a scale S attached to it. A small mercury pellet or a drop of coloured liquid (I) is introduced in the tube. This serves as an index since on pressing the membrance, the air inside gets compressed and pushes the index forward ; the larger the pressure on the membrane, greater will be the displacement of the index.

If the funnel is now introduced in a liquid and gradually lowered to greater depths, it will be found that the index steadily moves away from its initial position ; greater the depth to which the funnel is dipped, greater will be the displacement of the index in the same proportion. This proves that *the pressure increases continuously with the depth of the liquid* i.e., pressure at a point is proportional to its depth.

Keeping the funnel fixed at a certain depth below the liquid, if it is turned in different directions, the index will be found to remain stationary [Fig. 10·6(a)]. This shows that *liquid exerts pressure at a point equally in all directions.*

If the funnel is now displaced to different points at the same horizontal level [Fig. 10·6(b)], the position of the index will be found to remain unchanged. Thus, the *pressure exerted by a liquid at different points at the same horizontal level is the same.*



(a) (b)

Fig. 10·6

(iii) Hydrostatic Paradox : The thrust exerted by a liquid on the bottom of a containing vessel is independent of the total quantity of liquid contained or the shape of the vessel, but depends only on the height of the liquid column and the area of the base. This seems queer and the phenomenon is known a *hydrostatic paradox*.

Let us consider three vessels *A*, *B* and *C* of different shapes but having the same base area [Fig. 10·7]. If they are filled with a liquid to the same height, the pressure on the base of each is the same and since they all have equal areas, the thrust on the base of each will be the same.

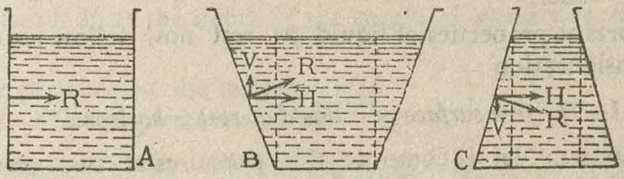


Fig. 10·7 : Hydrostatic Paradox

Now the thrust on the base of *A* is obviously exactly equal to the weight of water in *A*. The thrust on the base of *B* must, therefore, be less than the weight of water in *F* and the thrust on the base of *C* greater than the weight of water in *C*. Herein lies the paradox.

The seeming paradox can be easily explained if we remember that liquid always exerts normal thrust on the walls of the container and the walls in turn applies an equal and opposite reaction *R* on the liquid. In the case of *A*, this reaction *R* is everywhere horizontal and thus has no vertical component. The thrust on the base is thus only due to the weight of the liquid. In the case of B, the vertical component V of the reaction R supports the weight of liquid on the sloping side. The thrust on the base is thus only due to the weight of the vertical liquid column standing on the base of vessel. In the case of C, the vertical component acts downwards and is transmittted through the liquid to provide the extra thrust on the base.

Demonstration Experiment : Four vessels, A, B, C and D, known as *Pascal's vessels*, are taken [Fig. 10·8]. The vessels are open at both ends and have different shapes and sizes, but they have equal base areas. They can be screwed in turn to the socket S fitted on a platform. A metal plate E fixed at the end of a lever L can be pressed against the socket by placing weights on a pan attached to other end of the lever. One of the vessels (say A) is screwed to S; suitable weights are placed on the pan and water is then gradually poured into A, till the plate E just gets loose and water escapes. The height of water in the vessel is then marked by the pointer P. Repeating the experiment with other vessels, it will be found that for each vessel as the water rises to the same height, the resulting thrust is just able to lower the plate E when water begins to escape. This amply demonstrates that pressure depends only on the height of the liquid, not on the shape and size of the container.

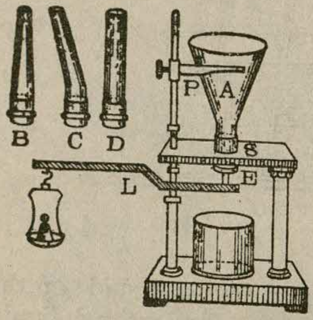


Fig. 10·8 : Demonstration of hydrostatic paradox

Two important properties of liquid at rest now follow as corollary to the above consideration :

Corollary I—*The free surface of a liquid at rest is horizontal.*

If this is not true, let us compute this pressures at two points A and B inside the liquid and in the same horizontal level [Fig. 10·9]. If the depths of the liquid at A and B are h_1 and h_2 respectively, the corresponding pressures at A and B are given by $h_1 \rho g$ and $h_2 \rho g$ where ρ is the density of the liquid. Since A and B lie on the same horizontal level, the pressures at these two points must be equal.

$$\therefore \quad h_1 \rho g = h_2 \rho g, \quad \text{or,} \quad h_1 = h_2.$$

It follows, therefore, that the free surface must be horizontal.

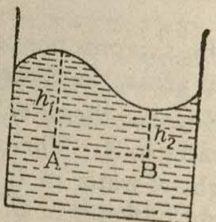


Fig. 10·9 : Free surface of a liquid is horizontal

Corollary II—*In communicating vessels, liquid stands at the same level.*

If a number of vessels of different shapes are interconnected as in [Fig. 10·10], it will be found that on pouring a liquid into them, the surfaces of the liquid stand at the same level in each of them irrespective of the size and shape of the vessels. This will be obvious if we remember that the pressures at the points A, B, C and D lying on the same horizontal level must be equal. Hence the depth of these points below the surfaces of the liquid must be equal. In other words, the level of the surface in each vessel must be the same. This fact is commonly expressed as *"liquid always finds its own level"*.

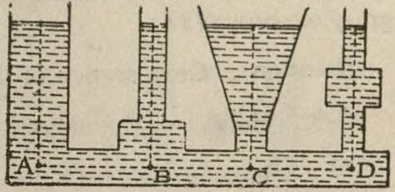


Fig. 10·10 : Liquid always finds its own level

10·6. Average pressure and total thrust on a surface immersed in a liquid.

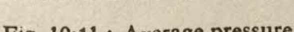
If a surface be immersed horizontally inside a liquid, then the pressure at each point of it will be the same and total thrust on the surface can be obtained simply by multiplying the pressure at any point by the area of the surface.

If, however, a surface be kept in a vertical or inclined position inside a liquid, then since different points of the surface lie at different depths, the pressure will vary from point to point. It can then be shown that *the average pressure exerted on the surface is given by the pressure at the centre of gravity of the surface.* The proof of the statement is beyond the scope of this book. The total thrust on the surface is then obtained by multiplying the average pressure with its area. If follows that the total thrust on a given surface depends not upon its inclination but on the depth of its centre of gravity. Thus if H be the depth of the centre of gravity of the surface, then

the average pressure on the surface $= H\rho g$

total thrust $=$ average pressure × area ... ... (10·5)

From this, we can easily calculate the total thrust on a rectangular surface immersed vertically in a liquid. Let the length and breadth of the surface be 'a' and 'b' respectively, its upper edge being at a depth h below the surface of the liquid [Fig. 10·11]. Since the centre of gravity of the surface is situated at the intersection of its diagonals, the depth of the c. g. below the surface of the liquid is given by,



Example 10·5. *What is the (i) total thrust and (ii) pressure on the bottom of a cylindrical vessel of diameter 28 cm. containing 30·8 litre of mercury? (Sp. gr. of mercury=13·6).*

Solution : Cross-section of the vessel$=\pi r^2=3·14\times 14^2=616$ cm^2.

$\therefore$ Height of the mercury column$=\dfrac{30·8\times 10^3}{616}=50$ cm.

$\therefore$ Pressure exerted$=h\rho g=50\times 13·6\times 980=66·64\times 10^4$ dyne/cm^2.

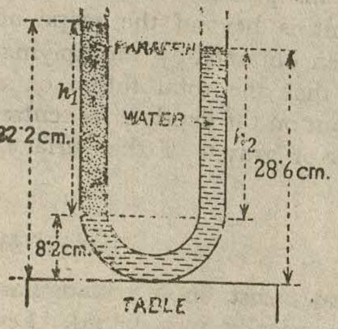
Thrust=pressure×area$=66·64\times 10^4\times 616=4·1\times 10^8$ dyne.

Example 10·6. *The area of the piston of a lift pump is 40 cm.^{2}. What force must be applied to raise water to a height of 150 metre?*

Solution : The applied force F must be equal to the thrust exerted by the water column of height 150 metre.

$$\therefore\quad F=Ah\rho g=40\times 150\times 10^2\times 1\times 980=5·88\times 10^8 \text{ dyne.}$$

Example 10·7. *A U-tube (a tube of uniform cross-section bent in the form of the letter U) containing paraffin oil in one limb and water in the other is placed vertically on the table. The top and the bottom of paraffin oil column from the table are respectively 32·2 cm. and 8·2 cm. The top of the water column is 28·6 cm. from the table. Calculate the specific gravity of paraffin oil.*



Solution : Height of the paraffin column above the surface of separation between water and paraffin is $h_1=32·2-8·2=24$ cm.

Height of the water column above that surface of separation is

$$h_2=28·6-8·2=20·4 \text{ cm.}$$

At the level of separation, the pressure exerted by the two liquids is the same. Therefore

$$h_1\rho_1 g=h_2\rho_2 g$$

$$\text{or,}\quad \frac{\rho_1}{\rho_2}=\frac{h_2}{h_1}$$

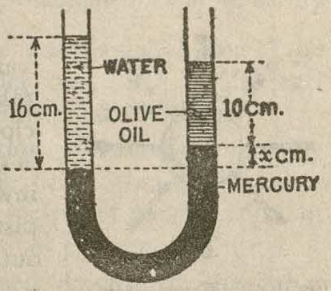
where ρ_1 and ρ_2 are the densities of paraffin and water respectively. Taking $\rho_2=1$ gm./c.c., we get,

$$\rho_1=\frac{h_2}{h_1}=\frac{20·4}{24}=0·85 \text{ gm./c.c.}$$

$\therefore$ sp. gr. of paraffin$=0·85$.

Example 10·8. *Two limbs of a U-tube contain water and olive oil separated by mercury. The length of the water column is 16 cm. while that of the oil is 10 cm. Find the difference in level between the surfaces of mercury in the two limbs. Given, sp. gr. of water=1·0, of olive oil=0·92, of mercury=13·6.*

Solution : Let the required difference in level be x cm. as shown in the diagram. Pressure of water column at the mercury surface in the left limb$=16\times1\times g$ dyne/cm^2.



Pressure at the same level in the right limb.
$=$Pressure of olive oil column$+$Pressure of mercury column of length x cm.
$=(10\times0{\cdot}92\times g+x\times13{\cdot}6\times g)$ dyne/cm^2.

Pressures in both the limbs at the same level are equal.

$$\therefore\ 16\times1\times g=10\times0{\cdot}92\times g+x\times13{\cdot}6\times g$$

$$\text{or,}\quad x=\frac{16-9{\cdot}2}{13{\cdot}6}=\frac{6{\cdot}8}{13{\cdot}6}=0{\cdot}5\text{ cm.}$$

Example 10·9. *The length of each side of a cube is 60 cm. It is completely immersed in a liquid of density 2·5 gm./c.c. in such a way that the upper face lies at a depth of 20 cm. Calculate the thrust on each face of the cube.*

Solution : $l=60$ cm., $\rho=2{\cdot}5$ gm./c.c. and depth of the upper face $=20$ cm.

$\therefore$ Thrust on the upper face$=20\times2{\cdot}5\times60\times60=18\times10^4$ gm. wt.
Thrust on the lower face$=(60+20)\times2{\cdot}5\times60\times60=72\times10^4$ gm. wt.
Average thrust on the side face$=(20+\frac{60}{2})\times2{\cdot}5\times60\times60$
$=45\times10^4$ gm. wt.

Example 10·10. *The gates of a certain canal lock are 20 metre wide. The water level on one side is 50 metre high and on the other side 24 metre. Calculate the resultant thrust on the lock gates.*

Solution : Total thrust on the side of the gate immersed in 50 m. of water$=$pressure at depth of 25 m.$\times$area of the gate immersed$=(25\times10^2\times1)\times(50\times20\times10^4)=25\times10^9$ gm. wt.

Total thrust on the opposite side$=$pressure at a depth of 12 m.$\times$area of the gate immersed$=(12\times10^2\times1)\times(24\times20\times10^4)=5{\cdot}76\times10^9$ gm. wt.

Since thrusts are in opposite directions.

$\therefore$ resultant thrust$=(25-5{\cdot}76)\times10^9=19{\cdot}24\times10^9$ gm. wt.

10·7. Transmission of Fluid Pressure ; Pascal's Law.

A change in pressure at any point of a fluid produces a corresponding and equal change in pressure at all other points. The fluid thus transmits the pressure. The phenomenon was first clearly formulated by the French scientist and philosopher Blaise Pascal in a famous law known as **Pascal's Law** after his name. The law states that :

The pressure applied at any place to an enclosed fluid is transmitted undiminished to every portion of the fluid.

Demonstration Experiment.

(1) A performed rubber ball is taken and filled up completely with

water. When the ball is pressed, water will be found spurting out equally and also normally from all the perforations [Fig. 10·12].

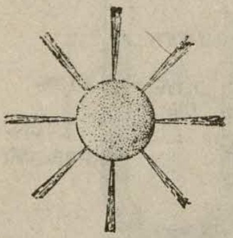


Fig. 10·12 : Transmission of fluid pressure

(2) A vessel with a number of openings of varied cross-sectional area viz. a, b, c, d etc. are taken [Fig. 10·13]. All the openings are kept closed by watertight movable pistons. The vessel is then completely filled up with water. When an inward pressure is applied on any one of the pistons, say A, the others will be found to move outwards. This shows that *pressure is transmitted in all directions.* If the force applied on the piston be F_a then since the cross-sectional area of it is a, the pressure applied on the liquid is F_a/a. Let the motion of the other pistons be prevented by applying additional forces F_b, F_c, $F_d \cdots$etc., on the pistons having cross-sectional areas b, c, d, etc., respectively. It will be found that,

$$\frac{F_a}{a}=\frac{F_b}{b}=\frac{F_c}{c}=\ldots$$

This proves that the pressure is transmitted with undiminished magnitude in all directions.

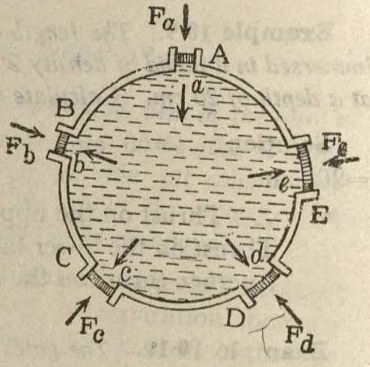


Fig 10·13 : Verification of Pascal's law

10·8. Principle of multiplication of thrust.

Two hollow cylinders of different cross-sectional areas 'a' and 'b' ($b>a$), comunicated by a tube, are filled with a liquid [Fig. 10·14]. Both cylinders are fitted with two water-tight and frictionless pistons of negligible masses.

If a small force F_a be now applied on the smaller piston, the pressure acting on the liquid is $p=F_a/a$. This pressure is transmitted undiminished through the liquid and the larger piston is thus pushed upwards with a force $p.b$. To keep it in equilibrium, a force F_b must be applied, where $F_b=p.b$.

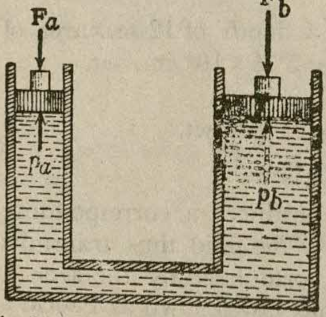


Fig. 10·14 : Multiplication of thrust

$$\text{or, } F_b=\frac{F_a}{a}\times b \qquad \ldots \qquad \ldots \qquad \ldots \qquad (10{\cdot}7)$$

$$=nF_a, \quad \text{where } \frac{b}{a}=n.$$

Hence, the thrust developed on the larger piston is multiplied n times, where $n=\frac{b}{a}$. This is known as the *principle of multiplication of thrust* by transmission of fluid pressure. For example, if the cross-sectional area of the larger piston be 200 times greater than the smaller one, then $n=200$ and the thrust developed on the larger piston is also multiplied 200 times.

Agreement with the principal of conservation of energy : It may appear at first sight that as a very large force is being obtained from a comparatively small one, hence in this case the principle of conservation of energy is violated. Actually this is not so, as we shall prove now.

Let us suppose that the smaller piston is pressed downwards through a distance l_1. In consequence, let the larger piston move up through a distance l_2. Assuming that water is perfectly incompressible.

Increase in the volume of water in the larger cylinder = decrease in the volume of water in the narrower cylinder

$$\text{or,} \quad bl_2=al_1$$

The work done by the force F_a on the smaller piston $=F_a\times l_1$

The work obtained from the force F_b on the larger piston $=F_b\times l_2$

$$\text{Now,} \quad F_b\times l_2=F_a\times\frac{b}{a}\times l_2$$
$$=F_a\times l_1$$

Hence, the work obtained = the work done.

which agrees with the principle of conservation of energy.

In the above discussion, we have neglected the effect of friction which produces dissipation of energy and thereby lowers the amount of work actually obtained.

It should be noted that as $F_b\gg F_a$, $l_2\ll l_1$. Hence the displacement of the larger piston is much smaller than that of the smaller one.

10·9. Application of Pascal's Law.

We have just explained how Pascal's law immediately leads to the very important principle of multiplication of thrust. This principle and hence Pascal's law have a wide number of practical applications, a few of which are discussed below :

[1] The Hydraulic or Bramah's Press.

A schematic diagram of hydraulic press is shown in the Fig. 10·15. It consists of two cylinders A and B connected by the tube T. The cylinders are filled up with a liquid. The tube contains a oneway valve V which allows liquid to flow from A to B, but does not allow a reverse flow. The cross-section of the cylinder B is many times larger than that of A. The cylinders are fitted with two water tight pistons P_1 and P_2. Force can be applied on the piston P_1 by means of a lever of the second class, which further increases the mechanical advantage. The piston P_2 carries a platform D on which the objects to be pressed is placed. To serve this purpose, a plate E fixed on rigid frame (not shown in Fig.) is kept above the platform D.

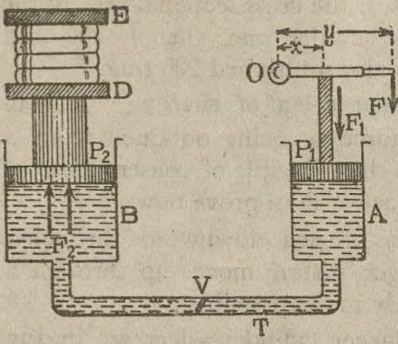


Fig. 10·15 : Hydraulic Press

When a force F_1 is applied on the piston P_1, then from the principle of multiplication of thrust, a much larger thrust F_2 is developed at the other piston P_2, given by,

$$F_2=F_1\times\frac{B}{A}$$

where A and B are the cross-sections of the respective cylinders. If the corresponding effort F be applied on the lever at a distance y from the fulcrum O, then taking moment about O,

$$F_1\times x=F\times y$$

Where x is the distance between the point at which the piston is attached to the lever and the fulcrum O.

$$\therefore\quad F_2=F\times\frac{y}{x}\times\frac{B}{A}\qquad\ldots\qquad\ldots\qquad(10{\cdot}8)$$

$\therefore$ Mechanical advantage of the machine

$$=\frac{\text{thrust generated}}{\text{effort applied}}=\frac{F_2}{F}=\frac{y}{x}\times\frac{B}{A}\qquad\ldots\qquad\ldots\qquad(10{\cdot}9)$$

The applied effort can thus be multiplied many times; as the piston P_1 descends under the action of the force F_1, the valve V opens and allows some liquid to enter into B and the piston P_2 and hence D rises under the action of the large thrust F_2 and presses the object against the fixed plate E. It should be noted that in the above discussions, we have neglected the effect of friction which obviously lowers the mechanical advantage gained.

The hydraulic press is utilised to serve wide variety of purposes. It is used in compressing bales of paper, cotton, jute etc., in extracting oil from seeds, for testing the strength of iron beams, for raising heavy loads or automobiles etc.

(2) Most road vehicles are now, equipped with hydraulic brakes which are the applications of Pascal's law. The system of breaking is shown in the Fig. 10·16. As the foot pedal F is pressed by the driver, the piston P in the master cylinder M moves inwards. The resulting pressure is transmitted by the fluid to the pair of pistons p_1, p_2, in the wheel cylinder W. These brake pistons move outward, pushing the brake

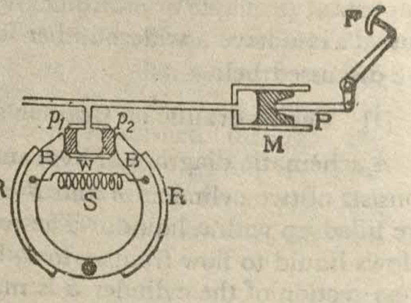


Fig. 10·16 : Hydraulic brake system

shoes B, B apart. The shoes get pressed against the rim R of the wheel, producing the breaking effect due to friction. A spring S pulls the shoes together again when the brakes are released.

Example 10·11. *The diameter of the neck and the bottom of a bottle completely filled up with water are 5 cm. and 25 cm. respectively. If a force of 6 kg. is applied on the stopper, how much thrust does the bottom of the bottle experience ?*

Solution : Diameter of the neck=5 cm. and that of bottom=25 cm. and force applied on the neck=6 kg. Hence from eqn. (10·7) the thrust developed at the bottom$=\frac{6}{5^2}\times 25^2=150$ kg.

Example 10·12. *The radius of the small piston of a hydraulic press is 8 cm. and that of the large piston is 96 cm. It is worked by a hand lever of which the ratio of the arms is 3 : 28. If a force of 27 kg. wt. is applied on the handle, what is the force developed by the large piston ?*

Solution : The ratio of the cross-sectional areas of the pistons $=\frac{B}{A}=\frac{96^2}{8^2}$, the ratio of the arms of the level$=\frac{y}{x}=\frac{28}{3}$ and the force applied on the handle of the lever$=F=27$ kg. wt. So from eqn. (10·8), force developed by the larger piston$=27\times\frac{28}{3}\times\frac{96^2}{8^2}=36288$ kg. wt.

10·10. Buoyancy.

If a body is immersed partly or fully in a liquid or a gas, it appears to become lighter. This occurs due to the fact that the liquid (or the gas) exerts an upward thrust or buoyant force upon the immersed object. As this acts in a direction opposite to the weight of the body, an apparent loss in its weight is felt. *The loss is not real* ; when immersed, a part of the weight of the body is simply being neutralised by the upthrust exerted by the liquid (or the gas). *The upward thrust which any liquid or gas exerts upon a body partly or fully submerged in it, is called its* **buoyancy.**

In order to get an explanation for the buoyancy of a liquid, let us imagine a rectangular solid block $ABCD$, immersed vertically in a liquid. The block is subjected to liquid thrust acting normally on all its sides as shown in the Fig. 10·17. The horizontal thrusts acting at all points on the sides AD and BC get mutually neutralised, because they are exactly equal in magnitude and opposite in sense. Thus only the vertical thrusts on the sides AB and DC should be taken into consideration. Let the depths of AB and DC from the free surface of the liquid be h_1 and h_2 respectively and the cross-sectional area of the face AB or DC be a. Then the pressure at a point on the face $AB=h_1\rho g$.

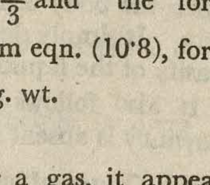


Fig. 10·17 : Buoyancy of a liquid

Hence, the thrust on $AB=h_1\rho ga$, acting vertically downwards. Similarly, the thrust on $DC=h_2\rho ga$, acting vertically upwards.

Since h_2 is greater than h_1, the upward thrust is greater than the downward thrust. Hence, the resultant upward thrust on the block

$$=g\rho a(h_2-h_1)=g\rho al$$

where $h_2-h_1=l$, the height of the block.

Now, al is the volume of the block and therefore, $al\rho g$ is the weight of an equal volume of the liquid. Since the block displaces and equal volume of liquid, we thus get,

the resultant upward thrust on the block or buoyancy=weight of equal volume of the liquid displaced by the block.

The above discusssion also holds true for a body of any shape and size.

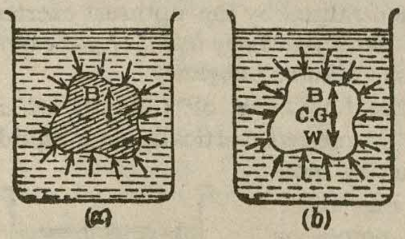
Thus, if W_1 be the weight of the body and W_2 be the weight of an equal volume of liquid, then on being fully immersed in the liquid an apparent loss W_2 in the weight of the body will take place. As a result, the apparent weight of the body is given by (W_1-W_2). This explains why it is easier to displace a heavy body when immersed in water, than when it is outside.

It is evident now, that the buoyancy does not depend on the depth to which the body is immersed, nor also on the shape, mass or material of the body. It simply depends on the immersed volume of the solid and the density of the liquid displaced.

It also follows from the above discussion, that the phenomenon of buoyancy is absent in a weightless condition, e.g. in an artificial satellite.

10·11. Centre of buoyancy.

The point at which the buoyancy of a liquid acts on a body is called the **centre of buoyancy** *or the centre of floatation.* To locate this point, we consider a body immersed in a liquid [Fig. 10·18 (*a*)]. The body will be subjected to normal thrust at each point of its boundary. These combine to produce the resultant upward thrust B on the body. If the body is now removed, its place will be occupied by an equal volume of the liquid. The same thrust acting round the same boundary will now act on this liquid [Fig. 10·18 (*b*)]. Hence the resultant of them will be the same i.e. B. Since the liquid is at rest, this resultant thrust must neutralise the weight W of the liquid within this boundary. Hence, the resultant thrust must act upward through the centre of gravity of the displaced liquid and must be equal in magnitude to the weight of the displaced liquid. *Centre of buoyancy thus coincides with the centre of gravity of the displaced liquid.*



(a) (b)

Fig. 10·18 : (a) Forces at boundary of the solid. (b) Same forces acting on same boundary enclosing the liquid

Reaction of the force of buoyancy : A body immersed in a liquid experiences an upward buoyant force exerted by the liquid. According to

Newton's third law, the body should also exert an equal and opposite (downward) force on the liquid. The existence of such a reaction can be demonstrated easily in the following experiment.

A beaker filled with water is placed on one of the scale pans of a common balance and suitable weights are added to the other pan to get accurate balance. If now, one end of a stick held by hand (or a piece of iron suspended by a string) is introduced in water without touching the beaker, the balance gets disturbed. The pan on which the beaker stands goes down. While the stick (or the iron piece) is buoyed up by the water, it exerts a downward reaction on water and thereby lowers the pan.

10·12. Archimedes' principle.

The fact that a body immersed in a fluid is buoyed up with a force equal to the weight of the displaced fluid was first discovered by the Greek scientist Archimedes in the third century B.C. He applied almost the same reasoning as in our preceding article. The principle, named after him as **Archimedes' Principle** states that

A body, partly or fully submerged in a fluid at rest, appears to lose a part of its weight which is equal to the weight of the fluid displaced.

It should be remembered that Archimedes' principle involves the weight of a body. Hence it does not hold when the body is in the state of free fall or when the body is in an artificial satellite moving in a circular orbit ; in both the cases the body is in weightless condition.

Demonstration experiments.

(1) A solid block of any substance heavier than and insoluble in water is taken and weighed by a spring balance. A can fitted with a spout is filled up with water up to the mouth of the spout. If the block is then lowered completely into the can of water without touching its sides, the water overflows. This overflowed water is collected carefully in another small container [Fig. 10·19] and weighed. The observed reading of the spring balance, when the block is immersed in the can of water, gives its apparent weight. The differences of weight of the block in air and its apparent weight, gives the upthrust or buoyant force on the submerged block. It will be found that this upthrust is just equal to the weight of thewater displaced by the block.

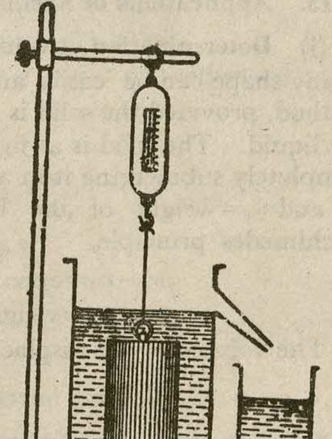
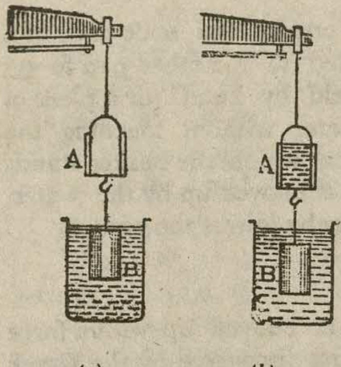


Fig. 10·19 : Demonstration of Archimedes' principle

(2) Archimedes' principle can also be verified by a hydrostatic balance

as shown in the Fig. 10·20. From the right hand side of the balance, two cylinders are hung together. One of them, *A*, is hollow and the other, *B*, is solid, such that the inner volume of *A* is equal to the outer volume of *B*. *B* is suspended from the hook provided in the bottom of *A*. Firstly, two cylinders together are made accurately balanced by placing a set of weights on the other pan of the balance. The solid cylinder is then submerged completely in water contained in a beaker, taking care that it does not touch the side of the beaker [Fig. 10·20 (*a*)]. The beam of the balance tilts towards the weight pan because of the upthrust or buoyant force of the water on the solid cylinder. From Archimedes' principle, the buoyant force is equal to the weight of a volume of water exactly equal to the volume of the cylinder *B*. Water is now slowly poured into the empty hollow cylinder *A*. When the cylinder *A* is just completely filled with water, the beam of the balance again becomes horizontal [Fig. 10·20(*b*)].



(*a*) (*b*)
Fig. 10·20 : Demonstration of Archimedes' principle

10·13. Applications of Archimedes' Principle.

(i) **Determination of volume of a solid :** The volume of a solid object of any shape can be easily and accurately determined by the following method, provided the solid is heavier than the liquid and is insoluble in the liquid. The solid is at first weighed in air and then again weighed by completely submerging it in a liquid. Thus if w_1=weight of the body in air and w_2=weight of the body in a liquid of density ρ, then from Archimedes' principle,

$$w_1 - w_2 = \text{apparent loss of weight of the body,}$$
$$= \text{weight of the liquid displaced.}$$

The volume of this displaced liquid is equal to the volume of the solid.

$$\therefore \text{ Volume of the solid, } V = \frac{w_1 - w_2}{\rho} \quad \ldots \quad \ldots \quad (10·10)$$

If the liquid be water, then since in C.G.S. system of units, the density of water approximately equals 1 gm./c.c.,

$$\text{volume of the solid, } V = (w_1 - w_2) \text{ c.c.} \quad \ldots \quad (10·11)$$

In F.P.S. system of units, the density of water is 62·5 lb/cu. ft.

$$\therefore \text{ volume of the solid, } V = \frac{w_1 - w_2}{62·5} \text{ cu. ft.}$$

For an irregular shaped body, this provides the easiest method for determining its volume. It is very difficult to measure the volume of an irregular body by the usual measuring instruments.

(ii) **Determination of density of a solid :** Let the weight of a solid object in air be w_1 and when completely immersed in a liquid of density ρ, its weight be w_2. Then, as before from Archimedes' Principle, we obtain that the volume of the solid,

$$V=\frac{w_1-w_2}{\rho}.\ \text{Since density of a solid, } D=\frac{\text{Mass}}{\text{Volume}}$$

We have, $D=\frac{w_1}{V}=\frac{w_1}{w_1-w_2}\times\rho$ ... ... (10·12)

If the body is submerged in water, we have as before,

$$D=\frac{w_1}{w_1-w_2}\text{gm./c.c. in C.G.S. units} \quad \ldots \quad \ldots \quad (10{\cdot}13)$$

$$\text{or,}\quad D=\frac{w_1}{w_1-w_2}\times 62{\cdot}5\ \text{lb./cu. ft. in F.P.S. units.}$$

(iii) **Determination of the amounts of two metals in an alloy :**

A piece of the alloy is at first weighed in air and again by completely submerging in water.

Let, w_1 gm. = weight of the alloy in air

w_2 gm. = weight of the alloy in water

Therefore, applying Archimedes' principle, we have from eqn. (10·11), volume of the alloy $=(w_1-w_2)$ c.c.

Also, let the amount of metal A in the alloy $=x$ gm.

$\therefore$ amount of metal B in the alloy $=(w_1-x)$ gm.

Now, if ρ_1 and ρ_2 be the densities of metals A and B respectively then,

$$\frac{x}{\rho_1}+\frac{w_1-x}{\rho_2}=w_1-w_2$$

$$\text{or,}\quad x\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right)=w_1-w_2-\frac{w_1}{\rho_2}$$

$$\text{or,}\quad x=\left(w_1-w_2-\frac{w_1}{\rho_2}\right)\times\frac{\rho_1\rho_2}{\rho_2-\rho_1}$$

Thus w_1, w_2, ρ_1 and ρ_2 being known, amount of metal A and hence that of metal B in the alloy can be determined.

10·14. Floatation : Conditions for equilibrium of floating bodies.

As previously discussed, if a body is partly or fully immersed in a liquid, two vertical forces simultaneously act on it. These are :

(i) the weight W of the body, acting downwards through the *centre of gravity of the body*,

(ii) the buoyancy equals to W' i.e., the weight of the displaced liquid, acting vertically upwards through the centre of buoyancy which coincides with the *centre of gravity of the displaced liquid.*

Depending upon the relative magnitudes of W and W', three cases may arise :

(*a*) If $W>W'$, i.e., *when the weight of the body is greater than the weight of the liquid displaced, the body sinks.* A net downwards force $W-W'$ acts on the body. For a homogeneous body of density ρ and volume V, its weight $W=V\rho g$. If ρ' be the density of liquid, then the weight of equal volume of liquid$=W'=V\rho' g$. The condition $W>W'$ thus implies that $\rho>\rho'$. Hence, *if the density of the body is greater than that of liquid, the body sinks.*

(b) *If* $W=W'$, i.e., *when the weight of the body equals the weight of the liquid it displaces, the body floats being wholly immersed anywhere in the liquid.* The net force on the body is zero and hence its apparent weight vanishes for all such positions. Evidently, $W=W'$, means $\rho=\rho'$; *the density of the body in this case equals that of the liquid.*

(*c*) If $W<W'$, i.e., *when the weight of the body is less than the weight of liquid displaced by it, the body does not sink. When completely submerged, the body rises.* A net upward force $W'-W$ acts on the body. As it reaches the surface, a portion of the body emerges from the liquid while the rest portion remains immersed ; thus smaller amount of liquid is displaced. Consequently W' decreases and as the body continues to rise, a time comes when W becomes equal to W'. The body then floats on the surface of the liquid being partly immersed in the liquid. The weight of the body is then equal to the weight of the liquid displaced by the submerged portion of the body. The apparent weight of the floating body is obviously zero. The condition $W<W'$ gives $\rho<\rho'$, so that for floating the density of the body must be smaller than that of the liquid.

10·15. Conditions of Equilibrium of a Floating Body.

A floating body is thus acted upon by two equal and oposite forces viz., its weight W acting downwards and the buoyant force which is equal to the weight W of the liquid displaced by the submerged portion of the body, acting upwards. These two forces must obviously act along the same straight line ; otherwise they will constitute a couple producing rotational motion of the floating body. Hence for equilibrium of floating bodies, the following two conditions must be satisfied :—

(*i*) **Condition of floatation :** The weight of the floating body must be equal to the weight of the liquid displaced.

(*ii*) **Condition of equilibrium :** The centre of gravity of the body and the centre of buoyancy i.e., the centre of gravity of the displaced liquid, must lie on the same vertical line.

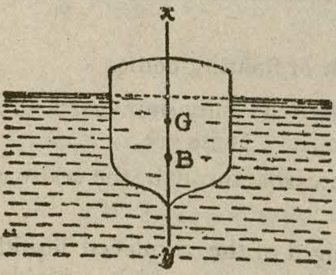


Fig. 10·21 : Floating body in equilibrium

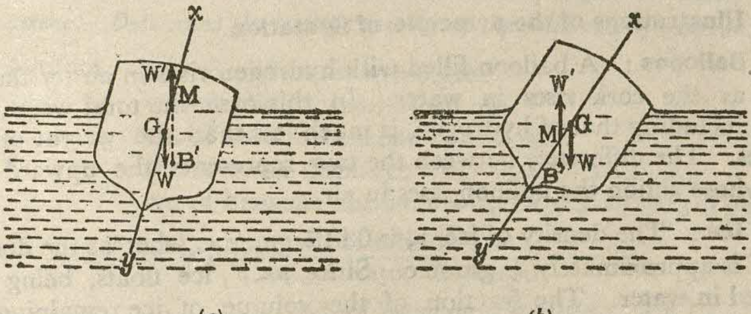
Fig. 10·21 shows a floating body in equilibrium where G represents the c. g. of the body and B, the centre of buoyancy. In this equilibrium position, the vertical line through the c. g. passes through the centre of buoyancy. This line (y) is known as the *centre line* of the body and is fixed with respect to the body.

10·16. Stability of floating bodies.

The equilibrium of a floating body may be either *neutral*, *stable* or *unstable*. This state of equilibrium of the floating body may be determined by slightly tilting it from its equilibrium position.

If a slight tilt of the floating body from its equilibrium position does not produce any shift of the centre of buoyancy, the body is said to be floating in *neutral equilibrium*. The c. g. of the body and the centre of buoyancy then always lie on the same vertical line. A floating sphere provides such an example.

In general, however, on tilting the body, the centre of buoyancy shifts to a new position B' towards the leaning side, because more liquid is now displaced on that side of the body. The forces of weight (W) and the buoyancy (W') no longer act on the same vertical line and hence produces a couple [Fig. 10·22]. If the couple formed tends to restore the body back to its original position, the equilibrium is said to be *stable* [Fig. 10·22 (*a*)]. If, however, the resulting couple tends to produce further displacement of the body from its original position thereby upsetting it, the equilibrium is said to be *unstable* [Fig. 10·22 (*b*)].



(*a*) (*b*)

Fig. 10·22 Stable and unstable equilibrium of a floating body

If we draw a vertical line through B', the point M where it cuts the centre line xy of the body is called the *meta centre*. It is obvious from the figure, that if the meta centre (M) lies above the c. g. (G) of the body, the equilibrium is stable. If it lies below, the equilibrium is unstable. Thus, for *stable equilibrium, the meta centre must lie above the centre of gravity of the body*. The distance GM is known as the *metacentric height*. Greater this metacentric height, greater will be the restoring couple tending to bring the body back to its original position and larger will be the degree of stability of the floating body. The c. g. of a ship is thus kept as low as possible by filling the bottom of the ship with ballast, thereby increasing its stability.

10·17. Two important relations for a floating body.

(*a*) Let us consider a body of volume V and density ρ, floating in a liquid of density ρ'. If a volume v of the body remains submerged in the liquid, then the volume of displaced liquid is also v. Hence, from the condition of floatation.

the weight of the body=weight of the displaced liquid

$$\text{or, } V\rho g = v\rho' g \quad \text{or, } \frac{v}{V} = \frac{\rho}{\rho'}$$

$$\therefore \frac{\text{Volume of the submerged portion of body}}{\text{Total volume of the body}} = \frac{\text{density of the body}}{\text{density of the liquid}} \quad \cdots \quad \cdots \quad (10{\cdot}14)$$

(b) If a fraction *n* of the floating body remains immersed in the liquid, then $\frac{v}{V} = n$

$$\therefore n = \frac{\rho}{\rho'} \quad \text{or, } \rho = n\rho' \quad \cdots \quad \cdots \quad (10{\cdot}15)$$

The density of the body is thus *n* times that of the liquid. If the body floats in water, then $\rho' = 1$; hence $\rho = n$. The density of the body is thus equal to the fraction of its volume remaining under water.

10·18. Illustrations of the principle of floatation.

(i) **Balloons :** A balloon filled with hydrogen rises in air for the same reason as the cork rises in water. In this case, the total weight of the balloon including that of hydrogen, is much less than the weight of air it displaces. The difference between the two represents the upward thrust under whose action the balloon rises in air.

(ii) **Ice :** The density of ice, ρ, is 0·917 gm./c.c, whereas the density of water ρ' is approximately 1 gm./c.c. Since $\rho < \rho'$, ice floats, being partly immersed in water. The fraction of the volume of ice remaining under water is $n = \rho = 0{\cdot}917 = \frac{11}{12}$ approximately. Ice thus floats in water with $\frac{11}{12}$ of its volume below the surface and $\frac{1}{12}$ above it. In polar regions, the sight of huge icebergs floating on the sea is very common. The density of sea water being about 1·08, an iceberg floats on the sea with about one-ninth of its volume lying above the level of water.

(iii) **Ships :** With a suitable choice of shape, a solid object heavier than a liquid may remain floated on the liquid. This is achieved by shaping the body in such a way, that the weight of the liquid displaced by the submerged portion of the body equals its weight. Thus, though an iron block readily sinks in water, a ship made of iron floats. The bottom of the ship is given a concave shape, so that its interior remains hollow. As the ship enters water, it displaces a large volume of water, whose weight equals the weight of the ship. The ship thus floats.

(iv) **Submarine :** A submarine contains a large number of tanks, called ballast tanks, fitted with valves or trap doors. Through these, the ballast tanks can be filled up with water. When the tanks are empty i. e., filled up with air, the submarine floats on the sea just like any other ship. When water is admitted into the ballast tanks through the trap doors, the submarine becomes slightly heavier than the displaced water and thus it sinks. To raise the submarine again to the surface, water is driven out of the ballast tanks by compressed air. These operations can be executed very swiftly.

Example 10·13. *A solid weighs 75 gm. in air and 60 gm. when immersed in a liquid of density 0·9 gm./c.c. Find the volume and density of the solid.*

Solution : Wt. of the solid in air $w_1=75$ gm.
Wt. of the solid in the liquid $w_2=60$ gm.
Density of the liquid $\rho=0{\cdot}9$ gm./c.c.

∴ Applying Archimedes' principle, we have from eqn. (10·10),

$$\text{Volume of the solid } V=\frac{w_1-w_2}{\rho}=\frac{75-60}{0{\cdot}9}=16{\cdot}67 \text{ c. c.}$$

$$\therefore \text{ Density of the solid } D=\frac{w_1}{V}=\frac{75}{16{\cdot}67}=4{\cdot}5 \text{ gm./c.c.}$$

Example 10·14. *A solid weighs 5 gm. in air, 3 gm. in water and 3·2 gm. in benzene. Determine the specific gravity of the solid and of benzene.*

Solution : wt. of solid in air $w_1=5$ gm.
wt. of solid in water $w_2=3$ gm.
wt. of solid in benzene $w_3=3{\cdot}2$ gm.
wt. of water displaced $=w_1-w_2=5-3=2$ gm.
wt. of benzene displaced $=w_1-w_3=5-3{\cdot}2=1{\cdot}8$ gm.

$$\therefore \text{ sp. gr. of solid}=\frac{\text{wt. of solid in air}}{\text{wt. of water displaced}}=\frac{5}{2}=2{\cdot}5.$$

$$\text{sp. gr. of benzene}=\frac{\text{wt. of benzene displaced}}{\text{wt. of water displaced}}=\frac{1{\cdot}8}{2}=0{\cdot}9.$$

Example 10·15. *A brass ring of weight 215 gm. is held by means of a string so as to be totally immersed in (a) water (b) olive oil. Find the tension in the string in each case. Density of brass is 8·6 gm./c.c. and of olive oil is 0·91 gm./c.c.*

Solution : (a) Sp. gr. of brass $=8{\cdot}6=\frac{215}{W}$ where W is the weight of equal volume of water. ∴ $W=25$ gm. Hence, the upthrust of water on the ring $=25$ gm. wt.

If T be the tension in the string, then $T=$weight of the ring$-$upward thrust of water $=215-25=190$ gm. wt.

(b) Sp. gr. of olive oil being 0·91, we have

$$0{\cdot}91=\frac{\text{weight of olive oil displaced by the ring}}{\text{weight of water displaced by the ring}}$$

∴ weight of olive oil displaced $=0{\cdot}91\times 25=22{\cdot}75$ gm.

∴ the upward thrust of olive oil on the ring $=22{\cdot}75$ gm. wt.

Hence, as before, the tension in the string $=215-22{\cdot}75=192{\cdot}25$ gm. wt.

Example 10·16. *A solid weighs 55 gm. in water and 70 gm. in a liquid of specific gravity 0·6. What is the mass of the solid in air and the specific gravity of the material of which it is made ?*

Solution : Let x be the weight of the solid in air, and hence from the given conditions we have,

$$\frac{x-70}{x-55}=0\cdot6, \quad \text{Solving we get } x=92\cdot5 \text{ gm.}$$

$$\therefore \text{ Sp. gr. of the solid}=\frac{92\cdot5}{92\cdot5-55}=2\cdot47.$$

Example 10·17. *An ornament weighs 11·2 gm. in air and 10·5 gm. when immersed in water. It is known that specific gravity of pure gold=19·3. (a) Is the ornament of pure gold ?*

(b) *If not, find the amount of gold in it. Assume the material of the ornament is an alloy of gold and quartz, the specific gravity of quartz being 2·6.*

Solution : (*a*) Here $w_1=11\cdot2$ and $w_2=10\cdot5$ gm.

∴ Sp. gr. of the material of the ornament

$$=\frac{w_1}{w_1-w_2}=\frac{11\cdot2}{11\cdot2-10\cdot5}=\frac{11\cdot2}{0\cdot7}=16\cdot0$$

It is less than that of pure gold. Hence the ornament is not of pure gold.

(*b*) Let mass of gold in the ornament=x gm.

∴ mass of quartz in the ornament=$(11\cdot2-x)$ gm.

Now, volume of the ornament=volume of gold+volume of quartz

$$\therefore \frac{11\cdot2}{16\cdot0}=\frac{x}{19\cdot3}+\frac{11\cdot2-x}{2\cdot6}$$

Solving, we get $x=11\cdot08$ gm.

Example 10·18. *A beaker containing water rests on a weighing machine which reads 700 gm. wt. A piece of glass is suspended from a spring balance which reads 20 gm. wt. If the specific gravity of glass is 2·5, what will be the readings of the spring balance and weighing machine when the glass piece is lowered into the water ?*

Solution : Volume of the glass piece=$\frac{20}{2\cdot5}$=8 c. c.

∴ Wt. of water equal in volume to glass piece=8 gm. wt.

∴ Upthrust on the glass piece when immersed completely in water =8 gm. wt.

Hence the spring balance now reads (20−8)=12 gm. wt.

By Newton's third law, upthrust on glass piece

=downthrust on weighing machine

∴ Extra downthurst on weighing machine=8 gm. wt.

Hence, weighing machine now reads (700+8)=708 gm. wt.

Example 10·19. *A piece of brass weighing 340 gm. floats in mercury with $\frac{3}{8}$th of its volume above the surface. Find the volume and density of brass (Density of mercury=13·6 gm./c.c.)*

Solution : Let the volume of the piece of brass be V and its density be ρ. Then, the volume of its submerged portion is $V-\frac{3}{8}V=\frac{5}{8}V$ c.c.

From eqn. (10·14), $\frac{\frac{5}{8}V}{V}=\frac{\rho}{13\cdot6}$ $\quad\therefore\ \rho=8\cdot5$ gm./c.c.

$\therefore$ Volume $V=\frac{340}{8\cdot5}=40$ c.c.

Example 10·20. *A piece of ice weighing 1·5 kg. floats in saline water. Calculate the volume of the submerged portion. Given that the density of saline water = 1·03 gm./c.c.*

Solution : Volume of the piece of ice $=\frac{1500}{\rho}$ c c., where ρ is the density of ice. From eqn. (10·14),

$$\frac{\text{Volume of submerged portion of the piece of ice}}{\text{Volume of the piece of ice}}=\frac{\text{Density of ice}}{\text{Density of saline water}}.$$

$\therefore$ Volume of the submerged portion $=\frac{\rho}{1\cdot03}\times\frac{1500}{\rho}=1456\cdot4$ c.c.

Example 10·21. *A hollow glass-stopper just sinks in water. Compare the volume of the cavity with that of the stopper. Sp. gr. of glass = 3.*

Solution : Let us suppose that the stopper possesses volume x c.c. and since it displaces x gm. of water, the upthrust of water $=x$ gm. wt. From the condition of floatation, weight of the stopper must be x gm. in order that it just sinks. Now, since the sp. gr. of glass is 3, the volume of glass $=\frac{x}{3}$ c.c.

Hence, the volume of the cavity = the difference of the volume of stopper and the volume of glass $=x-\frac{x}{3}=\frac{2}{3}x$.

$\therefore\ \frac{\text{volume of cavity}}{\text{volume of stopper}}=\frac{\frac{2}{3}x}{x}=\frac{2}{3}$; thus the volume of the cavity is $\frac{2}{3}$ of the volume of the stopper.

Example 10·22. *A wooden cube floats in water supporting an iron block of mass 1·2 kg. When the block is removed the cube rise 3 cm. Estimate the volume of the cube.*

Solution : Let each side of the cube be of x cm. length. From the condition of floatation, we have, weight of the block = weight of water displaced.

$\therefore\ x^2\times3=1200$; hence $x=20$ cm. and the volume of the cube $=8000$ c.c.

Example 10·23. *A piece of wood weighs 40 gm. in air ; when a piece of brass is fixed with it, the total weight is 52 gm. The combination floats, being wholly immersed in water. If the specific gravity of brass is 8·5, calculate the specific gravity of wood.*

Solution : The weight of the brass piece = 12 gm., and its volume $=\frac{12}{8\cdot5}$ c. c. If S = sp. gr. of wood, the total volume $=\left(\frac{12}{8\cdot5}+\frac{40}{S}\right)$ c.c.

$\therefore$ Weight of displaced water $=\left(\frac{12}{8\cdot5}+\frac{40}{S}\right)$ gm.

From the condition of floatation, $52=\frac{12}{8\cdot5}+\frac{40}{S}$; hence $S=0\cdot79$.

10·19. Air Pressure.

The earth's surface is covered with air which extends up to a height of about forty miles. This is known as the *atmosphere.* This air is really a mixture of well-known gases, about 77% nitrogen, 21% oxygen and 1% argon. The remaining portion consists of small quantities of such gases as carbon dioxide, water vapour, hydrogen, neon, krypton, helium, ozone and xenon.

Similar to the case of a liquid column exerting pressure due to its own weight, weight of air gives rise to a pressure on every object inside the atmosphere. This pressure, known as **atmospheric pressure,** is given by the weight of a column of air of unit cross-section and which extends from the point under consideration to the top of the atmosphere. Evidently, the pressure is greatest on the earth's surface and decreases as we go upwards.

10·20. Torricelli's Experiment.

About 300 years ago, the Italian physicist Torricelli first conclusively proved the existence of atmospheric pressure. He took a glass tube about one metre long, open at one end only and filled it completely with mercury, leaving no air bubble inside. Closing the open end with a finger, he inverted the tube over a dish of mercury keeping the end immersed in mercury. When the finger was removed, the mercury level in the tube dropped to a height of about 76 cm. or 30 in. from the surface of mercury in the dish [Fig. 10·23] and remained stationary. Obviously the pressure exerted by the mercury column at a point in level with the outside surface is being neutralised by the atmospheric pressure acting on the outside surface. If the tube is kept in an enclosure and the air within the enclosure is drawn out by a vacuum pump, it will be found that the height of the mercury column drops steadily. On taking it to high altitude, similar lowering of the mercury level in the tube is noticed. The experiment thus conclusively proves that *the atmospheric pressure is equal to the pressure due to a column of mercury 76 cm. or 30 in height.* The space in the tube above the mercury column contains practically nothing and is known **Torricellian vacuum.** Actually, the space contains a little mercury vapour whose pressure, however, is negligibly small.

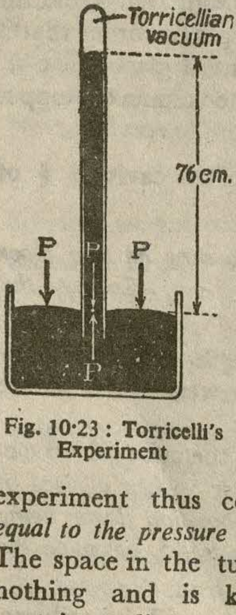


Fig. 10·23 : Torricelli's Experiment

10·21. Magnitude of the atmospheric pressure.

The pressure exerted by the mercury column is given by the usual formula ρgh, where ρ is the density of mercury, and h the vertical height of mercury column above the outside level.

Since $\rho=13\cdot6$ gm./c.c. and $h=76$ cm., we have

The atmospheric pressure $=P=76\times13\cdot6\times981$ dyne/cm^2

$=1\cdot014\times10^6$ dyne/cm^2.

In M.K.S. unit, the value is $P=1\cdot014\times10^5$ newton/m^2.

In F.P.S. unit, $h=30$ in., $\rho=13\cdot6\times62\cdot5$ lb./cu.ft.

$$\therefore\quad P=30\times\frac{13\cdot6\times62\cdot5}{12^3}\text{lb.wt./in.}^2=14\cdot75\text{ lb.wt./in}^2.$$

Sometimes the atmospheric pressure is expressed in bars and millibars.

1 bar $=10^6$ dyne/cm^2., 1 millibar $=1000$ dyne/cm.^{2}

On this basis, 1 atmosphere $=1\cdot014$ bar $=1014$ millibar.

Atmospheric pressure is usually expressed in terms of the height of the mercury column i.e., *in centimeter of mercury.*

10·22. Standard Atmospheric Pressure.

Due to air currents, varying quantities of water vapour and other irregularities, atmospheric pressure at a particular place does not remain constant. The atmospheric pressure also decreases with the height of the place. Due to variation in the value of acceleration due to gravity with latitude, the same atmospheric pressure is balanced at different places by different heights of the mercury column. Temperature is also a factor, since density of mercury decreases with increasing temperature. Hence it is necessary to define a standard value for atmospheric pressure. The **standard atmospheric pressure** *is defined to be the pressure exerted by a column of mercury 76 cm. high at 0°C. at 45° latitude and mean sea-level.*

Now, density of mercury at 0°C $=13\cdot596$ gm./c.c.

and, value of g at 45° latitude $=980\cdot6$ cm./sec.^{2}

$\therefore$ Standard atmospheric pressure $=76\times13\cdot596\times980\cdot6$

$=1\cdot013\times10^6$ dyne/cm.^{2}

$=1\cdot013$ bar.

10·23. Measurement of atmospheric pressure.

Atmospheric pressure is measured by an instrument, called the **barometer** and Torricelli's apparatus provides a simple barometer, We shall discuss only two types of barometer—(1) the **Fortin's barometer**, which is an improvement of the simple barometer and (2) the **aneroid barometer** in which no liquid is used.

(1) **Fortin's Barometer** : This essentially consists of a glass barometer tube B filled up with mercury and kept inverted over a cistern of mercury called the reservoir R [Fig. 10·24]. The bottom of the reservoir is flexible and usually made of chamois leather, L. A screw S presses against L, so that by turning the screw, the mercury level in the reservoir can be raised or lowered. A small ivory pin I is attached to the lid of the reservoir. In order that this pin and the level of mercury in the reservoir can be

observed, the upper part of the reservoir is made of glass. The barometric tube and the reservoir are enclosed in a metal tube T for safety. In the upper part of the tube, a wide vertical slit is made so that the upper level of the mercury column can be seen. Linear scales M in both centimeters and inches are engraved on the metal tube on both sides of the slit (only the centimeter scale is shown in the figure). A vernier scale V can be moved up and down in the slit by the knob P of a rack and pinion arrangement. The zero of the main scale coincides with the tip of pointer I.

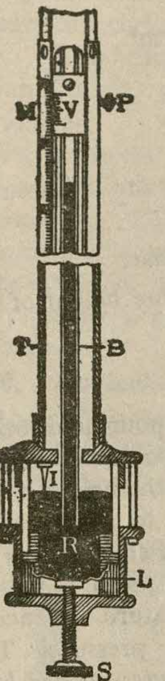


Fig. 10·24 : Fortin's Barometer

To read the barometer, the instrument is placed in a vertical position and the screw S is then turned until the level of mercury in the reservoir just touches the ivory pointer I. The vernier is then moved until its lower edge just touches the convex shaped mercury meniscus in the tube. Both the main scale and the vernier scale readings are then taken, which gives the barometric height in centimeters or inches of mercury. Temperature is recorded from a thermometer (not shown in the figure) attached to the tube T. For accurate work, temparature correction must be made to take into consideration the effects of expansion of the material of the scale and of the mercury, itself [c.f. Art 15·8].

(2) **Aneroid Barometer :** The aneroid barometer [Fig. 10·25] is of quite a different type and contains no liquid (Greek *a*, without, *neros*, liquid). It is more compact than mercury barometers, is portable and can be used in any position. It is frequently used as an altimeter and barometer combined. It consists essentially of a shallow partially evacuated metal box B with a thin, corrugated metal cover M. A stiff spring S pulls the cover upwards and prevents it from collapsing With slight decrease or increase in the atmospheric pressure, the cover moves up or down. The cover is attached to a system of levers which multiplies this movement. The end E of this lever system is connected to a small cable A which passes round a spindle L and is then connected to a spring H. A pointer R moving over a graduated scale is attached to this spindle.

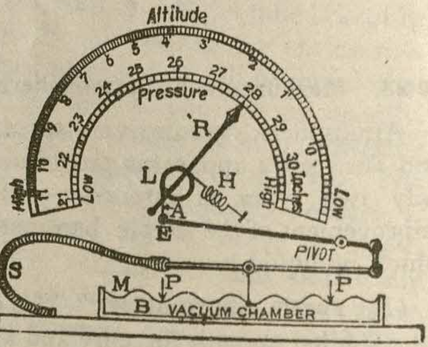


Fig. 10·25 : Aneroid Barometer

This scale must be graduated or calibrated by comparison with a standard mercury barometer.

With increase in atmospheric pressure, the cover *M* goes down resulting in a magnified upward movement of the end *E* of the lever system. The spring *H* draws the cable *A*, turning the pointer to the right. When pressure decreases, the reverse takes place.

Since atmospheric pressure decreases with altitude, a scale showing the altitude directly may be attached with the instrument. It is then called an altimeter which is used in every aeroplane.

Example 10·24. *A barometer is used to determine the height of a mountain. It was observed that the barometer reads 760 mm. at sea level and 667 mm. at the top of the mountain. Assuming the average density of air to be 1·24 gm./lt. and density of mercury to be 13·6 gm./c.c. calculate the height of the mountain.*

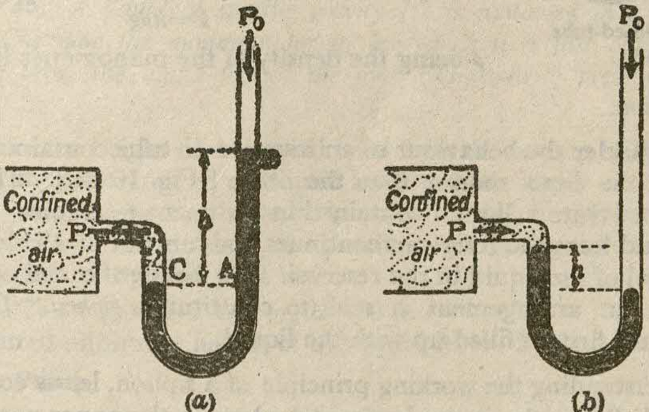
Solution : Let the height of the mountain be *H* cm. Pressure of a column of *H* cm. of air=Pressure of a column of $(76-66{\cdot}7)=9{\cdot}3$ cm. of mercury.

or, $H\times1{\cdot}24\times10^{-3}\times g=9{\cdot}3\times13{\cdot}6\times g$ $\left[\because\ 1{\cdot}24\text{ gm/lt.}=1{\cdot}24\times10^{-3}\text{ gm./c.c.}\right]$

or, $H=\dfrac{9{\cdot}3\times13{\cdot}6}{1{\cdot}24\times10^{-3}}\text{cm.}=102\times10^{3}\text{ cm.}=1020\text{ m.}$

10·24. Measurement of pressure of a confined volume of air.

Instrument used for measuring the pressure of a confined volume of air (or of any gas) is known as **manometer** or **pressure gauge**. For measurement of pressure which differs from the atomspheric pressure only by a small amount, an **open tube manometer** is used. This consists of a U shaped tube, open at both ends and containing a liquid, usually mercury [Fig. 10·26]. One end of the tube is connected to the vessel containing air, while the other end is open to the atmosphere.



(a) (b)

Fig. 10·26 : Open tube manometer

At first, let us assume that the pressure of air P in the vessel is greater than the atmospheric pressure P_0. The liquid then rises to a greater height in the open limb [Fig. 10·26 (*a*)].

We now consider a point A inside the liquid in the open limb and lying on the same horizontal level as C, the surface of the liquid in the closed limb. Then, since the pressure at all points at the same horizontal level in a liquid at rest is the same, the pressure at the points C and A must be equal. Thus,

Pressure of confined air = Atmospheric pressure + Pressure due to liquid column AB.

$$\text{or,}\quad P=P_0+\rho gh \quad \dots \quad \dots \quad (10{\cdot}16)$$

If the pressure of air in the vessel be less than the atmospheric pressure, liquid rises to a greater height in the closed limb [Fig. 10·26 (*b*)]. We then similarly get

$$P=P_0-\rho gh \quad \dots \quad \dots \quad (10{\cdot}17)$$

The manometer thus gives the pressure difference, but not the absolute pressure of the gas.

When the pressure of air in the vessel is small, a **closed tube manometer** is used. One end of the U-tube is then sealed, the space above the manometric liquid in that limb being evacuated and the other limb is connected to the vessel [Fig. 10·27]. If h be the difference in height between the liquid columns in the two limbs, the pressure of air in the vessel is then given by

$$P=h\rho g$$

ρ being the density of the manometric liquid.

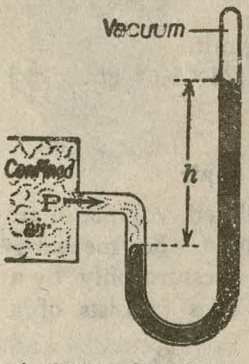


Fig. 10·27 : Closed-tube manometer

10·25. Siphon.

Let us consider the behaviour of an inverted U-tube containing a liquid and having one limb shorter than the other [Fig. 10·28]. The shorter limb is dipped into a liquid contained in an open reservoir. It is found that the liquid from the reservoir continues to run out of the longer limb until the level of the liquid in the reservoir falls below the end of the short limb. Such an arrangement is said to constitute a *siphon*. To start the siphon, it must first be filled up with the liquid.

For understanding the working principle of a siphon, let us consider two points B and C on the same horizontal plane in the upper portion of the limbs. Let the atmospheric pressure $=P$, the vertical height of the point B above the surface of the liquid in the reservoir $=h_1$, the vertical height of the point C above the open end D of the tube $=h_2$, the density of the liquid $=\rho$.

Thus, the pressure at $B=P-\rho gh_1$ and the pressure at $C=P-\rho gh_2$.

Now since $h_2>h_1$, the pressure at $B>$the pressure at C. Hence there is a resultant pressure tending to drive the liquid from the point B to C. As the liquid moves from B towards C, the vacancy created is filled up by atmospheric pressure forcing more liquid into the tube from the vessel. A continuous flow through the siphon thus takes place. From the above discussions, it is easy to formulate the following conditions for the working of siphon :

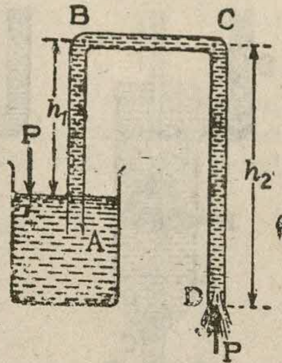


Fig. 10·28 : The siphon

(*i*) The siphon would work till the level of the liquid in the reservoir falls below the end A of the siphon arm dipped in liquid.

(*ii*) The vertical height (h_1) of the shorter limb above the liquid surface should not be greater than the barometric height for the liquid used ; otherwise the atmospheric pressure will not be able to raise the liquid to the point B.

(*iii*) The height h_2 should always be greater than the height h_1 ; otherwise the pressure at B will not be greater than the pressure at C and the siphon will cease to operate.

(*iv*) The siphon does not function in vacuum because due to absence of atmospheric pressure, the force pushing the liquid up the tube is non-existent and hence there would be no flow.

The principle of siphon is utilised in automatic flushes often fitted in latrines. It is also used in transferring liquid from one vessel to the other when it is not desirable to disturb the whole volume of the liquid or when the simple method of pouring is impossible.

Example 10·25. *A liquid of specific gravity 1·7 is contained in a tank of height 25 ft. Calculate the minimum height for which it is just possible by siphon action to bring the liquid out of the tank (atmospheric pressure = 30″ of Hg.).*

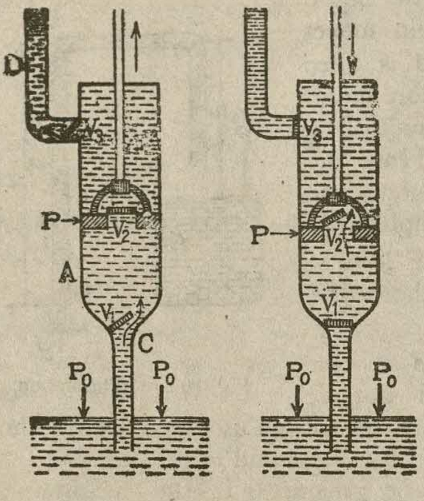
Solution : Let the height of the liquid level from the bottom of the tank be h ft. Hence, the difference in the liquid level and the upper edge of the tank $=25-h=H$ (say). The maximum value of H for which siphon action would be just possible is that when the pressure of the liquid column of height H equals the atmospheric pressure.

$\therefore\ H\times 1{\cdot}7\times g=\frac{30}{12}\times 13{\cdot}6\times g$; or, $H=20$ ft.

$\therefore$ required minimum height of the liquid level = 5 ft.

10·26. The Lift Pump.

The lift pump depends for its action on the pressure exerted by the atmosphere and can be used for raising water to any desired height. The pump consists of a vertical cylinder A, called the barrel, within which a piston P plays smoothly [Fig. 10·29]. The piston is made air-tight with a collar of soft leather or rubber strip. A pipe C is attached to the bottom of the barrel connecting it with the reservoir from which water is to be pumped out. Near the top of the barrel, a delivery tube D is joined through which water flows up out of the pump. The pump contains three valves V_1, V_2 and V_3, all of which open upwards. The valve V_1 is at the foot of the barrel, V_2 is on the piston and V_3 is at the junction of the delivery tube and the barrel.



(*a*) (*b*)

Fig. 10·29 : The lift pump

Action : The action of the pump is described in the following two steps—

(*i*) *The upstroke*—As the piston moves upwards, the air-pressure below it is reduced and so the atmospheric pressure acting from above closes the piston valve V_2. At the same time, the pressure within the barrel just below the piston being reduced, the resulting greater pressure of air in the tube C opens the valve V_1 and atmospheric pressure (P_o acting on the surface of the water in the reservoir, gradually forces the water up the pipe and through the valve V_1 into the barrel) [Fig. 10·29 (*a*)]. The valve V_3 is also opened at the same time and air (in later strokes, water) is driven through the delivery tube. This process continues until the piston reaches the top of its motion.

(*ii*) *The down stroke*—When the piston moves down, the resulting increase in pressure within the barrel below the piston closes the valve V_1, but opens the valve V_2, forcing water up through it [Fig. 10·29 (*b*)] in the space above the piston.

A few repetitions of the above two strokes result in a sufficient accumulation of water in the barrel in the space above the piston. At the next upstroke, water is forced into the delivery tube through the valve V_3. In the following down strokes, the pressure of water in the delivery tube keeps the valve V_3 closed. The pump thus produces an intermittent discharge of water ; water flows out of the delivery tube during the upstrokes only.

Limitation—Atmospheric pressure is approximately equivalent to that of a 10 m. water column. Since water is forced into the barrel by atmospheric pressure, the piston must not be above this height i. e., 10 m. from the water-level in the reservoir. In practice, it is usually arranged to be not more than 7 m. above the water level.

When it is not necessary to lift water to a large height a simple spout is connected to the barrel in place of the delivery tube and the valve V_3 is dispensed with. The pump is then called a *common pump* or *suction pump*, which is widely used in tubewells. The principle of action remains the same. Obviously, this pump can not raise water above a height of 10 m. from the water level of the source.

10·27. The Compression Pump.

This type of pump is used for compressing air into a closed container, called the *receiver*. It consists of a cylinder or barrel A, in which an air-tight piston P works [Fig. 10·30]. The piston and the end of the barrel contain valves V_1 and V_2 respectively, both of which open towards the receiver, R. There is a stopcock C provided at the mouth of the receiver, which may be closed when the desired amount of compression is attained.

Action—When the piston is moved outwards, the pressure of air in the barrel below the piston decreases ; the valve V_2 is closed by the pressure of air in the receiver and the valve V_1 is opened by the atmospheric pressure. Air thus rushes into the barrel which is therefore, filled up with air at the atmospheric pressure. This stroke is called the *suction-stroke.*

When the piston is next moved inward, the air within the barrel between the two valves gets compressed. Thus, the pressure in this region becomes higher than the atmospheric pressure. The valve V_1 is at once closed and V_2 is opened, and some air is forced into the receiver. This stroke is known as the *compression-stroke*. Repetitions of the above two strokes force more and more air into the receiver. The air inside it gets compressed and its density increases. The bicycle pump and football pump are common examples of the compression pump.

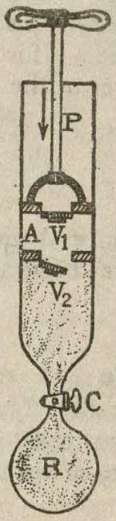


Fig. 10·30 : The compression pump

Determination of the degree of compression :

The degree of compression after the piston has executed n complete strokes (one complete stroke means one upstroke and one downstroke) is determined by the pressure or density of air in the receiver. To calculate this, let

The volume available within the receiver, the connecting tube and the part of the barrel upto the valve $V_2 = V$.

The additional volume available when the piston reaches the end of the upstroke $= v$

The initial pressure of air within the receiver $= P$

$= \text{The atmospheric pressure}$

When the piston completes the first upstroke, air at pressure P occupies both the volumes V and v. At the end of the next downstroke, this air is compressed to occupy the volume V. Let the pressure of this compressed air within the receiver be P_1. Assuming that temperature remains constant, we obtain from Boyle's law [vide Art. 16·2].

$$P_1V=P(V+v)$$

$$\therefore \quad P_1=P\left(1+\frac{v}{V}\right)$$

When the piston completes the second upstroke, air at pressure P occupies the volume v but the volume V remains filled up with air at pressure P_1. As the piston completes the subsequent downstroke, air within the volume v is forced into the receiver. So if the pressure of air within the receiver at the completion of this second complete stroke be P_2, then we have,

$$P_2V=Pv+P_1V$$

$$\therefore \quad P_2=P\frac{v}{V}+P_1=P\left(1+2\frac{v}{V}\right)$$

Hence at the end of n complete strokes, the pressure of air within the receiver becomes,

$$P_n=P\left(1+n\frac{v}{V}\right) \quad \dots \quad \dots \quad (10·18)$$

We shall see later that the density of a gas at constant temperature varies directly as its pressure (cf. Art. 16·13). Hence if D_n be the density of compressed air within the receiver after n complete strokes and D be the density of air at atmospheric condition, we have

$$D_n=D\left(1+n\frac{v}{V}\right) \quad \dots \quad \dots \quad (10·19)$$

Example 10·26. *Air is forced into a vessel by a compression pump the barrel of which is $\frac{1}{20}$ of the volume of the vessel. How many strokes will increase the pressure inside the vessel from one to three atmospheres ?*

Solution : Here, $P=1$ atmosphere.

$P_n=3$ atmosphere.

and $\frac{v}{V}=\frac{1}{20}$

We have from eqn. (10·18),

$$P_n=\left(1+n\frac{v}{V}\right)P$$

$$\text{or,} \quad 3=\left(1+\frac{n}{20}\right)\times 1 \qquad n=40$$

Hence, 40 strokes will be needed.

10·28. Vacuum Pump.

The device used to evacuate air out of a closed vessel is called a *vacuum pump* or an *exhaust pump*. Though various types of vacuum pumps are in use, we shall describe only two, viz, (i) the piston pump and (ii) the rotary pump.

(*i*) **Piston Pump.** This type of vacuum pump works on identically the same principle as the lift pump. The pump consists of a metal cylinder or barrel *B*, within which an air tight piston *P* can move freely [Fig. 10·31]. A valve V_1 opening outwards is fitted in the piston. The barrel is connected by a tube *T* to the vessel *C* containing air which is to be evacuated. Another valve V_2 also opening outwards is placed at the joint of the pipe and the barrel *B*. The two valves are made as light as possible ; generally oiled silk is used. To indicate the pressure in the vessel, a manometer may be connected to the pipe *T*.

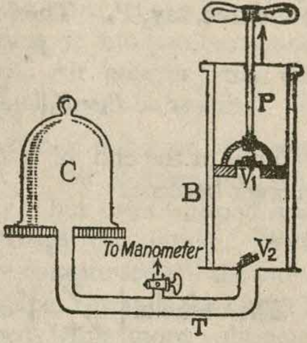


Fig. 10·31 : The piston type vacuum pump

In the upstroke i. e., on raising the piston, the pressure of air inside the barrel below the piston, gets lowered. The valve V_1 becomes closed due to the higher pressure above it. The pressure of air in the vessel *C* and the tube *T* raises up the valve V_2,.and some air enters the barrel through this valve. In the down stroke i.e., when the piston descends, the air below it is compressed and thereby its pressure rises. The valve V_2, therefore, closes and as the pressure goes on rising, it ultimately exceeds atmospheric pressure whereupon the valve V_1 opens. Some air then escapes into the atmosphere. Repetitions of the above two strokes produce continuous sucking out of air from the vessel, until the air inside it is too rarefied to lift the valve V_2. Pressure in the vessel can be reduced by this type of pump to about 1 or 2 mm. of mercury.

Obviously, the action of this pump is intermittent. To produce continuous action, two such pumps joined by a common tube to the vessel containing air, are used simultaneously. The pistons of them are so connected that while one goes down, the other moves up. This type of pump is known as the *double-barrel pump.*

Determination of the degree of exhaustion :

The degree of exhaustion is determined by the pressure or density of air remaining in the vessel *C*. To calculate this, let

The volume available within the vessel *C* and the connecting tube $T=V$.

The additional volume available when the piston reaches the end of the upstroke$=v$.

The initial pressure of air within the vessel $C=P$.

When the piston completes the first upstroke, air within the volume *V* expands and occupies the total volume $(V+v)$. Hence its pressure decreases and becomes, say, P_1. Assuming that no change in temperature takes place, we have from Boyle's law,

$$P_1\,(V+v)=PV$$

$$\therefore\ P_1=P\left(\frac{V}{V+v}\right)$$

At the end of the next down stroke, air within the volume v escapes but the air inside the vessel C and the connecting tube remains at the pressure P_1. When the second upstroke begins, this air will expand and ultimately again occupies the volume ($V+v$). Its pressure will then again decrease to a value, say, P_2. Then from Boyle's law,

$$P_2\,(V+v)=P_1 V$$

$$\therefore\quad P_2=P_1\left(\frac{V}{V+v}\right)=P\left(\frac{V}{V+v}\right)^2$$

Thus at the end of n complete strokes, the pressure of air within the receiver becomes

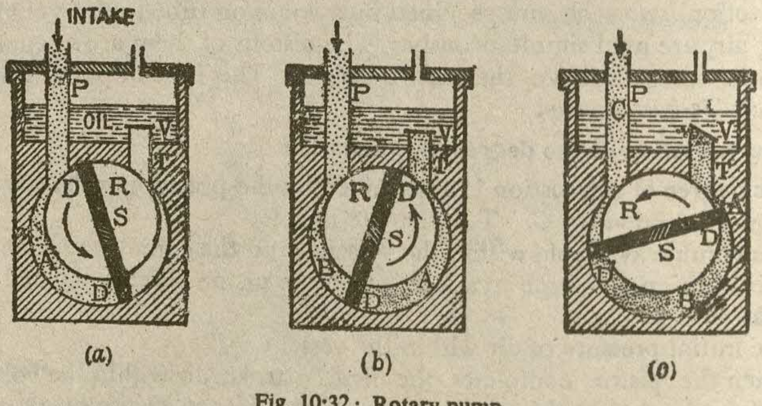
$$P_n=P\left(\frac{V}{V+v}\right)^n \qquad \ldots \qquad (10{\cdot}20)$$

The value of $\left(\frac{V}{V+v}\right)^n$ can never be equal to zero. Hence this eqn. shows that complete vacuum within the vessel C can not be achieved in practice however large the number of strokes may be.

If D_n be the density of air remaining within the vessel C after n complete strokes and D be the initial density, then since the density of a gas at constant temperature varies directly as its pressure, we have

$$D_n=D\left(\frac{V}{V+v}\right)^n \qquad \ldots \qquad \ldots \qquad (10{\cdot}21)$$

(ii) **Rotary Pump :** At present most vacuum work is done with this type of pump. In this pump, a certain volume of gas at the existing pressure in the vessel to be evacuated is trapped by the pump, compressed and then ejected to the atmosphere through an exhaust valve. A cylindrical pump barrel with its axis horizontal lies in a small tank of oil [Fig. 10·32]. From near the top of the barrel, a pipe P leads out to the vessel to be evacuated. This is known as the intake pipe. A horizontal cylinder R, called rotor, rotates about its own axis within the barrel, the two being in



(a) (b) (0)

Fig. 10·32 : Rotary pump

close contact at the top of the pump. The axis of the rotor does not coincide with that of the barrel. The rotor is driven electrically by a motor with the help of a shaft (not shown in the figure) passing through the side of the tank. The air is collected and driven out by a blade *DD'* attached to *R* and hence rotating with it. The ends of the blade are pressed against the side of the cylinder by dividing it into two parts and placing a compression spring *S* in between them. Air escapes from the barrel through the tube *T* fitted with a valve *V* which opens outwards.

The figures show the successive stages of operation of this type of pump. In the stage (*a*), as the rotor turns, some amount of air *A* flows down the intake to the barrel. In the next stage (*b*) this air *A* has been trapped and is getting compressed, while more air, *B* flows down the intake. With turning of the rotor, the air *A* gets more and more compressed till the valve *V* opens under its action and the air then escapes through the outlet *T* [stage (*c*)]. At this stage, the air *B* gets entrapped, while more volume of air, *C*, enters the barrel through the intake. As these cycles are repeated, the vessel gets quickly evacuated. The system is submerged into oil to prevent the leakage of air into the barrel. These pumps can quickly reduce the pressure to as low as a hundredth of a millimetre of mercury.

For obtaining still higher vacuum, a diffusion pump is used. With these pumps, the pressure can be reduced to an exceedingly low value of about 10^{-6} mm. of mercury. The discussion of the principle of such pumps is beyond the scope of this book.

Example 10·27. *Air is evacuated from a vessel by a vacuum pump, the barrel of which is* $\frac{1}{6}$ *of the volume of the vessel. How many strokes will decrease the pressure inside the vessel from one to* $\frac{216}{343}$ *atmosphere ?*

Solution : Here, $P=1$ atmosphere

$$P_n=\frac{216}{343} \text{ atmosphere}$$

$$\text{and } \frac{v}{V}=\frac{1}{6} \quad \text{or, } \frac{v}{V}+1=\frac{1}{6}+1 \qquad \therefore \quad \frac{V}{V+v}=\frac{6}{7}$$

Now, we have from eqn. (10·20),

$$P_n=\left(\frac{V}{V-v}\right)^n P$$

$$\text{or, } \frac{216}{343}=\left(\frac{6}{7}\right)^n\times 1 \qquad \therefore \quad n=3$$

Hence, 3 strokes will be required.

● EXERCISE ●

[A] Essay-type questions

1. Define specific gravity. How is it related to density is different systems of units ? What is the difference between 'density' and 'specific gravity' ? Show that relative density' and 'specific gravity' are equivalent.

The specific gravity of cane sugar is 1·59. What is its density in gm. per. cc. and in lb. per cu. ft ?

Why in the definition of specific gravity, water is taken at 4°C ?

2. Deduce the expression for the pressure at a point within a liquid at rest. Show that—(a) it is same in all directions at a point inside a liquid, (b) it is same at all points at the same horizontal level.

3. Distinguish between pressure and thrust as applied to a fluid. Calculate the thrust on a plane surface immersed in a liquid.

What do you mean by 'hydrostatic paradox' ? How is it explained ? Free surface of a liquid at rest always remains horizontal'—Justify the statement by enunciating the principle from which this follows as a corollary.

4. State and explain Pascal's law. How can you apply this law for obtaining multiplication of force ? Explain how does a hydraulic brake operate.

5. Describe with a neat diagram 'hydraulic press' and explain its action. Calculate its mechanical advantage and mention some of its application.

In a hydraulic press a given force is multiplied many times. Does this violate the principle of conservation of energy ?

6. What are meant by 'buoyancy' and 'apparent weight of a body' ? A cube remains immersed with its upper surface parallel to the surface of a liquid. Prove that the force of buoyancy acting on it is due to the difference in pressure exerted by the liquid at the top and the bottom surfaces of the cube. If the cube is taken to still greater depths inside the liquid, then does the force of buoyance acting on it increases, decreases or remains the same ?

'The ascent of a balloon in air illustrates the action of buoyancy.' Explain.

7. What is buoyancy ? On what factors does it depend ? How do you proceed to verify experimentally your answer ?

Prove that for a body immersed partly or fully in a liquid, the buoyancy acting on it is equal to the weight of equal volume of the liquid displaced by the body.

What is meant by 'centre of buoyancy'? Prove that it concides with the centre of gravity of the displaced liquid.

8. State Archimedes' principle. How can you demonstrate its truth in the laboratory ?

Apply the principle to determine the volume and density of a solid (a) which is irregular shaped and heavier than water, (b) which is insoluble and lighter than water.

How do you use the same principle to find out without melting whether a golden crown contains some base metal ?

9. How can you determine the amounts of different metals in an alloy by applying Archimedes' principle ?

A piece of cork is embedded inside an ice block which floats in water. What will happen to the level of water when all the ice melts ? *[I. I. T. '76]*

Does Archimedes' principle hold inside a freely falling lift ? Explain. Does this principle hold in a satellite moving in a circular orbit ? Explain.

10. What are the conditions for floatation and immersion of a body ? Prove that in case of a floating body, the fraction of volume submerged in the liquid is equal to the ratio of the density of the material of the body to that of liquid.

Why does an ordinary balloon rise to the ceiling when filled with hydrogen, but drops on the floor when filled with carbon-di-oxide ?

11. What are the conditions of equilibrium of a floating body ? Define 'meta centre'. Show how its position determine the stability of floatation of a body.

Explain why goods are loaded in the lower deck of a ship. A ship which just floats in the Bay of Bengal sinks when it enters the Ganges—Why ?

12. Discuss the action of a submarine. Sometimes a submarine which sinks down to a sea-bed of sand or clay can not raise itself again. Explain.

When a piece of nail is immersed in mercury and then released, it floats up. Will this happen in outer space ? Give reasons for your answer.

A solid wooden cylinder is placed in a container in contact with the base in such a manner that when water is poured into the container, no water can go beneath the solid. Will the cylinder float up ? Give reasons for your answer. [*H. S. '83*]

13. What is meant by 'atmospheric pressure' ? What is a standard atmospheric pressure ? Briefly explain why it is necessary to mention the temperature, latitude and height relative to sea level in this definition.

Calculate the magnitude of the atmospheric pressure in bar from the following data : barometric height$=76$ cm. of Hg., $g=981$ cm./sec^2, density of Hg$=13{\cdot}59$ gm./c.c.

14. Describe Torricelli's experiment to prove that air exerts pressure. What is Torricelli's vacuum ? Is it really a vacuum ?

In performing a Torricelli's experiment, it was suspected that some quantity of air has entered in the space above the mercury column. How do you ascertain whether it was really so ? Could the correct value of the atmospheric pressure be determined with this barometer ?

15. Describe a Fortin's Barometer and explain its working principle. Why a thermometer is always attached to it ? Does the reading of this barometer change if its tube does not remain in the vertical position ? How does the reading of a barometer change if it is within a vessel from which air is sucked out by an exhaust pump ?

16. Describe the functions of a closed tube manometer and an open tube manometer.

Show that the former gives the absolute pressure of the confined volume of air, while the latter indicates the pressure difference of the confined air with the atmospheric pressure.

What is the working principle of an aneroid barometer ?

How is it used as an altimeter ?

17. What is a 'siphon' ? Explain its action. What conditions must be fulfilled for working of a siphon ? State some of its uses.

Can a liquid be raised to a vessel placed at a higher level with the help of a siphon ? Explain why water in the bottom of a floating boat cannot be siphoned over the side. Can a siphon work in the moon ?

A small hole exists on the longer arm of a siphon. Show that whether the siphon will function or not is determined by the location of the hole.

18. Give a sectional diagram of a lift pump and explain its action. Can the pump draw from any depth ? Explain.

19. Explain the action of a compression pump. Obtain the pressure and density of air within the receiver after the end of n-complete strokes.

20. (a) Explain the construction and working principle of a vacuum pump of piston type. Mention the highest limit of vacuum that it can produce. Determine the degree of exhaustion of this pump.

(b) Describe the different parts of a rotary pump and explain its working principle. Mention the highest limit of vacuum that it can produce. To get still higher vacuum, which type of pump should be used ?

[B] Short answer type questions.

1. What are meant by real weight and apparent weight ? Which one of them is larger and why ?

2. A flat disc, a cube and a sphere, all made of iron and having the same mass, are immersed completely in water. Which one of them will experience the minimum buoyancy ?

3. Why is it necessary to make the dam of a water reservour thicker at the bottom than at the top ? [*H. S.* '82]

4. (a) 'A floating body apparently loses its whole weight'—Explain.

(b) If when floating in water you take a deep breath, will you float with more or less of your body out of water ? Explain.

5. A boy is carrying a bucket of water in one hand and a fish in the other. Is the weight carried by him less when he transfers the fish to the bucket without spilling any water during the transfer ? [*I. I. T.* '70]

6. A bucket of water is suspended from a spring balance. Does the balance reading change when a piece of iron suspended from a string is immersed in water ? If the string snaps, the iron piece will drop down to the bottom of the bucket. Will the reading of the spring balance then increase or decrease further or remain the same.

7. A wire cage in which a bird is sitting is weighed by a spring balance. Will the measured value of the weight be the same when the bird is flying inside the cage ? Give reasons for your answer. [*I. I. T.* '67]

8. A gas balloon rises upwards gaining both kinetic and potential energy. Does this imply a violation of principle of conservation of energy ? Give reasons for your answer.

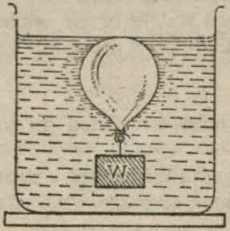
9. A glass bulb is balanced by a brass weight in a sensitive beam balance. State what will happen when the balance is covered by a ball jar which is then evacuated. Explain. [*I. I. T.* '72]

10. (a) How do you account for the facts that one kg. of cotton appears to be lighter than one kg. of lead ?

(b) Why is it easier to move a heavy stone when immersed in water ?

(c) Two identical spheres, one solid and the other hollow, are immersed completely in a liquid. Which one of them will experience greater upward thrust ?

(d) A balloon weighs same when empty as when filled with air at atmospheric pressure —Why ? Will the observation be similar if the weighings are done in vacuum ?



11. A balloon filled with air is weighted so that it barely floats in water as shown in figure. Explain, why it sinks to the bottom when it is submerged a short distance in water. [*I. I. T.* '73]

12 (i) Give reasons :

(a) It is easier to swim in sea-water than in the water of a river.

(b) A ship made of iron floats in water but a piece of nail sinks.

(c) A body may be cut into pieces easily with the sharp edge of a knife but not with its blunt edge.

(ii) Explain why a uniform wooden stick which will float horizontally, if it is not loaded will float vertically if enough weight is added to one end. [*Jt. Entrance* '72]

13. A boat floating in water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water-level ? [*I. I. T.* '79]

14. If the atmospheric pressure changes, will the rate of flow of liquid through a siphon undergo any change ? Explain your answer.

15. A beaker containing water is placed on the pan of a balance which shows a reading of M gms. A lump of sugar of mass m gms. and volume V c.c. is now suspended by thread in such a way that it is completely immersed in water without touching the beaker and without any overflow of water. What will be the reading of the balance just when the lump of sugar is immersed ? How will the reading change as time passes on ? [*I. I. T.* '78]

16. What would you conclude if you observe that—

(a) The height of the mercury column of a barometer is steadily increasing.

(b) The height of the mercury column decreased abruptly.

17. (a) A block of ice is floating in a liquid of specific gravity 1·2 contained in a beaker. When the ice melts completely, will the liquid level in the beaker change ? [*I. I. T. '74*]

(b) A piece of ice with a stone frozen in it floats on water kept in a beaker. Will the level of water increase, decrease or remain the same when the ice completely melts ? [*I. I. T. '73*]

18. Discuss the conditions under which it becomes safer for a vessel floating on water to take in more goods and load than to get rid of loads. [*Jt. Entrance '74*]

19. A hollow glass sphere, containing atmospheric air, is weighed in air by a balance. The sphere is then broken and the broken pieces are carefully collected (none being lost) and again weighed in air. Would there be any difference in the two weights ? Give reasons for your answer. [*Jt. Entrance '74*]

20. A man, sitting on a boat floating in a pool, drinks some water from the pool, so that water level of the pool gets down a little. Is it true ? Explain with reasons. [*I. I. T. '80*]

21. Two vessels are of identical volume. Both are filled with water upto the brim, but in one of them, a block of wood is floating. Can you guess which vessel has greater weight ? Justify your answer.

22. A piece of cork is floating on water in a beaker. The beaker is placed in a closed vessel and the air inside the vessel is compressed. Will the cork sink down or float up further ?

[C] Simple Problems

1. Acid in the car battery (accumulator) has specific gravity 1·285 and contains 38% H_2SO_4 by weight. Find the amount in gm. of H_2SO_4 in a litre of battery acid. [*Ans.* 488·3 gm.]

2. Ice, of mass 19·35 gm. floats in water in a beaker. Find the volume of the water formed when ice melts. By what fraction ice contracts on melting ? Is water level affected by the melting of ice ? Given, sp. gr. of ice=0·9. [*Ans.* 19·35 c.c. ; 0·1, No]

3. A glass capillary tube weighs 4·576 gm. A thread of mercury is drawn into the tube which then weighs 4·925 gm. The length of the mercury thread as measured by travelling microscope is 3·435 cm. Assuming the capillary bore to be uniform, find the diameter of the bore. Given, sp. gr. of Hg=13·6. [*Ans.* 0·9757 mm.]

4. A bottle contains 90 gm. of sulphuric acid of specific gravity 1·8. If 90 gm. of water is added to it, what should be the contraction in volume ? Given that the specific gravity of the mixture is 1·5. [*Ans.* 20 c.c. or $\frac{1}{7}$ of its original volume]

5. An alloy containing 80 parts of tin and 15 parts of antimony by weight weighs 190 gm. Find the volume of the alloy, given that specific gravities of tin and antimony are respectively 7·3 and 6·6. [*Ans.* 26·465 c.c.]

6. A bottle contains one litre milk whose mass is 1032 gm. The milk contains butter-fat to the extent of 4%by volume. What is the density of the 'fat-free' skimmed milk if the butter fat has specific gravity 0·865 ? [*Ans.* 1·04 gm./cm.^{3}]

7. A plate is coated with a thin layer of tin of thickness 10 millionths of a cm. How many square metres can be coated with 500 gm. of tin of specific gravity 7·3 ? [*Ans.* 685 m^2.]

8. The densities of three liquids are in the ratio 3 : 4 : 6. What ratio will be sustained by the densities of mixtures obtained by combining them in (a) equal volumes (b) equal weights ? [*Ans.* 4·33 : 4]

9. A vertical water pipe in a house provides a pressure gauge at the ground floor. When the gauge registers reading of 3 kg./cm.^{2}, at what height would water rise in the pipe ? [*Ans.* 30 m.]

10. At the vertical surface of a barrage, a hole of area 200 sq. cm. is found 2·5 m. below the water surface. What force is necessary to hold in position a light board used as a stop ? [*Ans.* 50 kg. wt.]

11. A cubical tank having each side 2 m. is filled with water. Calculate the force on the bottom and the force on a vertical side. [*Ans.* 8000 kg. wt. ; 4000 kg. wt.]

12. To what height should a cylindrical vessel be filled with a liquid so that the total force on the vertical surface of the vessel is equal to the force on the bottom ?
[*Ans.* height = radius of the vessel.]

13. A uniform U-tube is half full of water. Paraffin of specific gravity 0·83 is poured into one limb until the water rises by 5 cm. in the other limb. What height of paraffin is poured in ?
[*Ans.* 12·05 cm.]

14. A U-tube of which the diameter of the left limb is one-fourth of that of right limb contains some mercury. A column of water 35 cm. high is poured into the left limb. (a) How far the level of mercury in the right limb rises ? (b) How far the level of mercury in the left limb drops ? (c) By what height will the level of mercury in the left limb rise, if a column of water of same height is poured in the right limb ? Sp. gr. of mercury = 13·6.
[*Ans.* (a) 0·1515 cm. (b) 2·423 cm. (c) 2·423 cm.]

15. A vertical U-tube of uniform bore and limbs of equal length has mercury poured into it until each surface is 30 cm. from the open end. The limbs are then completely filled with water and kerosine oil respectively. Find the lengths of water and kerosine column. Given, sp. gr. of kerosine = 0·8, of mercury = 13·6.
[*Ans.* 30·236 cm. ; 29·764 cm.]

16. A bottle is completely filled with oil and corked. If the diameters of the neck and bottom of the bottle be one-half inch and 3 inch respectively, calculate the thrust on the bottom when the cork is pressed with a force of 5 lb.wt.
[*Ans.* 180 lb. wt.]

17. The diameter of the smaller piston of a hydraulic press is 1 inch and that of the larger one is 1 foot. Calculate the thrust developed on the larger piston when a force of 56 lb. is applied on the smaller one. The cross-sections of the pistons are circular.
[*Ans.* 8064 lb.] (*H. S. 71 comp*)

18. A solid sphere weighs 66 gm. in air, 41 gm. in water and 44 gm. in turpentine. Find (a) the volume of the sphere, (b) the specific gravity of its material and (c) specific gravity of turpentine.
[*Ans.* (a) 25 c.c. (b) 2·64 (c) 0·88]

19. A stone weighs 400 gm. in air and appears to weigh 300 gm. in water. What will it appear to weigh in (a) paraffin (sp. gr. 0·8) and (b) brine (sp. gr. 1·1) ?
[*Ans.* (a) 320 gm. (b) 290 gm.]

20. A man can support 77 kg. of iron in water. How many kg. of (a) iron, (b) lead can he support in air and (c) also of the latter in water ? Given, sp. gr. of iron 7·7, of lead = 11·4.
[*Ans.* (a) 67 kg., (b) 67 kg., (c) 73·45 kg.]

21. An ornament is suspected to be hollow. It weighs 288·75 gm. in air and 258·75 gm. in water. If the specific gravity of the material of the ornament be 10·5, calculate the volume of the cavity of the ornament.
[*H. S.* '69 (*Ans.* 2·5 c.c.)]

22. A piece of sugar weighing 48 gm. is coated with 7·60 gm. of wax of specific gravity 0·95. If the coated piece weighs 17·60 gm. in water, find the specific gravity of sugar.
[*Ans.* 1 6]

23. A block of wood weighs 22·4 gm. in air. A piece of aluminium weighs 10·3 gm. in water. The wood piece is attached to the aluminium piece and both together weighs 4 7 gm. in water. What is the sp. gr. of wood ?
[*Ans.* 0·8]

24. A specific gravity bottle completely filled with water, mercury and with copper sulphate solution weighs respectively 45 gm, 297 gm. and 49 gm. Calculate the density of the solution, that of mercury being 13·6 gm /c. c.
(*H. S. 1960*) [*Ans.* 1·2 gm./c.c]

25. A piece of cork weighing 19 gm. is attached to a bar of silver whose weight is 63 gm. and the combination is found to just float in water. Find the specific gravity of cork, assuming that of silver to be 10·5. Calculate also the weight of the combination in a liquid of specific gravity 0·8.
(*H. S* '67) [*Ans.* 0·25 ; 16·4 gm.]

26. A block floats in a liquid with 0 2 of its volume exposed and in water with 0·1 of its volume exposed. Find the density of the liquid.
[*Ans.* 1·125 gm./c.c.]

27. A buoy of volume 1000 litre and weighing 950 kg. is fully immersed in sea-water of sp. gr. 1 02 being anchored to the sea-bottom by a chain. What is the tension in the chain ? (Ignore the weight of the chain)
[*H.S.* '63 (*comp*)] [*Ans.* 70 kg. wt.]

28. The material of a balloon weighs 100 gm. It is filled with hydrogen at atmospheric pressure. If the volume of the balloon is 100 litre, calculate the tension in the cord with which the balloon is attached to the ground. Given, density of air at atmospheric pressure= 1·29 gm /lt., density of hydrogen at atmospheric pressure=0·09 gm./lt. [*Ans.* 20 gm. wt.]

29. A gold ring fitted with a stone weighs 5 gm. in air and 4·25 gm. in water. The specific gravities of gold and stone are 19 3 and 2·5 respectively. Calculate the amount of gold in the ring. [*Ans.* 3·59 gm.]

30. A piece of wood (sp. gr. 0·8) and a piece of metal (sp. gr. 9) are bound together. If the combination neither floats nor sinks in a liquid (sp. gr. 1·2), calculate the ratio of the masses of the metal and wood. [*Ans.* 26 : 15]

31. A cork of density 0·8 gm./c. c. is taken under water and then released. Show that it will come up with an acceleration $g/4$. [*H. S.* (*Comp*) *'65*]

32. The external and internal diameters of a hollow spherical ball are 4 cm. and 3 cm. respectively. It is found just to float in water. Find the density of the material of the ball. [*Ans.* 1·73 gm./c.c.]

33. Calculate the area of the smallest block of ice. 2 feet thick, floating in fresh water that will just support a man weighing 150 lb. (sp. gr. of ice=0·9). [*Ans.* 12 sq. ft.]

34. Lead shot is added to a uniform test tube until it floats upright in a liquid of density 1·1 gm./c.c. If the tube has cross-section 2 sq. cm. and it sinks to a depth of 11 cm., what is the weight of the tube and the shot ? To what depth would it sink in a liquid of density 0·8 gm./c.c. [*Ans.* 24·2 gm. ; 15·125 cm.]

35. A tank containing oil of specific gravity 0·8 rests on a weighing machine and weighs 5 kg. A cube of aluminium of side 5 cm. and specific gravity 2·7, is supported by a spring balance and completely immersed in the oil. Find (a) the reading of the spring balance and (a) the reading of the weighing machine. [*Ans.* (a) 237·5 gm. (b) 5·1 kg.]

36. A barometer reads 760 mm. at the ground floor of a tall building 102 m. high. What will it read at the top of the building ? (Take average density of air to be 1·24 gm./lt. and density of mercury to be 13·6 gm./c.c.) [*Ans.* 750·7 mm.]

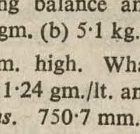
37. It is required to siphon gasoline (sp. gr.=0·7) over an obstacle. What must be the limiting height of the obstacle which will render siphoning just possible ? (Atmospheric pressure=76 cm. of mercury). [*Ans.* 14·77 m.]

38. A lift pump is used to pump oil of sp. gr. 0·6 from a lower into an upper tank. What is the maximum possible height of the pump above the lower tank when atmospheric pressure is 30 inches of mercury ? [*Ans.* 56·67 ft.]

39. The receiver of a compression pump has volume 25 times of that of its barrel. How many strokes will be needed to increase the pressure inside the receiver from 1 to 5 atmospheric pressure ? [*Ans.* 100]

40. If the volume of the barrel of a vacuum pump is one-third of the volume of the vessel to be exhausted, what fraction of the initial pressure will remain in the vessel after 5 strokes ? [*Ans.* 0·237]

41. A verlical U tube of uniform inner cross-section contains mercury in both its arms. A glycerine (density 1·3 gm./c.c.) column of length 10 cm. is introduced into one of the arms. Oil of density 0·8 gm./c.c is poured into the other arm until the upper surfaces of oil and glycerine are in the same horizontal level as shown in the figure. Find the length of the oil column. Density of mercury is 13·6 gm./c.c. (*I. I. T.* '72) [*Ans.* 9·61 cm.]



42. A piston of cross-sectional area 100 cm^2 is used in a hydraulic press to exert a force of 10^7 dyne on water. What is the cross-sectional area of the other piston which supports a truck having a mass 2000 kg ? [*I. I. T.* *'72* (*Ans.* 1·96 m^2)]

43. A balloon is descending with a uniform acceleration a ($<g$). Weight of the balloon with the busket and its contents is W. What weight ω of the ballast should be released so that the balloon will begin to rise with the same uniform acceleration a ? Neglect air resistance.

$$\left[\textit{Ans.}\ \frac{2Wa}{g+a} \right]$$

44. A large block of ice 5 metre thick has a vertical hole drilled through and is floating in the middle of a lake. What is the minimum length of a rope required to scoop up a bucket full of water through the hole. Density of ice=0·9 gm./c.c. [*I. I. T.* '75 (*Ans.* 50 cm.)]

45. A balloon filled with hydrogen has a volume of 1000 litres and its mass is 1 kg. What would be the volume of a block of a very light material which it can just lift ? One litre of this material has a mass of 91·3 gm. (Density of air=1·3 gm. per litre) (*I. I. T.* '75)
[*Ans.* 3⅓ lt.]

46. Two solid uniform spheres each of radius 4 cm. are connected by a light inextensible string and totally immersed in tank of water. If the specific gravities of the spheres are 0·4 and 1·8, find the tension in the string and the normal force between the bottom of the tank and the heavier sphere. [*Ans.* 1·576 N. ; 0·525 N]

47. A cubical block of wood 10 cm. along each side floats at the interface between an oil and water with its lower surface 2 cm. below the interface. The heights of the oil and water column are 10 cm. each. The density of the oil is 0·8 gm./c.c. (i) What is the mass of the block ? (ii) What is the pressure at the lower surface of the block ? (*I. I. T.* '77)
[*Ans.* (i) 840 gm. (ii) 10 cm. of water+Atmospheric pressure]

48. A cylindrical vessel is filled with equal amounts by weight of mercury and water. The overall height of the two layers is 29·2 cm. Determine the pressure of the liquid at the bottom of the vessel. The specific gravity of mercury is 13·6. [*Ans.* 4 cm. of Hg]

49. Calculate the percent error arising from the negligence of the buoyancy of air in weighing an object of density 12 gm./c.c. with brass weights of density 8 gm./c.c. by a physical balance. (Density of air during the experiment=$1{\cdot}2 \times 10^{-3}$ gm./c.c.) (*Jt. Entrance '81*)
[*Ans.* 0·005%]

[D] Harder Problems

1. The air inside a cement factory contains $2{\cdot}5 \times 10^{8}$ dust particles (sp. gr. 3·0) per m.³ of air. Assuming the particles to be spherical of diameter 2 micron., calculate the mass of dust (a) in 100 m.³ of factory air (b) inhaled in each average breath of 500 c. c. (one micron $=10^{-4}$ cm.) [*Ans.* (a) 0·314 gm. (b) $1{\cdot}57 \times 10^{-6}$ gm.]

2. A cube of side 10 cm. weighing 880 gm. floats with its base horizontal in brine (sp. gr. 1·1). Find the thrust on each surface of the cube.
[*Ans.* On base 880 gm. wt. ; on each vertical face 352 gm. wt.]

3. A piece of metal of mass 300 gm. and sp. gr. 7·5 is suspended by a thin string and completely immersed in a liquid of sp. gr. 1·2. (a) Determine the tension in the string. (b) If the string snaps under a load of 264 gm. wt. and the metal piece is pulled out of the liquid by means of it, calculate the fraction of the piece immersed when the string snaps.
[*Ans.* (a) 252 gm. wt. (b) 0·75]

4. A cubical tank with side 40 cm. is closed at the top. At 25 cm. from the base, a vertical pipe is joined to one side of the tank. The pipe contains water upto a length of 55 cm. If the cross-section of the tube is 100 cm², calculate the thrust on each side of the tank including the top. [*Ans.* 627·2 newton on top ; 1254·4 newton on bottom ; 940·8 newton on each vertical side without pipe ; 886·9 newton on vertical side with pipe.]

5. A hollow right circular cone of height h and semi angle θ rests with its base on a horizantal table. If the cone is filled with a liquid of density ρ and the weight of the empty cone is equal to the weight of the liquid it contains, find the thrust of the liquid on the base of the cone and the pressure exerted on the table. [*Ans.* $\pi \rho h^3 g \tan^2\theta : \frac{4}{3} h\rho g$]

6. Blocks of iron and lead each of mass 300 gm. are suspended respectively from two pans of a common balance. Arrangements are made so that both of them hang completely immersed in water. What weight must be added and in which pan to restore equilibrium ? Sq. gr. of Iron=7·9, of lead=11·4. [*Ans.* 11·66 gm. should be placed over iron pan.]

7. A tank containing water rests on a weighing machine which reads 40 kg. Determine its reading when a solid rod of diameter 6 cm. is held vertically in the water to a depth of 50 cm. [*Ans.* 41·413 kg.]

8. When an egg weighing 55 gm. and having volume 50 c.c. is dropped into a vessel containing 500 gm. of water, it sinks slowly to the bottom. How many gm. of common salt must be dissolved in the water to make the egg just float totally immersed ? [*Ans.* 50. gm.]

9. A U-tube, whose ends are open, whose section is one square inch, and whose vertical branches rise to a height of 33 inches, contains mercury in both branches upto a height of 6·8 inches. Find the greatest amount of water that can be poured into one of the branches, assuming the sp. gr. of mercury to be 13·6. (*London Univ.*) [*Ans.* 27·2 cu. in.]

10. A U-tube in which the cross-sectional area of the left limb is one-third of that of right limb contains some mercury. The level of mercury in left limb is 40 cm. below the upper end of the limb. By what height the level of mercury will rise in the right limb if the left limb is filled to the top with water ? (sp. gr. of mercury=13·6) [*Ans.* 7·78 mm.]

11. A piece of ice weighing 55 gm. floats in water in a cylindrical vessel having a cross-section of 25 cm^2. What happens to the level of water when the ice has melted ? What will be the difference in the result if the ice-piece is floating in brine instead of water ? Given, density of brine=1·1 gm./c.c. [*Ans.* No change ; rise in level by 0·2 cm.]

12. A solid floats on the boundary between two non-miscible liquids. The density of the upper liquid is ρ_1 ; that of the lower is ρ_2. The density of the material of the solid is $\rho(\rho_1<\rho<\rho_2)$. What fractions of the volume of the solid will be above and below the boundary of the liquids ?

$$\left[\ \textit{Ans.}\ \left(\frac{\rho_2-\rho}{\rho_2-\rho_1}\ ;\ \frac{\rho-\rho_1}{\rho_2-\rho_1}\right)\right]$$

13. A stone of density 2·5 gm./c.c completely immersed in sea water is allowed to sink from rest. Calculate the depth to which the stone would sink in two seconds. Neglect the effect of friction. (Specific gravity of sea water is 1·025. Acceleration due to gravity= 980 cm./sec.^{2}) (*I. I. T. '69*) [*Ans.* 11·56 m.]

14. A cubical block of steel (density 7·8 gm./cc.) floats on mercury (density 13·6 gm./c.c.) with its sides vertical. Assuming the side of the cube to be 10 cm. (a) what length of the block is above the mercury surface ? (b) If water is poured on the mercury surface, what will be the height of the water column when the water surface just covers the top of the steel block ?

(*I. I. T. '71*) [*Ans.* (a) 4·26 cm. (b) 4·6 cm.]

15. A body of density d is dropped from rest inside a liquid column of depth H. If the density of the liquid be ρ, then prove that the time taken by the body to reach the bottom of the liquid column is given by

$$t=\sqrt{\frac{2Hd}{g(\ \ -\rho)}}$$

16. A body when immersed in a liquid floats with $^1/_4$ of its volume above the liquid surface. This body is taken to depth of d cm. of the liquid and released. Show that the time of reaching the surface is $\sqrt{\frac{6d}{g}}$.

17. When equal volumes of two substances are mixed together, the sp. gr. of the mixture is 4·84. But when equal weights of the same substances are mixed together, the sp. gr. of the mixture is 2·28. Find the sp. gravities of the two substances. [*Ans.* 8·36 ; 1·32]

18. The weight of a body in air is 40 gm. The density of the body is 0·76 gm./c.c. that of the counter-poising weight is 8·4 gm./c.c. and that of air is 1·293 gm./litre. Find the true weight of the body. [*Ans.* 40·062 gm.]

19. Three ingots of equal weights consist of gold, silver, and a gold-silver alloy respectively. If the gold loses weight 14 gm. when immersed in water, the silver 26 gm., and the alloy 18 gm., what proportion by weight of the alloy consists of gold ? What is the density of the alloy ? (Density of silver=10·5 gm. per c. c.) [*Ans.* 2/3 ; 15·17 gm./c.c.]

20. A solid cubical block of side a, made of uniform material of specific gravity s (<1), is held just above the water surface in a large vessel, the lower horizontal face of the cube just touching the water surface. Prove that when the cube sinks down so that a depth x is immers-

ed, the P. E. of water and the block increases by $W\ (x^2-2xas)/2as$, where W is the weight of of the block. Show that the equilibrium position of the block when it is floating freely corresponds to a minimum value of this P. E.

21. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ. The height of the liquid in one vessel is h_1 and that in the other is h_2. The area of either base is A. What is the work done by gravity in equalising the levels when the two vessels are connected ? (*I. I. T,* '*81*)

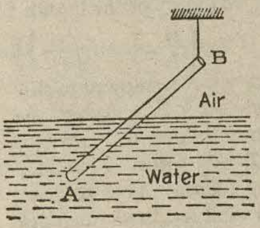
$$\left[Ans.\ \frac{A\rho g}{4}(h_1-h_2)^2 \right]$$

22. A straight plank of wood of uniform cross-section floats on the surface of water. Its one end is then raised above the surface by means of string attached to that end. If the specific gravity of wood is 0·80 find (i) what fraction of the length of the plank is immersed in that inclined position and (ii) the tension of the rope in terms of the weight of the plank.

[*Ans.* (i) 0·553 ; (ii) 0·31 × weight of the plank]

23. A rod of length 6 metres has a mass 12 kg. It is hinged at one end at a distance of 3 metres below a water surface. (i) What weight must be attached to the other end of the rod so that 5 metres of the rod are submerged ? (ii) Find the magnitude and direction of the force exerted by the hinge on the rod. (The specific gravity of the material of the rod is 0·5.)

(*I. I. T.* '*76*) [*Ans.* (i) 2·33 kg ; (ii) 5·67 kg. wt. acting vertically downwards through the hinge]

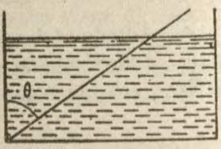


24. A uniform rod *AB* 4 m long, weighing 240 kg. is supported at the end *B* by light inextensible rope and weighted at the end *A* with a 120 kg lead weight. The rod floats as shown in the figure with one half its length submerged. The buoyant force on the lead weight may be neglected and the density of water may be taken as 1000 kg./m³. Find (a) the tension in the cord and (b) the total volume of the rod.

[*Ans.* (a) 40 kg. wt. (b) 0·64 m³]

25. A wooden plank of length 1 m. and uniform cross-section is hinged at one end to the bottom of a tank as shown in the figure. The tank is filled with water upto a height 0·5 m. The specific gravity of the plank is 0·5. Find the angle θ that the plank makes with the vertical in the equilibrium position. [Exclude the case $\theta=0°$]

(*I. I. T.* '*84*) [*Ans.* 45°]



26. Two cylinders of same cross-section and length l but made of two materials of densities ρ_1 and ρ_2 are cemented together to form a cylinder of length $2l$. If the combination floats in water with a length $l/2$ above the surface of water, show that if $\rho_1>\rho_2$ and the density of water be unity, then $\rho_1>\frac{1}{2}$. Show also that the equilibrium is unstable if the heavier part of the cylinder is at the top. [*Jt. Entrance* '*82*]

27. A bucket containing water is suspended by a rope which passes over a smooth pulley, the polley being sufficiently small, so that the other end of the rope to which is attached a ball hangs inside the bucket fully immersed in water. If W be the weight of the bucket plus the water contained in it and $s(>2)$ be the specific gravity of the ball, show that the weight of ball is $\frac{Ws}{s-2}$.

11 CHAPTER

SURFACE TENSION

11·1. Surface Tension.

If a glass rod is dipped into water and withdrawn, some of the water adheres to the rod and may hang from its end in the form of a drop. A question may arise in the mind of inquisitive students that since the drop is in equilibrium, what forces are balancing the downward weight of the drop ? Obviously, some other forces must be acting on the drop. These latter forces are traced to be residing on the surface of the drop and is attributed to a *special property* of liquids, known as **surface tension.**

A few simple but interesting experiments will help us to clearly grasp the nature of these forces :

Experiment 1. A soap bubble is blown at the end of a tube. If the bubble is then left to itself, it shrinks and air is driven out of the open end. This shows that the surface of the bubble behaves like a stretched membrane with a tendency to contract.

Experiment 2. A wire ring with a piece of thread tied loosely across it, is dipped into a soap solution. A film will cover the whole ring and the thread will remain loose but dipped in the film [Fig. 11·1(*a*)]. If now the film is broken on any side, the thread becomes taut and takes the form of an arc [Fig. 11·(*b*)].

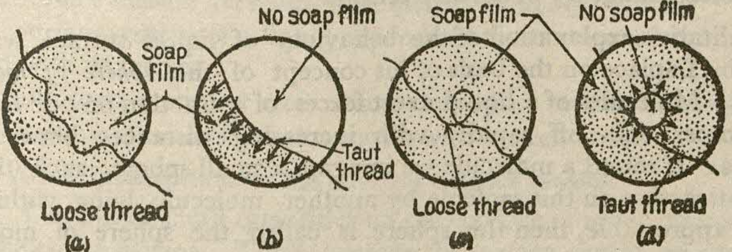


Fig. 11·1

This observation clearly suggests that soap film is exerting a tension on the thread. When soap films existed on both the sides of the string, equal and opposite tension acting on it from both sides cancelled each other ; as a result, the string remained loose.

Absence of soap film on one side of the thread, led to an unbalanced tension along the surface, acting perpendicularly at every point of the thread, as shown by arrows in the Fig. [11·1(*b*)]. The thread thus no longer remains loose and takes the shape of an arc.

Now, if we repeat the experiment with a loop in the thread [Fig. 11·1(*c*)] and break the film inside the loop, the thread spreads out in a circular shape [Fig. 11·1(*d*)]. The surfaces of the remaining soap film exert tensions which act radially outward on it.

These observations indicate that the surface of a liquid behaves at all times like a stretched membrane and the membrane is under tension. As a result, liquid surfaces always try to contract so as to possess smallest area. Due to this contractile property of a liquid surface, droplets of any liquid and soap bubbles in free state assume a spherical shape, because for any particular volume of matter, a sphere has the smallest surface area than any other geometrical figure. We thus conclude that if we imagine a line on the surface of a liquid, then due to its tendency to contract, the surface on either side of the line exerts a force on it [Fig. 11·2]. These forces lie in the plane of the surface and act perpendicular to the line. If the surface is plane and extends on both sides of the line, the force on the line in one direction is just equal in magnitude to that in the opposite direction. *This force per unit length of a line imagined on the surface of a liquid is called the* **surface tension** *of that liquid.* It is usually denoted by *T*. Thus if *F* be the force on any side of this imaginary line *AB* of length *L*,

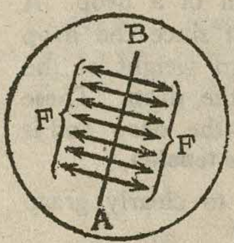


Fig. 11·2

$$T=F/L$$

Obviously, the unit of surface tension is dyne/cm.

11·2. Molecular theory of surface tension.

A qualitative explanation of the behaviour of liquid surface was put forward by Laplace on the basis of his concept of the *sphere of molecular attraction*. Molecules of a liquid exert forces of attraction on one another. These forces fall off rapidly with increasing distances between the molecules. If round a molecule we imagine a small sphere, such that the force of attraction on this molcule by another molecule lying within this sphere is appreciable, then this sphere is called the sphere of molecular attraction with respect to the molecule under consideration. According to Laplace's concept, each molecule attracts (and is also simultaneously attracted by) other molecules surrounding it when they lie within the sphere of attraction of the molecule. At a distance greater than the radius of this sphere, the molecular attraction practically vanishes. This *attraction between molecules of same kind is known as* **cohesion**.

A molecule, situated at a depth greater than the radius of the sphere of attraction, will be attracted equally in all directions and excepting its own weight, will have no net force acting on it [Fig. 11·3]. A molecule at the surface, on the other hand, has no other molecule of the liquid above it and has, therefore, no pull in the upward direction. A molecule also at a depth less than the radius of the sphere of attraction, has its upward pull less than the downward pull. Thus, there is a resulting downward pull on all molecules near the surface. Presence of this force causes the surface of the liquid to remain in a state of tension leading to the tendency to contract. This explains the phenomenon of surface tension.

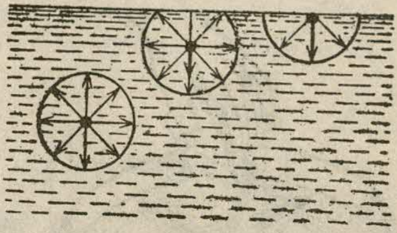
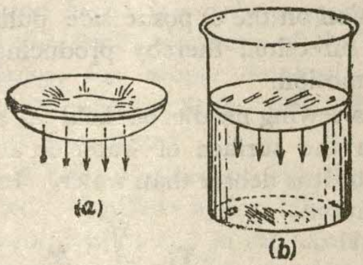


Fig. 11·3 : Origin of surface tension

Comparison with a simple example will clarify the concept. Let us now consider a horizontal rubber membrane fixed along its perimeter. If we now suppose that there is a downward uniform force on all points on the membrane, the membrane becomes stretched as shown in Fig. 11·4(*a*). The membrane is in a state of tension and evidently tends to contract. The free surface of a liquid, in a similar way [Fig. 11·4(*b*)], tends to contract. This is the ultimate cause of surface tension.


Stretched rubber membrane

Free surface of a liquid

Fig. 11·4

11·3. Dependence of surface tension on different factors.

Surface tension of a liquid is found to be influenced by the following factors :

1. Surface tension decreases with increase in temperature.
2. Surface tension of a liquid depends of the nature of the media in contact with its surface.
3. Oil, soap, camphor etc. reduces the surface tension of a liquid.
4. Surface tension of solution varies with concentration.

11·4. A few illustrations of the effect of surface tension.

(1) Oil, put in small quantities on the surface of water, spreads readily over the entire surface. This is because the surface tension of the portion of the surface on which the oil is dropped gets decreased. Greater surface tension of water in the remaining portion stretches the oil in all directions.

(2) Some match sticks are arranged likes spokes on a bowl of water. Now, if a soap (or a piece of camphor) is dipped at the centre, the sticks will move outwards [Fig. 11·5]. The soap going into solution causes a thin soap-film to spread across the surface arouud the centre. This reduces the surface tension and the sticks are pulled outwards by the larger surface

tension in the outer portion. A similar experiment can be performed if a piece of camphor, or soap, is attached to the back of a small plastic boat, and the assembly is put in clean water, so that the camphor, or soap, touches the surface of water. The boat will move forward.

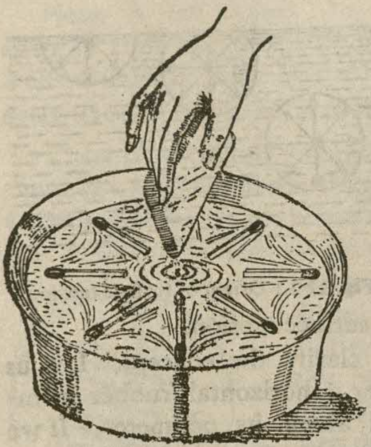


Fig. 11·5

If small camphor flakes are put on water surface, each of them is found to turn promptly and move rapidly in different directions on the surface of water. The camphor going into solution most rapidly at the pointed end of each flake, reduces surface tension more at that point than at any other. The greater surface tension on the opposite side pulls it in that direction, thereby producing the zigzag motion.

(3) If a sewing needle be held in a horizontal position and gently lowered on the surface of water in an open vessel, it will be found to float, though it is denser than water. Its weight (W) is then supported by the surface tension (T) of water, as shown in the Fig. 11·6. Due to the same reason, some light insects can actually walk on the surface of water.

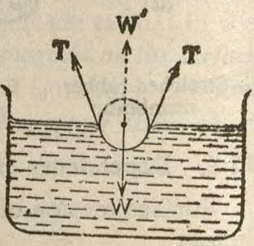


Fig. 11·6 : A floating needle

(4) Surface tension produces an interesting effect when the surface of a liquid comes in contact with the wall of its container. Water molecules are more strongly attracted by the molecules of glass than by other water molecules. The attraction between molecules of different kinds is called *adhesion*. The cohesive force of water molecules is less than the adhesive force between water and glass molecules. Due to this reason, water surface curves sharply upward where it touches a vertical glass surface [Fig. 11·7(*a*)]. The angle inside the liquid which the liquid surface makes with the solid at the point of contact is known as the angle of contact (θ). In this case, the angle of contact is obviously acute. Similar observations can be made with different liquids which wet glass, as for example, alcohol, ether, etc.

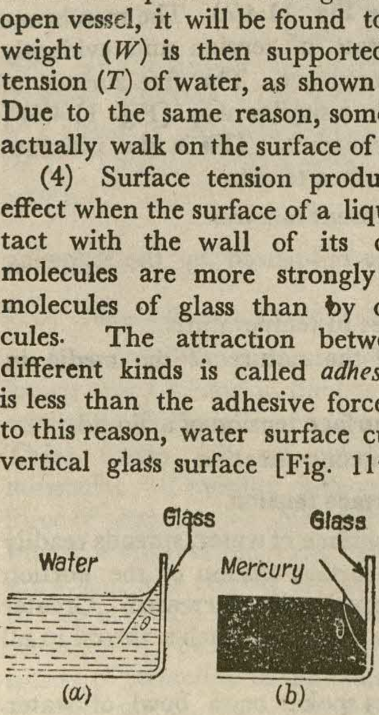
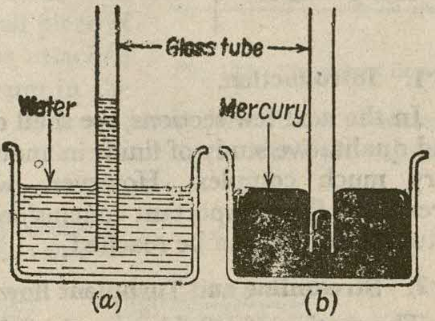


Fig. 11·7

On the other hand, with mercury which does not wet glass, the attraction between the glass and mercury molecules is less than that between mercury molcules themselves. In that case, the mercury surface cuts sharply downwards where it touches the vertical glass surface [Fig. 11·7(*b*)] ; the angle of contact is obtuse.

(5) If one end of a narrow glass tube is dipped into water, or in any liquid which wets glass, surface tension pulls the surface of water up into the tube above the level outside [Fig· 11·8(*a*)]. In upright uniform tubes, the action stops when the weight of water lifted is equal to the pull caused by surface tension. However, if the liquid e.g. mercury does not wet the tube material e.g. glass, it descends in the tube [Fig. 11·8(*b*)]. This elevation, or depression of liquids in capillary (hair-like) tubes is known as *capillarity*. The effect is found to be inversely proportional to the inside diameter of the tube. Hence, for proper demonstration, the diameter of the tube should be 1 mm. or less.



(*a*) (*b*)

Fig. 11·8 : Capillarity

A large number of effects which are connected with capillarity can be cited. Capillary action takes place in the use of wicks in oil lamps and patrol lighters and in the absorption of liquids by blotting paper or in the use of towel in drying our hands ; the liquid rises through the narrow channels between the fibres. Capillary action is one of the causes of the rise of water up the woody tissues of plants.

● EXERCISE ●

Questions.

1. Explain the term 'surface tension' and describe two experiments to illustrate it. Can gases or solids have the property ?

2. What is molecular theory of surface tension ? On what factors does surface tension depend ?

3. What are the units of surface tension ? What is capillarity ? State an example of capillary action. How a sewing neddle can be made to float on water ? Explain its underlying principle.

4. Explain the phenomena :

(a) Small insects walk on the water surface. (b) Oil put in small quantities on the surface of water spreads rapidly. (c) Small piece of camphor in water describes a zigzag motion. (d) Blotting paper readily absorbs ink. (e) Mercury surface in a capillary tube cuts sharply downwards where it touches the vertical glass surface.

12

CHAPTER

VISCOSITY

12·1. Introduction.

In the next few sections, we shall devote ourselves to a very elementary and qualitative study of fluids in motion. The subject in its actuality is very much complex. However, with the simplifying assumption of streamline flow, important conclusions, which approximate closely to the actual situation, can be reached.

12·2. Streamline and Turbulent flow.

The motion of a fluid may be of two types, viz., (i) streamline and (ii) turbulent flow.

In **streamline** or **orderly** flow, the path of any particle of the fluid is always directed along the line of motion of the fluid as a whole ; each particle in the fluid then travels in exactly the same path, called a streamline, as the particle preceding it. A streamline may be either straight or curved. In such a motion, the velocity of the fluid at a particular point remains constant in both magnitude and direction. It may, however, vary from point to point.

Fig. 12·1 represents a fluid in streamline motion, the lines showing the path followed by the particles of the fluid. Obviously, these represent the streamlines. Let us consider any particular streamiline AB and let at any instant of time, the velocities of the particles at the points a, b and c on the streamline be given by v_1, v_2, and v_3, respectively. Then as the fluid moves it will be found that at any later instant, the velocity of a fluid particle is still v_1 at a, v_2 at b and so no.

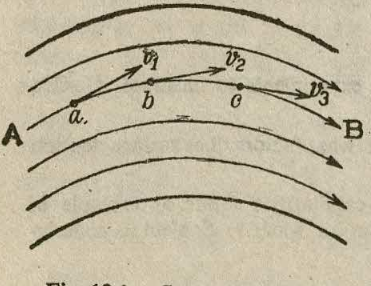


Fig. 12·1 : Streamline flow

When the paths of the particles of a moving fluid are completely irregular and random, the flow of the fluid is said to be **turbulent** *or* **disorderly** [Fig. 12·2]. Obviously, in such motion, the paths of fluid particles do not coincide, in general, with the line of motion of the fluid as a whole. The particles travel in zigzag paths, forming eddies and vortices.

Prof. Osborne Reynolds studied these two types of fluid motion in detail. He found that at relatively low velocities, fluid motion is streamlined. On increasing the velocity, when it exceeds a certain particular value, called the **critical velocity** of the fluid, the flow becomes turbulent. This critical velocity depends on the nature of the fluid and on temperature. For flow through a tube, it also depends on the radius of the tube.

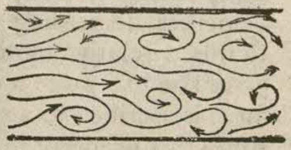
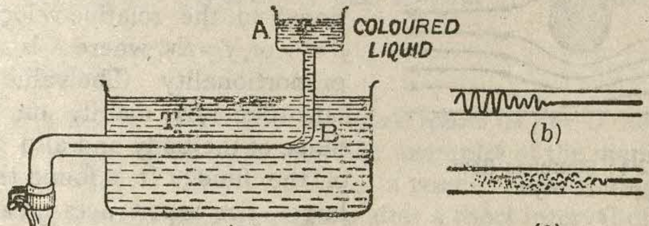


Fig. 12·2 : Turbulent flow

Demonstration Experiment.

One end of a glass tube *T* is introduced in a glass vessel containing water. The other end of the glass tube is connected to a small piece of rubber tubing to which a pinch-cock is attached [Fig. 12·3]. The velocity of flow of water in the tube may be increased or decreased by regulating the outflow of water with the help of the pinch-cock. Another tube *B* ending in a fine jet, allows coloured water from a tank *A* to pass into the tube *T*. At low velocities of flow, it is found that the jet of coloured water travels unbroken through the flowing water producing a coloured band, which appears to remain fixed in position [Fig. 12·3(*a*)], indicating streamline flow. As the speed of flow is increased, it is found that when a certain velocity is reached, the coloured band gets disrupted and the coloured liquid begins to move transversely across the tube [Fig. 12·3(*b*)]. At a slightly higher velocity, it gets mixed with the whole of the water in the tube [Fig. 12·3(*c*)]. The flow is then no longer streamlined but turbulent. The speed of flow at the transition point gives the critical velocity.



(a) (c)

Fig. 12·3

12·3. Motion in Fluid.

If an object moves through a previously stationary fluid, it experiences a resistance to its motion offered by the fluid, which is termed fluid friction. In discussing this phenomenon, it makes no difference if we picture the object as stationary and the fluid flowing past it. This means that we are concerned only with the relative motion between the object and the fluid. For example, in testing the design of an aeroplane, the prototype is kept fixed in a wind tunnel which is essentially a long tube and air is sent through the wind tunnel with high velocity.

If the body be at rest and the fluid in motion, the layer of the fluid in contact with the body remains at rest with respect to the body. Layers of the fluid at smaller distances from the body have velocities smaller than those of the layers at greater distances. If the body be in motion through a fluid at rest, the particles of the fluid in contact with the body are carried away with the body and different layers of the fluid acquire different velocities, decreasing with their respective distances from the body. Due to this, a relative motion exists between different layers of the fluid, which gives rise to a force opposing the relative motion. The origin of this resistive force is ascribed to a special property of fluids, known as *viscosity*. We shall study this property in somewhat greater detail in the next article.

Experimental observations show that when the relative velocity is small, the flow of fluid past the object is streamlined (Fig. 12·4). The streamlines bend round the object and converge just behind it. The force f, opposing the motion is then found to be proportional to the relative velocity v. Thus, $f \propto v$ or, $f=Kv$, where K is a constant of proportionality. The value of this constant depends mainly on the size and shape of the body and also on the nature of the fluid. It is found that a body of larger size experiences greater resistance to its motion through the fluid. While moving through a fluid in a direction perpendicular to its area, a disc having same cross-sectional area as a spherical body is subjected to a larger resistive force than the latter. Water offers greater resistance to the motion of a body than that offered by air.

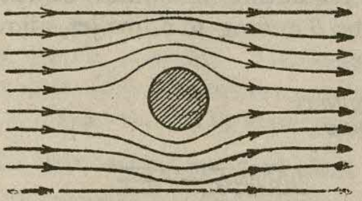


Fig. 12·4 : Stremline flow past an object

As the relative velocity increases beyond the critical velocity, the flow is no longer streamlined ; turbulence sets in and in the region just behind the body, the fluid moves in whirls and eddies (vortices). Formation of these vortices [Fig. 12·5] leads to a large dissipation of energy. If the body is in motion, a large part of its K. E. is thus lost, so that its velocity gets decreased. In other words a considerable increase in resistance to its motion takes place.

It is found experimentally that the force f opposing the motion is then proportional to the square of the relative velocity v, i.e., $f \propto v^2$, or $f=kv^2$, where k is a constant of proportionality. As in the case of streamline flow, the constant of proportionality k depends mainly upon the size and shape of the body and on the nature of the fluid.

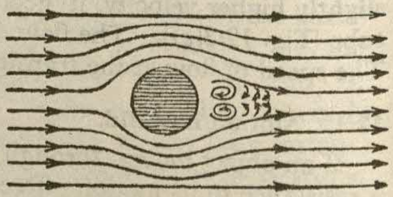


Fig. 12·5 : Turbulent flow past an object

In order to minimise this resistance to relative motion between the object and the fluid, the object is shaped as the streamlines of the fluid through which it is moving [Fig. 12·6]. For such a *streamlined* object, the cross-section gradually decreases tapering towards the rear. It must also possess a smooth surface having no sharp corners or edges. Giving the body a streamlined shape reduces greatly the tendency of formation of vortices behind it, thereby allowing it to move through the fluid facing minimum resistance. This is particularly important at high velocities, where otherwise much energy will be dissipated due to the resulting turbulent flow of the fluid. This explains the shape of the bodies of aeroplanes, torpedos and modern high speed trains and cars. A natural example is provided by the streamlined shape of a fish, so that it can glide through water at ease.

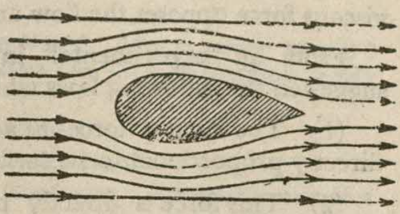


Fig. 12 6 : Fluid flow past a streamlined object

12·4. Viscosity.

The property by virtue of which a fluid resists its flow is known as **viscosity.** Many common phenomena provide examples of the manifestations of this property of fluids. When water in a vessel, after stirring, is left to itself, the swirling motion comes to a stop after a short interval of time. Internal forces of friction, called *viscous forces,* opposing the relative motion of different layers of water come into play and bring them to rest. Thus viscosity is also called internal friction. This is an inherent property of all fluids, varying only in degree from one to other. It is easier for water to pass through a tube than glycerine. Glycerine is said to be more *viscous* than water. Liquids are found to be more viscous than gases.

Let us consider the streamline flow of a fluid over a fixed horizontal surface. The layer of the fluid in contact with the surface remains at rest relative to the surface. The next layer of fluid moves slowly over the first, the third layer moves slowly over the second and so on. The velocity of the fluid layers increases uniformly with distance from the fixed surface as shown by the arrows [Fig. 12·7]. If the velocity of a fluid layer at a distance l from the fixed surface be v, then v/l gives the rate of change of velocity with distance in a direction at right angles to the flow and is called the *velocity gradient.* In a streamline flow this is usually uniform.

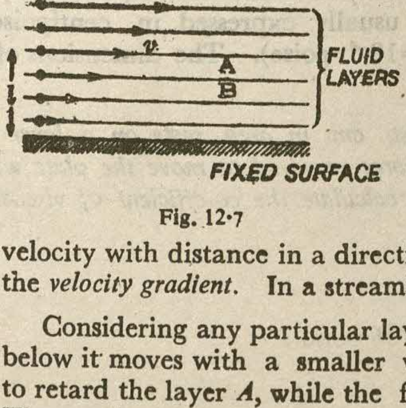


Fig. 12·7

Considering any particular layer A of the fluid, the layer B immediately below it moves with a smaller velocity. The slower moving layer B tries to retard the layer A, while the faster one, A tries to accelerate the layer B. The two layers thus tend to destroy their relative motion ; this suggests that a tangential force opposing the motion, called viscous force act on the layers. Therefore to maintain the flow, an external force must be applied to overcome this viscous force. In absence of such an external force, this viscous force opposes the flow and brings the fluid to rest.

From an experimental investigation of this phenomenon, Newton suggested the following laws which hold only for streamline flow :—

(1) The viscous force F acts tangentially to a layer of the fluid and is directly proportional to its surface area A i.e., $F \propto A$.

(2) This force is directly proportional to the velocity gradient of the fluid layers. If the velocity of a layer at a distance l from the stationary layer be v, then $F \propto (v/l)$.

Combining we get, $F \propto A \dfrac{v}{l}$ or, $F = \eta \dfrac{Av}{l}$ ... (12·1)

This constant of proportionality, represented by the Greek latter η (*eta*), is called the **co-efficient of viscosity**, or simply the *viscosity*. For fluids which flow readily like water, its value is small, while for heavy fluids like glycerine, treacle etc., its value is larger. Obviously, its value for gases is much smaller than that for liquids. Its value depends markedly on temperature ; as the temperature is increased, the value of η increases for gases, but decreases for liquids.

The absolute C.G.S. unit of co-efficient of viscosity is called the '*poise*' after Poiseuille, a poineer research worker in this field. From equation (12·1), we have, $\eta = \dfrac{Fl}{Av}$ ... ... (12·2)

$$\therefore \; 1 \text{ poise} = \frac{1 \text{ dyne} \times 1 \text{ cm.}}{1 \text{ cm.}^2 \times 1 \text{ cm./sec.}} = 1 \text{ dyne sec./cm.}^2$$

Hence the co-efficient of viscosity of a fluid is 1 poise, if a tangential force of 1 dyne for each square centimeter of surface is necessary to maintain a relative velocity of 1 cm./sec. between two layers of the fluid separated by a distance of 1 cm. Small viscosities are usually expressed in centipoises. (1 cp$=10^{-2}$ poise) or micropoises ($1 \mu p = 10^{-6}$ poise). The dimensions of η are $ML^{-1}T^{-1}$.

Example 12·1. *A metal plate 250 sq. cm. in area, rests on a layer of glycerine 5 mm. thick. If the horizontal force required to move the plate with a speed of 22 cm./sec. be 9.3 × 10⁴ dyne, calculate the co-efficient of viscosity of glycerine.*

Solution : From eqn. (12·2),

$$\eta = \frac{Fl}{Av} = \frac{9{\cdot}3 \times 10^4 \times 0{\cdot}5}{250 \times 22} = 8{\cdot}46 \text{ poise.}$$

● EXERCISE ●

Questions.

1. Distinguish between streamline and turbulent flow of fluid. What is critical velocity ? Indicate its importance in fluid flow. On what factors does the value of critical velocity depend ?

2. What do you mean by fluid friction ? How does it arise ?

'The shape of the fast moving bodies must be streamlined'.—Explain.

3. Define viscosity. What is viscous force and how dose is arise ? On what factors does is depend and how ? What is 'poise' ?

4. Define 'coefficient of viscosity'. Mention its unit and dimension. What is the effect of temperature on it ? 'Viscosity is a special property of fluid only'. Can you agree ? Why ?

5. 'Coefficient of viscosity of glycerine is 8·4 poise'—Explain. Coefficient of viscosity of liquid *A* is double than that of liquid *B*. Which liquid will flow more readily ?

Problem.

1. A flat metal plate of area 100 sq. cm. is separated from a large plate by a layer of castor oil 2 mm. thick. If the coefficient of viseosity of castor oil be 15·5 poise, calculate the horizontal force required to move the plate with the speed of 3 cm./sec.

[*Ans.* $2{\cdot}325 \times 10^4$ dyne]

3

HEAT

13
CHAPTER

HEAT AND TEMPERATURE
(*Recapitulation*)

13·1. Introduction.

With this chapter we begin our study of the branch of physics, known as Heat. In aiming to describe the *thermal phenomena*, we shall primarily explain two physical quantities, called *heat* and *temperature*. *Heat is a form of energy* and can be expressed in energy units defined in mechanics. But temperature cannot be defined in terms of the three fundamental quantities viz., length, mass and time. Thus, *temperature is a new physical quantity of a type different from the above fundamental quantities.* Starting from the discovery of a thermometer by Galileo in the closing end of the 16th century, the long history of thermometry and the establishment of the concept of temperature is an interesting story of its own ; but we may simply understand the temperature to be the scale reading of a suitable thermometer placed in close contact with the object.

13·2. Heat and temperature.

All bodies in nature contain heat. *Heat energy is associated with molecular motion within the substance* (vide Art. 20·2). It is perceptible only when it passes from one body to another. As a result of heat communication, the temperature of the recipient increases and the temperature of the donor decreases. Thus, the quantity of heat that a substance either releases or receives, can only be measured ; its absolute content of heat is not measurable. In principle, heat gained or lost by a substance is measured by the *calorimetric principle* (vide Art. 17·6). The unit of heat, called the *gramme-calorie* or simply *calorie*, is the quantity of heat required to raise the temperature of 1 gm. of water by 1°C. Other units of heat are set out in table 13·1.

TABLE 13·1 : HEAT UNITS

Unit	*Mass of water*	*Temp. change*	*Gm-calorie or calorie*
Gramme-calorie	1 gm.	1°C.	1
Large calorie	1 kg.	1°C.	1000
B. Th. U.	1 lb.	1°F.	252
Therm	10^5 lb.	1°F.	$2{\cdot}52 \times 10^7$
Pound-calorie	1 lb.	1°C.	453·6

Temperature *is a measure of the degree of hotness or is the intensity of sensible heat.* Whenever two bodies at different temperatures are brought into contact, heat flows from one at higher temperature to other at lower temperature.

Thus, *temperature can be defined as the thermal condition of a body which determines the direction of heat flow i.e., it determines whether the body will receive heat from another body or it will give heat to the same.*

The human body is quite sensitive to changes in temperature. But the sense of touch is merely qualitative, while scientific precision requires that every physical quantity should be measurable in numerical terms. Instruments devised for the purpose of measurement of temperature are called *thermometers.*

Discussion on the methods of measuring temperature forms the subject matter of *thermometry*. Any property of matter which has one to one correlation with temperature i.e. which has its magnitude changed in a regular way by a change of temperature can be used to measure temperature. The said property is called the *thermometric property* and the corresponding substance is known as the *thermometric substance.* Some most common thermometric substances and their properties are listed in the following table :—

Thermometric substance	*Thermometric property*	*Thermometer*	*Approximate range* °C
1. Mercury	Volume	Mercury-in-glass thermometer	−38 to 350
2. Alcohol	Do	Alcohol thermometer	−80 to 60
3. Gas (e.g. hydrogen)	Do	Constant-pressure gas thermometer	−183 to 600
4. Gas	Pressure	Constant-volume gas thermometer	−260 to 1600
5. Bimetallic strip	Relative expansion of two different metals	Bimetallic thermometer	−40 to 500
6. Liquid-vapour	Vapour pressure	Vapour pressure thermometer	−150 to −272
7. Platinum	Resistance of a wire	Platinum resistance thermometer	−200 to 1200
8. Closed circuit of two dissimilar metals	Thermo e.m.f.	Thermocouple	−200 to 1600
9. Metallic wire (e.g. of tungsten)	Brightness	Radiation pyrometer	800 up

13·3. Distinction between heat and temperature.

The following are the points of difference between heat and temperature :

(*i*) Heat and temperature are quite separate physical entities. The former is the cause and the latter is its effect. Generally, when a body receives some quantity of heat, its temperature rises and when a body releases heat, its temperature decreases.

(*ii*) While heat is a form of energy, the temperature defines the thermal condition of a body which signifies the direction of heat flow.

The relation between heat and temperature, is something like the relation between the total quantity of liquid in a vessel and its level. The direction of flow of liquid is determined by the height of liquid level in two connecting vessels. Liquid is always found to flow from the vessel with the higher level to the other, though the second may actually contain greater quantity of liquid. Similarly, heat flows from the body at higher temperature to another at lower temperature, irrespective of whether the latter contains smaller or larger quantity of heat.

(*iii*) Even if two bodies are at the same temperature, they may contain different amounts of heat. Also, the rise in temperature of two bodies may be different even when the same amount of heat is communicated to both of them.

(*iv*) The temperature of a substance remains unchanged during change of state but its total heat content varies.

13·4. Scales of temperature.

Thermometer based on the expansion of a liquid viz., mercury-in-glass-thermometer, was invented early in the seventeenth century. At that time completely arbitrary scales were used, so that readings of any two thermometers had wide disparity. This inconsistency was however, removed when common scales of temperature were devised in the first half of the eighteenth century.

In defining the common temperature scales, two conveniently reproducible temperatures, called fixed points, are used :

The **upper fixed point** *(or the steam point) is the temperature of steam from water boiling under an external pressure of 76 cm. of mercury at sea level and 45° latitude.*

The **lower fixed point** *(or the ice point) is the temperature of pure melting ice under the same conditions.*

The *Centigrade scale* was first suggested by Anders Celsius. It is also called the *Celsius scale,* in which the lower and upper fixed points are chosen as 0°C. and 100°C. respectively. This scale is in common use in most of the countries of the world. There is another important scale of temperature. It is called *the Fahrenheit scale* and was devised by Gabriel Daniel Fahrenheit. On this scale, the fixed points are 32°F. and 212°F. For ready reference, and also for brevity, the fixed points of some useful temperature scales are shown in the Table 13·2.

TABLE 13·2 : CONVENTIONAL SCALES OF TEMPERATURE

Scale	*Lower fixed point*	*Upper fixed point*	*Symbol*
Celsius (Centigrade)	0	100	C
Fahrenheit	32	212	F
Reaumur	0	80	R
Absolute (Kelvin)	273	373	K

The interval between the fixed point marks in a thermometer is called the **fundamental interval.** It is divided into 100 equal spaces for the Celsius scale, or 180 for the Fahrenheit scale, to give the individual degrees. Above and below the fixed points, the scale may be extended by marking off degrees of the same size.

13·5. Relation between the Centigrade and Fahrenheit scales of temperature.

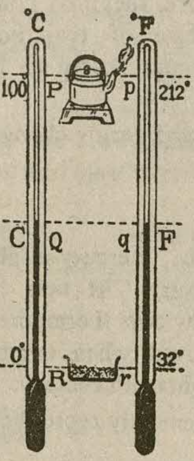


Fig. 13·1 : Centigrade and Fahrenheit scales

Fig. 13·1 shows a comparison of the Centigrade with the Fahrenheit scales. Since there are 100 Celsius or Centigrade degrees and 180 Fahrenheit degrees between the two fixed points, we see that

100C. degree = 180F. degree, i.e., 100 divisions on the centigrade scale = 180 divisions on the Fahrenheit scale.

∴ *1 division on the Centigrade scale* $=\frac{9}{5}$ *division on the Fahrenheit scale.* ... (*a*)

and also, 1 division on the Fahrenheit scale $=\frac{5}{9}$ division on the Centigrade scale. ... (*b*)

Let us suppose that an unknown temperature reads C and F on the Centigrade and Fahrenheit scales respectively. Now, the reading F corresponds to the movement of (F—32) divisions of the mercury thread from the lower fixed point (Fig. 13·1), while the reading C corresponds to a movement of C divisions from the said point.

Hence, (F—32) divisions on the Fahrenheit scale = C divisions on the Centigrade scale.

But from *(b)*, (F—32) divisions on the Fahrenheit scale

$=\frac{5}{9}$(F—32) divisions on the Centigrade scale

Hence,
$$C=\frac{5}{9}(F-32)$$

i.e.,
$$\frac{C}{5}=\frac{F-32}{9} \quad \ldots \quad \ldots \quad (13\cdot1)$$

We can arrive at the relation (13·1) more simply as follows : Suppose C corresponds to the mercury level at Q, and F to the level q. The corresponding levels in the fixed points may be denoted by R, P and r, p on the Centigrade and Fahrenheit scale respectively (Fig. 13·1). Evidently, we have,

$$\frac{RQ}{RP}=\frac{rq}{rp}$$

from which it follows that,

$$\frac{C}{100}=\frac{F-32}{180} \text{ i.e., } \frac{C}{5}=\frac{F-32}{9}$$

By this relation, it is possible to convert a temperature reading from Centigrade scale to Fahrenheit scale and vice versa.

Example 13·1. *Over a period of a year, the extreme temperatures in Calcutta have differed by 30 centigrade degrees. Express this range in Fahrenheit degrees.*

Solution : Since 1 division on the centigrade scale $=\frac{9}{5}$ division of the Fahrenheit scale, we have, the range of

$$30 \text{ C. degree} = \frac{9}{5} \times 30 \text{ F. degree} = 54 \text{ F. degree.}$$

Example 13·2. *Mercury freezes at −39°C. and boils at 357°C. at one atmospheric pressure. Convert these temperatures to the Fahrenheit scale.*

Solution : We have from the relation (13·1),

$$\frac{-39}{5} = \frac{F-32}{9}, \qquad \therefore \quad F = -38{\cdot}2^\circ$$

and,

$$\frac{357}{5} = \frac{F-32}{9}, \qquad \therefore \quad F = 674{\cdot}6^\circ$$

Example 13·3. *The temperature of dry ice at normal pressure is −109°F. Is this hotter or colder than the temperature of boiling ethane which is −88°C. ?*

Solution : Let −109°F. correspond to x°C. We have from the relation (13·1),

$$\frac{x}{5} = \frac{-109-32}{9}$$

or,

$$x = -78{\cdot}3^\circ\text{C}$$

Hence, the dry ice is hotter than the boiling ethane.

Example 13·4. *At what temperature will the centigrade thermometer read twice as much as the Fahrenheit thermometer ?*

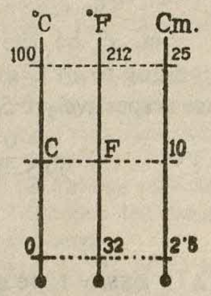
Solution : Let x be the temperature on the Fahrenheit thermometer and $2x$ on the centigrade thermometer.

∴ We have from the relation (13·1), $\frac{2x}{5} = \frac{x-32}{9}$

or, $18x = 5x - 160$ or, $x = -12{\cdot}3$

Hence, the required temperatures are −12·3°F. and −24·6°C.

Example 13·5. *The stem of a mercury thermometer is of uniform bore and a scale graduated in centimetres is attached to it. The lower fixed point reads 2·5 cm. and the upper fixed point 25 cm. What is the temperature (a) in °C., (b) in °F., when the top of the mercury thread indicates 10 cm. ?*



Solution : Length of the scale of the unknown thermometer between the two fixed points i.e. its fundamental interval $=(25-2{\cdot}5)=22{\cdot}5$ cm.

Length of scale between the lower fixed point and the top of the thread $=(10-2{\cdot}5)=7{\cdot}5$ cm. ∴ We have from the adjacent figure,

$$\frac{C}{100} = \frac{F-32}{180} = \frac{7{\cdot}5}{22{\cdot}5}$$

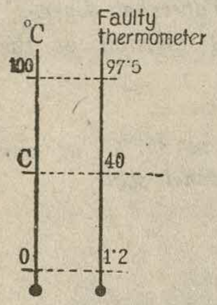
Hence,

$$\frac{C}{100} = \frac{7{\cdot}5}{22{\cdot}5} \qquad \therefore \quad C = 33{\cdot}3^\circ$$

and,

$$\frac{F-32}{180} = \frac{7{\cdot}5}{22{\cdot}5} \qquad \therefore \quad F = 92^\circ$$

Example 13·6. *A faulty thermometer reads 1·2° when placed in melting ice and 97·5° in steam at one atmospheric pressure. Find the correct temperature when the thermometer reads 40° in the same pressure, the bore of the thread and graduations being uniform.*



Solution : Let the correct temperature in the centigrade scale be C corresponding to that (40°) registered in the faulty thermometer. We have, therefore, from the adjacent figure

$$\frac{C}{100}=\frac{40-1{\cdot}2}{97{\cdot}5-1{\cdot}2} \qquad \therefore \quad C=40{\cdot}3^\circ$$

Example 13·7. *A thermometer A has got its fundamental interval divided into 50 equal parts and another B into 100. If the lower fixed point of A is marked 0° and that of B 20°, what is the temperature recorded by A when it is shown to be 100° by B ?*

Solution : Suppose the thermometer A registers $x_1°$, while B registers $x_2°$. Hence, we can write,

$$\frac{x_1-(0)}{50}=\frac{x_2-20}{100}$$

Since, $x_2=100$, $\quad \frac{x_1}{50}=\frac{100-20}{100} \qquad \therefore \quad x_1=40^\circ$

Example 13·8. *A centigrade thermometer reads 0·5°C. in melting ice and 98°C. in dry steam at 748 mm. of Hg. pressure. Find the correct temperature when this thermometer reads 14°C., given boiling point of pure water at 736 mm. of Hg. pressure is 99°C.*

Solution : We know that the boiling point of water = 100°C. at 760 mm. of Hg. pressure.

Given that boiling point of water = 99°C. at 736 mm. of Hg. pressure.

∴ for the variation of (760 − 736) or 24 mm. of Hg. pressure the boiling point changes through 1°C.

Thus, for the variation of (748 − 736) or 12 mm. of Hg. pressure the boiling point should change by 0·5°C.

∴ The true boiling point at 748 mm. of Hg. pressure = 99 + 0·5 = 99·5°C.

Let $x°$ be the correct temperature on the centigrade thermometer corresponding to 14°C on the faulty thermometer whose lower and upper fixed points are respectively 0·5°C. and 98°C.

$$\therefore \quad \frac{x-0}{99{\cdot}5-0}=\frac{14-0{\cdot}5}{98-0{\cdot}5} \quad \text{or,} \quad x=\frac{13{\cdot}5\times 99{\cdot}5}{97{\cdot}5}=13{\cdot}77^\circ\text{C}$$

● EXERCISE ●

[A] Essay type questions

1. Define temperature. Distinguish between heat and temperature. What physical properties are taken into account in the design of thermometers ? Mention the name of a few types of thermometers.

2. Define Calcius and Fahrenheit scales. In what direction would the liquid column in a thermometer move if the expansion of the liquid is smaller than that of the solid containing it? What are the advantages and disadvantages of using mercury as a thermometric substance? What is the disadvantage if the tube of a thermometer is not of uniform bone throughout?

3. Define upper fixed point and lower fixed point. What is meant by 'Fundamental Interval'? Find its value in different scales of temperature. Find the relation between the Calcius and Fahrenheit scales of temperature. Why a small bulb is provided at the top of the tube of a thermometer?

4. Why is it necessary to note the barometric height when determining the upper and lower fixed points of a thermometer?

'The relation between heat and temperature is like that between total quantity of liquid in a vessel and its level'. —Explain.

[B] Short Answer type questions

1. There are two thermometers of which one has the larger bulb and the other the finer bone. Explain the advantages and disadvantages in each case.

2 Two thermometers are hanging in a room; one registers 10° and the other 50°. If both the thermometers are correct, what is the reason for such differences?

3. Can a clinical thermometer be employed to find (i) maximum and minimum temperature of atmosphere in a day of 24 hours, and (ii) the boiling point of water. Give reasons for your answer. [*Jt. Entrance*, '73]

4. A nurse cleaned a clinical thermometer using boiling water and the thermometer became useless. Explain its reason.

5. How could a thermometer be used to find whether the atmospheric pressure was above or below the normal? [*H. S. 1960* (*comp.*)]

6. Two thermometers are constructed in the same way except that one has a spherical bulb and the other an elongated cylindrical bulb. Which one will respond quickly to temperature changes? [*I. I. T.*, '73]

[C] Simple problems

1. (a) How many degrees will the mercury in a Centigrade thermometer rise when the Fahrenheit thermometer rises by 63°?

(b) How many degrees will the Fahrenheit thermometer rise when the Centigrade thermometer rises by 65°? [*Ans.* (a) 35°C ; (b) 117°F.]

2. A Fahrenheit thermometer indicates a temperature of 5°F. What is the corresponding reading on the centigrade scale? [*Ans.* —15°C.]

3. The normal temperature of a human body is 98·6°F. Express it in centigrade unit.
[*Ans.* 37°C.]

4. Calculate the temperature at which the readings (a) of a fahrenheit and a centigrade thermometer are the same (b) of a fahrenheit thermometer is five times that of the centigrade thermometer? [*Ans.* —40° ; 50°F.]

5. The difference in the readings of the temperature of a bath determined by a fahrenheit thermometer and a centigrade thermometer is 40°. What is the temperature in °C and °F?
[*Ans.* 10°C., 50°F. or, –90°C., –130°F.]

6. A centigrade thermometer is placed in milk and reads 56°C. whilst a fahrenheit thermometer placed in water reads 202°F. What would the thermometers read if they were interchanged? [*Ans.* 94·4°C. ; 132·8°F.]

7. When a thermometer is dipped into a warm liquid the level of the mercury rises one quarter of the distance between the ice point and the steam point. Calculate the rise in temperature of the thermometer in (i) Centigrade degrees and (ii) Fahrenheit degrees.
[*Ans.* (i) 25°C, (ii) 45°F]

8. While determining the temperature of the same object, the readings of a Centigrade and a Reaumur thermometer differ by 2°. What is the temperature in °C and °R?
[*Ans.* 8°R, 10°C or, –8°R, –10°C.]

9. (a) The boiling point of mercury is 357°C. Express it on Kelvin's absolute scale.

(b) What is the temperature which is numerically the same on both the Fahrenheit and Kelvin's absolute scales ? [*Ans*., (a) 630°K, (b) 574·25°]

10. If the lower fixed point and upper fixed point of a thermometer are marked 20 and 140 respectively, what reading would this thermometer indicate for a temperature of 92°F. (*H. S. '62*) [*Ans.* 60°]

11. An accurate, centigrade thermometer registers 15·5°C., while a faulty Fahrenheit thermometer, hanging beside it, registers 61·5°F. What is the correction to be applied to this latter reading ? [*Ans.* −1·6°F]

12. A thermometer reads 5° in melting ice and 99° in dry steam at normal pressure. Find the temperature on the fahrenheit scale when the thermometer reads 52°. (*Jt. Entrance '68*) [*Ans.* 122°F]

13. The lower and upper fixed points of a thermometer are marked 30° and 150° respectively. What will be the reading on this thermometer for a temperature of 60°C ? [*Ans.* 102°]

14. A reading of 15·6 mm. is found from the scale attached to an ungraduated thermometer when it is taken in ice and the reading of 235·2 mm. when in steam at standard pressure. What will the thermometer read on a day when the temperature is 25°C ? [*Ans.* 70·5 mm.]

15. When the fixed points of a centigrade thermometer are verified, it reads 0·5°C at the melting point of ice and 99·2°C at the boiling point of water at normal pressure. What is the correct temperature when it reads 15° ? At what temperature will it give exactly correct reading ? [*Ans.* 14·69°C, ; 38·46°C]

16. Dr. Das's thermometer has got its fundamental interval divided into 70 equal parts and another thermometer belonging to Dr. Chatterjee into 110. If the lower fixed point of Dr. Das's thermometer is marked 0° and that of Dr. Chatterjee's 30°, what is the temperature by Dr. Das's thermometer when it is 100° by Dr. Chatterjee's ? [*Ans.* 44·54°]

17. A thermometer in melting ice indicates —2°, whilst the reading is 101·4° in the steam from water boiling at a pressure of 760 mm. of Hg. What temperature does the thermometer register in the vapour of boiling methylated spirit (66·9°C) ? [*Ans.* 67·16°]

[D] Harder problems

1. The freezing point on a Fahrenheit thermometer is correctly marked and the bore of the tube is uniform but it indicates 103·5° when a standard centigrade thermometer reads 40°C. What is the reading of boiling point on the Fahrenheit thermometer ? [*Ans.* 210·75°]

2. A centigrade thermometer reads 1·5°C in pure melting ice and 98·5°C in steam from water boiling at a pressure of 747 mm. of Hg. What is the correct temperature in Fahrenheit when the thermometer indicates 20°C ? Given, boiling point of water at 734 mm. of Hg. is 99°C. [*Ans.* 66·16°F]

3. A faulty centigrade thermometer reads −0·5°C as the ice point and 100·5°C as the steam point at 754 mm. of Hg. pressure. What should be the correct temperature when this thermometer reads 49·5°C. Given the true boiling point of pure water at 748 mm. of Hg. pressure is 99°C. [*Ans.* 49·26°C]

4. A thermometer immersed in water of mass 10 gm. undergoes temperature rise of 15° and indicates a temperature 35°. What is the temperature of the water prior to the measurement ? The water equivalent of the thermometer is 10 gm. (**Hint :** Use the relation, Heat lost = Heat gained.) [*Ans.* 50°]

14 CHAPTER

THERMAL EXPANSION OF SOLIDS

14·1. Thermal expansion of solids.

In general, the dimensions of all substances increase with increase in temperature. This phenomenon is known as *thermal expansion.* When cooled, the substances contract and regain their original dimensions. We shall see in a later chapter (vide Chapter 20) how the explanation of this phenomenon is provided by the kinetic theory of matter.

In the present chapter, we shall restrict ourselves to the study of the thermal expansion of solids only. When heated, the dimensions of a solid get expanded in all directions. The increase in length is called its *linear expansion.* The increase in its surface area is known as the *surface* or *superficial expansion,* while the increase in volume is called the *volume* or *cubical expansion.* The amount of increase is, however, very small. With the same rise in temperature, different solids expand by different amounts. As for example, copper expands more than iron even if they are heated to the same temperature. In addition to these effects produced, expansion or contraction gives rise to another very important effect. A very large force is exerted by the solid when the expansion or contraction is opposed. The corresponding stress generated is known as *thermal stress.*

The above discussion is not universally true for all solids. Crystals generally expand unequally in different directions. There are a few crystals which even contract along certain directions when heated. A stretched rubber band is found to contract when heated. Much more recently the Swiss physicist C.E. Guillaume invented an alloy of iron and nickel called invar. *With the rise in temperature, the expansion of this alloy is practically nil.*

14·2. Demonstration Experiments.

The simple and easily demonstrable experiments described below, illustrate vividly the different characteristics of thermal expansion of solids :—

(A) Solids expand on heating.

(*i*) *Rod and Gauge Experiment* : The apparatus consists of a cylindrical metal rod *A* and a gauge *G* (a thick metal piece) provided with a length-gap *B* and a hole *C* [Fig. 14·1]. At room temperature, the rod *A* just fits the gap *B* when inserted in it longitudinally ; it also exactly fits the hole *C* when inserted transversely. If the rod held in a wooden handle is heated in a flame, it will be found that the rod now fits neither

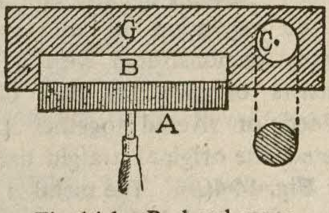


Fig. 14·1 : Rod and gauge experiment

the gap B nor the whole C. This shows that due to linear and superficial expansion, the length as well as the cross-section of the rod have increased appreciably. On cooling, the rod A is found to fit again into both B and C ; this shows the contraction of the rod with fall in temperature.

(*ii*) *Ball and ring experiment* : The apparatus consists of a metallic ball and ring such that the metal ball at room temperature passes through the snugly fitting ring. When heated, the ball fails to pass through the ring [Fig. 14·2]. This is due to the expansion of volume on heating.

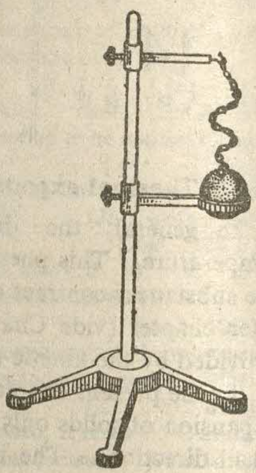


Fig. 14·2 : Ball and ring experiment

(B) Different solids expand by different amounts.

(*i*) *Ferguson's experiment* : A metal rod R is clamped at one end with its free end resting on a cylinder N of small diameter. The cylinder is free to roll on a glass plate G as shown in the Fig. 14·3. It carries a pointer P moving over a vertical circular scale S. The rod is then heated along its length by Bunsen burners. It expands causing the cylinder to rotate, so that the pointer gets deflected, the amount of deflection being proportional to the expansion in the length of the rod. The deflection is found to increase with rise in temperature of the rod. This indicates that the amount of linear expansion undergone by the rod increases with temperature. On repeating the experiment taking identical rods of different materials, and keeping the rise in temperature same for each of them, the deflection of the pointer will be found to be different. This shows that when heated to the same temperature, different materials expand by different amounts.

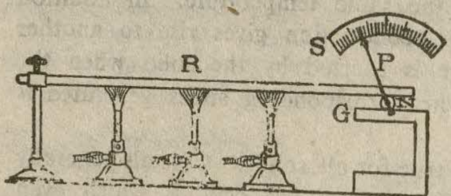


Fig. 14·3 : Ferguson's experiment

(*ii*) *Bending of Bimetallic strip* : The inequality in the expansion of different materials can be simply demonstrated with a bimetallic strip which consists of two flat bars of different materials welded or riveted together [Fig. 14·4 (*a*)]. When heated, the original straight bar bends as shown in the Fig. 14·4(*b*). The metal A, say brass, expands more than metal B, say iron ; so the strip bends with the metal A on the outside.

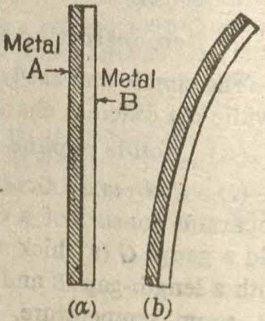


Fig. 14·4 : Bending of bimetallic strip

(C) Forces brought into play by expansion or contraction of solids.

An iron or a brass bar B passes through a strong cast-iron yoke Y as shown in Fig. 14·5. Through two holes P_1 and P_2 near one end of the bar, two short cast-iron pins may be inserted. These pins may be pressed very firmly against the shoulders of the yoke by tightening the large nut N at the other end of the bar. At first, introducing a pin in the hole P_1, the bar is heated and then clamped tightly with the help of the nut N so that the pin in the hole P_1 presses against the yoke. As the whole system is allowed to cool, the pin snaps *under the action of the large force developed due to contraction of the bar.*

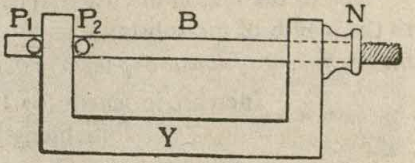


Fig. 14·5 : Demonstration of force developed due to expansion and constraction

A similar *force arising due to the expansion of the bar on heating* may be demonstrated by introducing the remaining pin in the hole P_2 and by tightening the nut when the bar is cold, so that the pin presses against the yoke. On heating the bar, the pin snaps.

14·3. Coefficient of linear expansion.

Experimental investigations show that the *increase in length of a rod due to heating is very nearly proportional to (i) the original length of the rod, and (ii) to the rise in temperature. The increase also depends upon the material of the rod.*

Let us suppose that a rod has initial length l_1 at $t_1°$ and when it is heated to $t_2°$, its length increases to l_2. We may, therefore, state that increase in length

$(l_2-l_1)\propto l_1$, *for fixed rise in temperature,*

and, $(l_2-l_1)\propto(t_2-t_1)$ *for fixed initial length of the rod,*

i.e., $(l_2-l_1)\propto l_1(t_2-t_1)$ *when both vary*

$\therefore\quad l_2-l_1=\alpha l_1(t_2-t_1)$

where, α is a constant of proportionality, different for different solids. This constant is called the *coefficient of linear expansion* of solids,

$$\therefore\quad \alpha=\frac{(l_2-l_1)/l_1}{(t_2-t_1)} \quad \ldots \quad \ldots \qquad (14·1)$$

$$\text{or,}\quad l_2=l_1\{1+\alpha(t_2-t_1)\} \quad \ldots \quad \ldots \qquad (14·2)$$

Hence, from eqn. (14·1),

$$\text{Coefficient of linear expansion}=\frac{\text{Increase in length/Original length}}{\text{Rise in temperature}}$$

$$=\frac{\text{Fractional change in length of the rod}}{\text{Rise in temperature}}$$

Thus, the **coefficient of linear expansion** *is defined as the fractional increase in length of the solid per degree rise in temperature.*

In the foregoing discussions, we have tacitly assumed that the value of α remains constant over different ranges of temperature. This does not hold accurately and the value of α is found to vary with temperature. The value defined by the eqn. (14·1) gives really the mean value of α between $t_1°$ and $t_2°$. The variation is, however, so small that in practice it is neglected without any appreciable error.

Due to the reason discussed above, the value of α is defined with respect to the length of the substance at 0° when high accuracy is required. Hence, *the coefficient of linear expansion is strictly defined as*

$$\alpha = \frac{\text{Increase in length for } 1° \text{ rise in temperature}}{\text{Initial length at } 0°}$$

14·4. Units of Coefficient of Linear Expansion.

In the expression for the coefficient of linear expansion, the numerator is simply the ratio of the two lengths and is, therefore, a pure number. Since the denominator represents a rise in temperature, α has thus the dimension of the reciprocal of temperature and its magnitude depends only on the scale of temperature adopted. Hence, *the units of coefficient of linear expansion are simply 'per °C' or 'per °F' without mention of a length.*

The statement that "the coefficient of linear expansion of brass is 0·000018 per °C" means that 1 cm. of a brass rod raised in temperature by 1°C. expands by 0·000018 cm. If the rod is 1 yard or 1 foot in length, rise of temperature by 1°C. will make it expand by 0·000018 yard or 0·000018 foot respectively.

Since a change of 1°C. is equivalent to a change of $(\frac{9}{5})$°F., the coefficient of linear expansion of a solid per degree Fahrenheit is $\frac{5}{9}$ times the corresponding value expressed on Centigrade scale. For example, brass has $\alpha = 0{\cdot}000018$ per °C ; on Fahrenheit scale, $\alpha = (0{\cdot}000018 \times \frac{5}{9})$ per °F$=0{\cdot}000010$ per °F.

Example 14·1. *A copper rod is 150 cm. long at 20°C. What is the increase in length when it is heated to 40°C. ? The coefficient of linear expansion of copper is 17×10^{-6} per °C.*

Solution : $l_1 = 150$ cm., $t_1 = 20$°C., $t_2 = 40$°C., and $\alpha = 17 \times 10^{-6}$ per °C. Then from eqn. (14·1), increase in length

$$l_2 - l_1 = l_1 \alpha (t_2 - t_1) = 150 \times 17 \times 10^{-6}(40 - 20) = 0{\cdot}051 \text{ cm.}$$

Example 14·2. *A hole of diameter 6 cm. is bored in an iron plate at the temperature of 25°C. Find the temperature to which the plate must be heated in order that a ball of diameter 6·05 cm., just passes through the hole (the coefficient of linear expansion of iron is 12×10^{-6} per °C.).*

Solution : The diameter of the hole must obviously increase by $6{\cdot}05 - 6{\cdot}0 = 0{\cdot}05$ cm. Let the required temperature to which the plate is heated be t°C. ; then from eqn. (14·1),

$$\text{the increase of diameter} = 6 \times 12 \times 10^{-6} \times (t - 25)$$

$$\text{or,} \quad 0{\cdot}05 = 6 \times 12 \times 10^{-6} \times (t - 25) \qquad \therefore\ t = 719{\cdot}5°\text{C.}$$

Example 14·3. *An iron rod and a zinc rod are respectively 30·05 cm. and 30 cm. long at 0°C. At what temperature will they be just equal in length ? (Coefficient of linear expansion of iron is 12×10^{-6} per °C. and that of zinc is 30×10^{-6} per °C.)*

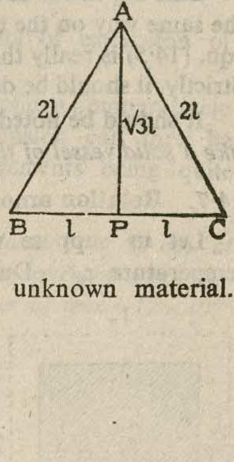
Solution : Let the rods be heated to t°C., then from eqn. (14·2),
$l_2=30{\cdot}05(1+12\times10^{-6}t)$, for the iron rod ;
and $l'_2=30(1+30\times10^{-6}t)$, for the zinc rod.
But, since $l_2=l_2'$, we have

$$30{\cdot}05(1+12\times10^{-6}t)=30(1+30\times10^{-6}t) \text{ whence } t=92{\cdot}7°\text{C}.$$

Example 14·4. *Two copper and one aluminium rods of the same length are oined together at their ends to form an equilateral traingle ; another rod of unknown material connects the vertex to the mid-point of the base rod of the traingle which is the aluminium rod. Show that for a small rise in the temperature of the system there will be no tendency for its sides to buckle provided that the material of the unknown rod is iron whose coefficient of linear expansion is $12{\cdot}7\times10^{-6}$ per °C. (Given the coefficient of linear expansion of copper and aluminium are 16×10^{-6} and 26×10^{-6} per °C. respectively).*

Solution : In the Fig., AB and AC are the copper rods and BC the aluminium one. Let the length of each be $2l$. AP is the rod of unknown material. Its length is obtained from the relation

$$AP^2=AB^2-BP^2=4l^2-l^2=3l^2 \quad \therefore \quad AP=\sqrt{3}l$$



With rise in temperature, if the sides do not buckle then this means that the shape of the triangle remains unchanged and the above relation still holds. So for a rise in temperature through t°C,

the new length of $AB=2l(1+16\times10^{-6}t)$
" " " of $BP=l(1+26\times10^{-6}t)$
" " " of $AP=\sqrt{3}l(1+\alpha t)$

where α is the coefficient of linear expansion of the unknown material. Therefore, for no buckling

$$[\sqrt{3}l(1+\alpha t)]^2=[2l(1+16\times10^{-6}t)]^2-[l(1+26\times10^{-6}t)]^2$$

Neglecting the higher order terms, we have,

$$3(1+2\alpha t)=4(1+32\times10^{-6}t)-(1+52\times10^{-6}t)$$
$$\text{or,}\quad 3+6\alpha t=4+128\times10^{-6}t-1-52\times10^{-6}t$$
$$\text{or,}\quad 6\alpha t=76\times10^{-6}t$$
$$\therefore \quad \alpha=12{\cdot}7\times10^{-6}/°\text{C}.$$

Hence, the unknown material is iron.

14·5. Coefficient of Superficial Expansion.

The coefficient of superficial (or surface) expansion of a solid is defined as the ractional change in surface area of the solid per degree rise in temperature.

Thus if S_1 be the initial surface area of a solid substance at t_1° and S_2 be that at $t_2°(t_2>t_1)$, the coefficient of superficial expansion, denoted by β, is given by

$$\beta=\frac{(S_2-S_1)/S_1}{(t_2-t_1)} \quad \ldots \quad \ldots \quad (14{\cdot}3)$$

$$\text{or,}\quad S_2=S_1\{1+\beta(t_2-t_1)\} \quad \ldots \quad \ldots \quad (14{\cdot}4)$$

As in the case of linear expansion, the units of β are independent of the units of surface area. However, they depend on the units of temperature.

Thus the value of β per °F is $\frac{5}{9}$ times the corresponding value expressed in the Centigrade scale.

The value of β, defined by eqn. (14·3), actually gives its mean value over the temperature interval (t_2-t_1). As before, for accurate work β should be defined by considering the initial surface area at the temperature of 0°.

In this connection it may be mentioned that *a ring expands in area with rise in temperature just like a disc of the same area as the ring.*

14·6. Coefficient of Cubical Expansion.

The coefficient of cubical (or volume) expansion of a solid is defined as the fractional increase in volume of the solid per degree rise in temperature. Thus, if V_1 and V_2 be the volumes at $t_1°$ and $t_2°$ $(t_2>t_1)$ respectively, and γ be the coefficient of cubical expansion, then,

$$\gamma=\frac{(V_2-V_1)/V_1}{(t_2-t_1)} \qquad \cdots \qquad \cdots \qquad (14\cdot5)$$

$$\text{or,}\quad V_2=V_1\{1+\gamma(t_2-t_1) \qquad \cdots \qquad \cdots \qquad (14\cdot6)$$

Like the linear and superficial expansion coefficients of solids, γ depends in the same way on the temperature scale only. Also the value of γ obtained from eqn. (14·5) is really the mean value over the corresponding temperature range. Strictly, it should be defined with reference to the initial volume of the solid at 0°.

It should be noted that *a hollow vessel expands with rise in temperature just like a solid vessel of the same volume.*

14·7. Relation among the three coefficients of expansion.

Let us suppose that a square plate of length l_1 at $t_1°$ is heated to a temperature $t_2°$. Due to this, the length of each side of the square will increase, the new length being $l_2=l_1(1+\alpha t)$, where α is the coefficient of linear expansion of the material of the plate and $t=(t_2-t_1)$ [Fig. 14·6]. The area of the plate is then given by

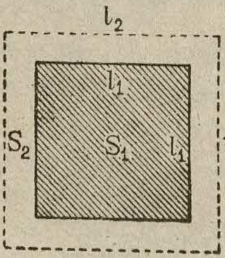


Fig. 14·6: Superficial expansion

$$S_2=l_2^2=l_1^2(1+\alpha t)^2$$

Since the initial area is $S_1=l_1^2$, we have

$$S_2=S_1(1+\alpha t)^2=S_1(1+2\alpha t+\alpha^2t^2).$$

The term containing α^2 can be neglected because α itself is very small, being of the order of 10^{-5}. Hence,

$$S_2=S_1(1+2\alpha t) \qquad \cdots \qquad \cdots \qquad (14\cdot7)$$

If β be the coefficient of superficial expansion of the material of the plate, then,

$$S_2=S_1(1+\beta t) \qquad \cdots \qquad \cdots \qquad (14\cdot8)$$

Comparing eqns. (14·7) and (14·8) we get

$$\beta=2\alpha \qquad \cdots \qquad \cdots \qquad (14\cdot9)$$

Again let us consider a cube of the solid, having each side of length l_1 at $t_1°$. When heated to a temperature of $t_2°$, the length of each side will increase, the new length being given by $l_2=l_1(1+\alpha t)$, where $t=(t_2-t_1)$ [Fig. 14·7]. The new volume of the cube is then

$$V_2=l_2^3=l_1^3(1+\alpha t)^3=l_1^3(1+3\alpha t+3\alpha^2t^2+\alpha^3t^3)$$

Since the initial volume of the cube is given by $V_1=l_1^3$ and the terms containing α^2 and α^3 are negligibly small, we have

$$V_2=V_1(1+3\alpha t) \quad \ldots \quad (14{\cdot}10)$$

If γ be the coefficient of cubical expansion of the material of the cube, then

$$V_2=V_1(1+\gamma t) \quad \ldots \quad (14{\cdot}11)$$

Comparison of eqns. (14·10) and (14·11) gives

$$\gamma=3\alpha \quad \ldots \quad (14{\cdot}12)$$

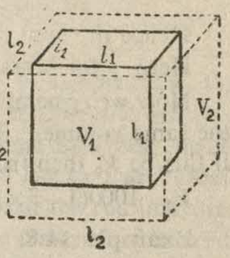


Fig. 14·7 : Cubical expansion

Hence, from eqns. (14·9) and (14·12) we obtain

$$\alpha=\frac{\beta}{2}=\frac{\gamma}{3} \quad \ldots \quad \ldots \quad (14{\cdot}13)$$

which is the required relation among the three coefficients of expansion of solids.

Measurement of the coefficient of linear expansion of a solid is comparatively easier. The coefficients of superficial and cubical expansion can then be obtained from the above relation, their actual measurements being quite unnecessary.

Example 14·5. *The base of an iron saucepan has a diameter of 20 cm. at 15°C. What will be the increase in area of the base of the saucepan when it is filled with boiling water ? Given coefficient of linear expansion of iron*$=12\times10^{-6}$ *per °C.*

Solution : Radius of the base$=20/2=10$ cm.

We have from the relation (14·4),

$$S_2-S_1=S_1\times\beta\times(t_2-t_1)$$

where $S_1=\pi\times10^2$ sq. cm., $\beta=2\alpha=2\times12\times10^{-6}$ per °C., and $t_2-t_1=100-15=85$°C.

∴ The required increase in area

$$=\pi\times10^2\times24\times10^{-6}\times85=0{\cdot}6404 \text{ cm.}^2$$

Example 14·6. *A shop window is made of glass plate 2 m. by 2 m., an allowance of 25 sq. cm. being kept for expansion. Find the maximum variation of temperature which this allowance anticipates. The coefficient of linear expansion of glass is* 9×10^{-6} *per °C.*

Solution : 25 sq. cm.$=25\times10^{-4}$ sq. m.

Here, $S_1=2\times2=4$ sq. m. ; and $\beta=2\alpha=2\times9\times10^{-6}$ per °C.

∴ From eqn. (14·4), the increase in area

$$S_2-S_1=S_1\beta(t_2-t_1)$$

or, $25\times10^{-4}=4\times2\times9\times10^{-6}(t_2-t_1)$ or, $(t_2-t_1)=34{\cdot}7$°C.

which is the maximum variation of temperature anticipated.

Example 14·7. *The volumes of a hollow and a solid brass sphere at 0°C are both equal to 1000 c.c. What will be their volumes on being heated to 86°F? The coefficient of linear expansion of brass is 19×10^{-6} per °C.*

Solution : We have $\frac{C}{5}=\frac{86-32}{9}$ or, $C=30°C$.

Hence the final temperature is 30°C.

Here, γ for brass $=3\times19\times10^{-6}$ per °C.

Now we remember that a hollow vessel expands just like a solid vessel of the same volume. So the volumes of both spheres will be the same at 30°C. If this be V_2 then from eqn. (14·6),

$$V_2=1000(1+3\times19\times10^{-6}\times30)=1001{\cdot}71 \text{ c.c.}$$

Example 14·8. *A flask of volume 500 c.c. at 35°C. is made of glass having $\alpha=4{\cdot}5\times10^{-6}$ per °C. Find (a) the contraction in volume when it is placed in ice-cold water at 0°C. and (b) temperature at which its capacity is 500·3 c.c.*

Solution : Here, γ for glass $=3\alpha=3\times4{\cdot}5\times10^{-6}$ per °C.

(*a*) From eqn. (14·6), contraction of 500 c.c., when cooled by $(35-0)=35°C$. is

$$V_2-V_1=V_1\gamma(t_2-t_1)$$
$$=500\times3\times4{\cdot}5\times10^{-6}\times35=0{\cdot}236 \text{ c.c. (nearly).}$$

(*b*) A contraction of 0·236 c.c. corresponds to a cooling of ~~the~~ flask by 35°C.

∴ An expansion of 0·3 c.c. will be caused by a rise in temperature

$$=\frac{0{\cdot}3\times35}{0{\cdot}236}=44{\cdot}5°C \text{ (nearly)}$$

Hence the flask must be heated to 35°+44·5°=79·5°C.

14·8. Variation of density of a solid with temperature.

As the volume of a solid changes with temperature, its density also changes in a corresponding way, because the mass of the solid remains the same. Let us consider a solid of mass m. If its volume at $t_1°$ be V_1, then the corresponding density is given by,

$$\rho_1=\frac{m}{V_1}$$

If its volume at temperature $t_2°$ be V_2, then

$$V_2=V_1\{1+\gamma(t_2-t_1)\}=V_1(1+\gamma t)$$

where, $t=(t_2-t_1)$. Its density is then given by

$$\rho_2=\frac{m}{V_2}=\frac{m}{V_1(1+\gamma t)}=\frac{\rho_1}{1+\gamma t}=\rho_1(1+\gamma t)^{-1}$$

Expanding by the bionomial theorem and neglecting the higher order terms containing γ, we get

$$\rho_2=\rho_1(1-\gamma t)=\rho_1\{1-\gamma(t_2-t_1)\} \quad \cdots \quad \cdots \quad (14{\cdot}14)$$

Thus, if ρ_t and ρ_0 are the densities of the solid at $t°$ and 0° respectively, then

$$\rho_t=\rho_0(1-\gamma t) \quad \cdots \quad \cdots \quad (14{\cdot}15)$$

Example 14·9. *The density of iron at 0°C. is 7·6 gm. per c.c. What is its density at 200°C? (Coefficient of linear expansion of iron $=10{\cdot}2\times10^{-6}$ per C.)*

Solution : The coefficient of volume expansion of iron

$$\gamma = 3 \times 10{\cdot}2 \times 10^{-6} = 30{\cdot}6 \times 10^{-6} \text{ per } ^\circ\text{C}.$$

$$\rho = 7{\cdot}5 \text{ gm. per c.c., } t = 200^\circ\text{C}.$$

Hence from eqn. (14·15), the density of iron at 200°C. is

$$\rho_t = \rho_0(1 - \gamma t) = 7{\cdot}5(1 - 30{\cdot}6 \times 10^{-6} \times 200) = 7{\cdot}46 \text{ gm. c.c.}$$

14·9. Thermal Stresses.

If the ends of a rod are rigidly fixed so that with change in temperature, the expansion or contraction of the rod is prevented, tensile or compressive stresses, known as *thermal stresses*, of a very large magnitude will come into play inside the rod. To calculate its magnitude, let us consider a rod of length l and cross-section A. When the temperature increases by t°, the length of the rod increases by $l\alpha t$, α being the coefficient of linear expansion of the material of the rod. If the temperature now decreases to the initial value, the rod contacts and attains its original length. This contraction due to cooling can be prevented by a pull F on the rod, which must be sufficient to produce the same fractional change in length. From Hooke's Law, we have for Young's modulus.

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Force per unit area}}{\text{Change in length per unit length}} = \frac{F/A}{l\alpha t/l}$$

$$\therefore \quad F = AY\alpha t \quad \ldots \quad \ldots \quad \ldots \quad (14{\cdot}16)$$

$$\text{Hence, the stress in the rod is } \frac{F}{A} = Y\alpha t \quad \ldots \quad \ldots \quad (14{\cdot}17)$$

Example 14·10. *The ends of a copper rod exactly 3 sq. cm. in cross-sectional area are held between two fixed points at a temperature of 30°C. What force would the rod exert when its temperature is raised to 50°C ? (Young's modulus, Y for copper* $=10 \times 10^{11}$ *dyne per sq. cm., coefficient of linear expansion of copper* $=16{\cdot}7 \times 10^{-6}$ *per °C.)*

Solution : Here, rise in the temperature $= 50 - 30 = 20^\circ$C.

$\alpha = 16{\cdot}7 \times 10^{-6}$ per °C., cross-section $A = 3$ sq. cm., and Y for copper $= 10 \times 10^{11}$ dyne per sq. cm. Therefore, from eqn. (14·16),

$$\text{Force developed, } F = AY\alpha t = 3 \times 10 \times 10^{11} \times 16{\cdot}7 \times 10^{-6} \times 20$$

$$= 10{\cdot}02 \times 10^{8} \text{ dyne.}$$

Example 14·11. *A steel wire of cross-sectional area 1 square mm. is held between two fixed supports. The tension in this wire is negligible and is just taut at a temperature of 30°C. Calculate the tension when the temperature falls to 0°C., assuming that the distance between the supports remains the same, the coefficient of expansion of steel is* 12×10^{-6} *per °C., and the Young's modulus of steel is* $2{\cdot}1 \times 10^{12}$ *dyne/sq. cm.*

Solution : As the temperature decreases, the steel wire contracts and being prevented in doing so by the supports, it exerts pull on them with a large force. The tension in the wire equals this force on the supports and is given by,

$$F = AY\alpha t$$

Here $A = 1$ sq. mm. $= 10^{-2}$ sq. cm.

$$Y = 2{\cdot}1 \times 10^{12} \text{ dyne/cm}^2., \quad \alpha = 12 \times 10^{-6}/^\circ\text{C.}, \quad t = 30^\circ\text{C}.$$

$$\therefore \quad \text{Tension} = 10^{-2} \times 2{\cdot}1 \times 10^{12} \times 12 \times 10^{-6} \times 30 = 75{\cdot}6 \times 10^{5} \text{ dyne}$$

$$= 75{\cdot}6 \text{ Newton.}$$

14·10. Some Applications of Thermal Expansion of Solids.

Many instances can be cited in which expansion serves useful purpose, while in others it is quite undesirable. We shall now briefly discuss some of them.

(A) **Disadvantages of thermal expansion and their remedies.**

(*i*) A scale gives correct reading only at the temperature at which graduation is made. While measuring length with the aid of the scale at some other temperature, allowance has to be made for the fact that the scale expands or contracts with variation in temperature. Knowing the fluctuation of temperature and the coefficient of linear expansion of the material of the scale, a correction must, therefore, be applied. In order to minimise this error, the scale should be made up of a metal whose coefficient of expansion is small. In common measurements where a high degree of accuracy is not required, wooden scales are used in preference to metallic scales.

(*ii*) A glass vessel may crack when it is suddenly heated. The outer layer becomes heated, but glass being a bad conductor, the inside of the vessel remains cold. The resulting inequality in expansion sets up internal stresses which result in fracture. The same phenomenon is observed when boiling water is poured quickly into a thick-walled glass tumbler. In this case, the inner layer becomes hot before the outer. For this reason, thin walled glass vessels are used. Special glassware of low expansion coefficient (e.g., pyrex-glass) is now available for kitchen and laboratory uses.

(*iii*) In laying railway lines, a small gap is left between two successive rails to allow for expansion of the lines when heated. Otherwise, the huge force developed would buckle the lines. In each gap, a fish plate (Fig. 14·8) is joined with the help of iron bolts. Bolt holes are made oval shaped to allow for the expansion of the rails in the direction of their lengths.

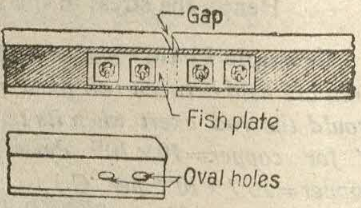


Fig. 14·8

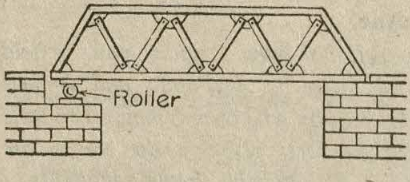


Fig. 14·9

Due to similar reason, the metal beams supporting a long bridge are not firmly fastened to each pier [Fig. 14·9]. One end of the bridge is fastened while the other end is mounted on rollers to permit the beam to expand and contract.

(*iv*) **The compensated pendulums.**

In pendulum clocks, the time period of the oscillating pendulum varies directly as the square root of its effective length, which is the distance between the point of suspension and the centre of gravity of the bob of the pendulum. In summer, the length of the pendulum increases effecting a corresponding increase in its period. Hence the clock loses i.e., goes slow. In winter, just the reverse phenomenon occurs and thus the clock gains i.e., runs faster.

In order to avoid this, pendulum rods of some clocks are made of invar which has a very small coefficient of expansion. In accurate clocks, the effect of expansion is, however, completely nullified by using specially constructed pendulums, so that its effective length always remains the same. Such pendulums, are called *compensated pendulum.* Two types are in use :

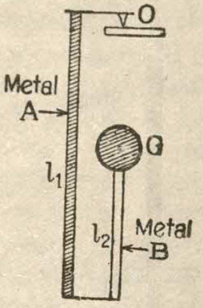


Fig. 14·10 : Compensated pendulum

(*a*) *Compensating rod type* : In this type, the pendulum bob is held on rods of two different materials, say *A* and *B* [Fig. 14·10]. The downward expansion of the rod of metal *A* is exactly compensated by the upward expansion of the other rod of metal *B.* Thus the effective length *OG,* where *O* is the point of suspension and *G*, the centre of gravity of the bob of the pendulum, remains the same.

If l_1, l_2 be the lengths of the rods and α_1, α_2 be the coefficients of linear expansion of metals *A* and *B* respectively, then for a rise in temperature *t*, the expansion of the rod *A* is given by $l_1 \alpha_1 t$, while that of the rod *B* by $l_2 \alpha_2 t$. For exact compensation,

$$l_1 \alpha_1 t = l_2 \alpha_2 t$$

$$\text{or, } \frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1} \quad \dots \quad \dots \quad (14\cdot18)$$

Thus, if the rod *A* be of iron ($\alpha_1 = 0{\cdot}000012/°\text{C.}$) and the rod *B* be of brass ($\alpha_2 = 0{\cdot}000018/°\text{C}$), then

$$\frac{l_1}{l_2} = \frac{0{\cdot}000018}{0{\cdot}000012} = \frac{3}{2}$$

Harrison's gird-iron pendulum utilises this principle. It consists of five steel rods and four brass rods arranged alternatively as shown in the Fig. 14·11. The downward expansion of the steel rods is exactly compensated by the upward expansion of the brass rods. It should be noted that all the rods, except the central one, are arranged in pairs. Thus, the *effective* expansion for each pair is equal to that of one of the rods only. So the effective expansion of the five steel rods is that of three of them. Similarly, for the four brass rods, the effective expansion is that of two of them. For exact compensation, following relation is, therefore, satisfied :

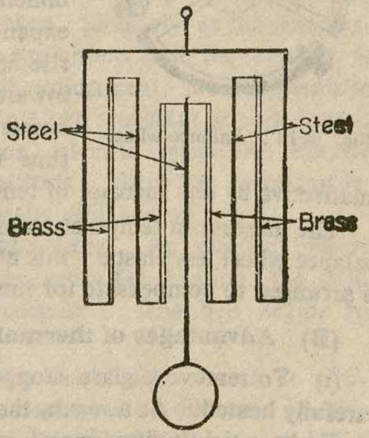


Fig. 14·11 : Harrison's gird-iron pendulum

$$\frac{\text{Total effective length of steel}}{\text{Total effective length of brass}} = \frac{\text{Coefficient of linear expansion of brass}}{\text{Coefficient of linear expansion of steel}}$$

$$= \tfrac{3}{2} \text{ (approx.)} \qquad (14\cdot19)$$

(*b*) *Mercury type*: This type of compensated pendulum (Fig. 14·12), invented by Graham, works on the same principle as the grid-iron pendulum. With rise in temperature, the steel rod expands so as to shift the centre of gravity downwards. But at the same time the mercury expands upwards and raises the centre of gravity by exactly the same amount. The effective length thus remains constant.

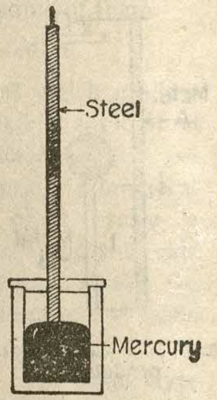


Fig. 14·12 : Graham's pendulum

(*v*) **Balance wheel**: In watches, there is no pendulum. The rate at which the watch runs depends on the balance wheel. Bimetallic strips are used in constructing compensated balance wheels of good watches. The rim of the balance wheel consists of bimetallic strips of brass and steel ; the brass placed in outside layer has a larger expansion coefficient than that of the inner layer of steel. The rim is not continuous, but is divided in segments supported by spokes as shown in the Fig. 14·13. Small screw weights are attached to the free ends of the segments. Such a balance wheel is *compensated* against temperature changes. In warm weather, the spokes expand in length ; in an uncompensated rim, this would increase the diameter of the wheel and cause the watch to beat slower and to lose time. With the bimetallic arrangement, however, the outer strip expands more than the inner strip for the same rise of temperature ; the effect is to bend the rim inwards at the gaps, so that the screw-weights move nearer to the axis. The effective radius of the ring thus remains constant, keeping the time period unaffective by any increase of temperature.

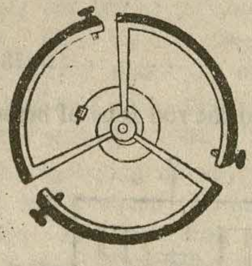


Fig. 14·13 : Balance-wheel

The increase in temperature also makes the hair spring connected to the balance wheel less elastic ; this also causes the watch to slow down. The wheel is arranged to compensate for this effect also.

(B) Advantages of thermal expansion and some applications.

(i) To remove a glass stopper fixed tightly, the mouth of the bottle is carefully heated. As a result, the mouth expands, so that the stopper gets loose. Similarly, a tightly fitted metal cap is opened by slightly heating the cap itself.

(ii) Metal tyres of cart wheels are heated before they are put on the wheel. As they cool, they shrink and fit on the wheel tightly. The method is also used to fit wheels on axles or for gun barrels, so that rigid attachment is made without the use of set-screws.

(iii) Rivets are heated red hot before inserting them into the holes bored on the metal plates to be riveted together. The rivets are then hammered and left to cool. On contracting, the rivets tightly hold the plates together.

(iv) Sometimes walls spread apart and threaten to fall. To straighten them, a long iron is put through the walls. On heating the rod, its ends are attached tightly with the wall by screwing the nuts at its ends. As the rod contracts upon cooling, the walls are pulled together.

(v) **Bimetallic thermometer** : Fig. 14·14 shows such a thermometer, made up of a bimetallic strip of brass and invar bent in the form of an arc. One end *A* of the strip is fixed, while the other end *B* is attached to a pointer *C* which can move over a scale graduated in degrees. Brass having larger coefficient of linear expansion lies on the outside of the strip, so that when heated the strip bends more and the pointer moves to the right. On the other hand, if the temperature decreases, the bending of the strip decreases and the pointer moves towards left.

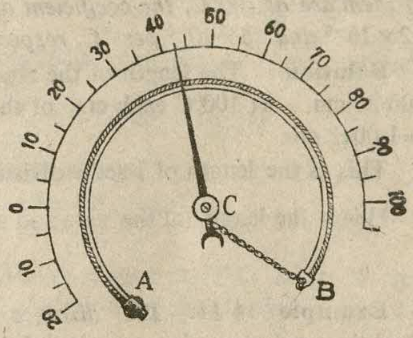


Fig. 14·14 : Bimetallic thermometer

(vi) **Thermostat and Fire alarm** : Thermostats, fire-alarms etc., are designed with bimetallic strips. A simple form of thermostat is illustrated in the Fig. 14·15. The bimetallic strip usually consists of brass and invar, the brass-side of the strip facing the contact to the electric circuit of the heater. With rise in temperature, the strip bends outwards and the contact is broken. Similarly, as the temperature falls, the strip straightens and the contact is restored. By suitable adjustment, the temperature therefore can be maintained constant at any desired value. Electric ovens, electric irons and electric water-heaters etc. are fitted with such thermostats ; when the temperature reaches the desired value, the supply is automatically switched off.

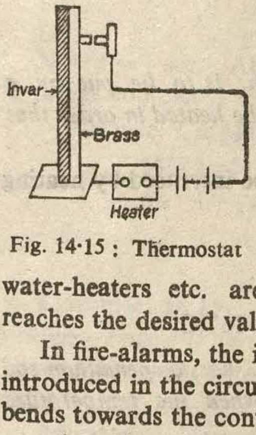


Fig. 14·15 : Thermostat

In fire-alarms, the invar side is kept facing the contact. An electric bell is introduced in the circuit and the contact is kept broken. On heating, the strip bends towards the contact and the circuit is completed. The bell begins to sound an alarm.

Example 14·12. *The graduations on a zinc scale are correct at 0°C. (a) What would be the true length of a steel wire measured as 600 mm. by this scale at 20°C ? (b) What would be the exact length of the wire at 0°C ? (Given, coefficient of linear expansion of zinc*$=26\times10^{-6}$ *per°C, and of steel*$=12\times10^{-6}$ *per°C.)*

Solution : (*a*) As the zinc scale is correct at 0°C. therefore, at 20°C. 1 scale division of the zinc scale is not exactly 1 mm., but a little greater than 1 mm. Thus, 1 scale division at 20°C.$=(1+26\times10^{-6}\times20)$ mm. So the correct length of 600 mm. of the zinc scale at 20°C.$=600\,(1+26\times10^{-6}\times20)=$ 600·312 mm., which is the true length of the steel wire.

(*b*) If the length of the steel wire at 0°C. be l_0, then

$$l_0[1+12\times10^{-6}\times20]=600·32 \qquad \therefore \quad l_0=600·168 \text{ mm.}$$

Example 14·13. *An iron scale is correct at 0°C. and the length of a zinc rod as measured by the scale is 100 cm. when both the scale and the rod are at 0°C. What will be the length of the rod as measured by the iron scale when both of them are at 100°C, the coefficient of linear expansion of iron and zinc being 12×10^{-6} and 26×10^{-6} per °C, respectively ?*

Solution : The length of the zinc rod at 100°C. is $100(1+26\times10^{-6}\times100)=$ 100·26 cm. At 100°C each cm. of the iron scale becomes $(1+12\times10^{-6}\times100)$ $=1·0012$ cm.

This is the length of 1 scale division at 100°C.

Hence the length of the zinc rod as measured by the scale $=\dfrac{100·26}{1·0012}$

$$=100·14 \text{ cm.}$$

Example 14·14. *The distance between two railway stations is 234 km. Find the total space that must be left between the rails to allow for a change of temperature from 3°C, in winter to 48°C. in summer. (Coefficient of linear expansion of iron$=12\times10^{-6}$ per °C.)*

Solution : The total space to be left=expansion of the iron rail 234 km. long for $(48-3)=45$°C. rise of temperature.

$$=234\times12\times10^{-6}\times45=0·1264 \text{ km.}=126·4 \text{ m.}$$

Example 14·15. *A steel tyre of inner diameter 99·8 cm. is to be put on a wheel of diameter 100 cm. Find by how much the tyre must be heated in order that it may just slip on. (α for steel$=12\times10^{-6}$ per °C.)*

Solution : The inner diameter of the steel tyre must be increased by heating to 100 cm. $\therefore$ Increase in length required$=0·2$ cm.

From eqn. (14·1), $l_2-l_1=l_1\alpha(t_2-t_1)$

$$\text{or,} \quad 0·2=99·8\times12\times10^{-6}\times(t_2-t_1)$$

$\therefore$ $t_2-t_1=167$°C. Hence, the tyre must be heated by 167°C.

Example 14·16. *A clock which beats seconds at 20°C., has a pendulum rod made of iron. How many seconds will it lose per day when the temperature is 40°C ? (α for iron$=12\times10^{-6}$ per °C).*

Solution : Let l_1 be the length of pendulum at 20°C. and l_2 be its length at 40°C. Since time for a swing i.e., for half an oscillation is $t=\pi\sqrt{\dfrac{l}{g}}$, we have

$$\frac{t_2}{t_1}=\sqrt{\frac{l_2}{l_1}}=\sqrt{\frac{l_1(1+12\times10^{-6}\times20)}{l_1}}$$

But $t_1=1$ sec., $\therefore$ $t_2=\sqrt{1+12\times10^{-6}\times20}=1·00012$ sec.

$\because$ 1 day$=86400$ sec.,

$\therefore$ Number of swings per day at 40°C.$=\frac{86400}{1·00012}=86389·6$

Hence, the clock loses $(86400-86389·6)$ or 10·4 sec. a day.

Example 14·17. *An iron pendulum makes 86395 oscillations in one day. At the end of the next day the clock has gained 5 seconds. Find the change in temperature (*α *for iron*$=12\times10^{-6}$ *per °C.)*

Solution : Let l_1, t_1 and l_2, t_2 give the lengths and the time periods for the 1st day and the second day respectively. Let the variation in temperature be θ°C. Since the pendulum makes 86395 oscillations on the first day, therefore

$$t_1=\tfrac{86400}{86395}\text{ sec.}$$

Next day pendulum gains 5 sec., so it makes 86400 oscillations. Thus $t_2=1$ sec.

$$\frac{t_1}{t_2}=\frac{86400}{86395}=\sqrt{\frac{l_2}{l_1}}=\sqrt{\frac{l_1(1+\alpha\theta)}{l_1}}=\sqrt{1+\alpha\theta}=1+\tfrac{1}{2}\alpha\theta$$

where α is the coefficient of linear expansion of iron.

$$\therefore\quad 6\times10^{-6}\theta=\frac{86400}{86395}-1 \qquad\text{or,}\quad \theta=\frac{5\times10^6}{518370}=9{\cdot}6\text{°C.}$$

Example 14·18. *A second pendulum which is correct at 15°C. loses 20 sec. per day. Find the day temperature if* α *for the metal of the pendulum rod be* $18\times10^{-6}/$°C.

Solution : Let the day temperature be t°C. Let l_1 and t_1 be the length and the time for a swing of the pendulum at 15°C. and let l_2 and t_2 be the corresponding quantities at t°C.

$$\therefore\quad \frac{t_2}{t_1}=\sqrt{\frac{l_2}{l_1}}=\sqrt{\frac{l_1[1+18\times10^{-6}\times(t-15)]}{l_1}}$$

Now $t_1=1$ sec. Also since the pendulum loses 20 sec. in a day, the number of swings completed$=(86400—20)=86380$.

$$\text{Hence, } t_2=\tfrac{86400}{86380}\text{ sec.}$$

$$\therefore\quad \tfrac{86400}{86380}=[1+18\times10^{-6}\times(t-15)]^{1/2}$$

$$\text{or,}\quad 1+\tfrac{20}{86380}=1+9\times10^{-6}\times(t-15)$$

$$\text{or,}\quad t-15=\frac{2\times10^6}{9\times8638}=25{\cdot}7 \qquad \therefore\quad t=40{\cdot}7\text{°C.}$$

Example 14·19. *A compensated pendulum is made of 7 iron rods and 6 copper rods. If each iron rod is of length 150 cm., calculate the length of each copper rod. Coefficients of linear expansion of iron and copper are* 12×10^{-6} *per°C. and* 17×10^{-6} *per °C. respectively.*

Solution : Here the total effective length of iron rods$=4\times150$ cm. and the total effective length of copper rods$=3\times l$ cm., where l is the length of each copper rod in cm. From the relation (14·19),

$$\frac{\text{total effective length of iron}}{\text{total effective length of copper}}=\frac{\text{coefficient of linear expansion of copper}}{\text{coefficient of linear expansion of iron}}$$

$$\text{or,}\quad \frac{4\times150}{3\times l}=\frac{17\times10^{-6}}{12\times10^{-6}} \qquad \therefore\quad l=141{\cdot}2\text{ cm.}$$

14·11. Determination of coefficient of linear expansion of solid.

Determination of the coefficient of linear expansion of a solid can be carried out by measuring how much a rod of known length l of the substance expands in length when heated through $t°$. The coefficient of expansion is then calculated from the relation

$$\alpha = \frac{x}{lt}$$ where x is the increase in length.

In the above relation the quantities l and t can be measured fairly accurately but the expansion in length (x) is very small. For a rod of length 1 metre raised through 100°C. the expansion is of the order of 1-2 mm. Hence the accuracy of the determined value of α will depend largely on the accuracy with which x can be measured. Several methods have been devised but we shall discuss here a laboratory method due to Pullinger in which x is measured by a micrometer screw.

Pullinger's method : The experimental substance is taken in the form of a long rod E and is inserted in a steam jacket J [Fig. 14·16]. The steam jacket is fixed on a wooden frame W. The rod E stands vertically, its lower end resting on a glass or marble plate P. Any linear expansion of the rod in the downward direction is thereby prevented. The upper end of the jacket is fitted with a cork C having a central hole. The upper part of the rod E passes through this hole so that a very small length of it projects outside the cork. A glass plate G provided with a central gap is placed horizontally on the frame. A spherometer S stands on the plate. The central leg of the spherometer can be lowered to touch the upper end of the rod E. Steam may be inserted into the jacket through a hole in its upper part as shown in the figure and goes out through another hole in the lower part of the jacket. The jacket is provided with two more holes fitted with bored corks through which thermometers T_1 and T_2 are inserted. Usually the outside of the jacket is covered with heat insulating substances like felt or asbestos.

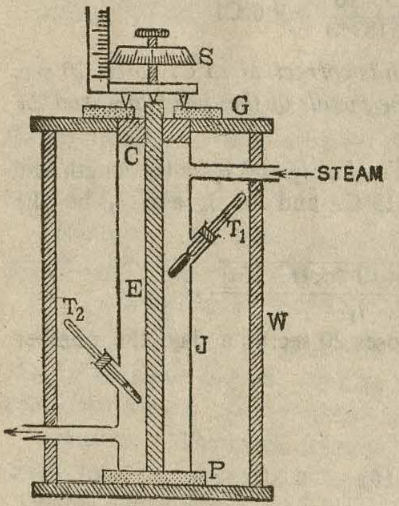


Fig. 14·16 : Pullinger's apparatus

In the actual experiment, the length (l) of the rod E is first accurately measured. It is then inserted into the jacket. After waiting for a few minutes, the readings of the thermometers T_1 and T_2 are noted. The mean of the two readings gives the initial temperature (t_1). The spherometer is then placed in position and its central leg is lowered to touch the upper end of the rod. The reading of the spherometer is noted. The central leg of the spherometer is then raised through an appropriate distance so that enough space is left for the rod to expand in the upward direction.

Steam is then sent through the jacket, whence the temperatures indicated by T_1 and T_2 will rise. The rise continues for some time and finally the readings of the thermometers become steady. The readings are noted. The mean gives the final temperature (t_2). The central leg of the spherometer is then lowered again to touch the upper end of the rod. The reading of the spherometer is noted. The difference of the two readings of the spherometer gives the linear expansion (x) of the rod for a rise of temperature (t_2-t_1). The coefficient of linear expansion (α) of the material of the rod is then calculated from the relation

$$\alpha=\frac{x}{l(t_2-t_1)}$$

The greatest source of error lies in the measurement of x which is a very small quantity. Hence the spherometer readings should be taken very carefully. The uppermost part of the rod lies outside the jacket and hence does not attain the final temperature t_2. To minimise this error, the length of the rod projecting outside the jacket is kept as small as possible.

● EXERCISE ●

[A] Essay type questions

1. Describe experiments to show that a solid can have longitudinal, surface and volume expansion when heated. Give two examples each where expansion on heating causes inconvenience and where it serves useful purposes.

2. Describe an experiment to show that different solids expand by different amounts for the same temperature rise. Discuss a few practical applications of this inequality in the expansion of different materials.

3. Define coefficient of linear expansion of a solid. Show that it does not depend on the unit of length, but depends upon scale of temperature. *[H. S. '83]*

State the units of the coefficient of linear expansion in C. G. S. and F. P. S. systems and derive the relation between the two.

4. Describe a laboratory method for determining the coefficient of linear expansion of a solid. In this method, which quantity should be measured most accurately to get correct result ? Explain.

5. What is meant by the statement that the coefficient of linear expansion of steel is 12×10^{-6} per degree C. ? Will the coefficient of linear expansion be different (a) if inch is used as the unit of length rather than the centimeter ? (b) if centigrade scale is used rather than the Fahrenheit scale ? Justifies your answer.

What would be the coefficient of linear expansion of steel in the above two cases ?

6. Define coefficient of superficial expansion and coefficient of cubical expansion of a solid. Show that (i) the coefficient of superficial expansion is approximately twice that of the linear expansion and (ii) the coefficient of volume expansion is approximately three times that of the linear expansion.

7. Describe an experiment to demonstrate the existence of forces brought into play by expansion or contraction of solids. Calculate its magnitude for a bar of cross-section A.

8. Deduce an expression for the variation of density of a solid with temperature.

The coefficient of linear expansion of iron is smaller than that of brass. Two iron and brass bars of same sizes are riveted together. Explain whut would happen if the bimetallic strip is heated.

Describe one practical application of a bimetallic strip.

9. What is a 'compensated pendulum' ? Discuss the working principle of Harrison's grid-iron pendulum.

10. Explain why a wall clock goes fast in winter and slow in summer. How this clock is compensated against variation of temperature in all seasons ?

Describe the action of a compensated balance wheel used in a watch.

11. Explain the actions of (a) Graham's Pendulum, (b) Bimetallic thermometer and (c) Thermostat.

[B] Short-answer type questions

1. An isosceles triangle is formed with three zinc rods. What will be the change of the base angle if there is a change of temperature ? Give reasons for your answer. *[H. S. '83]*

2. Explain the following statements :

(a) Scales should be made up of a metal whose coefficient of expansion is very small.

(b) Their walled glass vessels are preferred when used to heat water.

(c) In railway lines small gaps are left between successive rails.

(d) Two rails are joined by a fish plate with the help of iron bolts. The bolt holes are made oval-shaped.

(e) A tightly fitting metal cap is opened by heating it slightly.

(f) A platinum wire can be sealed in a glass rod very easily, but not a copper wire.

3. State whether each of the following characteristics of an iron ring increases, decreases or remains unchanged when it is heated : (i) internal diameter (ii) volume (iii) mass (iv) density.

4. Some thick glass vessels crack when hot liquid is poured into them.—Explain.

Why do the following not crack when so treated : (i) Thin glass beakers (ii) Thick vessels made of certain kind of glass ?

5. What is 'thermal stress' ? What is its relation with Young's modulus ?

6. Two identical rectangular plates of copper and steel are riveted together to form a bimetallic strip. If it is heated, then it will (i) remain straight, (ii) bend with the copper plate at the convex side, (iii) bend with the steel plate at the convex side, (iv) get twisted. Choose the correct alternative with justification. Given α for copper$=1{\cdot}7\times10^{-5}/°C$ and α for steel $=1{\cdot}1\times10^{-5}/°C$.

[I. I. T. '75]

7. An iron rod is joined along a diameter to the opposite sides of a circular iron hoop. If the system is heated, will the hoop remain circular ? Why ?

8. The difference between the length of a certain brass rod and that of a steel rod is claimed to be constant at all temperatures. Is this possible ? *[I. I. T. '74]*

9. A brass disc fits tightly into the bore of a steel plate. Should you apply heat in order to remove the disc from the bore ? Given that α for brass$=19\times10^{-6}/°C$ and that of steel $=12\times10^{-6}/°C$.

[I. I. T. '73]

[C] Simple problems

1. A 10 m. beam increases in length by 0·24 cm. when the temperature changes from 30°C. to 50°C. Find the coefficient of linear expansion of the material of the beam.

[*Ans.* $12\times10^{-6}/°C$]

2. An iron rod, the length of which at 0°C is 2·5 metre, is heated to 250°C ; what will be its length now ? At what temperature will its length be 250·6 centimeter ? (Coefficient of linear expansion for iron is 12×10^{-6} per °C.

[*Ans.* 250·75 cm. ; 200°C.]

3. A rod of iron and a rod of zinc are both 2 metre long at 0°C. and both are heated equally. At 50°C. the zinc rod is found to be longer than the iron rod by 0·181 cm. Find the coefficient of linear expansion of iron while that of zinc is 0·0000298/°C. *(C.U. 1927)*

[*Ans.* $11{\cdot}7\times10^{-6}$ per °C.]

4. If the coefficient of linear expansion of a metal be 0·000027/°C., the length being measured in cm. what would be its value for a Fahrenheit degree, if the length be measured in yards ?

[*Ans.* 0·000015/°F.]

5. A square metal plate, each side 100 cm. long at 0·C has a circular hole of diameter 40 cm. in the middle of it. At what temperature will the sides be 101 cm. long and what will then be the diameter of the hole ? (Coefficient of linear expansion of the metal$=12{\cdot}5\times10^{-6}/°C$).

(Cambridge) [*Ans.* 800°C. ; 40·4 cm.]

6. A compound strip of brass and iron 10 cm. long at 20°C. is held horizontally with the iron uppermost. When heated from below with a bunsen flame, the temperature of the brass is 820°C. and of the iron 770°C. Calculate the difference in lengths of the iron and the brass (α of brass$=19\times10^{-6}$/°C. and of iron$=12\times10^{-6}$/°C) (*London*) [*Ans.* 0·62 mm.]

7. A rod of iron and a rod of zinc are connected in such a way that while one expands upwards, the other expands downwards. They are heated through 100°C. and are found to expand equally. Find the ratio of their original lengths, if the coefficient of linear expansion of iron and zinc are 12×10^{-6} and 30×10^{-6} per °C. respectively. [*Ans.* 5 : 2]

8. The Eiffel tower in France is 335 metre high. Its extreme temperature rises from 0°F. in winter to 100°F. in summer. The tower is made of steel of coefficient of linear expansion equal to 12×10^{-6}/°C. How taller is the tower in summer than in winter.

[*Ans.* 22·23 cm.] (*H.S.* '63)

9. Find the ratio of lengths of two rods of different metals *X* and *Y* at 0°C, if the difference of their lengths is the same at all temperatures. The coefficient of linear expansion of the metals *X* and *Y* are 20×10^{-6} and 40×10^{-6} per °C. respectively. [*Ans.* 2 : 1]

10. The difference in length between a brass and an iron rod is 21 cm. at 20°C. What must be the length of the iron rod for this difference to remain at 21 cm. when both rods are heated to 80°C? Given α for brass$=19\times10^{-6}$/°C for iron$=12\times10^{-6}$/°C. [*Ans.* 57 cm.]

11. Calculate the length of a zinc rod which will expand in length to the same extent as a platinum rod 130 cm. in length when both are heated through the same range of temperature Given α of zinc$=26\times10^{-6}$/°C. and of platinum $=9\times10^{-6}$/C. [*Ans* 45 cm.]

12. What must be the length of a zinc rod at 59°F. if its length is to increase by 5 mm. when the temperature is raised to 100°C.? α for zinc $=29\times10^{-6}$/°C. (*H.S.* '60)

[*Ans.* 202·8 cm.]

13. The length of a iron rod at 100°C is 300·36 cm. and at 150°C. it is 300·54 cm. Find the length at 0°C. and the coefficient of linear expansion of iron. [*Ans.* 300 cm. ; 12×10^{-6}/°C]

14. A platinum wire of coefficient of linear expansion 9×10^{-6} per °C. and a zinc wire of the coefficient 25×10^{-6} per °C. are found to be 25·55 cm. and 25·5 cm. long respectively at 0°C. If they are free to expand, at what temperature will they have the same length?

[*Ans.* 122·7°C]

15. Three zinc rods form an equilateral triangle. Show that the base angles of this triangle remain unaltered even if it is heated to a fairly high temperature.

16. A thin steel sheet at 20°C, has the same surface area as a thin copper sheet at 30°C. At what common temperature, if any, will the two sheets have the same area? Given α of steel and copper$=11\times10^{-6}$/°C. and 17×10^{-6}/°C. respectively. [*Ans.* 48·33°C]

17. A sheet of aluminium has an area of 24 square yards at 80°F. Find the area when the temperature is raised to 116°F. Coefficient of linear expansion of aluminium$=25\cdot5\times10^{-6}$/°C.

[*Ans.* 24·0245 sq. yd.]

18. A sheet of steel of length 50 cm. and breadth 8 cm. is at 0°C. Its area increases by 88 sq. mm. at 100°C. Find the coefficient of linear expansion of steel. [*Ans.* 11×10^{-6}/°C]

19. A silica rod has a diameter of 0·75 cm. at 20°C and just fits into an aluminium tube at this temperature. Find the temperature at which the cross-sectional area of the space between the rod and the tube is 0·50 mm². The coefficient of linear expansion of silica and aluminium are $0\cdot50\times10^{-6}$ and $25\cdot5\times10^{-6}$ per °C. respectively. [*Ans.* 246·26°C.]

20. An aluminium piston whose diameter is 20 cm. at 0°C. is placed in a steel cylinder. The clearance (difference in radii) at this temperature is 0·02 cm. At what temperature of of the piston and cylinder will the piston just fit? Given, the coefficient of linear expansion of aluminium and steel are 24×10^{-6} and 11×10^{-6} per °C. respectively. [*Ans.* 154°C.]

21. The length of a steel span of a bridge is 0·5 kilometre and it has to withstand temperature from 44°F. to 116°F. What allowance should be kept for its expansion if the coefficient of linear expansion of steel is 10^{-5}/°C. (*Jt. Entrance* '62) [*Ans.* 20 cm.]

22. If the volume of an aluminium kettle at 18°C. is four gallons, how much will it expand when the temperature is raised to 100°C? (The coefficient of linear expansion for aluminium is 22×10^{-6} per/°C) [*Ans.* 0·02165 gal.]

23. A rectangular glass baking dish has a coefficient of linear expansion of 8×10^{-6} per°C. When taken from ice water and placed in an oven at 160°C. the dish expands 0·0386 cm. in length. How long was the dish before it was placed in the oven ? [*Ans.* 30·16 cm.]

24. The density of platinum at 10°C. is 21·4 gm./c.c. What is the density at 500°C ? Given that the coefficient of linear expansion of platinum $= 9 \times 10^{-6}$/°C ? [*Ans.* 21·12 gm./c.c.]

25. The ends of a steel rod exactly 1·5 cm.^{2} in cross-sectional area are held rigidly between two fixed points at a temperature of 33°C. Determine the pull in the rod when the temperature drops to 23°C. Young's modulus Y for steel is 2×10^{12} dyne/cm.^{2} and coefficient of linear expansion of steel is 11×10^{-6}/°C. [*Ans.* $3{\cdot}3 \times 10^{8}$ dyne]

26. The ends of a steel rod of cross-sectional area 3 cm.^{2} are supported between two rigidly fixed, massive steel plates. What will be the total force on each plate if the temperature of the rod rises by 50°C ? The Young's modulus Y for steel is $2{\cdot}1 \times 10^{6}$ kg. wt/cm.^{2} and the co-efficient of linear expansion has the value of $1{\cdot}1 \times 10^{-5}$/°C. [*Ans.* 3465 kg. wt.]

27. Calculate the value of the linear compression stress that must be applied to the ends of a steel rod in order to prevent it from expanding longitudinally when the temperature is raised through 10°C. The linear coefficient of steel is $1{\cdot}2 \times 10^{-5}$ per °C. and Young's modulus is 2×10^{12} dyne per sq. cm. [*Ans.* $2{\cdot}4 \times 10^{8}$ dyne/cm.^{2}]

28. A brass scale is correct at 20°C. The length of a rod measured by it at 45°C is 50 cm. Calculate, giving reasons, the true length of the rod. Given that the coefficient of linear expansion of brass$= 18 \times 10^{-6}$/°C. [*H. S.* '65] [*Ans.* 50·0225 cm.]

29. The diameter of a steel tyre is 91·06 cm. at 20°C. The tyre just slips on a wheel of diameter 91·44 cm., when the tyre is heated to 369°C. Find the coefficient of linear expansion of steel. [*Ans.* $11{\cdot}96 \times 10^{-6}$/°C]

30. A measurement of the length of a rod with a steel scale gave the value 150 mm. The temperature during the measurement was 10°C. What error would be made if the divisions on the steel scale were made at a temperature of 25°C ? Given α of steel is 11×10^{-6}/°C. [*Ans.* 0·025 mm.]

31. Railway lines are laid with gaps to allow for expansion. If the gaps between steel lines 2 metre long be 0·1 cm. at 10°C. at what temperature will the lines just touch ? [Coefficient of linear expansion of steel is 11×10^{-6} per °C.] [*Ans.* 55·45°C.]

32. A zinc rod is measured with a copper scale which gives correct measurement at 0°C. At 10°C. the length of the zinc rod is 100·01 cm. What is the true length at 0°C ? Given the coefficient of linear expansion for zinc and copper are 29×10^{-6} and 19×10^{-6} per °C. [*Ans.* 100 cm.

33. A wheel is 200 ft. in circumference. An iron tyre measures 198 ft. around its inner face. How much must the temperature of the tyre be raised in order that it may just slip on to the wheel ? α of iron$= 12 \times 10^{-6}$/°C. [*Ans.* 841·8°C]

34. A clock which has an uncompensated pendulum made of steel keeps correct time at 15°C. How much it would lose per day at 25°C ? Given, coefficient of linear expansion of steel$=$0·000011/°C. [*Ans.* 4·8 sec. (nearly)]

35. The pendulum of a clock of length l_0 at a temperature t_0 gives the time accurately. The coefficient of linear expansion of the pendulum material is $1{\cdot}85 \times 10^{-5}$ per °C. How much will the clock gain or lose per day if the ambient temperature is 10°C. higher that t_0 ? [*Ans.* The clock loses 8 sec.]

36. A compensated grid-iron pendulum has 5 iron rods each 1 m. long and 4 brass rods. Calculate the length of each brass rod if the coefficient of expansion of iron$=$0·000012/°C. and the coefficient of expansion of brass$=$0·000019/°C. [*Ans.* 0·95 m.]

37. The pendulum of a clock going fast makes 86405 swings per day. At the end of the day, however, the clock loses 10 seconds. Find the change in day temperature. α for the metal of the pendulum$= 16 \times 10^{-6}$/°C. [*Ans.* 21·7°C]

38. The brass scale of a barometer gives correct reading at 0°C. Coefficient of thermal expansion of brass is 0·00002 per °C. The barometer reads 75 cm. at 27°C. What is the atmospheric pressure at 0°C ? (*I. I. T.* '77) [*Ans.* 75·0405 cm.]

39 A brass sphere of 10 cm. radius is heated at the rate of 2·5°C. temperature rise per second. At what rate is it increasing in (i) diameter (ii) surface area and (iii) volume ? Given α for brass $= 18 \times 10^{-6}/°C$.

[*Ans.* (i) 0·0009 cm./sec ; (ii) 0·1131 cm.²/sec. ; (iii) 0·5655 cm.³/sec.]

40. A uniform pressure P is exerted on all sides of a solid cube at temperature t°C. By what amount should the temperature of the cube be raised in order to bring its volume back to the value it had before the pressure was applied ?

Given : The coefficient of volume expansion of the cube is α and the bulk modulus of elasticity is B. (*I. I. T. '78*) $\left[\textit{Ans.}\ \frac{P}{B\alpha} \right]$

[D] Harder Problems

1. The temperature of a cylindrical steel work piece undergoing machining on a turning lathe rises to 60°C. Its diameter at a temperature of 10°C. should be 10 cm. The allowable tolerance on the specified dimension is 100μ. Should a correction be made to the measurements during the machining operation to allow for the thermal expansion of the work piece ? Coefficient of linear expansion for steel is 11×10^{-6} per °C. [*Ans.* No.]

2. An iron wire is stretched inside a rectangular brass frame work, both ends of the wire being rigidly attached to the frame. The length of the wire at 0°C. is 2 metre and the diameter of the wire is 10 mm. What extra tension will be set up in the stretched wire when the temperature of the system is raised to 40°C ?

(Given, the coefficient of linear expansion of iron is 12×10^{-6} per °C., and that of brass is 18×10^{-6} per °C. ; Young's modulus for iron is $2{\cdot}1 \times 10^{12}$ dyne/sq. cm.) [*Ans.* $3{\cdot}954 \times 10^{8}$ dyne.]

5. Two equal bars, 50 cm. long one of brass and the other of iron, are joined together at one end and a needle 1 mm. in diameter and carrying a pointer is clipped between their free ends. When the bars are heated the needle rotates through 10°. What is the temperature interval through which they were heated ? The coefficient of linear expansion of brass and iron are $1{\cdot}8 \times 10^{-5}$ and $1{\cdot}2 \times 10^{-5}$ per °C. respectively. (*Oxford and Cambridge*)

[*Ans.* 29·1°C.]

4. A steel wire is cooled from 60°C. to 0°C. Its length at 60°C is 100 metre. What is its length at 0°C. ? If the diameter of the wire at 60°C. is 5 mm. what is its volume at 0°C. ? (Coefficient of expansion for steel=0·000012/°C) (*I.I.T.* 1963) [*Ans.* 99·928 metre ; 1957 c.c.]

5. A wire 3 metre in length and 1 mm. in diameter at 30°C is kept in a low temperature chamber at —170°C and is stretched by hanging a 10 kg. mass at one end. Calculate the change in the length of the wire. Coefficient of linear expansion$= 1{\cdot}2 \times 10^{-5}/°C$. and Young's modulus$= 1{\cdot}995 \times 10^{12}$ dyne/cm². [*Ans.* 0·188 cm.]

6. A screw of pitch 0·5 mm. is mounted such that its tip can be moved against the end of a rod of length 100 cm. The other end of the rod is fixed. A circular scale attached to the screw has 100 divisions and when rotated fully moves against one small division of a linear pitch scale, each division of length 0·5 mm. At 20°C. the pitch scale reading is a little above zero and the circular scale reading is 92 divisions. When the temperature of the rod is raised to 100°C. the reading on the pitch scale is a little above 4 divisions and the reading of the circular scale is 72 divisions. Calculate the coefficient of linear expansion of the material of the rod. (*I.I.T.* 1964) [*Ans.* $23{\cdot}75 \times 10^{-6}/°C$.]

7. It is required to prepare a steel metre scale such that the millimetre intervals are to be accurate within 0·0005 mm. at a certain temperature. Determine the maximum temperature variation allowable during the ruling of the millimetre mark. (Coefficient of linear expansion of steel$= 13{\cdot}22 \times 10^{-6}$ per/°C.) (*I.I.T.* 1967) [*Ans.* 37·8°C.]

8. Two strips of different metals are riveted together forming a straight bar of thickness 0·1 cm. The temperature is raised by 100°C. What is the radius of curvature of the bent bar ? Given α for the metals $18 \times 10^{-6}/C$ and $12 \times 10^{-6}/°C$. [*Ans.* 167 cm.]

9. A uniform solid brass cylinder of mass 500 gm. and radius 3 cm. is placed on frictionless bearings and rotated with an angular velocity 60 radians/sec about the cylinder axis at 20°C. If now the temperature of the cylinder is increased to 100°C. without mechanical contact, will there be any change in its angular velocity, angular momentum and kinetic energy of rotation ? Explain your answer. Calculate the percentage changes of the above quantities, if any, assuming the mean coefficient of linear expansion of brass to be $2\times10^{-5}/^\circ C$. (*Jt. Entrance'83*)

[*Ans.* yes, no, yes ; 0·32, 0, 0·32]

10. A piece of metal weighs 46 gm. in air. When it is immersed in liquid of specific gravity 1·24 at 27°C, it weighs 30 gm. when the temperature is raised to 42°C, the metal piece weighs 30·5 gm. Specific gravity of the liquid at 42°C is 1·20. Calculate the co-efficient of linear expansion of the metal. (*I.I.T.* '74) [*Ans.* $2{\cdot}31\times10^{-5}/^\circ C$.]

11. A composite rod is made by joining a copper rod, end to end, with a second rod of different material but of the same cross-section. At 25°C the composite rod is 1 metre in length, of which the length of the copper rod in 30 cm. At 125°C the length of the composite rod increases by 1·91 mm.

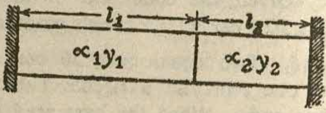
When the composite rod is not allowed to expand by holding it between two rigid walls, it is found that the lengths of the two constituents do not change with the rise of temperature.

Find the Young's Modulus and co-efficient of linear expansion of the second rod.

Given, $\alpha_{cu}=1{\cdot}7\times10^{-5}/^\circ C$; $Y_{cu}=1{\cdot}3\times10^{11}$ Newton/m². (*Jt. Entrance '84*)

[*Ans.* $1{\cdot}105\times10^{11}$ Newton/m² ; $2\times10^{-5}/^\circ C$]

12. Two rods of different materials having the same area of cross-section A, are placed end to end between two massive walls as shown in the Fig. The first rod has a length l_1 coefficient of linear expansion α_1 and Young's Modulus Y_1. The corresponding quantities for the second rod are l_2, α_2 and Y_2. The temperature of both the rods is now raised by T degrees. (i) Find the force with which the rods act on each other (at the higher temperature) in terms of the given quantities (ii) Also find the length of the rods at the higher temperature.



Assume that there is no change in the cross-sectional area of the rods and the rods do not bend. There is no deformation of the rods. (*I.I.T.* '75)

$$\left[\text{Ans. (i) } \frac{ATY_1Y_2(\alpha_1l_1+\alpha_2l_2)}{(l_1Y_2+l_2Y_1)},\ \text{(ii) } l_1\left\{1+\alpha_1T-\frac{Y_2T(\alpha_1l_1+\alpha_2l_2)}{(l_1Y_2+l_2Y_1)}\right\}\right.$$

$$\left.\text{and } l_2\left\{1+\alpha_2T-\frac{Y_1T(\alpha_1l_1+\alpha_2l_2)}{(l_1Y_2+l_2Y_1)}\right\}\right]$$

13. An iron and a copper rod differ in length by 2 cm. at 50°C, as well as at 450°C. Find the lengths of the rods at 0°C. if the co-efficient of linear expansion for iron$=12\times10^{-6}$ deg C^{-1} and that of copper$=17\times10^{-6}$ deg.C^{-1}. (*Jt. Entrance '84*)

[*Ans.* 6·796 cm and 4·796 cm.]

15
CHAPTER

THERMAL EXPANSION OF LIQUIDS

15·1. Expansion of liquids.

Unlike solids, liquids have no definite shape ; they take up the shape of the containing vessel. Their lengths and surface areas are therefore not definite. Thus in the case of liquids, we are mainly concerned with their volume expansion with rise in temperature. Generally, liquids expand much more than solids when they are heated through the same temperature range, the rates of expansion being slightly different at different temperatures. The liquid 'water' exhibits a particular behaviour ; in a certain range of temperature it is found to *contract* with rise in temperature (vide Art. 15·9).

Real and apparent expansion of liquids.

Measurement of the expansion of a liquid is complicated by the fact that when a liquid is heated, the containing vessel is also heated and it expands. For this reason, the observed expansion of a liquid is always less than its real expansion. The observed expansion of liquid is, therefore, apparent. *When the expansion is measured by ignoring the expansion of the containing vessel, it is called the* **apparent expansion of the liquid.** But often it becomes necessary to take into account the effect of the expansion of the container ; *on adding the correction necessary for the expansion of the vessel to the apparent expansion of the liquid, we get the* **real expansion of a liquid.**

The distinction between the real and apparent expansions of a liquid will be clear from the following experiment.

The apparatus consists of a glass flask provided with a rubber stopper and a long glass tube of uniform bore passing through the stopper as shown in the Fig. 15·1. The bore of the tube should be narrow. A scale stands vertically on the side of the tube to record the rise of liquid in it. After filling the flask and the part of the tube upto the mark *O* with a properly coloured liquid, the flask is suddenly immersed in a hot water bath. A sudden drop in the liquid level from *O* to *A* results, followed by a considerable greater steady rise of liquid inside the tube and ultimately the liquid meniscus stands at *B*. The sudden drop in the liquid level is due to the fact since glass is a bad conductor of heat, the liquid inside

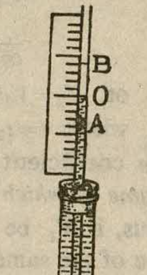


Fig. 15·1 : Expansion of a liquid

the flask does not become immediately heated so that at first only the flask expands in volume. This increase in the volume of the container produces the sudden drop in the liquid level.

An instant later, the liquid gets heated and the steady rise of the liquid level beyond the original indicates that the expansion of the liquid is greater than that of the vessel. Since the initial drop is sudden and short-lived, to a casual observer the liquid appears to expand from O to B. This is why OB is called the observed or apparent expansion. AB, however, is the real (or absolute) expansion of the liquid. Now from the Fig. 15·1,

$$AB=OB+OA$$

$$\text{i.e.}\quad \begin{matrix}\text{Real}\\ \text{expansion}\end{matrix} = \begin{matrix}\text{Apparent}\\ \text{expansion}\end{matrix} + \begin{matrix}\text{Expansion}\\ \text{of the vessel}\end{matrix} \quad \cdots \quad (15{\cdot}1)$$

15·2. Coefficient of real and apparent expansion of liquids.

We have just observed that when temperature rises, both liquid and the containing vessel increase in volume. This phenomenon leads us to define two coefficients viz., real and apparent coefficients of expansion of liquids.

The **coefficient of apparent** (*or observed*) **expansion** *of a liquid is the fraction of its volume by which the liquid appears to expand per degree rise in temperature.*

Thus if V_1 be the volume of a given mass of liquid at $t_1°$ and V_2 be its *apparent* volume at $t_2°(t_2>t_1)$, then the apparent fractional increase in the volume of the liquid$=\dfrac{V_2-V_1}{V_1}$

∴ The coefficient of apparent expansion of the liquid denoted by γ_a is given by

$$\gamma_a=\frac{V_2-V_1}{V_1(t_2-t_1)} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (15{\cdot}2)$$

$$=\frac{\text{apparent increase in volume}}{\text{original volume}\times\text{rise in temperature}}$$

$$\text{or,}\ V_2=V_1(1+\gamma_a t) \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (15{\cdot}3)$$

where $t=t_2-t_1$

The **coefficient of real** (*or absolute*) **expansion** *of a liquid is the fraction of its volume by which it actually expands per degree rise in temperature.*

Thus, if V_1 be the volume of a given mass of a liquid at $t_1°$ and V_2 the *real* volume of the same mass at $t_2°$, then γ_r the real coefficient of expansion of the liquid is given by

$$\gamma_r=\frac{V_2-V_1}{V_1(t_2-t_1)} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (15{\cdot}4)$$

$$=\frac{\text{real increase in volume}}{\text{original volume}\times\text{rise in temperature}}$$

$$\text{or,}\ V_2=V_1(1+\gamma_r t) \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (15{\cdot}5)$$

where $t=t_2-t_1$

As in the case of expansion of solids, the units of expansion coefficients of liquids depend only on the units of temperature, and not on the units of volume, because volumes appear in both the numerator and denominator of the expressions (15·2) and (15·4). Thus the value of γ_a or γ_r obtained in the Fahrenheit scale is $\frac{5}{9}$ times of the corresponding value in the Centigrade scale. The value of γ_r is a constant for a particular liquid at a particular temperature but the value of γ_a depends on the coefficient of the expansion of the material of the containing vessel.

In these definitions of coefficient of expansion of a liquid, the initial volume was taken to be the volume at any temperature in any part of the scale of temperature. Since the value of γ is very small, the error which creeps in is negligible. However, for very accurate work, we should alway refer to the initial volume of the liquid at 0°C.

Also in these discussion we have tacitly assumed that value of γ remains constant over different ranges of temperature. This is not true exactly and the value of γ is found to vary with temperature. The values defined by the above equations give really the mean values of γ between t_1° and t_2°. The variation is, however, so small that in practice it is neglected without any appreciable error.

15·3. Relation between expansion coefficient of liquids.

In obtaining the relation between the real and apparent coefficients of a liquid, we make use of the experimental fact that a hollow container expands just as if the container is solid.

Let V_o be the initial volume of the portion of the container occupied by a liquid at 0° and V_t be its final volume at temperature t°. The volume of the liquid is obviously V_o at 0°. If the observed volume of the liquid at temperature t° be V_a and its real volume be V_r, then,

apparent expansion of the liquid $= V_a - V_o$,
real expansion of the liquid $= V_r - V_o$, and
expansion of the container $= V_t - V_o$.

Hence, from eqn. (15·1) $(V_r - V_o) = (V_a - V_o) + (V_t - V_o)$

$$\text{or,}\quad V_r - V_a = V_t - V_o \qquad \cdots \qquad (15{\cdot}6)$$

Now, from eqn. (15·2), we get,
$$\gamma_a = \frac{V_a - V_o}{V_o t} \qquad \cdots \qquad (15{\cdot}7)$$

and from eqn. (15·4),
$$\gamma_r = \frac{V_r - V_o}{V_o t} \qquad \cdots \qquad (15{\cdot}8)$$

Subtracting (15·7) from (15·8), we get
$$\gamma_r - \gamma_a = \frac{V_r - V_a}{V_o . t} = \frac{V_t - V_o}{V_o . t} \qquad \text{[From 15·6]}$$
$$= \gamma_g$$
where γ_a represents the coefficient of volume expansion of the material of the container.

$$\therefore \quad \gamma_r = \gamma_a + \gamma_g \qquad \cdots \qquad \cdots \qquad (15{\cdot}9)$$

Thus

Coeff. of real expansion of a liquid = Coeff. of apparent expansion of the liquid + Coeff. of volume expansion of the material of the vessel

Alternative Proof.

Let us suppose that the liquid is contained in a graduated vessel. The volume occupied by the liquid can then be read off directly from the graduations which give correct readings, say, at 0°. At other temperatures, corrections will have to be made on account of the expansion of the vessel.

Let the volume of the liquid at 0° be indicated by the graduations as V_o. If the temperature be increase to t°, let the volume of the liquid as indicated by the graduated scale be V_1. This must be its apparent volume ; thus,

$$V_1 = V_o(1+\gamma_a t) \quad \cdots \quad (15\cdot10)$$

where γ_a is the coefficient of apparent expansion of the liquid.

Due to temperature rise, the volume of the vessel itself has expanded. Therefore, the volume V_1 as indicated by the graduated scale is actually equal to $V_1(1+\gamma_g t)$, where γ_g is the coefficient of volume expansion of the material of the vessel. So the real volume occupied by the liquid at temperature t is this volume, $V_1(1+\gamma_g t)$. If γ_r be the coefficient of real expansion of the liquid, then this is also equal to $V_o(1+\gamma_r t)$

$$\therefore \quad V_1(1+\gamma_g t) = V_o(1+\gamma_r t) \quad \cdots \quad (15\cdot11)$$

Substituting for V_1 from (15·10) in (15·11) and cancelling the common factor V_o, we get $(1+\gamma_a t)(1+\gamma_g t) = (1+\gamma_r t)$ or, $\gamma_r = \gamma_a + \gamma_g + \gamma_a \gamma_g t$

But the last term involving the product of two small terms γ_a and γ_g is too small to be considered and therefore,

$$\gamma_r = \gamma_a + \gamma_g$$

similar to the relation expressed in (15·9).

15·4. Variation of density of a liquid with temperature.

With rise in temperature the volume of a given mass of liquid increases and hence the density of the liquid decreases. A simple relation exists between the change in density of liquid with temperature and its real coefficient of expansion γ_r. This relation is useful in direct experimental determination of the value of γ_r.

Let the real volume and density of a given mass m of a liquid at t_1° be V_1 and ρ_1 respectively. The corresponding values for the same mass of the liquid at a higher temperature t_2° are V_2 and ρ_2 respectively. Then $m = V_1\rho_1 = V_2\rho_2$.

$$\therefore \quad \frac{\rho_1}{\rho_2} = \frac{V_2}{V_1}$$

But from the equation (15·5), $V_2 = V_1(1+\gamma_r t)$ where $t = t_2 - t_1$

$$\therefore \quad \rho_1 = \rho_2(1+\gamma_r t) \quad \cdots \quad \cdots \quad (15\cdot12)$$

Since the value of γ_r is small, we can write,

$$\rho_2 = \rho_1(1+\gamma_r t)^{-1} = \rho_1(1-\gamma_r t) \text{ approximately} \quad \cdots \quad \cdots \quad (15\cdot13)$$

Thus, in general, the density of a liquid is greater at a lower temperature than at a higher one.

Example 15·1. *A glass vessel is completely filled with 60 c.c. of mercury at 20°C. What volume of mercury will flow out of the vessel if the temperature is raised to 40°C. ? Given that the coefficient of linear expansion of glass* $= 9 \times 10^{-6}$ *per°C. and the coefficient of volume expansion of mercury* $= 18 \times 10^{-5}$ *per°C.*

Solution : The coefficient of volume expansion of glass

$$\gamma_g = 3\times 9\times 10^{-6} \text{ per}^\circ\text{C.}$$

Let V_1 be the initial volume of the vessel at 20°C. and γ_r and γ_g be the coefficient of volume expansion (real) of the liquid and of vessel respectively.

Now, volume of mercury which overflows = volume expansion (real) of mercury—volume expansion of the vessel.

$$= V_1\times\gamma_r\times\text{rise in temp.} - V_1\times\gamma_g\times\text{rise in temp.}$$
$$= (\gamma_r-\gamma_g)\times V_1\times\text{rise in temp.}$$
$$= (18\times 10^{-5} - 3\times 9\times 10^{-6})\times 60\times 20 = 0{\cdot}184 \text{ c.c.}$$

Example 15·2. *The coefficient of apparent expansion of a liquid is* $18\times 10^{-5}/^\circ C.$ *when an iron vessel is used and* $14{\cdot}46\times 10^{-5}/^\circ C.$ *when an aluminium vessel is used. If the coefficient of linear expansion of iron be* $12\times 10^{-6}/^\circ C.$, *find that of aluminium.*

Solution : The coefficient of volume expansion of iron

$$= 3\times 12\times 10^{-6} = 3{\cdot}6\times 10^{-5}/^\circ\text{C.}$$

The coefficient of real expansion of the liquid is given by

$$\gamma_r = 18\times 10^{-5} + 3{\cdot}6\times 10^{-5} = 21{\cdot}6\times 10^{-5}/^\circ\text{C.}$$

If the coefficient of linear expansion of aluminium be α, then

$$21{\cdot}6\times 10^{-5} = 14{\cdot}46\times 10^{-5} + 3\alpha$$

$$\therefore \quad \alpha = \frac{7{\cdot}14}{3}\times 10^{-5} = 2{\cdot}38\times 10^{-5}/^\circ\text{C.}$$

Example 15·3. *A capillary glass tube of uniform bore contains a thread of mercury one metre long at 0°C. When temperature is to 100°C., the thread is found to be 1·53 cm. longer. If the coefficient of real expansion of mercury be* $18\times 10^{-5}/^\circ C.$, *calculate the coefficient of linear expansion of glass.*

Solution : If the glass tube be of cross-sectional area a, the volume of the mercury thread at 0°C. is $V_o = (a\times 100)$ c.c. and at 100°C. is $V_{100} = (a\times 101{\cdot}53)$ c.c.

From eqn. (15·2), the coefficient of apparent expansion of mercury is

$$\gamma_a = \frac{V_t - V_o}{V_o\times 100} = \frac{a(101{\cdot}53-100)}{a\times 100\times 100} = 15{\cdot}3\times 10^{-5} \text{ per}^\circ\text{C.}$$

Now from eqn. (15·9), we have $18\times 10^{-5} = 15{\cdot}3\times 10^{-5} + \gamma_g$ where γ_g is the coefficient of volume expansion of glass.

$$\therefore \quad \gamma_g = 2{\cdot}7\times 10^{-5} \text{ per}^\circ\text{C.}$$

Hence the coefficient of linear expansion of glass.

$$= \frac{\gamma_g}{3} = 9\times 10^{-6} \text{ per}^\circ\text{C.}$$

Example 15·4. *The bulb and the stem of a mercury-in-glass thermometer contain up to the zero mark 1 c.c. of mercury. If the internal diameter of the stem be 0·3 mm., what is the length of a degree centigrade on the scale ? [Coefficient of real expansion of mercury*$=18\times 10^{-5}/^\circ C.$ *and coefficient of linear expansion of glass*$=8\times 10^{-6}/^\circ C.$

Solution : Coefficient of cubical expansion of glass $= 3\times 8\times 10^{-6}$

$$= 2{\cdot}4\times 10^{-5}/^\circ\text{C.}$$

$\therefore$ Coefficient of apparent expansion of mercury

$$= 18\times 10^{-5} - 2{\cdot}4\times 10^{-5} = 15{\cdot}6\times 10^{-5}/^\circ\text{C.}$$

Now, volume of mercury at 0°C. $=1$ c.c. Expansion of 1 c.c. of mercury at 0°C., when heated to 1°C. $=1\times 15{\cdot}6\times 10^{-5}\times 1$ c.c. $=15{\cdot}6\times 10^{-5}$ c.c. Hence the volume of stem between 0°C. mark and 1°C. mark $=15{\cdot}6\times 10^{-5}$ c.c.

Since, the internal cross-section of the stem $=\pi(0{\cdot}03/2)^2=70{\cdot}65\times 10^{-5}$ sq. cm.

∴ Length of a degree centigrade on the scale

$$=\frac{15{\cdot}6\times 10^{-5}}{70{\cdot}65+10^{-5}}=0{\cdot}221 \text{ cm.}=2{\cdot}21 \text{ mm.}$$

Example 15·5. *Show how to preserve the free volume in a glass vessel at a value independent of temperature by inclusion of an appropriate amount of mercury. Assume the coefficient of linear expansion of the glass to be 8×10^{-6} per°C. and the coefficient of cubical expansion of mercury to be $1{\cdot}82\times 10^{-4}$ per°C.*

Solution : Let the volumes of the glass vessel and of the mercury at any arbitrary temperature be V_o and v_o respectively. If t be the rise in temperature, and the vessel has volume V and of this v is occupied by mercury, then the space available at temperature t is

$$V-v=V_o(1+3\times 8\times 10^{-6}t)-v_o(1\times 1{\cdot}82\times 10^{-4}t)$$
$$=(V_o-v_o)+(V_o\times 24\times 10^{-6}t-v_o\times 1{\cdot}82\times 10^{-4}t)$$

In order that this free volume remains unchanged, we have the condition

$$V-v=V_o-v_o$$
$$\therefore\quad V_o\times 24\times 10^{-6}t-v_o\times 1{\cdot}82\times 10^{-4}t=0$$
$$\text{or,}\quad \frac{v_o}{V_o}=\frac{24}{182}=0{\cdot}13$$

Thus the mercury must occupy 0·13th of the volume of the glass vessel.

Example 15·6. *Calculate the density of mercury at 70°C., given the density at 0°C. is 13·6 gm./c.c and the coefficient of real expansion of mercury is $18{\cdot}2\times 10^{-5}$/°C.*

Solution : Here ρ_1=density at 0°C. $=13{\cdot}6$ gm./c.c., ρ_2=density at 70°C., $t=(70-0)=70$°C., and $\gamma_r=18{\cdot}2\times 10^{-5}$/°C.

Hence from eqn. (15·12), $\rho_1=\rho_2(1+\gamma_r t)$.

$$\text{or,}\quad \rho_2=\frac{13{\cdot}6}{1+18{\cdot}2\times 10^{-5}\times 70}=13{\cdot}43 \text{ gm./c.c. (nearly)}$$

Example 15·7. *The density of mercury at 15°C. is 13·55 gm. per c.c. At what higher temperature will this figure be in error by 2 percent ? Given, the coefficient of real expansion of mercury is $18{\cdot}2\times 10^{-5}$/°C.*

Solution : Let us suppose that the required temperature be t°C. At this higher temperature, the density of mercury is less than 13·55 gm./c.c. by 2%.

$$\therefore\quad \text{Density } \rho_t \text{ at } t\text{°C.}=13{\cdot}55-\tfrac{2}{100}\times 13{\cdot}55$$
$$=13{\cdot}55-0{\cdot}271=13{\cdot}279 \text{ gm./c.c.}$$

But $\rho_{15}=\rho_t\,[1+\gamma_r(t-15)]$

or, $13{\cdot}55=13{\cdot}279\,[1+18{\cdot}2\times 10^{-5}(t-15)]$

$$\therefore\quad t-15=\frac{0{\cdot}271}{13{\cdot}279\times 18{\cdot}2\times 10^{-5}}=112{\cdot}1$$

$$\therefore\quad r=127{\cdot}1\text{°C.}$$

15·5. Determination of the coefficient of apparent expansion of a liquid.

The coefficient of apparent expansion of a liquid may be determined with the help of a weight thermometer. This consists of a small glass bulb fitted with a bent capillary tube as shown in the figure 15·2.

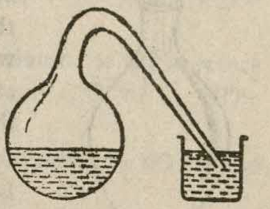


Fig. 15·2 : Weight thermometer

The weight thermometer is first weighed empty. The open end of the capillary tube is immersed in the experimental liquid and the bulb of the thermometer is gently heated. The air inside will expand and therefore a part of it leaves the vessel. On cooling, some liquid will be drawn into the bulb. A few alternate heating and cooling will be sufficient to completely fill the thermometer.

Keeping the open end of the capillary tube dipped into the liquid, the thermometer is allowed to return to the room temperature, say t_1°C. Its outside is dried and thermometer is then weighed. The thermometer is now raised to some temperature t_2°C. by placing it in water maintained at this temperature. This causes the liquid to expand and a portion of the liquid comes out of the thermometer. This excess liquid is removed. Obviously the thermometer is now completely filled up with the liquid at temperature t_2°C. The thermometer is then cooled and weighed again.

Let, the mass of the empty weight thermometer $= m_1$ gm.

the mass of the completely filled up weight thermometer at t_1°C $= m_2$ gm.

the mass of the completely filled up weight thermometer at t_2°C. $= m_3$ gm.

$\therefore\ m_2 - m_1 = M_1$ (say) = Mass of liquid filling the thermometer at t_1°C.

$m_3 - m_1 = M_2$ (say) = Mass of liquid filling the thermometer at t_2°C.

$\therefore\ M_1 - M_2 =$ mass of the expelled liquid.

Let the volume of the weight thermometer be V c.c. *If we ignore the expansion of the thermometer,* then we say that

$V =$ Volume of M_1 gm. of liquid at t_1°C.

$=$ Volume of M_2 gm. of liquid at t_2°C.

$\therefore$ Volume of M_1 gm. of liquid at t_2°C. $= \dfrac{V}{M_2} \times M_1$

Hence the coefficient of apparent expansion of liquid is given by

$$\gamma_a = \frac{\frac{VM_1}{M_2} - V}{V(t_2 - t_1)} = \frac{M_1 - M_2}{M_2(t_2 - t_1)} \quad \ldots \quad \ldots \quad (15\cdot14)$$

$$= \frac{\text{mass of the expelled liquid}}{\text{mass of liquid remaining at } t_2\text{°C.} + \text{rise in temperature}}$$

It should be noted that the coefficient of apparent expansion is determined in this method by measuring weights instead of volumes. Hence the name weight thermometer. The term 'thermometer' is due to the fact that if γ_a is known, the method can be employed for determination of unknown temperatures.

We can use a specific gravity bottle as a weight thermometer to determine the coefficient of apparent expansion. It consists of a small flat bottomed glass bottle provided with a tightly fitting ground glass stopper [Fig. 15·3]. A capillary bore runs centrally through the stopper. If the bottle is completely filled up with a liquid, then with the insertion of the stopper, the excess liquid will escape through this bore. The bottle is, thereby, filled up with a definite volume of the liquid.

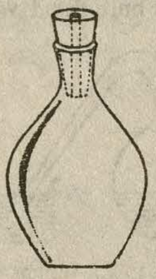


Fig. 15·3 : Specific gravity bottle

The empty dry bottle with its stopper is first weighed. Let this be m_1 gm. The bottle is then completely filled up with the experimental liquid at room temperature (t_1°C.). No air bubble should remain inside. The outer surface of the bottle is dried and its weight (m_2 gm.) is determined. The bottle is then dipped in water maintained at a higher temperature, say t_2°C. The liquid inside will expand. Some liquid will therefore be expelled from the bottle through the bore in the stopper. When the process is completed, the bottle is cooled. Again the outer surface of the bottle is dried and its weight (m_3 gm.) is measured. The value of γ_a is then calculated as above from eqn. (15·14).

15·6. Determination of the coefficient of real expansion of a liquid.

The coefficient of real expansion of a liquid may be obtained by adding the coefficient of volume expansion of the material of the container with the coefficient of apparent expansion of the liquid.

Dulong and Petit developed a method in which the expansion of the container has no effect on the observations and the coefficient of real expansion of the liquid is obtained directly.

Dulong and Petit's method : The liquid is taken in a U tube $ABCD$ bent in the form shown in the figure 15·4. One of the arm, say AB, passes through a jacket J_1 through which ice-cold water is circulated. The other arm CD is surrounded by the jacket J_2 through which steam is passed. The thermometers T_1 and T_2 are inserted into the jackets J_1 and J_2 respectively and indicate their temperatures. The horizontal part BC of the tube is covered with wet blotting paper to minimise the transmission of heat along it.

The liquid in the arm CD being at a higher temperature than that in the arm AB, the density of the latter is greater than the former. Hence at steady state, the liquid columns in the two arms will be of different heights.

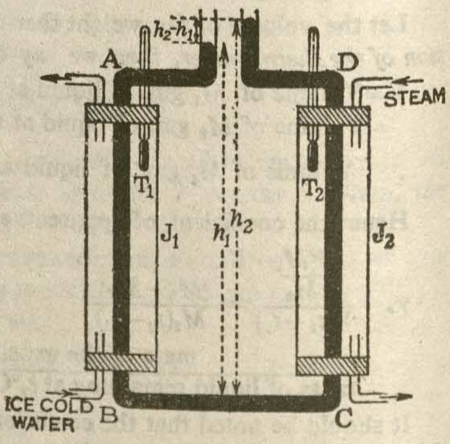


Fig. 15·4 : Dulong and Petit's apparatus

The liquid in the arm *CD* being lighter will stand at a greater height than that in *AB*. With the help of a cathetometer telescope moving along a vertical scale, this difference in height and the height of the liquid column at lower temperature are measured. The coefficient of real expansion (γ_r) of the liquid can thereby be obtained as follows :

At steady state, let the height of the cold liquid column at temperature t_1°C. be h_1 and that of the hot column at temperature t_2°C. be h_2. The pressures exerted by these columns must be equal.

If ρ_1 is the density of the liquid in the cold column and ρ_2 is that of the hot column, we have

$$h_1\rho_1 g = h_2\rho_2 g$$

where g is the acceleration due to gravity.

$$\therefore \quad h_1\rho_1 = h_2\rho_2$$

But $\rho_1 = \rho_2[1+\gamma_r(t_2-t_1)]$

Hence $h_2 = h_1[1+\gamma_r(t_2-t_1)]$

$$\text{or,} \quad \gamma_r = \frac{h_2-h_1}{h_1(t_2-t_1)} \quad \cdots \quad \cdots \quad (15{\cdot}15)$$

$$= \frac{\text{Difference in height}}{\text{Height of the cold column of liquid} \times \text{Difference of temperature}}$$

All other quantities being known, γ_r can be calculated out.

It should be noted that upper parts of the tubes were bent inwards so as to bring the upper surfaces of the columns close together. This enabled the direct measurement of the difference in height.

Example 15·8. *A specific gravity bottle weighs 25 gm. when empty and 64·65 gm. when full of alcohol at 0°C. It was placed in a water-bath at 40°C., then removed and allowed to cool. When cool it was carefully wiped and again weighed ; its weight was found to be 62·82 gm. Find the coefficient of apparent expansion of alcohol. If the coefficient of cubical expansion of glass is* $2{\cdot}5\times 10^{-5}$ *per °C., what is the real coefficient of expansion of alcohol ?*

Solution : Weight of alcohol at 0°C. to fill the bottle completely

$$=64{\cdot}65-25=39{\cdot}65 \text{ gm.}$$

Weight of alcohol at 40°C. to fill the bottle completely

$$=62{\cdot}82-25=37{\cdot}82 \text{ gm.}$$

$\therefore$ Weight of alcohol expelled $=39{\cdot}65-37{\cdot}82=1{\cdot}83$ gm.

Rise in temperature $=40$°C.

Thus, from the relation (15·14) we have for the coefficient of apparent expansion of alcohol,

$$\gamma_a = \frac{1{\cdot}83}{37{\cdot}82\times 40} = 12{\cdot}09\times 10^{-4} \text{ per °C.}$$

Again, from the relation (15·9), the coefficient of real expansion of alcohol,

$$\gamma_r = 12{\cdot}09\times 10^{-4} + 0{\cdot}25\times 10^{-4}$$
$$= 12{\cdot}34\times 10^{-4} \text{ per °C.}$$

Example 15·9. *A weight thermometer contains 85 gm. of liquid at 20°C. and on heating it to 95°C., 5 gm. of the liquid is expelled. Find the coefficient of real expansion of the liquid, if the coefficient of linear expansion of glass is* $0{\cdot}9\times10^{-5}$ *per °C.*

Solution : Mass of liquid expelled = 5 gm.
Mass of liquid left = 85 − 5 = 80 gm.
Rise in temperature = 95 − 20 = 75°C.

Hence, from the relation (15·14), we have the coefficient of apparent expansion of liquid.

$$\gamma_a=\frac{5}{80\times75}=0{\cdot}000833 \text{ per °C.}$$

Also, from the relation (15·9),

$$\gamma_r=8{\cdot}33\times10^{-4}+3\times0{\cdot}9\times10^{-5}$$
$$=8{\cdot}33\times10^{-4}+0{\cdot}27\times10^{-4}=8{\cdot}6\times10^{-4} \text{ per °C.}$$

Example 15·10. *A weight thermometer contains 200 gm. of mercury at 20°C. When placed in an oil bath, 5 gm. is found to overflow. Find the temperature of the bath. Coefficients of cubical expansion of mercury and glass are* $1{\cdot}8\times10^{-4}$ *per °C. and* $2{\cdot}6\times10^{-5}$ *per °C. respectively.*

Solution : Mass of liquid expelled = 5 gm.
Mass of liquid left = 200 − 5 = 195 gm.
Let the temperature of the bath = t°C.

∴ Rise in temperature = $(t-20)$°C.

∴ The coefficient of apparent expansion of mercury as obtained from the relation (15·14) is

$$\gamma_a=\frac{5}{195\times(t-20)}$$

But, $\gamma_a=1{\cdot}8\times10^{-4}-0{\cdot}26\times10^{-4}=1{\cdot}54\times10^{-4}/°\text{C}.$

$$\therefore\quad 1{\cdot}54\times10^{-4}=\frac{5}{195\times(t-20)}\qquad\text{or, } t=186{\cdot}5°\text{C}.$$

Example 15·11. *A U-tube contains a liquid, its limbs being at 0°C. and 100°C. respectively. If the height of the cold and the hot columns be 50 cm. and 52 cm. respectively, what would be the coefficient of real expansion of the liquid ?*

Solution : Difference in the heights of the columns = 52 − 50 = 2 cm.
Difference in temperature = 100 − 0 = 100°C.

$$\therefore\quad \gamma_r=\frac{2}{50\times100}=4\times10^{-4}/°\text{C}.$$

Example 15·12 *In an experiment performed by Dulong and Petit's method, the heights of the cold and hot columns of mercury are found to be 90 cm., and 91·6 cm. respectively. If the first column is at 1°C., find the temperature of the other. Given coefficient of real expansion of mercury is* 18×10^{-5} *per °C.*

Solution : Let t°C. be the required temperature. Then we have, the difference in the heights of two columns = 91·6 − 90 = 1·6 cm., the height of the cold column = 90 cm.

$$\gamma_r=18\times10^{-5} \text{ per °C.}$$

Hence, from the relation (15·15), we have,

$$18\times10^{-5}=\frac{1\cdot6}{90\times(t-1)}$$

$$\text{or,}\quad t-1=\frac{1\cdot6}{90\times18\times10^{-5}}=98\cdot8$$

$$\therefore\quad t=99\cdot8°\text{C}.$$

15·7. Apparent loss of weight of a solid immersed in a liquid at different temperatures.

Let the weight of a solid in air be W. When immersed completely in a liquid at temperature t_1°C., let its weight be W . Then the apparent loss of weight of the body is

$$W-W_1=M_1 \text{ (say)}.$$

If V_1 be the volume of the solid at temperature t_1°C and ρ_1 be the density of the liquid, then from Archimedes' principle

$$M_1=V_1\rho_1$$

If the weight of the immersed solid when the temperature of the liquid is t_2°C. ($t_2>t_1$) be W_2, then similarly

$$W-W_2=M_2$$

$$\text{and}\quad M_2=V_2\rho_2$$

where V_2 and ρ_2 are the volume of the solid and the density of the liquid at t_2°C. If γ_r be the coefficient of real expansion of the liquid and γ be the coefficient of volume expansion of the solid, then putting $t_2-t_1=t$, we have

$$\rho_1=\rho_2\,(1+\gamma_r t)\text{ and }V_2=V_1\,(1+\gamma t)$$

$$\begin{aligned}\therefore\quad M_2&=V_1\rho_1\,(1+\gamma t)\,(1+\gamma_r t)^{-1}\\&=M_1(1+\gamma t))1-\gamma_r t)\\&=M_1[1-(\gamma_r-\gamma)t]\qquad\cdots\qquad\cdots\qquad(15\cdot16)\end{aligned}$$

Hence, $M_1>M_2$ i.e., $W_1<W_2$. Thus the apparent weight of an immersed solid increases with rise in temperature of the liquid.

15·8. Temperature correction for barometer reading.

We have mentioned earlier that the reading of a barometer should be corrected for temperature. We shall now explain how this is done.

Let us suppose that a barometer is read at t°C. and the barometric height as measured from the scale be H cm. *True barometric height* is given by the height of a mercury column at 0°C. which would exert the same pressure as the observed column.

To find the true barometric height, corrections should be introduced due to (i) the expansion of the scale and (ii) expansion of mercury.

(i) *Correction for expansion of scale* : Let us suppose the scale is graduated at 0°C. Each division of the scale will then be exactly 1 cm. at 0°C. But at t°C., the length of a scale division will be $(1+\alpha t)$, where α is the coefficient of linear expansion of the material of the scale (usually brass). Hence the correct height of the mercury column is

$$h=H(1+\alpha t)$$

(ii) *Correction for expansion of mercury :* If ρ be the density of mercury at t°C. then the pressure exerted by the column of mercury in the barometer is $h\rho g$, where g is the acceleration due to gravity.

If H_o be the true barometric height and ρ_o be the density of mercury at 0°C., then from definition,

$$H_o\rho_o g=h\rho g$$

$$\text{or,}\quad H_o=h\frac{\rho}{\rho_o}$$

If γ be the coefficient of real expansion of mercury, then

$$\rho_o=\rho(1+\gamma t)$$

$$\therefore\quad H_o=\frac{h}{1+\gamma t}$$

$$=H\frac{1+\alpha t}{1+\gamma t}\qquad\ldots\qquad\ldots\qquad(15·17)$$

which is the required expression.

The expression can be simplified as follows :

$$H_o=H(1+\alpha t)(1+\gamma t)^{-1}$$

$$=H(1+\alpha t)(1-\gamma t)\text{ neglecting higher order terms.}$$

$$=H[1-(\gamma-\alpha)t]\text{ neglecting }\alpha\gamma t^2\qquad\ldots\qquad(15·18)$$

Example 15·13. *A metallic bob weighs 50 gm. in air. If it is immersed in a liquid at a temperature of 25°C. it weighs 45 gram. When the temperature of the liquid is raised to 100°C., it weighs 45·1 gram. Calculate the coefficient of cubical expansion of the liquid assuming the linear expansion of the metal to be* 12×10^{-6} *per °C.*

[*I.I.T. 1973*]

Solution : Apparent loss in weight at 25°C $=M_1=50-45=5$ gm.
Apparent loss in weight at 100°C. $=M_2=50-45·1=4·9$ gm.
$t_2-t_1=100-25=75$°C.

$$\gamma=3\times12\times10^{-6}=36\times10^{-6}\text{ per°C.}$$

Hence from the relation (15·16),

$$4·9=5[1-(\gamma_r-36\times10^{-6})75]$$

$$\text{or,}\quad \gamma_r-36\times10^{-6}=\frac{5-4·9}{5\times75}=\frac{0·1}{5\times75}=2·67\times10^{-4}$$

$$\therefore\quad \gamma_r=3·03\times10^{-4}/°\text{C}.$$

Example 15·14. *The brass scale of a mercury barometer was graduated at 0°C. At a temperature of 20°C., the barometer indicated a pressure of 765 mm. Find the true barometric height. The coefficient of linear expansion of brass is* $19\times10^{-6}/C.$ *and the coefficient of cubical expansion of mercury is* $18\times10^{-5}/°C.$

Solution : From the relation (15·18),

$$H_o=H[1-(\gamma-\alpha)t]$$

We have,

$$H_o=765[1-(18-1·9)\times10^{-5}\times20]$$

$$=765-765\times16·1\times2\times10^{-4}$$

$$=765-2·46=762·54\text{ mm.}$$

15·9. Anomalous expansion of water.

In general, liquids expand when they are heated. We have also observed in the preceding articles that volumes of liquids increase and hence their densities decrease with increasing temperature. But the behaviour of water is rather peculiar within a certain range of temperature. If water is cooled from room temperature, it contracts fairly uniformly until the temperature reaches 4°C. ; below this temperature instead of contraction, water expands with fall in temperature till 0°C., when it freezes into ice. This phenomenon is called *the anomalous expansion of water*. Thus, *the volume of a given mass of water is least at 4°C. and hence its density is greatest at this temperature.* Accurate determination gave the value of this temperature of maximum density of water at a pressure of one atmosphere as 3·98°C. With change of pressure this temperature also slightly changes. Fig. 15·5 illustrates the variation of density of water with temperature between 0°C. and 10°C.

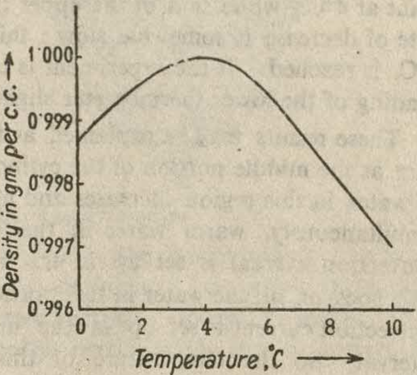


Fig. 15·5 : Variation of density of water with temperature

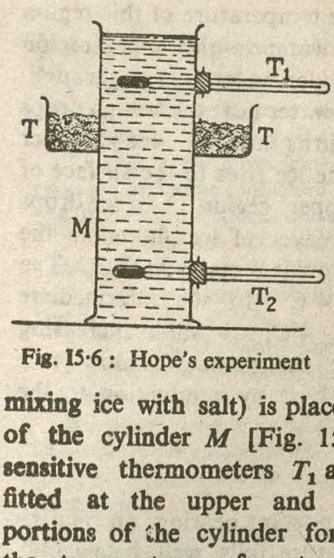


Fig. 15·6 : Hope's experiment

Demonstration experiment. The simple but classic experiment to demonstrate this maximum density of water was devised by Thomas Charles Hope and is commonly known as Hope's experiment after him. A hollow glass cylinder *M* is almost filled up with water. Freezing mixture (prepared by mixing ice with salt) is placed in an annular trough *T* round the middle portion of the cylinder *M* [Fig. 15·6]. Two sensitive thermometers T_1 and T_2 are fitted at the upper and the lower portions of the cylinder for recording the temperature of water in these regions. Readings of both thermometers are taken before placing the freezing mixture and are found to be the same. On introducing the freezing mixture, the readings of both of them are recorded at regular intervals. Graphs of temperature against time are then plotted for readings of both T_1 and T_2. The nature of the graphs thus obtained are shown in the Fig. 15·7. The

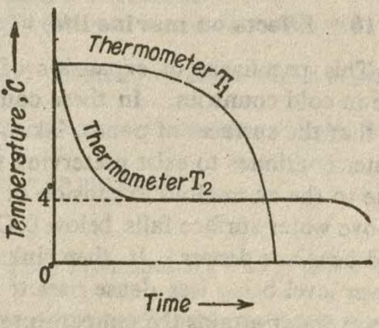


Fig. 15·7 : Thermometer readings in Hope's experiment graphs show that at first the reading of lower thermometer falls till it reaches about 4°C. ; the reading of the upper thermometer at the same time remains almost unaffected. The reading of the lower thermometer then remains constant at 4°C., while that of the upper thermometer begins to fall. Initially the rate of decrease is somewhat slow ; this however quickly becomes rapid until 0°C. is reached. If the experiment is continued for a sufficiently long time, the reading of the lower thermometer slightly falls from 4°C.

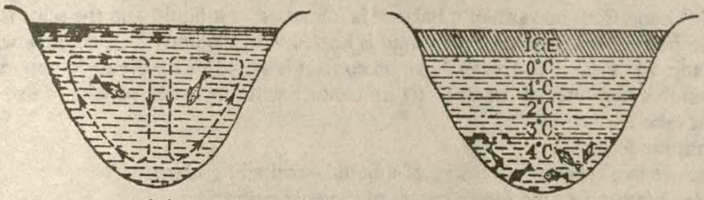
These results may be explained as follows. Due to the lowering of temperature at the middle portion of the cylinder near the freezing mixture, the density of water in this region increases and hence this cold water sinks to the bottom. Simultaneously, warm water in the bottom of the vessel rises up ; thus a convection current is set up in the region of the cylinder below the trough. This goes on till the water in this portion reaches the temperature of 4°C. No convection current is set up in the upper region of the cylinder during this interval. So the temperature of this region remains practically unchanged. Further cooling of the layer of water near the trough below 4°C reduces its density. This layer thus becomes less dense than the water below it and hence would not sink to the bottom any further. So the convection current in the lower region of the cylinder gets stopped and the temperature of this region remains fixed at 4°C. However the temperature of water in the upper region decreases slowly due to thermal contact with the colder water near the trough. With still more cooling, the temperature of the layer of water near the trough drops to 0°C and after sometime, the water of this region turns into ice. Ice is lighter than water. Hence immediately after formation, the ice rises to the surface of water. From now on, the temperature of the upper region of water drops quickly and eventually gets fixed at 0°C. So finally a layer of ice floats at the top and the temperature of water immediately below it remains at 0°C. The water at the bottom of the cylinder remains still at 4°C. In the intermediate region, the temperature remains between 0°C and 4°C, its value increasing downwards. The fall in the reading of the lower thermometer below 4°C. after sufficiently long time takes place due to loss of heat by conduction to the upper regions.

15·10. Effects on marine life.

This peculiarity of expansion of water has an important bearing on marine life in cold countries. In these countries the temperature falls well below 0°C. so that the surfaces of ponds, lakes, rivers etc. freeze, while below the surface, water continues to exist preserving the marine life. This phenomenon happens due to the anomalous expansion of water. As temperature of the surroundings above water surface falls below 0°C. the water layer in the surface also cools and becomes denser. It then sinks to the bottom while the warm water in the lower level being less dense rises to the surface. This process continues till the entire water attains the temperature of 4°C. [Fig. 15·8 (*a*)].

On further lowering of the temperature at the surface, the uppermost layer of water becomes lighter and hence remains at the top till it freezes into ice.

Since ice is lighter than water, the layer of ice so formed also remains at the top Moreover, both ice and water being bad conductors of heat, heat lost due to



(a) (b)

Fig. 15·8 : Marine life in cold countries

conduction by water below the layer of ice to the surroundings at a lower temperature will be very small. Water thus continues to exist below the surface layer of ice and the temperature of water increases slowly downwards from 0°C. to 4°C. [Fig. 15·8 (*b*)]. Consequently, marine life is able to survive the severe winter of these countries.

• EXERCISE •

[A] Essay-type questions

1. Distinguish between real and apparent expansions of a liquid. Establish a relation between them and the expansion of the vessel which contains the liquid.

Describe a simple method of determining the coefficient of apparent expansion of a liquid.

2. Define the coefficient of real and apparent expansion of a liquid. Why are these two types of coefficients considered ? On which factors do their numerical values depend ? Which one of these is a characteristic of the liquid ? Find the relation between them

3. Explain the statement—"The coefficient of real expansion of mercury is 0·00018 per °C. Describe Dulong and Petit's method of determining the coefficient of real expansion of a liquid.

4. What do you mean by anomalous expansion of water ? How do you demonstrate it ? Discuss its one usefulness.

5. (a) Find the relation between the densities of a liquid at different temperatures.

(b) Describe a demonstration experiment to show that the density of water at 4°C is maximum.

6. Which type of expansion coefficient of a liquid is measured by the weight thermometer ? Describe its working principle. Why is this device called a 'thermometer' ?

7. (a) Show how the apparent weight of a solid immersed in a liquid changes with rise in temperature.

(b) What is establised from Hope's experiment ? Explain your answer with a description of the experiment.

8. Explain with the help of a curve how the volume of a given mass of water changes between 0°C. and 10°C.

Discuss the effects of anomalous expansion of water on marine life.

[B Short Answer type question

1 An observation made on the expansion of liquid contained in a bulb or tube does not give the true expansion of the liquid. Explain.

2. A tube of small bore is attached to a thin walled metallic vessel and water is poured into it until it fills the bulb completely and the tube partly. Now the bulb is heated (i) suddenly or (ii) slowly. What changes would seem to occur in the level of water in the tube ?

3. A block of wood is floating on water at 0°C with a certain volume V above water-level. The temperature of the water is slowly raised from 0°C to 20°C. How does the volume V change with the rise of temperature ? *[I. I. T. '74]*

4. A body hung from one arm of a balance is immersed in a liquid and the scale is balanced. Will the scale become unbalanced if the liquid is heated with the body ? If so, how will it tilt ?

5. The top of a lake is frozen. Air in contact is at –15°C. What do you expect to be the maximum temperature of water : (i) in contact with the lower surface of ice, (ii) at the bottom of the lake ? *[I. I. T. 73]*

6. Explain the following :

(a) There are two types of expansion of a liquid—real and apparent.

(b) Marine life survives the severe winter of the cold countries.

(c) When the bulb of a thermometer is introduced suddenly in a hot water bath, its reading first falls and then rises.

(d) The barometric height measured in room temperature needs correction.

7. A beaker filled with water at 4°C overflows if temperature is increased or decreased. Explain. *[I. I. T '69]*

8. A piece of ice is floating in a beaker completely filled with a liquid of density 1·5 gm./c.c. Will there be any overflow of liquid when the ice melts ? Explain. *[I. I. T. '80]*

9. Water is poured in a beaker containing a piece of ice. If the temperature of water before pouring be (i) 0°C, (ii) 4°C or (iii) 31°C, will there be any change, in the level of water in each case when the ice melts completely ? Justify your answer.

[C] Simple Problems

1. 100 c.c. of turpentine at 0°C are raised in temperature to 60°C. Calculate the new volume and hence the new density. Given, density of turpentine at 0°C=0·87 gm. per c.c. and the coefficient of volume expansion $=9{\cdot}4\times10^{-1}$/°C. [*Ans.* 105·64 c.c. ; 0·82 gm. per c.c.]

2. A one litre flask made of glass contains some mercury. It is found that at different temperatures the volume of air inside the flask remains the same. What is the volume of mercury in the flask ? [Given, the coefficient of linear expansion of glass$=9\times10^{-6}$/°C. and coefficient of real expansion of mercury$=1{\cdot}8\times10^{-4}$/°C.] (*I.I.T. '73*) [*Ans.* 150 c.c.]

3. To eliminate the influence of the change in the volume of a glass vessel when heated during the observation of the thermal expansion of liquids, part of the vessel is filled with an alloy having a coefficient of cubical expansion of 8×10^{-5} per °C. Determine what portion of the volume of the vessel should be filled with the alloy for the thermal expansion of the vessel to be fully compensated. The coefficient of cubical expansion of glass is 3×10^{-5} per °C.

[*Ans.* $\frac{3}{8}$ of the initial volume]

4. A thermometer has a bulb of volume 0·5 c.c., and the mercury in the stem moves 2 mm for 1°C. rise in temperature. If the coefficient of cubical expansion of mercury is 0·00018 per °C., calculate approximately the diameter of the circular bore of the stem, neglecting the effect of the expansion of the glass. [*Ans.* 0·24 mm.]

5. The bulb of a mercury thermometer measures 0·25 c.c. and the capillary stem is 25 cm. long and 0·2 mm. in diameter. If at 0°C. mercury occupies the whole of the bulb, find the maximum temperature for which the thermometer can be used. (Coefficient of apparent expansion of mercury$=18\times10^{-5}$/°C.) [*Ans.* 174·4°C.]

6. A glass weight thermometer filled with mercury at 0°C. is heated to 100°C. 7·785 gm. of mercury overflows and 450 gm. remains in the thermometer. Using glycerine in place of mercury, the corresponding figures are 2·173 gm. and 41 gm. The coefficient of real expansion of mercury is $18{\cdot}3\times10^{-5}$ per °C. Determine (a) the coefficient of linear expansion of glass, (b) the coefficient of real expansion of glycerine.

[*Ans.* (*a*) $3{\cdot}33\times10^{-6}$ per °C ; (*b*) 54×10^{-5} per °C]

7. A specific gravity bottle weighs 26·71 gm. when empty and 66·36 gm. when full of alcohol at 0°C. It was placed in a water bath at 40°C. then removed and allowed to cool ; the bottle now weighs 64·53 gm. If the coefficient of cubical expansion of glass is 25×10^{-5} per °C what is the coefficient of real expansion of alcohol ? [*Ans.* $12{\cdot}35\times10^{-4}$ per °C.]

8. A weight thermometer weighs 40 gm. when empty and 510 gm. when filled with mercury at 0°C. On heating it to 100°C., 7·01 gm. of mercury overflows. Calculate the coefficient of cubical expansion of glass; given, coefficient of real expansion of mercury is 18×10^{-5} per °C. [*Ans.* $28{\cdot}5\times10^{-6}$ per °C.]

9. A weight thermometer contains 43 gm. of mercury at 0°C. When placed in an oil bath, 1 gm. is found to overflow. Find the temperature of the bath. Given, the coefficient of cubical expansion of mercury and the glass are 18×10^{-5} and 26×10^{-6} per °C. respectively. [*Ans.* 154·6°C.]

10. An empty glass weight thermometer has a mass of 15·2 gm. When completely filled with mercury at 100°C., the mass of the weight thermometer and contents is 465·4 gm. Calculate the internal volume of the weight thermometer at 0°C. Given, density of mercury at 0°C. =13·6 gm./c.c., coeff. of real expansion of mercury=$18{\cdot}3\times10^{-5}$/°C., coeff. of linear expansion of glass =8×10^{-6}/°C. [*Ans.* 33·64 c.c.]

11. A glass weight thermometer is found to contain 32·5 gm. of a liquid which just fills it at 30°C. If the coefficient of volume expansion of glass is 27×10^{-6}/°C. and that of the liquid 67×10^{-5}/°C. find the mass of liquid the weight thermometer will retain if its temperature is raised to 70°C. [*Ans.* 31·7 gm.]

12. A glass bottle with a fine stem when immersed in melting ice just contains 300 gm. of mercury. Calculate the amount of mercury that will overflow if the bottle is kept a sufficiently long time in boiling water, the barometric pressure being 76 cm. Would the amount of mercury be different if the pressure had been considerably lower? (Coefficient of app. exp. of mercury =1/6500) (*C. U.* '43) [*Ans.* 4·55 gm.; Yes]

13. A glass vessel contains 100 c.c. of alcohol and is full at a temperature of 10°C. How many c.c. will spill out when the temperature is raised to 30°C? Coeff. of linear expansion of glass=8×10^{-6}/°C., coeff. of real expansion of alcohol=11×10^{-4}/°C. [*Ans.* 2·15 c.c.]

14. A weight thermometer contains *m* gm. of a liquid at 0°C. If γ be the coefficient of apparent expansion of the liquid, prove that if the liquid is heated to t°C., the mass of liquid expelled is $\gamma mt\,(1-\gamma t)$ gm.

15. A specific gravity bottle contains 51·65 gm. of a liquid when it is full at 0°C. and 49·3 gm. of the same liquid when it is full at 40°C. If the coefficient of linear expansion of the glass is 10×10^{-6} per °C., find: (a) the ratio of the liquid densities at 0°C. and 40°C.; (b) the coefficient of real expansion of the liquid. [*Ans.* (a) 1·0488; $1{\cdot}22\times10^{-3}$ per °C]

16. A quartz weight thermometer of negligible expansion coefficient is filled with water at room temperature and heated. Compare the weights of water expelled between 20°C. and 40°C. and between 40°C. and 60°C. as accurately as the following data permit.

Mean coefficient of volume expansion of water from 20°C. to 40°C. =30×10^{-5} per °C., from 40°C. to 60°C=45×10^{-5} per °C. [*Ans.* 0·673]

17. The weights of 1 c.c. of water at 0°C. and 4°C. are respectively 0·999874 gm. and 1 gm. Find the coefficient of real expansion of water between temperatures 0°C. and 4°C. (*H. S.* '68) [*Ans.* $3{\cdot}15\times10^{-5}$/°C.]

18. The density of mercury at 0°C. and 100°C. are 13·6 gm./c.c. and 13·35 gm./c.c. respectively. Calculate the coefficient of real expansion of mercury. [*Ans.* $1{\cdot}87\times10^{-4}$/°C.]

19. The density of mercury at 20°C is 13·55 gm./c.c. At what higher temperature will this figure be in error by 1 percent? Given, coeff. of real expansion of mercury is $18{\cdot}2\times10^{-5}$/°C. [*Ans.* 74·94°C]

20. The density of alcohol at 0°C. is 0·83 gm./c.c. What is its density at 20°C. and —20°C? The coefficient of real expansion of alcohol=11×10^{-4}/°C. [*Ans.* 0·812 gm./c.c. 0·848 gm./c.c.]

21. A metal piece weighing 100 gm. in air is found to weigh 55·4 gm. in a certain liquid at 20°C. At 95°C., its apparent weight in the same liquid is 57·9 gm. Find the coefficient of real expansion of liquid. (Given, coefficient of cubical expansion of metal=27×10^{-6}/°C.) [*Ans.* $8{\cdot}2\times10^{-4}$/°C.]

22. A block of metal weighs 66·25 gm. in air, 45·75 gm. when immersed in a certain liquid at 20°C., and 46·63 gm. when immersed in the same liquid at 80°C. Calculate the coefficient of linear expansion of the metal, given that the coefficient of real expansion of the liquid is $7 \cdot 74 \times 10^{-4}$ per °C. [*Ans.* $8 \cdot 43 \times 10^{-6}$ per °C.]

23. A sinker made of silica (whose expansion may be neglected) weighs 110·505 gm. in air, 60·515 gm. in water at 0·C. and 61·000 gm. in water at 50°C. Determine the mean coefficient of expansion of water between 0°C and 50°C. [*Ans.* $1 \cdot 96 \times 10^{-4}$/°C]

24. A column of mercury at 100°C. is balanced by a column at 20°C. Their lengths are 50·97 cm. and 50·25 cm. Calculate the coefficient of real expansion of mercury.

[*Ans.* $1 \cdot 79 \times 10^{-4}$/°C.]

25. The heights of the mercury columns in equilibrium in the two vertical limbs of a U-tube kept at 100°C and 0°C. respectively are 76·35 cm. and 75 cm. respectively. Calculate the coefficient of real expansion of mercury. [*Ans.* $1 \cdot 8 \times 10^{-4}$ per °C]

26. In a Dulong and Petit's experiment using mercury, the height of the cold column, which was at room temperature, was 60·2 cm. and the difference in the heights of the two columns was 0·9 cm. The hot column was surrounded by steam at atmospheric pressure. Find the temperature of the room, the coefficient of real expansion of mercury being 18×10^{-5} per °C. [*Ans.* 16·94°C]

27. The brass scale of a mercury barometer was adjusted at 0°C. At a temperature of 20°C. the barometer indicated a pressure of 750 mm. Reduce the reading of the barometer to a temperature of 0°C. The coefficient of linear expansion of brass is 18×10^{-6} per °C. and the coefficient of real expansion of mercury is 18×10^{-5} per °C. [*Ans.* 74·757 cm.]

28. A glass vessel of internal volume V c.c. at 20°C is completely filled up 3/20 part of it by mercury and the rest by water. The vessel with its contents was raised to a uniform temperature of 40°C. Calculate the volume of water at 40°C. that will overflow as a fraction of its initial volume. (Coefficient of linear expansion of glass $= 9 \times 10^{-6}$ per °C., coefficient of real expansion of mercury and water $= 18 \times 10^{-5}$ and $30 \cdot 2 \times 10^{-5}$ per °C. respectively.)

[*Ans.* 0·006]

29. A long cylindrical vessel having a linear coefficient of expansion α is filled with a liquid upto a certain level. On heating, it is found that the level of the liquid in the cylinder remains the same. What is the volume coefficient of expansion of the liquid ?

(*I. I. T.-76*) [*Ans.* 3α]

30. In a mercury thermometer, the mercury just fills its bulb of volume V_0 at 0°C. Derive an expression for the length l of the murcury thread in its capillary at a temperature of t°C assuming its area of cross-section A remains unchanged. Coefficients of volume expansion of mercury and glass are γ_m and γ_g respectively. [*Ans.* $l = \frac{V_0}{A}(\gamma_m - \gamma_g)\,t$]

31. Cubical expansivities of benzene and wood are $1 \cdot 2 \times 10^{-3}$/°C and 1·[illegible] $\times 10^{-4}$/°C and their densities at 0°C are $9 \cdot 0 \times 10^2$ kg/m³ and $8 \cdot 8 \times 10^2$ kg/m³ respectively. At what temperature will wood just sink in benzene ?

(*Jt. Entrance '84*)

[*Ans.* 21·1°C]

[D] Harder Problems

1. A long glass tube of uniform capillary bore contains a thread of mercury 50 cm. long at 0°C. When the temperature is raised to 150°C., the thread of mercury is found to be 12·3 mm. longer. If the coefficient of absolute expansion of mercury be $18 \cdot 3 \times 10^{-5}$/°C., calculate the coefficient of linear expansion of glass. [*Ans.* $9 \cdot 3 \times 10^{-6}$/°C]

2. A sphere of diameter 7 cm. and mass 266·5 gm., floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature of 35°C. If the density of the liquid at 0°C is 1·527 gm./cm.³, find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere. (*I.I.T. 1962*) [*Ans.* $8 \cdot 08 \times 10^{-4}$ per °C.]

3. A piece of iron floats on mercury with 0·575 of its volume submerged when the temperature is 0°C. What fraction of the volume will be submerged when the temperature of the whole system is raised to 220°C. ? Given, the coefficient of linear expansion of iron $=1{\cdot}2\times10^{-5}/°C$, and the coefficient of real expansion of mercury is $1{\cdot}82\times10^{-4}/°C$. [*Ans.* 0·5965]

4. A hollow aluminium bulb of mass 29·3 gm. is floated in benzene at 0°C. at which temperature the external volume of the bulb is 35 cm^3 and the density of benzene 0·88 gm./cm^3. The liquid and the bulb are then warmed in such a way that their temperatures are always uniform and equal. Estimate the temperature at which the bulb just sinks. Given that the coefficient of linear expansion of aluminium is 25×10^{-6} per °C. and the coefficient of real expansion of benzene is $12{\cdot}4\times10^{-4}$ per °C. [*Ans.* 41·8°C.]

5. A submarine weighing 10^7 gm. has to take in 10^6 gm. of water in order to submerge when the temperature of the sea is 20°C. How much less water will she have to take in when the sea is at 30°C ? Coefficient of volume expansion of sea water$=2\times10^{-4}/°C.$, coefficient of linear expansion of steel$=1{\cdot}2\times10^{-5}/°C$. [*Ans.* $1{\cdot}66\times10^4$ gm.]

6. A vertical glass tube, closed at the bottom, contains mercury to a height of 100 cm. at a temperature of 15°C. Find the height of the mercury column at 45°C. Coefficient of real expansion of mercury$=18\times10^{-5}/°C.$, coefficient of linear expansion of glass$=9\times10^{-6}/°C$.]*Ans.* 100·49 cm.]

7. A vessel contains 8·450 gm. of water at 0°C. when the remainder of the vessel is filled with paraffin. When the water is turned to ice at 0°C., 0·620 gm. of paraffin is expelled. The specific gravity of paraffin at 20°C. is 0·800 and its coefficient of expansion 0·00090/°C. Determine the specific gravity of ice from the above data. (*London Univ.*) [*Ans.* 0·917]

8. A barometer with a brass scale reads 75·91 cm. at 20°C. The scale is graduated to be correct at 15°C What is the height of the barometer reduced to 0°C. if the real expansion of mercury is 18×10^{-5} per °C., and the linear coefficient of brass is 19×10^{-6} per °C. [*Ans.* 75·64 cm.]

9. Determine the approximate height of mercury required in Graham's compensated pendulum, as a fraction of total length of the pendulum, assuming this to be iron of negligible weight with a coefficient of linear expansion 12×10^{-6} per °C. The coefficient of real expansion of mercury is 18×10^{-5} per °C. [*Ans.* 0·15]

10. A compensated pendulum consists of a steel rod of negligible weight, 150 cm. long, which carries a thin cylindrical vessel containing mercury to a depth h cm., the base of the vessel being fixed to the lower end of the rod. Taking the coefficient of linear expansion of steel to be 11×10^{-6} per °C. and the coefficient of real expansion of mercury to be $18{\cdot}2\times10^{-5}$ per °C. find the value of h that will make the effective length of the pendulum independent of temperature. [*Ans.* 20·6 cm.]

11. Two thermometers A and B are made of same kind of glass containing the same liquid. The bulb of both the thermometers are spherical. The internal diameter of the bulb in A is 7·5 mm. and the radius of cross-section of the tube is 1·25. ; the corresponding figures for B are 6·2 mm. and 0·9 mm. Compare approximately the length of a degree of A with that of B. (*Jt. Entrance* '70) [*Ans.* 0·92 (nearly)]

12. A mercury thermometer is immersed completely in boiling water and gives the reading of 100°C. The stem of the thermometer is then drawn out of water and kept in such a way that the average temperature of the upper part of the stem from the 0°C. mark equals 20°C. The thermometer then indicated a reading of 98·8°C. Find the coefficient of apparent expansion of mercury. [*Ans.* $15{\cdot}2\times10^{-5}$ °C.]

13. Two completely identical thermometers are filled at 0°C. with equal volumes of mercury and alcohol. Find the ratio of the length l of a division corresponding to 1° on the mercury thermometer scale to the length l_1 of a division on the alcohol thermometer scale. The volumetric expansion coefficient of mercury is α, of alcohol α_1 and the linear expansion of coefficient of glass is β. $\left[\textit{Ans.}\ \frac{l}{l_1}=\frac{\alpha-3\beta}{\alpha_1-3\beta}\right]$

14. A closed glass bulb contains some mercury leaving an empty space of 1·00 c.c. at 0°C. On raising the temperature to 40°C. the volume of the air space is 0·98 c.c. What is the volume of the mercury if the coefficient of linear expansion of glass is $7{\cdot}2 \times 10^{-6}$ and the coefficient of cubical expansion of mercury $1{\cdot}81 \times 10^{-4}$ per degree centigrade ?

(*London Univ.*) [*Ans.* 3·27 c.c.]

15. A cylinder of iron contains mercury to a depth of 10 cm at 0°C. Determine the change in the depth of mercury if the cylinder with its contents has its temperature raised to 100°C. α of iron=0·0000118/°C, γ of Hg=0·000182/°C. [*Ans.* increases by 0·16 cm.]

16. A vessel is filled completely with 500 gm of water and 1000 gm of mercury. When 21,200 calories of heat are given to it, water of mass 3·52 gm overflows. Calculate the co-efficient of volume expansion of mercury. Expansion of the vessel may be neglected. (Coefficient of volume expansion of water $=1{\cdot}5 \times 10^{-4}$/°C ; Density of mercury =13·6 gm/c.c ; Density of water =1 gm/c.c. ; Specific heat of mercury =0·03 cal/gm/°C. (*I. I. T. '76*)

[*Ans.* $1{\cdot}8 \times 10^{-4}$/°C]

17. On a centigrade thermometer the distance between the readings 0°C and 100°C is 30 cm. and the internal area of cross-section of the stem is $1{\cdot}5 \times 10^{-3}$ cm². Find the total volume of mercury in the thermometer at 0°C. Coefficient of linear expansion of glass is 9×10^{-6}/°C and the coefficient of real expansion of mercury in 18×10^{-5}/°C. [*Ans.* 2·94 c.c.]

16

CHAPTER

EXPANSION OF GASES

16·1. Expansion of gases.

Change of pressure does not produce any appreciable effect on the volumes of solids and liquids. In the discussions of thermal expansion of solids and liquids, the effect of pressure on their volumes was, therefore, neglected. The volumes of gases are, on the other hand, considerably affected by small changes of pressure. It is, therefore, meaningless to specify the volume of a given mass of a gas unless its pressure is defined. The term pressure means either the pressure exerted by the gas itself or the external pressure acting on the gas because these two are equal in magnitude under equilibrium conditions. In fact, while studying the behaviour of a gas the pressure, volume and temperature of the gas must be taken as three variables. We shall first examine the laws governing the behaviour of gases, obtained by keeping in turn any one of these three factors constant and studying the relationship between the remaining two.

16·2. Expansion of gases at constant temperature : Boyle's Law.

Robert Boyle, an English philosopher, gave in 1660 an important relation expounding the variation of the pressure of a gas with its volume, temperature remaining constant. It states that *the pressure of a fixed mass of a gas at constant temperature is inversely proportional to its volume*. This is known as **Boyle's Law**.

Thus, if P is the pressure and V, the volume of a given mass of gas, Boyle's law states that temperature remaining constant,

$$P \propto \frac{1}{V}$$

$$\text{or, } P=\frac{k}{V} \quad \text{or, } PV=k \quad \cdots \quad (16{\cdot}1)$$

where k is the proportionality constant which depends on (i) the mass of the gas taken, (ii) nature of the gas, (iii) temperature of the gas and (iv) the units in which P and V are expressed.

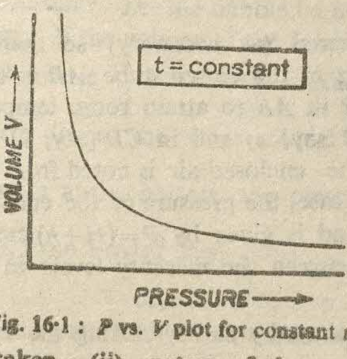


Fig. 16·1 : P vs. V plot for constant t

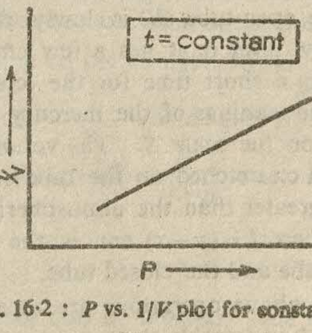


Fig. 16·2 : P vs. $1/V$ plot for constant t

If the volume V of a given mass of a gas at constant temperature is plotted against the corresponding pressure P, a curve (a rectangular hyperbola) is obtained [Fig. 16·1]. It is better to plot the reciprocal of volume against pressure when a straight line is obtained [Fig. 16·2]. These results verify Boyle's law.

16·3. Verification of Boyle's Law.

For the verification of Boyle's law, an apparatus known as Boyle's law apparatus is used. It consists essentially of two uniform glass tubes AB and CD [Fig. 16·3] capable of sliding up and down along two metal uprights MN and PQ fixed to an wooden base. They can be kept fixed at any position on MN and PQ. The upper end of AB is provided with a stopcock, the lower end being open. Both the ends of CD are open. The lower ends of AB and CD are connected by a rubber pressure tubing. Part of the tubes AB and CD and the whole of the connecting tube are filled with pure and dry mercury. The rest of the tube AB usually contains air. The volume of air can be noted from the scale in c.c. etched on the tube AB. The mercury levels in AB and CD can be read off from the vertically attached scale S.

In some forms of the instrument, the tube AB is permanently closed at the upper end and is not provided with any scale in c.c. etched on it. Since the tube is of uniform cross-section, the volume of air enclosed in it is directly proportional to the length of the air column.

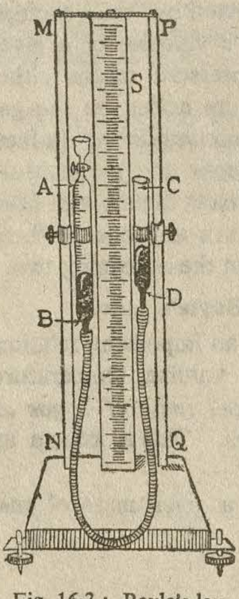


Fig. 16·3 : Boyle's law apparatus

Before the experiment, the atmospheric pressure H in cm. of mercury is noted from the barometer. The observation is also repeated after the experiment and mean H is calculated.

The open tube CD is slowly raised or lowered (as necessary) so that the mercury level in it lies a few cm. above that in the closed tube AB. After allowing a short time for the compressed air in AB to attain room temperature, the readings of the mercury levels in AB (say, x) and in CD (say, y) are noted on the scale S. The volume V of the enclosed air is noted from the scale in c.c. etched on the tube AB. In this case, the pressure of the enclosed air is greater than the atmospheric pressure and is given by $P=(H+h)$ cm. of Hg. where $h=(y-x)$ cm. is the difference between the mercury levels in the open tube and the closed tube.

The above operations are repeated a few times by slowly raising the open tube CD in suitable steps.

The tube CD is then lowered slowly until the mercury level in it stands a few cm. below that in the closed tube AB. After allowing a short time, as before, the readings of the mercury levels in AB and CD and the volume V of the enclosed air are noted. In this case, the pressure of the enclosed air is less than atmospheric pressure and is given by $P=(H-h)$ cm. of Hg where $h=(x-y)$ cm. These operations are also repeated a few times by slowly lowering the tube CD in suitable steps.

For each set of observations, the product $P\times V$ is calculated and is found to be constant. A graph may be drawn by ploting V against P. It is found to be a rectangular hyperbola [Fig. 16·1]. [Alternatively, P may be plotted against $1/V$, whence a straight line is obtained (Fig. 16·2)]. These results verify Boyle's law.

Precautions : (i) The enclosed air must be perfectly dry.

(ii) When the enclosed air is compressed or rarefied rapidly its temperature changes. Hence the raising or lowering of the tube CD should not be rapid and the readings should be taken after waiting for a few minutes. This enables the enclosed air to attain the room temperature, which remains practically constant during the experiment.

(iii) Care must be taken against any possible leakage of air through the stop-cock at the top of the tube AB. It should be properly greased.

(iv) The readings of the mercury levels should be taken at their convex meniscus avoiding parallax.

Example 16·1. *Find the volume of a mass of gas at a pressure 100 cm. of Hg., if the volume of the same mass of the gas is 5 litre at a pressure 76 cm. of Hg. Assume the temperature of the gas to be constant.*

Solution : Here, $P_1=76$ cm. of Hg., $V_1=5$ lt, $P_2=100$ cm. of Hg. Let V_2 be the required volume of the gas. Since the temperature of the gas remains constant, applying Boyle's law we get,

$$P_1V_1=P_2V_2$$

$$\text{or}\quad 76\times 5=100\times V_2$$

$$\therefore\quad V_2=3{\cdot}8 \text{ lt.}$$

Example 16·2. *The volume of a bubble of air increases 3 times in rising from the bottom to the surface of water in a lake. Calculate the depth at which the bubble was formed, when the atmospheric pressure is 76 cm. of mercury.*

Solution : Let the bubble be formed at a depth of h cm. below the water surface. If the volume of the bubble at the surface be v c.c. then at the bottom, its volume is $\frac{v}{3}$ c.c. At the surface, the pressure on the bubble $=76$ cm. The pressure due to the water column of height h cm. $=\frac{h}{13{\cdot}6}$ cm. of mercury.

Hence at the bottom, the pressure on the bubble $=\left(76+\frac{h}{13{\cdot}6}\right)$ cm.

So, from Boyle's law, $\left(76+\frac{h}{13{\cdot}6}\right)\frac{v}{3}=76\,v.$

or, $h=2067{\cdot}2$ cm. $=20{\cdot}67$ m.

Example 16·3. *On introducing an air bubble into a barometer tube of 10 sq. cm. cross-section, the mercury level falls to 66 cm. from 76 cm. If the length above mercury column before introduction of the bubble was 5 cm., what is the volume of the bubble under normal atmospheric pressure ?*

Solution : Since the mercury level falls by $76-66=10$ cm., the space occupied by air$=10+5=15$ cm. in length and thus it occupies the volume $=15\times10=150$ c.c. Evidently, the pressure of this air is equal to 10 cm. of mercury. If V be the volume of the bubble under normal atmospheric pressure which is equal to 76 cm. of mercury, then from Boyle's law,

$$V\times76=150\times10 \quad \text{or,} \quad V=19{\cdot}7 \text{ c.c.}$$

Example 16·4. *A flask whose capacity is 1 litre is dipped into a water reservoir with its mouth downwards. How far must it be dipped for 200 c.c. of water to enter into the flask ? Given, atmospheric pressure=76 cm. of mercury, and density of mercury=13·6 gm. per c.c.*

Solution : Let us suppose that when the flask is dipped through h cm. below the water surface, 200 c.c. of water enters into it. If the pressure of the confined air at that position be P, we have from Boyle's law, $1000\times76=P\times800$ or, $P=95$ cm. of mercury.

Hence, the excess pressure at the depth of h cm. over that acting on the free surface of water$=(95-76)=19$ cm. of mercury. Since the excess pressure is caused by the water column of height h cm., we have, $h\times1\times g=19\times13{\cdot}6\times g$ or, $h=258{\cdot}4$ cm.

Example 16·5. *A vertical cylinder of total length 1 metre is closed at the lower end and is fitted with a movable, frictionless, gas-tight plug at the other end. Inside the cylinder an ideal gas has been entrapped forming 94 cm. gas column when there exists equilibrium between the gas and the atmosphere. Mercury is then poured on the top of the plug until it overflows and the plug descends through 30 cm. Find the atmospheric pressure, assuming the temperature of the system remains constant and neglecting the weight of the plug.*

Solution : Let P cm. of mercury be the atmospheric pressure. Initial volume of the gas entrapped into the cylinder at the atmospheric pressure is $94\times\alpha$ c.c. where α cm.^{2} is the area of cross-section of the cylinder.

When mercury is poured, the volume of the gas entrapped at the pressure $(P+30+6)$ or $(P+36)$ cm. of mercury$=(94-30)\times\alpha=64\times\alpha$ c.c.

Since the temperature of the gas in constant, from Boyle's law,

$$P\times94\times\alpha=(P+36)\times64\times\alpha$$

whence, $P=76{\cdot}8$ cm. of mercury.

Example 16·6. *In the middle of a narrow horizontal tube sealed at both ends is a column of mercury 12 cm. in length. Both halves of the tube contain air under a pressure of 76 cm. of mercury. By what distance will the mercury column be displaced if the tube is held vertically ? The length of the tube is 1 m.*

Solution : We denote by P_0 the pressure of air in both halves of the tube in horizontal position, by l_0 the length of tube occupied by the gas in each half of the tube in the horizontal position, by P_1 and P_2 the gas pressure in the upper and lower halves of the tube after being placed in a vertical position, by h the length of the mercury column and by x the displacement of the mercury column.

Applying Boyle's law,

$l_0P_0=(l_0+x)\,P_1$ for the upper half

$l_0P_0=(l_0-x)\,P_2$ for the lower half

and, $P_2=P_1+h$

Here, $l_0=\dfrac{100-12}{2}=44$ cm., $P_0=76$ cm. of Hg., $h=12$ cm. We get, on solving above equations,

$$x=3{\cdot}5 \text{ cm.}$$

Example 16·7. *A simple barometer contains a little air in the Torricellian space above the mercury column. It reads 73·5 cm. when the true atmospheric pressure is 76 cm., and 72·4 cm. when, the true pressure is 74·7 cm. Find (i) the length of the barometer tube itself, measured from the free surface of the mercury in the bowl (ii) the true atmospheric pressure when this barometer reads 75 cm., assuming the temperature to be constant.*

Solution : (*i*) Let the length of the barometer tube measured from the free surface of the mercury in the bowl$=l$ cm. In the first case, the air in the Torricellian space is under $(76-73{\cdot}5)=2{\cdot}5$ cm of mercury pressure.

The volume occupied by the air$=(l-73{\cdot}5)\times\alpha$ c.c.,

where α is the area of cross-section of the tube.

In the second case, the pressure of air in the Torricellian space$=(74{\cdot}7-72{\cdot}4)$ $=2{\cdot}3$ cm. of mercury, when the volume of air in the space$=(l-72{\cdot}4)\times\alpha$ c.c.

Since temperature remains constant, we have, from the Boyle's law,

$$2{\cdot}5\times(l-73{\cdot}5)\times\alpha=2{\cdot}3\times(l-72{\cdot}4)\times\alpha$$

whence, $l=86{\cdot}15$ cm.

(*ii*) When the barometer reads 73·5 cm. the volume of the air enclosed$=$ $(86{\cdot}15-73{\cdot}5)\times\alpha=12{\cdot}65\times\alpha$ c.c. and the pressure acted upon it$=(76-73{\cdot}5)=$ 2·5 cm. of mercury.

When the barometer reads 75 cm., the volume of the air enclosed$=$ $(86{\cdot}15-75)\times\alpha=11{\cdot}15\times\alpha$ c.c.

Let P be the true atmospheric pressure, so that the pressure of air in the enclosed space$=(P-75)$ cm. of mercury.

Hence, from Boyle's law,

$$(P-75)\times11{\cdot}15\times\alpha=2{\cdot}5\times12{\cdot}65\times\alpha$$

or, $P=77{\cdot}84$ cm. of mercury.

16·4. Expansion of gases at constant pressure ; Charles' Law.

In 1787 Jacques Charles, a French scientist, carried out an investigation into the expansion of a given mass of a gas at constant pressure. He found that *equal volumes of all gases expand ordinarily by the same amount for the same rise of temperature provided that the pressure remains constant.* In 1802, Joseph Louis Gay Lussac reached the almost similar conclusion independently. He found that *at constant pressure the functional change in volume of a given mass of a gas at 0°C. per degree centigrade rise in temperature is approximately equal for all gases.* Regnault in 1842, found experimentally that *the fractional change in volume of gas at 0°C. is almost equal to* $\frac{1}{273}$ *or 0·00366 per °C. rise of temperature.*

Thus when Gay Lussac's and Regnault's contributions are taken into account, the Charles' law in the complete form stands as :

The pressure remaining constant, the volume of a given mass of any gas increases (or decreases) by the constant fraction $\frac{1}{273}$ of its volume at 0°C. for every degree centigrade increase (or decrease) of temperature.

Let V_0 denote the volume of a given mass of a gas at 0°C. and V_t be the volume of the same mass of the gas at t°C., the pressure in both cases being the same ; then from Charles' law, the increase in volume of the gas

$$=\frac{\text{volume at } 0°\text{C}}{273}\times \text{rise in temperature}$$

$$\therefore\ V_t-V_0=\frac{V_0}{273}\times t$$

$$\text{or,}\quad V_t=V_0\left(1+\frac{t}{273}\right)\quad \dots\ (16{\cdot}2)$$

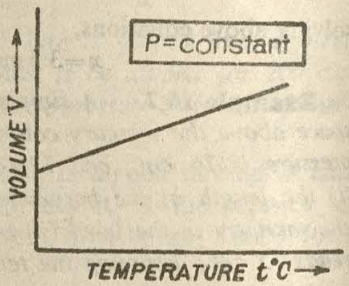


Fig. 16·4 : V vs. t plot at constant P

A plot of V against t at constant pressure is a straight line as shown in the Fig. 16·4.

Example 16·8. *A certain mass of air at 60°C. occupies 170 c.c. What volume will it occupy (a) at 0°C., (b) at 15°C., the pressure remaining unaltered ?*

Solution : (a) Let the volume of air at 0°C. be V_0 ; then from eqn (16·2),

$$170=V_0\left(1+\tfrac{60}{273}\right) \text{ or, } V_0=139{\cdot}4 \text{ c.c.}$$

(b) Let the volume of air at 15°C. be V_t. Since the volume at 0°C. is 139·4 c.c., hence from eqn. (16·2),

$$V_t=139{\cdot}4\left(1+\tfrac{15}{273}\right) \text{ or, } V_t=147{\cdot}1 \text{ c.c.}$$

16·5. Pressure Law.

On keeping the volume of a given mass of a gas constant, if the temperature be varied, the pressure of the gas changes. The relation giving the variation of pressure of a given mass of a gas at constant volume with temperature is known as the Charles' law of pressure or simply **Pressure Law.**

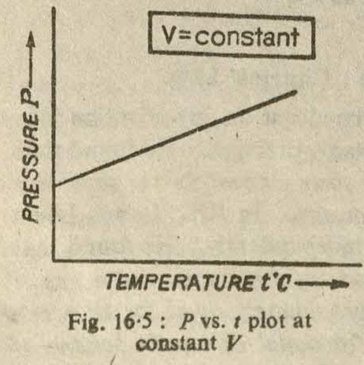


Fig. 16·5 : P vs. t plot at constant V

The law states that *volume remaining constant, the pressure of a given mass of any gas increases (or decreases) by the constant fraction $\frac{1}{273}$ of its pressure at 0°C. for each degree centigrade increase (or decrease) of temperature.*

If P_0 and P_t are the pressures of a given mass of gas at 0°C., and t°C. respectively, then volume remaining constant,

$$P_t=P_0\left(1+\frac{t}{273}\right)\quad \dots\ (16{\cdot}3)$$

A plot of P against t at constant volume is, therefore, a straight line as shown in the Fig. 16·5.

16·6. Volume and Pressure coefficients of a gas.

In the case of gases, since with a change in temperature both the volume and pressure of the gas may vary, it is necessary to distinguish between two coefficients : (1) the volume coefficient, γ_p, at constant pressure and (2) the pressure coefficient, γ_v, at constant volume.

The **volume coefficient** γ_p *of a gas may be defined as the fractional increase in volume of the gas at 0°C. for each degree centigrade rise in temperature, the pressure being kept constant.* If V_o and V_t be the volumes of a given mass of a gas at 0°C. and t°C. respectively, then

$$\gamma_p = \frac{\text{fractional increase in volume}}{\text{rise of temperature}}$$

$$= \frac{(V_t - V_o)/V_o}{t}$$

$$\text{or,}\quad V_t = V_o\,(1 + \gamma_p t) \qquad \dots \qquad \dots \qquad (16{\cdot}4)$$

This relation is equivalent to Charles' law as stated in equation (16·2) and on comparing we see that γ_p is equal to $\frac{1}{273}$ or 0·00366 per °C.

The **pressure coefficient** γ_v *of a gas may be defined as the fractional increase in pressure of the gas at 0°C. for each degree centigrade rise in temperature, the volume being kept constant.* If P_o and P_t be the pressures of a given mass of a gas at 0°C. and t°C. respectively, then

$$\gamma_v = \frac{\text{fractional increase in pressure}}{\text{rise of temperature}}$$

$$= \frac{(P_t - P_o)/P_o}{t}$$

$$\text{or,}\quad P_t = P_o\,(1 + \gamma_v t) \qquad \dots \qquad \dots \qquad (16{\cdot}5)$$

The relation is just the same as that stated in pressure law by equation (16·3) and γ_v is equal to $\frac{1}{273}$ or 0·00366 per °C.

The following important points are worth mentioning in the connection.

(i) The values of the two coefficients are the same for all the gases. This is an important distinction between the property of gases and that the solids and liquids where the coefficients of expansion vary from one to other.

(ii) In studying the expansion of a gas, it must always be enclosed in a container as in the case of a liquid. However, unlike liquids we did not make any distinction between the apparent and real coefficients of expansion of a gas. This is so because the expansion of a gas is about 100 times that of a solid and hence the expansion of the container may be neglected except for a very accurate work.

(iii) While defining the coefficients for a gas, we always referred to the initial volume or pressure at 0°C. in contrast to the case of solids and liquids, where the initial lower temperature may lie anywhere in the temperature scale. The reason behind is that the coefficient of expansion of a gas is not very small like solids and liquids. So if the initial temperature be not definite, then in the case of gases, a large error creeps in ; the corresponding errors in the case of solids and liquids are negligibly small.

The following example will help to clarify this point. Let us suppose that at a certain pressure, a given mass of gas occupies a volume 546 c.c. at the temperature 0°C. Then pressure remaining constant, its volumes at the temperatures 100°C. and 200°C. are obtained from Charles' Law as

$$V_{100}=V_0\left(1+\frac{100}{273}\right)=546\left(1+\frac{100}{273}\right)=746 \text{ c.c.}$$

$$V_{200}=V_0\left(1+\frac{200}{273}\right)=546\left(1+\frac{200}{273}\right)=946 \text{ c.c.}$$

If, however, the initial volume is taken to be that at 100°C., then the volume at 200°C. would have been

$$V_{200}=V_{100}\left[1+\frac{(200-100)}{273}\right]=746\left(1+\frac{100}{273}\right)=1019{\cdot}2 \text{ c.c.}$$

which differs by a large amount from the previously calculated correct value. Hence the importance of taking the initial values always at 0°C. in dealing with the expansion of gases.

Example 16·9. *A certain quantity of air occupies a volume of 500 c.c. at 0°C. and 1 litre at 273°C. Find the coefficient of expansion of the air at constant pressure.*

Solution*: Here $V_0=500$ c.c.

$V_t=1000$ c.c.

$t=273$°C

Applying eqn. (16·4), we get

$$1000=500(1+\gamma_p.273)$$

$$\text{or,}\quad 273\gamma_p=1 \qquad \text{or,}\quad \gamma_p=\frac{1}{273} \text{ per °C.}$$

16·7. Equality of pressure and volume coefficients (theoretical proof)

In the foregoing article, the values of both γ_p and γ_v are taken to be the same, viz. $\frac{1}{273}$, which followed as the consequence of experimental observations. However, the equality of the two coefficients can be justified from the theoretical standpoint for gases obeying Boyle's law and Charles' law.

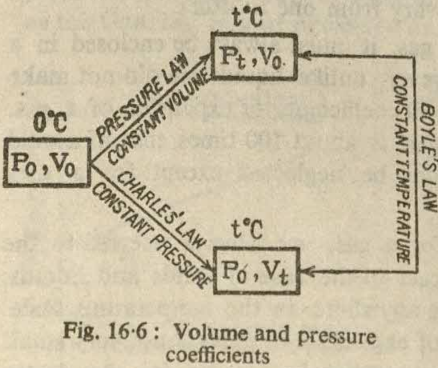


Fig. 16·6 : Volume and pressure coefficients

Let us suppose that a given mass of a gas at 0°C. has a pressure P_0 and a volume V_0, and that it is heated to a temperature t°C. This change of temperature can be effected in two alternative ways viz. *at constant volume*, when the pressure changes to a value P_t, or *at constant pressure*, when the volume changes to a value V_t. These two alternative ways are illustrated diagrammatically in the Fig. 16·6.

When the temperature of the gas changes at constant volume, the final pressure of the gas is given by the Pressure law as

$$P_t=P_0(1+\gamma_v t) \qquad \ldots \qquad (16{\cdot}6)$$

Similarly, on changing the temperature at constant pressure, **the final volume** is given by Charles' law as

$$V_t = V_o\,(1+\gamma_p t)$$

$$\text{or,}\quad \frac{V_t}{V_o} = 1+\gamma_p t \qquad \cdots \qquad \cdots \qquad (16{\cdot}7)$$

Since the final temperature t°C. is the same in either of the two **cases, it** then follows from Boyle's law,

$$P_o V_t = P_t V_o$$

$$\text{or,}\quad \frac{P_t}{V_o} = \frac{V_t}{V_o} \qquad \cdots \qquad \cdots \qquad (16{\cdot}8)$$

Combining the equations (16·7) and (16·8), we get

$$\frac{P_t}{P_o} = 1+\gamma_p t$$

$$\text{or,}\quad P_t = P_o(1+\gamma_p t) \qquad \cdots \qquad \cdots \qquad (16{\cdot}9)$$

Comparison of equation (16·9) with equation (16·6), shows

$$\gamma_p = \gamma_v$$

16·8. Verification of Charles' Law.

The apparatus consists of a U-tube of glass having a bulb B at one **end,** the other end A being open to the atmosphere [Fig. 16·7]. The bulb and the part of the tube near it are graduated in c.c. Some quantity of dry air **is** enclosed in the bulb, while the lower part of the U-tube is filled up with concentrated sulphuric acid. The volume of enclosed air can be read off directly from the graduations. The sulphuric acid acts as a pressure gauge. A short tube provided with a stopcock C is fitted at the bend of the U-tube. The sulphuric acid in the two limbs of the U-tube can be maintained at the same level by adjusting the quantity of acid. This is done either by opening the stopcock carefully so that some acid drips out or by pouring in fresh acid at A. The entire U-tube is immersed in water contained in the glass jacket J. The temperature of water can be raised to a desired level by allowing steam from a boiler to pass through the copper tube D. Water may be stirred by means of the stirrer S. The whole apparatus is mounted on a vertical wooden frame. A thermometer T is so arranged that its bulb lies very close to the bulb B.

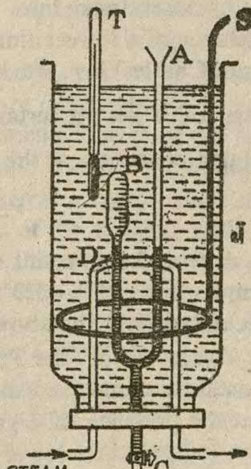


Fig. 16·7

At first, the temperature indicated by the thermometer is read off after a thorough stirring of water. The acid levels in the two limbs of the tube are adjusted to be same. Consequently, the pressure of this enclosed air becomes equal to the atmospheric pressure. Its volume is then noted.

Steam is then passed through the tube D. The water is stirred briskly. As the air in the bulb is heated, it pushes down the acid level in the closed limb. When the temperature has risen by a suitable amount, the steam supply is regulated to maintain the temperature steady at this higher value which is noted. The acid levels in the two limbs are adjusted to be same and the volume of air is recorded. This process is repeated to obtain different values for the volume of the air at different temperatures but at constant pressure (i.e., atmospheric pressure).

A graph is drawn with temperature as the abscissa and the volume as the the ordinate. The graph is found to be a straight line as shown in Fig. 16·4. This verifies Charles' law.

During the experiment, the atmospheric pressure may change producing error. Thus the barometric height must be recorded both before and after the experiment to check its constancy. Error also creeps in due to the volume expansion of the glass bulb B. The necessary correction is a very small one and may be calculated.

N.B. From the graph, the value of the volume coefficient γ_p of the gas may be found out. By extrapolation, the volume V_o at 0°C. is found out. From any other point of the graph V_t and t are obtained. Hence the value of γ_p may be calculated by using the equation (16·4).

16·9. Absolute scale of temperature.

A closer look into Charles' law provides a very important concept of the existence of a lowest limit to the value of temperature realisable in practice. From Charles' law, we know that at constant pressure, the volume of a given mass of a gas at certain temperature t°C. is given by, $V_t = V_o\left(1+\frac{t}{273}\right)$. The volume of the gas at the temperature -273°C. is thus,

$$V_{-273} = V_0\left(1+\frac{-273}{273}\right) = 0$$

A remarkable result showing that volume of a gas reduces to zero at the temperature of -273°C. At a temperature lower than this, the volume as calculated from the above equation, turns out to be negative, which is absurd.

Again from the Pressure law, the pressure of a given mass of a gas at constant volume decreases by $\frac{1}{273}$ of its pressure at 0°C. Thus it follows similarly that the pressure of a gas at constant volume will become zero at -273°C. At any temperature lower than this, the pressure exerted by the gas will become negative, which is impossible.

Hence at temperature -273°C., neither the volume nor the pressure of a given mass of a gas does exist. This temperature at which the calculated volume or pressure of a gas becomes zero is thus the lowest temperature attainable and is called the **absolute zero of temperature.** Accurate observations, however, give the absolute zero as $-273·16$°C.

It should be noted that in arriving at the conclusion, we have assumed that a gas continues to follow the Charles' law and the Pressure law, no matter how far the temperature is lowered. Actually, however, all gases liquefy before the temperature −273°C. is reached and then the gas laws are no longer applicable. Hence it is physically impossible to reduce the pressure and the volume of a gas to zero. But the concept is theoretically valid and we may restate it as—*if the law expressing the variation of volume or pressure of a gas with temperature remains valid, then the volume or pressure of a gas disappears at −273°C.*

The above concept of absolute zero of temperature can also be arrived at from the volume-temperature (Fig. 16·4) and pressure-temperature (Fig. 16·5) graphs. If the straight-line graphs representing the behaviour of the gas at moderate temperatures are extrapolated, they will cut the temperature axis at some point, marked *A* as shown in the Fig. 16·8. Moreover, if different sets of graphs are drawn with different samples of gas, the extrapolated curves will cut in each case the temperature axis at the same point *A*. This point is found to lie at −273°C. Since the corresponding volume or pressure is zero, this temperature, therefore, gives the absolute zero of temperature.

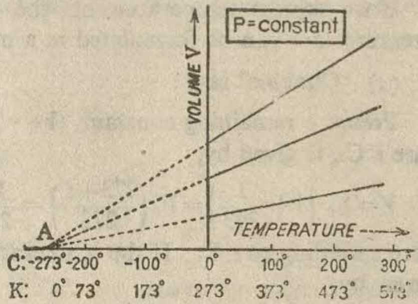


Fig. 16·8 : Existence of Absolute zero of temperature

So it is seen that the concept of absolute zero of temperature is true regardless of the kind, amount and initial pressure or volume of the gas. Thus, unlike centigrade zero, which depends on the behaviour of a particular material such as water, the *absolute zero is more universal and more fundamental.*

A new scale of temperature can be introduced in which the zero of the scale coincides with the absolute zero of temperature. This scale is called the **absolute scale of temperature**. This scale is also known as the *kelvin scale of temperature* in honour of Lord Kelvin who first laid theoretical basis of the absolute scale.

In the absolute centrigrade scale of temperature, there are 100 divisions between the ice point and the steam point. Thus each degree in this scale equals that in the centigrade scale. Therefore, *the scale of temperature in which temperatures are measured from the absolute zero (−273°C.) and in which each degree interval is equal to a degree interval on the centigrade scale, is called the absolute centigrade scale of temperature.* The readings on this scale are expressed in "degrees Absolute (°A)" or "degrees Kelvin (°K)". The ice-point temperature in this scale is 273°K., the steam-point temperature is 373°K. and in general, the absolute temperature T°K. corresponding to a temperature t°C. is given by,

$$T°\text{K.} = 273 + t°\text{C.} \qquad \cdots \qquad \cdots \qquad (16{\cdot}10)$$

On the Fahrenheit scale the value of the absolute zero can be obtained from the relation (13·1). Putting C=−273°, we get

$$-\frac{273}{5}=\frac{\text{F}-32}{9}, \quad \text{or, F}=-459{\cdot}4°$$

Thus, on the Fahrenheit system the value of the absolute zero is equal to $-459{\cdot}4$°F. Another scale, known as Rankine scale, starts with this temperature ($-459{\cdot}4$°F.) as its zero and on this scale each degree interval equals the degree interval on the Fahrenheit scale. Hence any temperature T°R. on the Rankine scale is given by,

$$T^\circ\text{R.}=459{\cdot}4+t^\circ\text{F.} \qquad \ldots \qquad \ldots \qquad (16{\cdot}11)$$

16·10. Charles' law and Pressure law in terms of Absolute Temperature.

If we express temperature on the absolute scale, both Charles' law and Pressure law can be formulated in a much simple manner as shown below.

(*a*) **Charles' law :**

Pressure remaining constant, the volume V of a fixed mass of gas at temperature t°C., is given by,

$$V=V_0\left(1+\frac{t}{273}\right)=V_0\left(\frac{273+t}{273}\right)=\frac{V_0}{273}T,$$ where T is the absolute temperature corresponding to t°C. If the temperature changes to t'°C., the new volume is given by,

$$V'=V_0\left(1+\frac{t'}{273}\right)=V_0\left(\frac{273+t'}{273}\right)=\frac{V_0}{273}T'$$

T' being the absolute temperature corresponding to t'°C.

$$\therefore \quad \frac{V}{V'}=\frac{T}{T'} \text{ or, } \frac{V}{T}=\frac{V'}{T'} \qquad \ldots \qquad \ldots \qquad (16{\cdot}12)$$

i.e., $\frac{V}{T}=$ a constant, when P is constant. Hence, $V \propto T$, when P is constant.

In other words, *at constant pressure, the volume of a given mass of any gas is directly proportional to its absolute temperature.*

(*b*) **Pressure law :**

If the pressure of a given mass of gas at temperature t°C. and t'°C. be P and P' respectively, then volume remaining constant, we have from Pressure law (eqn. 16·3),

$$P=P_0\left(1+\frac{t}{273}\right)=\frac{P_0T}{273}; \qquad P'=P_0\left(1+\frac{t'}{273}\right)=\frac{P_0T'}{273}$$

where T°K. and T'°K. are the absolute temperatures corresponding to the temperature t°C. and t'°C. respectively.

Thus, $$\frac{P}{P'}=\frac{T}{T'} \text{ or, } \frac{P}{T}=\frac{P'}{T'} \qquad \ldots \qquad \ldots \qquad (16{\cdot}3)$$

i.e., $\frac{P}{T}=$ a constant, when V is constant. Hence $P \propto T$, when V is constant.

In other words, *volume remaining constant, the pressure of a given mass of any gas is directly proportional to its absolute temperature.*

Example 16·10. *At what temperature will the volume of a given mass of gas is double of what is at 30°C., if the pressure remains constant ?*

Solution : Let t°C be the required temperature. Then applying eqn. (16·12), we have,

$$\frac{V}{V'}=2=\frac{273+t}{273+30} \quad \text{or,} \quad 2=\frac{273+t}{303}$$

$$\text{or,} \quad 273+t=606 \quad \text{or,} \quad t=333°\text{C}.$$

Example 16·11. *At constant pressure, 10 litre of a certain gas expands to 11·27 litre when heated from 0°C. to 34·7°C. Calculate from this data the value of the absolute zero on the centigrade scale.*

Solution : Let the value of the absolute zero be T_0°C. Now from Charle's law [eqn. 16·12],

$$\frac{V_1}{V_2}=\frac{-T_0+t_1}{-T_0+t_2}$$

Now, $t_1=0°\text{C}$, $V_1=10$ litre

and $t_2=34·7°\text{C}$, $V_2=11·27$ litre

$$\therefore \frac{10}{11·27}=\frac{-T_0+0}{-T_0+34·7}$$

$$\text{or,} \quad -1·27T_0=347 \quad \therefore \quad T_0=-273·2°\text{C}.$$

Example 16·12. *Five litres of air at N.T.P. are heated in such a way as to keep its volume unchanged. Calculate the temperature of the air when its pressure becomes 5 atmosphere.*

Solution : Here, $P_1=1$ atmos., $T_1=273$°K. ; $P_2=5$ atmos. Since the volume is constant, applying Pressure law (eqn. 16·13) we get,

$$\frac{P_1}{T_1}=\frac{P_2}{T_2} \quad \text{or} \quad \frac{1}{273}=\frac{5}{T_2} \quad \therefore \quad T_2=1365°\text{K}.$$

∴ The required temperature $=1365-273=1092$°C.

Example 16·13. *A cylinder closed at both ends is divided into two parts by a thermally insulated piston. Both halves of the cylinder contain equal masses of a gas at a temperature of 37°C. and a pressure of one atmosphere. What distance from the centre of the cylinder will the piston move if the gas in one section is heated to 67°C.? The initial length of each section of the cylinder is 40 cm.*

Solution : Making use of the fact that (i) the new value of the pressure in both halves of the cylinder will be equal after the piston has shifted by a distance, say x, and (ii) that both halves contain the same mass of the gas we may apply eqn. (16·12),

$$\frac{(l-x)\alpha}{T}=\frac{(l+x)\alpha}{T'}$$

where l is the initial length of each section of the cylinder, T and T' are the initial and the final temperatures in Kelvin scale, and α is the cross-section of the cylinder.

Solving for x, we get

$$x=\frac{(T'-T)\,l}{T+T'}$$

Here $T=273+37=310$°K $T'=273+67=340$°K $l=40$ cm.

$$\therefore \quad x=\frac{30\times40}{650}=1·85 \text{ cm.}$$

16·11. Equation of state of an ideal gas.

A gas which strictly obeys Boyle's law and Charles' law is known as an **ideal gas** *or a* **perfect gas.** For the sake of simplicity we shall assume that all gases behave like ideal gas.

Let P, V and T denote the pressure, volume and absolute temperature respectively of a given mass of gas.

Then, from Boyle's law, $V \propto \frac{1}{P}$, when T is constant,

and from Charles' law, $V \propto T$, when P is constant.

$\therefore \quad V \propto \frac{T}{P}$, when T and P both vary.

$$\text{or,} \quad \frac{PV}{T} = K \qquad \text{i.e.,} \quad PV = KT \qquad \ldots \qquad \ldots \qquad (16{\cdot}14)$$

where K is a constant of proportionality. The value of this constant depends on the mass of the gas as well as on the units in which P, V and T are expressed.

It is obvious that for a fixed mass of the gas, if P_1, V_1, T_1 are the initial and P_2, V_2, T_2 the final values then

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad \ldots \qquad \ldots \qquad (16{\cdot}15)$$

The relation (16·14) connecting the three variables viz., pressure, volume and temperature of an ideal gas is called the **characteristic equation** or **equation of state** of an ideal gas. Any physical state of a given mass of an ideal gas is determined if any two of the three quantities P, V and T be known ; the third can be found out by using the equation (16·14). Hence the name of the equation.

To use this equation, the following points must be remembered. Firstly, P and V represent the pressure and volume of a fixed mass of a gas. Secondly, T is the absolute temperature ; any temperature specified in °C. or °F. should be converted to the absolute temperature.

We should, however, remember that real gases do not obey the ideal gas equation perfectly. Under ordinary temperature and pressure, the discrepancy is usually very small. The variation is found to be considerable at extremely high pressures or at a low temperature near the temperature of liquefaction of the gas.

Example 16·14. *A mass of nitrogen occupies 500 c.c. at 10°C. under a pressure of 76 cm. of murcury. Determine its volume at 30°C., the pressure being 80 cm. of mercury.*

Solution : Since the mass is constant, we use the eqn. (16·15),

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{76 \times 500}{(10+273)} = \frac{80 \times V_2}{(30+273)} \text{ or, } V_2 = 508{\cdot}6 \text{ c.c.}$$

Example 16·15. *A sample of gas occupies 250 c.c. at 20°C. and a pressure of 77 cm. of mercury. Find, (a) the volume at N.T.P., and (b) the temperature at which the volume will be 200 c.c., the pressure being 76 cm. of mercury.*

Solution : (*a*) Here $P_1=77$ cm., $P_2=76$ cm.
$T_1=273+20=293°K.$
$T_2=273°K.$
$V_1=250$ c.c., $V_2=$the required volume.

Applying eqn. (16·15), we have

$$V_2=V_1\left(\frac{T_2}{T_1}\right)\left(\frac{P_1}{P_2}\right)$$
$$=\frac{250\times273\times77}{293\times76}=236 \text{ c.c.}$$

(*b*) Here, since the pressure remains same as in Part (*a*) of the problems, therefore for the volume to shrink in the ratio 236 : 200, the absolute temperature must fall in like ratio. So applying eqn. (16·12),

$$\frac{200}{236}=\frac{T}{273}, \text{ whence } T=273\times\frac{200}{236}=231{\cdot}4°K.$$

The required temperature is, therefore, $231{\cdot}4-273=-41{\cdot}6°C.$

This calculation preassumes, of course, that the gas in question is still well away from the conditions for its liquefaction.

Example 16·16. *An empty bottle is corked when the air inside it is at 10°C. and the barometer reads 75 cm. of Hg. If the cork blows out when the the pressure inside the bottle exceeds the atmospheric pressure by 100 cm. of Hg., calculate the temperature to which the bottle must be heated to blow the cork. Assume that the volume of the bottle remains constant when heated.* [*I. C. S. 1965*]

Solution : Here $P=75$ cm. of Hg. ; $T=273+10=283°K$
$P'=75+100=175$ cm. of Hg.
Let T' be the required temperature.

Since the volume of air is constant, applying eqn. (16·13), we have

$$T'=\frac{P'T}{P}=\frac{175\times283}{75}=660{\cdot}3°K$$

$\therefore$ **The required temperature**$=660{\cdot}3-273=387{\cdot}3°C.$

Example 16·17. *An air bubble gradually rises from a depth of 10 metre in a pond where the temperature is 10°C. to the surface at which the temperature is 25°C. Compare the radii of the bubble in the two positions. Assume the atmospheric pressure to be 76 cm. of mercury, and the density of mercury to be 13·6/c.c.*

Soluion : Let r_1 and r_2 be the radii and V_1 and V_2 be the volumes of the bubble at the bottom and at the surface respectively.

$$\therefore\quad V_1=\frac{4}{3}\pi r_1^3 \text{ and } V_2=\frac{4}{3}\pi r_2^3 \quad\text{or,}\quad \frac{V_2}{V_1}=\left(\frac{r_2}{r_1}\right)^3$$

If P_1, P_2 be the pressures and T_1, T_2 be the absolute temperatures at the bottom and at the surface, then

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \text{ or, } \frac{V_2}{V_1}=\frac{P_1T_2}{P_2T_1}$$

$$\text{or, } \left(\frac{r_2}{r_1}\right)^3=\frac{P_1T_2}{P_2T_1} \qquad \cdots \qquad \cdots \qquad (1)$$

But $P_1=(76\times 13\cdot 6+1000)\times g$ dyne/cm.², $P_2=76\times 13\cdot 6g$ dyme/cm²., $T_1=10+273=283^\circ$K., $T_2=25+273=298^\circ$K. Hence from (1), we get,

$$\left(\frac{r_2}{r_1}\right)^3=\frac{(76\times 13\cdot 6+1000)\times 298}{76\times 13\cdot 6\times 283}\ \text{or,}\ \frac{r_2}{r_1}=1\cdot 275.$$

16·12. Universal gas constant.

In the preceding article, we have mentioned that the value of the constant K depends on the mass of the gas. This is obvious because if on keeping P and T same, the mass of the gas is doubled, a two-fold increase in its volume will take place. Hence the value of the constant will become twice the previous value. The constant K thus varies proportionally with the mass of the gas.

When one gram-molecular weight of the gas is taken, the constant is usually denoted by R. Since from Avogadro's hypothesis *one gram-molecule of all gases occupy the same volume under identical conditions of temperature and pressure, the value of R is the same for all gases.* For this reason R is known as **universal gas constant.** It is also sometimes called *gram-molecular gas constant.* Thus for 1 gram-molecule of any gas, $PV=RT$ ... (16·16)

If n gram-molecules of the gas be considered, then

$$PV=nRT \quad \cdots \quad \cdots \quad (16\cdot 17)$$

If the corresponding mass of the gas be m and its molecular weight be M, then $n=m/M$

$$\therefore\ PV=\frac{mRT}{M} \quad \cdots \quad \cdots \quad (16\cdot 18)$$

As the gas constant for any mass of a gas is denoted by K, we have from the equation (16·18), $K=nR=\dfrac{mR}{M}$ ... (16·19)

For 1 gm. of the gas, the gas constant $=R/M$ ... (16·20)

This constant is known as *specific gas constant.*

Value of R.

From Avogadro's hypothesis, one gram-molecule of any gas at N.T.P.* occupies a volume of 22·4 litre. Thus, volume of 1 gm. mole of the gas at N.T.P., $V_0=22\cdot 4$ litre$=22400$ c.c. normal pressure, $P_0=76\times 13\cdot 6\times 980$ dyne/cm.² and normal temperature, $T_0=273^\circ$K.

$$\therefore\ R=\frac{P_0V_0}{T_0}=\frac{76\times 13\cdot 6\times 980\times 22400\cdot\dfrac{\text{dyne.}}{\text{cm.}^2}\,\dfrac{\text{c.c.}}{\text{gm. mole}}}{273^\circ\text{K}}$$

$$=8\cdot 31\times 10^7\ \text{dyne cm. per}\ ^\circ\text{K per mole.}$$

$$=8\cdot 31\times 10^7\ \text{erg per}\ ^\circ\text{K per mole.}$$

Example 16·18. *A bulb A of volume 1 litre is connected to a second bulb B of volume 400 c.c. and the whole is initially full of air at N.T.P. If bulb A is now raised in temperature to 100°C., find : (a) the new pressure in the system, and (b) the mass of air which is transferred from one bulb to the other during the heating. Given density of air at N.T.P. is 1·29 gm. per litre.*

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* N.T.P. means normal temperature and pressure. Normal temperature is the temperature of melting ice under a pressure of one standard atmosphere. It is equal to 0°C. or 273°K. Normal pressure is one standard atmospheric pressure i. e., equal to the pressure of 76 cm. of mercury at 0°C.

Solution : (a) Representing the initial pressure and absolute temperature as P_1 and T_1 and the final pressure as P_2 the fact that the total mass of air is constant and is expressed by applying eqn. (16·15),

$$\frac{P_1(V_A+V_B)}{T_1}=P_2\left(\frac{V_A}{T_1+100}+\frac{V_B}{T_1}\right)$$

$$\text{So that, } P_2=P_1\,\frac{(V_A+V_B)(T_1+100)}{T_1V_A+(T_1+100)V_B}$$

$$=76\times\frac{1400\times373}{273\times1000+373\times400}$$

$$=93·99 \text{ cm. of mercury.}$$

(b) The fraction of the original mass in A that stays in the bulb is, vide eqn. 16·18,

$$\left(\frac{P_2V_A/T_2}{P_1V_A/T_1}\right)=\frac{P_2T_1}{P_1T_2}=\frac{93·99}{76}\times\frac{273}{373}=0·905$$

since molecular weight and the gas constant are constant.

Thus, $1-0·905=0·095$ of the original mass in A is transferred. Since 1000 c.c. of air at N.T.P. has a mass of 1·29 gm., this transfer amounts to $0·095\times1·29=0·12$ gm. approximately.

Example 16·19. *The density of hydrogen at N.T.P. is 0·0896 gm./ litre. Find (a) the value of R for 1 gm. molecule of this gas, and also (b) obtain the value of the constant for 10 gm. of the gas.*

Solution : (*a*) 1 gm-mole of hydrogen contains 2 gm. of hydrogen, since molecular weight of hydrogen is 2. Now, since 0·0896 gm. of hydrogen occupies the volume of 1 litre at N.T.P., therefore,

2 gm. of hydrogen occupies the volume of $2\times\dfrac{1}{0·0896}=22·321$ litre.

But, normal pressure $P_0=76\times13·6\times980=1·013\times10^6$ dyne/sq. cm.

The normal temperature $V_0=273°\text{K}$. Hence we have,

$$R=\frac{P_0V_0}{T°}=\frac{(1·013\times10^6)\times22·321\times10^3}{273}$$

$$=8·28\times10^7 \text{ erg/°K/mole.}$$

(b) From eqn. (16·19), $K=\dfrac{mR}{M}=\dfrac{10\times8·28\times10^7}{2}$

$$=4·14\times10^8 \text{ erg/°K.}$$

Example 16·20. *One litre of air at N. T. P. weighs 1·293 gm. Calculate the value of the constant K in the ideal gas equation PV=KT for 1 gm. of air.*

Solution : Volume of 1 gm. of air at N.T.P. $=\dfrac{1000}{1·293}$ c.c.

Hence, K for 1 gm. of air is given by

$$K=\frac{PV}{T}=\frac{1·013\times10^6\times\frac{1000}{1·293}}{273}=2·869\times10^6 \text{ erg per °K per gm.}$$

16·13. Relations among the pressure, temperature and density of a gas.

Let $P, V, T,$ and ρ denote respectively the pressure, volume, absolute temperature and density of the fixed mass of a gas. Then from ideal gas equation, $PV/T=K$, where K is a constant.

But mass being fixed, $V \propto \frac{1}{\rho}$ $\quad \therefore \quad \frac{P}{\rho T}=$ a constant ... (16·21)

Hence, if for fixed mass of a gas P_1, T_1, ρ_1 are the initial and P_2, T_2, ρ_2 the final values, then

$$\frac{P_1}{\rho_1 T_1}=\frac{P_2}{\rho_2 T_2} \quad \ldots \quad \ldots \quad (16{\cdot}22)$$

(i) If the pressure is kept constant and the temperature is changed, then from eqn. (16·21), $\rho T=$constant ... (16·23)

Hence, *density of a gas at constant pressure varies inversely as its absolute temperature.*

(ii) If the temperature is kept constant and the pressure is varied, then from eqn. (16·21),

$$\frac{P}{\rho}=\text{constant} \quad \ldots \quad \ldots \quad (16{\cdot}24)$$

Therefore, *the density of a gas at constant temperature varies directly as its pressure.*

Example 16·21. *One litre of the inert gas argon is found to weigh 1·62 gm. at 15°C. and under a pressure of 74 cm. of mercury. Find the density at N.T.P. and deduce the molecular weight of argon.*

Solution : Here $P_1=74$ cm. ; $T_1=273+15=288°$K ;

$\rho_1=1{\cdot}62$ gm./lt.

$P_2=76$ cm. ; $T_2=273°$K. Let ρ_2 be the required density.

From eqn. (16·22),

$$\rho_2=\frac{P_2 T_1}{P_1 T_2}\cdot\rho_1=\frac{76}{74}\times\frac{288}{273}\times 1{\cdot}62 \text{ gm./litre.}$$

$$=1{\cdot}755 \text{ gm./litre.}$$

Gram molecular volume=22·4 litre at N.T.P.

The mass in gm. of argon occupying this volume is its molecular weight, which is $1{\cdot}755\times 22{\cdot}4=39{\cdot}31$.

Example 16·22. *The density of air is 1·29 gm. per litre at N.T.P. Calculate the mass of 5 litre of air at 182°C. and at a pressure of 6 atmospheres.*

Solution : Here $P_1=1$ atmosphere, $T_1=273°$K.

$\rho_1=1{\cdot}29$ gm./litre and $P_2=6$ atmosphere, $T_2=182+272=455°$K.

$\therefore$ Fron eqn. (16·22), $\rho_2=\frac{P_2}{P_1}\cdot\frac{T_1}{T_2}\cdot\rho_1=\frac{6}{1}\times\frac{273}{455}\times 1{\cdot}29=4{\cdot}645$ gm./litre

Hence, the required mass$=5\times 4{\cdot}645=23{\cdot}23$ gm.

Example 16·23. *An electric bulb of volume 300 c.c. was sealed off during the manufacture at a pressure of 10^{-6} cm. of Hg. at 30°C. Compute the number of air molecules contained in the bulb. Avogadro's number$=6{\cdot}02\times 10^{23}$ per gm. molecule.*

Solution : Let n be the required number of air molecules. Since 1 gm. mole of a gas at N.T.P. occupies 22·4 lt. and contains N (Avogadro's Number) molecules, we have from eqn. (16·22),

$$\rho_2 = \rho_1 \cdot \left(\frac{T_1}{T_2}\right)\left(\frac{P_2}{P_1}\right)$$

where, $P_1 = 76$ cm., $\rho_1 = \dfrac{6{\cdot}02 \times 10^{23}}{22{\cdot}4 \times 10^3}$ $T_1 = 273°\text{K}$.

$$P_2 = 10^{-6} \text{ cm.}, \quad \rho_2 = \frac{n}{300}, \quad T_2 = 273 + 30 = 303°\text{K}.$$

$$\therefore \quad \frac{n}{300} = \frac{6{\cdot}02 \times 10^{23} \times 273 \times 10^{-6}}{22{\cdot}4 \times 10^3 \times 303 \times 76}$$

or, $n = 9{\cdot}561 \times 10^{13}$

● EXERCISE ●

[A] Essay type questions

1. State and explain Boyle's Law. How is this law verified experimentally ?

2. State and explain Charles' law. Describe an experiment for it's verification.

3. Define 'Absolute Zero'. Calculate its value in both Centigrade and Fahrenheit scales. Why the absolute zero is treated as more universal and more fundamental than the centigrade zero.

4. State (a) how the volume of a gas increases when it is heated at constant pressure and (b) how its pressure increases when it is heated at constant volume. Derive the relation $\frac{PV}{T}$ = constant.

5. Define volume and pressure coefficients for a gas ? Why is it necessary to define two coefficients for a gas ? Show that for a perfect gas, the volume coefficient of expansion is equal to the pressure coefficient. Write down the units of these two coefficients in C.G.S. and F.P.S. systems.

6. What is meant by ideal gas ? Under what conditions a real gas may be treated as an ideal gas ? Give reasons for your answer.

What is an equation of state ? Derive it for an ideal gas.

7. Show from Charles' Law that the volume of a gas should become zero at a certain temperature. How is it utilised in defining the absolute scale of temperature ? Why is this scale termed absolute ?

8. What do you understand by the absolute scale of temperature ? How is the centigrade scale related to it ? How do you arrive at the idea of an absolute scale of temperature ?

[*Jt. Entrance '78*]

9. Establish Charles' Law in terms of absolute temperature. How does expansion of a gas differ from that of a solid and liquid ?

10. What do you mean by 'universal gas constant' ? Why is it so called ? Calculate the value of the constant for 1 gm. mole of any gas at N. T. P.

11. (a) Deduce an equation to show how the density of a gas varies with its pressure and temperature.

(b) State and explain 'Pressure Law'.

(c) Describe an experiment to determine 'the volume coefficient' of a gas.

[B] Short-answer type questions

1. 'In the study of the expansion of gases, volume, temperature and pressure are taken into account as variables'. Explain the significance of the statement.

2. Both gas and liquid are enclosed in a container to study their expansions. But, unlike liquid, gas has no apparent expansion coefficient. Explain the statement.

3. A container filled with oxygen is brought to the surface of the moon from the earth's surface. What would happen if the container is (i) a steel cylinder, and (ii) a rubber balloon? [*Model Questions, H. S. Council, '78*]

4. Why do we always take the initial volume and pressure at 0°C., while we define co-efficient of expansion of a gas in contrast to the case of solids and liquids?

5. Can we ever attain any temperature lower than the absolute zero of temperature? Why?

[C] Simple Problems

1. An open glass tube is immersed in mercury so that a length of 13 cm. extends above the mercury level. The open end of the tube is then closed and is raised further by 35 cm. Find the length of the air column above mercury in the closed tube. Assume that the atmospheric pressure=76 cm. of Hg. [*Ans.* 20·41 cm.]

2. A column of air in a uniform tube, closed at one end, occupies 27 cm. at atmospheric pressure. When subjected to the pressure of the water supply, the column of air occupies 20 cm. Calculate the pressure of the water supply, the atmospheric pressure being 76·5 cm. of mercury. [*Ans.* 103·3 cm. of Hg.]

3. A diving bell of internal volume 6 cubic metre is lowered into sea water of density 1·03 gm. cm^{-3} until the volume of the contained air is reduced to 5 cubic metre. Assuming that the temperature is unchanged, calculate how far the surface of the water in the bell is below the surface of the sea, the pressure of the atmosphere being equal to that due to 75 cm. of mercury. Density of mercury=13·6 gm. cm^{-3}. [*Ans.* 1·98 metre]

4. A cylindrical vessel of length 100 cm. with its open mouth downward is lowered to the bottom of a tank. The water is found to rise 30 cm. in it. Find the depth of the tank. Given that the atmospheric pressure at the surface of the tank is 900 cm. of water and the temperature is constant. [*Ans.* 415·7 cm.]

5. An air filled cylinder at atmospheric pressure is urged to immerse into water with its mouth downwards until water occupies 1/3 rd part of the whole volume of the cylinder. How much further it has to be dipped into water so that water occupies 2/3 rd of its volume? Given, atmospheric pressure=76 cm. of Hg. and density of Hg.=13·6 gm./c.c [*Ans* 15·5 metre]

6. A tyre of a motor car has 1·5 metre length and 14 cm. diameter. How much air has to be introduced into the tyre at atmospheric pressure so that the pressure inside it becomes 3 atmosphere. [*Ans.* 69·3 litre]

7. A barometer reads 76 cm. Its torricellian space is 9 cm. long. What volume of air measured at atmospheric pressure should be introduced into the space to cause the mercury column to drop to 57 cm.? The cross-section of the barometer tube is 1 sq. cm. [*Ans.* 7 c.c.]

8. A faulty barometer contains some air in its torricellian space. It reads 75 cm. when the true atmospheric pressure is 76 cm. What will be the true atmospheric pressure when the barometer reads 74·5 cm.? Given, length of the tube measured from the free surface of mercury in the reservoir is 90 cm. [*Ans.* 75·47 cm.]

9. A barometer reads 75 cm. and the torricellian space above the mercury column in 3 cm. If some air which could occupy the entire space at atmospheric pressure has been introduced, what should be the new reading of the barometer? [*Ans.* 61·42 cm.]

10. A uniform narrow tube closed at one end contains some air confined by a mercury 15 cm. long. When the tube is held vertically with its closed end downwards, the air column is 20 cm. long but when it is held horizontally the air column is 24 cm. long. Calculate the atmospheric pressure. [*Ans.* 75 cm. of Hg.]

11. A narrow uniform glass tube contains some dry air which is sealed off by a thread of mercury 15 cm. long. When the tube is held vertically, with the closed end at the bottom, the air column is 40 cm. long; but when it is inverted, the air column is 60 cm. long. Calculate the atmospheric pressure. [*Ans.* 75 cm. of Hg.]

12. A narrow uniform tube contains some air entrapped by a mercury column of length 25 cm. While the open end of the tube is held vertically upwards the air column is of length 5 cm., but if the tube is inverted, this length becomes 10 cm. What would be the length of the air column when the tube is held horizontally? [*Ans.* 6·67 cm.]

13. A uniform capillary tube, 125 cm. long and closed at one end, contains air confined by a mercury thread. When the tube is held vertically with its closed end downwards the air occupies a length of 40 cm. and the mercury thread is 85 cm. long. The barometric height is 76 cm. of mercury. What length of the thread will run out if the tube is slowly inverted ?
[*Ans.* 68·4 cm.]

14. In an experiment the pressure of a mass of air at constant volume was found to increase from 75 cm. of mercury at 15°C. to 91 cm. of mercury at 75°C. From these results what would be (a) the pressure of air at 0 °C. (b) the coefficient of increase of pressure of air at constant volume ? [*Ans.* (a) 71 cm. of Hg., (b) $3{\cdot}756\times10^{-3}$/°C.]

15. Three litre of air at 0°C, and a pressure of 1 atmosphere are heated in such a way as to keep its volume unchanged. Calculate the temperature of the air when its pressure becomes 5 atmosphere.

The temperature is now kept constant and the pressure reduced to 1 atmosphere. What will be the resulting volume ?

If finally this last pressure is maintained, calculate the temperature for which the volume will be reduced to 2 litre. (*London*)
[*Ans.* 1092°C. ; 15 litre ; —91°C.]

16. A gas at 15°C. has its temperature raised so that its volume is doubled, the pressure remaining the same. What is its final temperature ? [*Ans.* 303°C.]

17. A mass of gas is at 20°C. Find the temperature at which its pressure will be tripled if it is heated at constant volume. At what temperature its volume will be tripled, if the pressure is kept constant ? [*Ans.* 606°C., 606°C.]

18. A flask containing air is corked up at 30°C., the pressure being 15 lb./sq. inch. Calculate the temperature at which the cork will be blown out if this occurs when the pressure inside the flask is 25 lb./sq. inch. [*Ans.* 232°C.]

19. The volume of a gas at 30°C. temperature and 108 cm. pressure is 256 c.c. What will be the volume of the same gas when the temperature is 0°C. and the pressure is 76 cm. ?
(*H.S. comp. '71*) [*Ans.* 327·8 c.c.]

20. A motor car tyre has a volume of 15 litre and contains air at 2·5 times atmospheric pressure when the temperature is 17°C. What will be the pressure inside the tyre in lb./inch^2 when the temperature rises to 37°C. and volume of the tyre increases to 15·5 litre ?
(*Jt. Entrance '62*) [*Ans.* 38·15 lb./inch^2]

21. A narrow uniform tube closed at one end contains some dry air which is sealed off by a mercury thread of length 10 cm. The tube stands vertically with its open end at the top and the length of the air column is 25 cm. at 12°C. To what temperature must the tube be heated for the mercury to rise 5 cm. ? [*Ans.* 69°C.]

22. A pressure vessel contains 20 litre of a gas at 13°C. and the pressure gauge attached to the vessel reads 15 atmosphere. The vessel is opened and then closed. The pressure gauge now reads 11 atmosphere. What volume of gas at atmospheric pressure escapes ? Assume that there has been no temperature change during the process.

The gas remaining in the vessel is then cooled to 0°C. What will then be the reading of the gauge ? [*Ans.* 80 litre ; 10·5 atmos.]

23. An open glass flask of 0·5 litre at 27°C. is heated uniformly to 167°C. allowing expulsion of air and corked. What is the new pressure of air inside the flask if the atmospheric pressure is 76 cm. of Hg. and the flask is cooled to 27°C. ? [*Ans.* 51·82 cm. of Hg.]

24. A barometer tube 1 metre long, contains some air above the mercury. The reading is 74 cm. when the true atmospheric pressure is 76 cm. and the temperature is 17°C. If the reading is observed to be 75·5 cm. on a day when the temperature is 7°C., what is the true pressure ? [*Ans.* 77·55 cm.]

25. The measurement of a room is 15 metre × 10 metre × 5 metre. If the temperature of the room is increased from 27°C. to 37°C. calculate what percentage of the original volume of air will be expelled from the room, the pressure remaining constant ? [*Ans.* 3·33%]

26. A vessel 500 c.c. in volume contains air at a pressure of 10 atmosphere and temperature 50°C. Another vessel 750 c.c. in volume contains air at a pressure of 12 atmosphere and temperature 30°C. If the two vessels are connected together and maintained at a temperature of 97°C., calculate the pressure inside the vessels. [*Ans.* 13·38 atmosphere]

27. One litre of air at 0°C. expands to 1·5 litre at 136·5°C. What is the value of absolute zero? [*Ans.* —273°C.]

28. What will be the volume of 1 gm. mole of a gas at a pressure of 100 atmosphere and at a temperature of -173°C.? [*Ans.* 82 c.c.]

29. Calculate the value of the gas constant in erg per gm. mole per °C. Given that the coefficient of expansion of the gas=0·00366/°C., volume of a gm. mole of the gas at N.T.P.= 22·414 litre, density of mercury=13·6 gm./c.c. and g=981 cm./sec.^{2} [*Ans.* $8{\cdot}318\times10^7$]

30. At a place a barometer stands at 75 cm. and the temperature is 27°C. At another place the respective readings are 70 cm. and 17°C. Compare the densities of air at the two places. [*Ans.* 29 : 28]

31. 500 c.c. of air at N.T.P. weighs 0·6 gm. Find the mass of the air required to produce at -23°C. a pressure of 3 atmosphere in a volume of 50 c.c. [*Ans.* 0·197 gm.]

32. The density of Hydrogen at 0°C. and standard pressure is 0·09 gm./lt. Find the volume occupied by 1 gm. of hydrogen at 200°C. and standard pressure. (*Oxford*) [*Ans.* 19·25 lt.]

33. Find the density of methane at 27°C. and 3 atmospheric pressure. Given that the molecular weight of methane is 16, and the molar gas constant is 0·082 litre atm. °K.$^{-1}$ mole^{-1}. [*Ans.* 1·95 gm./litre]

34. An electric vacuum tube was sealed off during manufacture at a pressure of $1{\cdot}2\times10^{-5}$ mm. at 27°C. Its volume is 100 cm^3. Compute the number of gas molecules remaining in the tube. [*Ans.* $3{\cdot}862\times10^8$]

35. The air in a vessel with volume 5 lt has a temperature of 27°C and is subject to a pressure of 20 atmosphere. What mass of air must be released from the vessel so that its pressure drops to 10 atmosphere? The density of air at N. T. P=1·29 gm/lt. [*Ans.* 58·7 gm]

36. How many molecules are there in a litre of air at atmospheric pressure and room temperature? Take room temperature =27°C, one atmospheric pressure =10^6 dyne/cm^2, and $R=8{\cdot}31\times10^7$ erg/(°K mole) [*Ans.* $2{\cdot}45\times10^{22}$]

37. Two glass bulbs of volume 3 litres and 1 litre respectively are connected by a capillary tube. Air at a pressure of 76 cm at 30°C is contained in the apparatus which is then hermetically sealed. If the 3 litre bulb be immersed in steam at 100°C the other remaining at 30°C, what would be the pressure of the air in the bulbs.

(Neglect the increase in volume of the 3 litre bulb) (*Jt. Entrance* '81) [*Ans.* 88·4 cm of Hg]

[D] Harder Problems

1. A sample of gas occupies 100 c.c. at 0°C. and 72 cm. pressure, and a volume of 200 c.c. at 68·25°C. and 45 cm. pressure. Calculate the coefficient of expansion, assuming that the gas obeys Boyle's law and expands uniformly at constant pressure. [*Ans.* $\frac{1}{273}$ per °C]

2. A narrow cylindrical tube 80 cm. in length and open at both ends is submerged to half its length in mercury. The upper end of the tube is then closed and the tube is lifted out of the mercury. The length of the column of mercury remaining in the tube is 23 cm. What is the atmospheric pressure? [*Ans.* 77·13 cm. mercury]

3. A cylinder closed at both ends has a capacity of 22·4 litre and it contains 4 gm. of hydrogen at 0°C. Calculate the pressure when the cylinder is raised to 60°C. If the hydrogen is replaced by 14 gm. of nitrogen at 0°C., what will be the pressure exerted by the gas at 100°C.? (*Jt. Entrance* '68) [*Ans.* 2·44 atmos.; 0·683 atmos.]

4. A narrow cylindrical pipe closed at one end contains air which is separated from the outer atmosphere by a column of mercury. If the pipe is turned with the closed end uppermost, the air inside it occupies a length x mm.; whereas when the open end is uppermost, the length occupied is x' $(<x)$ mm. The length of the mercury column is h mm. Find the atmospheric pressure. [*Ans.* $\dfrac{(x'+x)h}{x-x'}$ mm. of Hg.]

5. A barometric tube is immersed in a deep vessel containing mercury so that the levels of the mercury in the tube and the vessel are the same. The air in the tube occupies a column of length x cm. The tube is now raised x' cm. Prove that the mercury rises in the tube by $\frac{1}{2}[P+x+x'-\sqrt{(P+x+x')^2-4x'P}]$ cm., where P is the atmospheric pressure measured in cm. of Hg.

6. A cylindrical glass tumbler is lowered upside-down into water and floats vertically, with the internal surface of its bottom level with the surface of the water in the vessel. The weight of the tumbler is 500 gm.; the area of the bottom is 10 cm^2. The air pressure in the glass before submersion was 76 cm. of mercury. What portion of the tumbler will the air occupy after submersion? [*Ans.* 0·9535 of the initial volume]

7. An air bubble starts rising from the bottom of a lake. Its diameter is 3·6 mm. at the bottom and 4 mm. at the surface. The depth of the lake is 2·5 metre and the temperature at the surface is 40°C. What is the temperature at the bottom of the lake? (Assume that the variation of density of water with depth is negligible. Atmospheric pressure is 76 cm. of mercury and acceleration due to gravity is 980 cm./sec^2.) (*I.I.T.* '66) [*Ans.* 10·2°C]

8. A balloon will carry a total load of 175 kg. when the temperature and pressure are normal. What load will be balloon carry on rising to a height at which the barometric pressure is 50 cm. of mercury and the temperature is —10°C., assuming that the envelope maintains a constant volume? (*London*) [*Ans.* 119·5 kg.]

9. A cylindrical pipette of length x cm. is half submerged in mercury. It is closed by a finger and withdrawn. Part of the mercury flows out. Prove that the length of mercury column that remains in the tube is $\frac{P+x-\sqrt{P^2+x^2}}{2}$ cm. where P is the atmospheric pressure in cm. of Hg.

10. In a mercury barometer with a regular cylindrical barometric tube, the distance from the mercury level in the cup to the soldered end of the tube is x mm. An air bubble is introduced into the tube at normal barometric pressure P and temperature t_1, as a result of which the length of the mercury column diminishes and becomes equal to h_1 mm. Show that the expression for the correction p_1 to be added to the barometer reading h mm. in order to be able to use it at any temperature t and any height h of the mercury column is $p_1=\frac{1+\alpha t}{1+\alpha t_1}\cdot\frac{x-h_1}{x-h}(P-h_1)$ mm. where α is the expansion coefficient of the gas.

11. Two thermally insulated vessels of volumes 1 litre and 3 litre are connected by a pipe with a tap. Prior to opening the tap, the first vessel contains nitrogen at 0°C. and pressure 0·5 atm., and the second vessel argon at 100°C. and pressure 1·5 atm. When the tap is opened the temperature established in the gas mixture is 79°C. Calculate the pressure. [*Ans* 1·223 atm.]

12. A mercury column of length 15 cm. encloses an air column of 10 cm. at 27°C. in a capillary tube when the tube with its open end upward is held vertically. Now the tube is inverted and the length of the air column becomes 15 cm. What is the temperature of the air in the tube while it encloses air column of length 20 cm. at its inverted position? [*Ans.* 127°C.]

13. Find the percentage increase of pressure in tyres of a bicycle taken out of the shade (15°C.) into the sun (35°C.) disregarding the expansion of rubber. [*Ans.* 6·94%]

14. It is found that the volume of a certain gas increases in the ratio 1·035 : 1 between 15°C. and 25°C. Calculate the absolute zero on the centigrade scale for this gas. [*Ans.* —270·7°C.]

15. Use the ideal gas equation $PV=nRT$, where $R=0·0821$ litre atmosphere per °K per mole, to solve each of the following:

(i) At 17°C. and 765 mm. of mercury pressure, 1·3 litre of an ideal gas weighs 3 gm. Compute the molecular weight of the gas.

(ii) Compute the volume occupied by 10 gm. of oxygen at 27°C. and 750 mm. of mercury. The molecular weight of oxygen is 32.

(iii) Find the density of argon at 20°C. and 5 atmos. Molecular weight of argon is 40.

[*Ans.* (i) 54·58 gm./mole; (ii) 7·798 litre; (iii) 8·314 gm./litre.]

16. The density of argon is 1·6 gm. per litre at 17°C., at a pressure of 76 cm. of mercury. What is the mass of argon in an argon filled electric lamp bulb of volume 150 c.c. if the pressure inside is 76 cm. of mercury when the average temperature of the gas is 127°C.?

[*Ans.* 0·174 gm.]

17. The density of a gas may be determined as follows. A large bulb of capacity 1 litre is filled with the gas under test to a pressure 74 cm. of Hg and weighed. Let its weight be 0·71 gm. Part of the gas is now removed, so that the pressure falls to 70 cm. of Hg. The new weight of the bulb is 0·61 gm. What is the density of the gas at atmospheric pressure?

[*Ans.* $1·9 \times 10^{-3}$ gm./c.c.]

18. A glass capillary tube, sealed at both ends is 100 cm long. It lies horizontally with middle 10 cm containing mercury. The two ends of the tube (which are equal in length) contain air at 27°C and at a pressure of 76 cm of mercury. The tube is kept in a horizontal position, such that the air column at one end is at 0°C and the other end is maintained at 127°C. Calculate the length of the air column which is at 0°C and its pressure. Neglect the change in volume of mercury and glass. (*I.I.T. '75*) [*Ans.* 36·5 cm ; 85·3 cm of Hg]

17

CHAPTER

CALORIMETRY

17·1. Calorimetry.

The term *calorimetry* relates to the measurement of *quantity of heat*, i.e. of quantity of thermal energy in the process of transfer from one body to another. When several bodies at different temperatures are placed in thermal contact with each other, the hotter bodies become cooler and the cooler bodies become warmer. Ultimately, all bodies arrive at the *same temperature*. They are then said to be in *thermal equilibrium*. In principle, the object of all the calorimetric measurements is to know this final temperature, called the *equilibrium temperature* and hence to derive quantitatively the heat gained or lost by the substances from the knowledge of their known thermal properties.

17·2. Quantity of heat.

The quantity of heat possessed by a body depends on several factors. An idea of these factors can be obtained from the following experiments.

Experiment 1 : The apparatus consists simply of a copper vessel, half filled with water. A finely graduated thermometer and an electric immersion heater, which supplies heat at a uniform rate, is introduced into the mass of the water. Keeping the water continually stirred, time taken to raise the temperature through say, 5°C., 10°C., 15°C, etc. are noted very carefully. Those times are found to be roughly in the ratio* 1 : 2 : 3 etc. This means that the quantities of heat supplied by the heater and hence that absorbed by the given mass of water (as well as the vessel) are in the ratio 1 : 2 : 3. Since the rise in temperature also bear the same ratio, we get that *for a fixed mass of water, the heat supplied is proportional to the rise in temperature.* The same result will be obtained for other substances also.

Experiment 2 : Different amounts of water such as 200, 400, 600 gm. etc., are taken successively in the vessel. The time taken for a selected rise in temperature, say 10°C., to occur is noted ; these are found to be in the ratio 1 : 2 : 3 etc., in the same proportion as the mass of water taken. We, therefore, conclude that *for a fixed temperature rise, the heat supplied is proportional to the mass of water taken.* The same conclusion holds for other substances as well.

Combining these experimental results, we immediately get that *quantity of heat supplied to a substance ∝ mass of the substance × rise in temperature.*

If we represent the quantity of heat supplied by the letter Q, the rise in temperature by t and mass of substance taken by m, then

$$Q \propto mt, \quad \text{when both } m \text{ and } t \text{ vary,}$$

$$\text{i.e., } Q = smt \qquad \ldots \qquad (17{\cdot}1)$$

where s is the constant of proportionality depending on the *nature of the substance.*

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* The slight departure of the observed times from this ratio occurs due to the heat absorbed by the vessel, heat lost during experiment etc.

17·3. Thermal Units.

Units of the quantity of heat are defined in such a way that for water, the above constant of proportionality equals unity. Thus, for water eqn. (17·1) becomes $Q=mt$ ... (17·2)

When $m=1$, $t=1$, we get $Q=1$. So we define *unit quantity of heat* (on any system of units) as the quantity of heat required to raise the temperature of unit mass of water through one degree.

Choice of particular system of units for expressing mass and temperature of a substance leads to the definition of different thermal units as follows :

Calorie : *One calorie is the quantity of heat which must be given to 1 gm. of water to raise its temperature through 1°C.*

British Thermal Unit : *One British thermal unit (B. Th. U.) is the quantity of heat which must be given to 1 lb. of water to raise its temperature through 1°F.*

Kilogram Calorie : *One kilogram calorie is equal to 1000 calorie i.e., it is the quantity of heat which must be given to 1000 gm. of water to raise its temperature by 1°C.*

Centigrade Heat Unit : *Centigrade heat unit (C.H.U), also called pound calorie, is the quantity of heat which must be given to 1 lb. of water to raise its temperature by 1°C.*

It should be noted that the thermal units defined above, vary with the location of the degrees. This means that the thermal energy required to raise the temperature, say, from 5°C. to 6°C., is different from that required for a rise from, say, 20°C to 21°C. But the differences are so small, amounting in some instances to 1 or 2 part in a thousand, that for most practical purposes it is neglected. However, for exact purposes the thermal units must be defined for some definite temperature of water. *The calorie is precisely defined as the amount of heat required to raise the temperature of 1 gm. of water from 14·5°C to 15·5°C.* It is called **15°C calorie.** For an accurate definition of B. Th. U., the said temperature interval is taken from 63°F. to 64°F.

Sometimes, in experiments involving large changes of temperature, **'the mean-calorie'** is used. It is defined as the $\frac{1}{100}$th. of the amount of heat required to raise the temperature of 1 gm. of water from 0°C. to 100°C.

Relation among different thermal Units.

1 B. Th. U. = Quantity of heat supplied to raise the temperature of 1 lb. of water by 1°F.
= 1 lb. × 1°F. [by eqn. 17·2]
= 453·6 gm. × $\frac{5}{9}$°C. [∵ 1 lb. = 453·6 gm. and 1 F. degree = $\frac{5}{9}$C. degree]
= 252 calorie = 0·252 kg. calorie.

1 C. H. U. = 1 lb. × 1°C. = 453·5 gm. × 1°C. = 453·6 calorie = 0·4536 kg. calorie.

In England, for commercial purposes a larger unit called **therm,** is used.

1 therm = 10^5 B. Th. U. = $2·52 \times 10^7$ calorie.

17·4. Specific heat.

In equation (17·1) the constant s depends on the nature of the substance. Hence the quantity of heat required to effect a definite rise in temperature of a given mass of different substances must be different. Similarly, when a body loses heat, the amount of heat lost is found to depend likewise upon the nature of the substance. The following demonstration experiment brings out this fact clearly.

Demonstration experiment : In this experiment, first devised by Tyndall, four balls, as shown in the Fig. 17·1, all of same mass but of different materials are heated in boiling water to the temperature of 100°C. They are then simultaneously placed on a thick slab of wax. As the balls melt their way through the slab, it will be found that different balls penetrate to different depths [Fig. 17·1]. Obviously the ball that gives up more heat melts more wax and penetrates deeper. This shows that the heat given out by equal mass of copper and brass for the same drop in temperature is considerably greater than that given out by tin and lead.

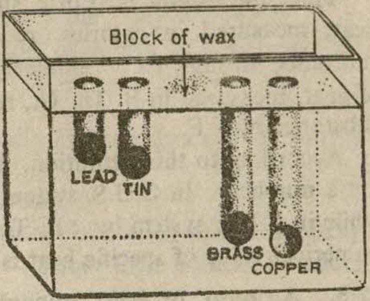


Fig. 17·1 : Tyndall's experiment

Hence the amount of heat lost by a body does not depend only upon its mass and temperature, but also upon the nature of the substance.

By simple experiments we can similarly show that the amount of heat required to raise the temperature of a substance depends also upon its nature. This dependence of the heat lost or heat gained by a substance on its nature is ascribed to a specific property of the substance, called its *specific heat.*

Definition of specific heat : The specific heat of a substance is usually defined in two ways :

(1) *The* **specific heat** *of a substance is the ratio of the qnantity of heat required to raise the temperature of a certain mass of the substance through a certain range of temperature to the quantity of heat required to raise the temperature of the same mass of water through the same range of temperature.*

According to this definition, specific heat (s) of a substance

$$=\frac{\text{Heat required to raise a given mass of substance by } t^\circ.}{\text{Heat required to raise the same mass of water by } t^\circ.}$$

$$=\frac{\text{Heat required to raise unit mass of a substance by } 1^\circ}{\text{Heat required to raise unit mass of water by } 1^\circ.}$$

But heat required to raise the temperature of unit mass of water by $1^\circ = 1$ calorie or B. Th. U.

Thus the heat required to raise the temperature of unit mass of a substance by $1^\circ = s$ calorie or B. Th. U. It should be noted that, by definition, the specific heat of water is unity.

Since the specific heat is the ratio between two quantities of heat, it is a number. Obviously, *the value of specific heat does not depend on the system of units chosen, and is the same in all systems.*

Thus the statement that, the specific heat of aluminium is 0·21 means that to raise the temperature of 1 gm. of aluminium through 1°C. requires 0·21 calorie of heat or that to raise the temperature of 1 lb. of aluminium through 1°F. requires 0·21 B. Th. U. of heat.

(2) *The* **specific heat** *of a substance is the quantity of heat required to raise the temperature of unit mass of it by one degree.*

Thus, the specific heat of a substance in C. G. S. system is the quantity of heat, measured in calories, required to raise the temperature of 1 gm. of the substance through 1°C. Similarly in F.P.S. system, specific heat is the quantity of heat, measured in B. Th. U., required to raise the temperature of 1 lb. of the substance by 1°F.

According to this definition, the specific heat has some definite unit ; it is not a number. In C.G.S. system, it is expressed by "calorie per gm. per °C," while in F.P.S. system by "B. Th. U. per lb. per °F." As stated earlier, the numerical value of specific heat is the same in both the systems.

17·5. Amount of heat absorbed or given out by a body for a rise or fall of temperature.

From the two definitions of specific heat, it follows that if s be the specific heat of a substance—

The amount of heat required to raise 1 gm. of the substance through 1°C$=s$ calorie.

The amount of heat required to raise m gm. of the substance through 1°C$=ms$ calorie.

So the amount of heat required to raise m gm. of the substance through t°C.$=m.s.t.$ calorie.

Similarly, to raise the temperature of m lb. of a substance through t°F., heat required$=m.s.t.$ B. Th. U.

Symbolically $Q=m.s.t.$ ... (17·3)

∴ **Heat gained = mass × specific heat × temperature rise**

Similarly, *Heat lost = mass × specific heat × temperature fall.*

17·6. Thermal Capacity.

The thermal capacity of a substance is the quantity of heat required to raise the temperature of it by 1°

Thus, if m be the mass of the substance and s its specific heat, the thermal capacity of the substance is according to the definition$=ms$ units of heat.

∴ Thermal capacity of a substance = Its mass × its specific heat.

In C.G.S. system, thermal capacity of a substance is expressed in "calorie per °C" and in F.P.S. system "B. Th. U. per °F".

It is obvious that the specific heat of a substance is equal to its thermal capacity per unit mass.

17·7. Water equivalent.

The water equivalent of a body is defined as the mass of water which will be heated through 1° by the amount of heat that raises the temperature of the body itself through 1°

Let the mass of the body of specific heat s be m gm. To raise its temperature by 1°C., ms calorie of heat will be required. Since the heat required to raise 1 gm. of water by 1°C. is 1 calorie, this amount of heat will raise ms gm. of water by 1°C.

∴ Water equivalent of the body, $W = ms$ gm.

Similarly in F.P.S. system, the water equivalent of the body $W = ms$ lb.

It is obvious that the expression for heat lost or gained by a body can be written as,

$$Q = Wt \quad \cdots \quad (17{\cdot}4)$$

∴ Heat gained or lost = Water equivalent of the body × rise or fall in temperature.

Comparison of thermal capacity and water equivalent :

(i) Both thermal capacity and water equivalent of a body are numerically equal to the product of its mass and specific heat. Hence their numerical values are equal.

(ii) By thermal capacity we mean some amount of heat ; on the other hand, water equivalent signifies some amount of water.

(iii) They are expressed in different units. The unit of thermal capacity is cal./°C. or B. Th. U./°F. But the unit of water equivalent is gm. or lb.

Example 17·1. *Calculate what should be the thermal capacity and water equivalent of 10 gm. of aluminium of specific heat 0·21. How much heat would be necessary to raise its temperature from 0°C. to 30°C. ?*

Solution : Mass = 10 gm., Specific heat = 0·21

∴ Thermal capacity of 10 gm. of aluminium

$= 10 \times 0{\cdot}21 = 2{\cdot}1$ cal./°C.

Hence, water equivalent of 10 gm. of aluminium = 2·1 gm.

∴ The quantity of heat required to raise the temperature of 10 gm. of aluminium from 0°C. to 30°C. $= 2{\cdot}1 \times 30 = 63$ cal.

Example 17·2. *(a) Find the amount of heat required to raise 150 gm. of copper from 20°C. to 100°C. (b) If the same quantity of heat is added to 250 gm. of lead at 20°C., will it be hotter than the piece of copper ? (Sp. heat of Cu = 0·093, of Pb = 0·03).*

Solution : (a) From eqn. (17·3), heat required

$= 150 \times 0{\cdot}093 \times (100 - 20) = 1116$ cal.

(b) Rise in temperature of the lead piece $= \dfrac{1116}{250 \times 0{\cdot}03} = 148{\cdot}8$°C.

Hence, the temperature of the lead piece $= 148{\cdot}8 + 20 = 168{\cdot}8$°C. i.e., the lead piece becomes hotter.

Example 17·3. *Two substances have their densities in the ratio 3 : 4 and specific heats 0·13 and 0·09 respectively. Compare their thermal capacities per unit volume.*

Solution : If $3x$ be the density of one substance, that of the other will be $4x$. Let the thermal capacities of the substances per unit volume be H_1 and H_2. Since density is the mass per unit volume, therefore, $H_1=3x\times 0·13$ and $H_2=4x\times 0·09$.

$$\therefore \quad \frac{H_1}{H_2}=\frac{13}{12}.$$

Example 17·4. *A copper calorimeter of water equivalent 20 gm. contains 500 gm. of water at 25°C. Calculate the time required to raise the temperature of water to the boiling point, when a burner supplying 1 B. Th. U. per second is employed to heat the water.*

Solution : Heat gained by water $=500\times(100-25)=37500$ cal.

Heat gained by the calorimeter $=20\times(100-25)=1500$ cal.

Hence the total heat supplied by the burner $=39000$ cal.

Since 1 B. Th. U. $=252$ cal. therefore, the time spent for raising water to its boiling point $=\frac{39000}{252}$ sec. $=2$ min. 34·8 sec.

Example 17·5. *A steam boiler made of copper weighs 200 kg. The boiler contains 100 kg. of water. How much coal is required to raise the temperature of the boiler filled with water from 25°C. to 100°C., if 3% of the heat is useful ? (Sp. heat of copper = 0·09 ; heat of combustion of coal = 7 × 10⁶ cal./kg.).*

Solution : Heat required by the boiler $=200\times 10^3\times 0·09\times(100-25)$

$$=1·35\times 10^6 \text{ cal.}$$

Heat required by water $=100\times 10^3\times 1\times(100-25)=7·5\times 10^6$ cal.

$\therefore$ Total heat required $=(1·35+7·5)\times 10^6$ cal. $=8·85\times 10^6$ cal.

$\therefore$ Total heat to be supplied by coal $=8·85\times 10^6\times\frac{100}{3}=2·95\times 10^8$ cal.

Amount of coal required $=\dfrac{2·95\times 10^8}{7\times 10^6}=42·14$ kg.

17·8. Calorimeter.

Calorimetric measurements are performed in a special type of metallic vessel, called *calorimeter*. In its simplest form, it consists of a thin walled copper can C of definite capacity, partly filled up with a liquid [Fig. 17·2]. A thermometer T and stirrer S of the same material as that of the calorimeter, are introduced inside the liquid ; the former registers the temperature of the liquid and the latter is used to stir up the liquid contained in the calorimeter.

17·9. Fundamental principle of calorimetry.

Most of the calorimetric measurements are done by introducing a heated body of definite mass and temperature into a calorimeter. Transfer of heat from this heated body to the calorimeter and its contents takes place since the latter are in a lower temperature. This transference of heat continues until all the substances acquire a common temperature. A condition of thermal equilibrium is then said to have been reached.

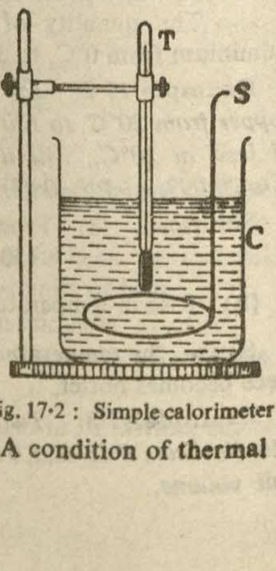


Fig. 17·2 : Simple calorimeter

We make the ideal assumptions that (i) during this interval, no external heat enters the calorimeter from outside and no heat leaves the calorimeter and goes outside and (ii) no chemical reaction takes place between the different substances involved.

Then it follows from the principle of conservation of energy,

$$\textbf{Heat lost by the hot bodies} = \textbf{Heat gained by the cold bodies} \qquad (17\cdot5)$$

This is the *fundamental principle of calorimetry.*

17·10. Prevention of heat transfer from or to the calorimeter.

In the above discussions, we have assumed that during experiment no heat enters or leaves the calorimeter. Failure to fulfil this condition will lead to error in the measurements done by application of the principle. The loss of heat from the calorimeter may take place by three processes, viz. (i) conduction, (ii) convention, and (iii) radiation (vide Chapter 21). For accurate work, these losses must be minimised as far as possible. In order to reduce the conduction loss, the calorimeter is placed on spikes of bad conductors like cork, card-board, wood etc. [Fig. 17·3]. To minimise the heat loss due to convection, the calorimeter is placed in a container which surrounds the calorimeter with a layer of *still-air.* Small holes bored in the lid of the container allow thermometer and stirrer to pass. Round the container itself, insulating materials like dry cotton, wood or felt is placed. Sometimes, it is surrounded by a constant temperature water jacket, so that the heat losses become regular. Radiation loss is minimised by polishing or by nickel-plating the outer surface of the calorimeter.

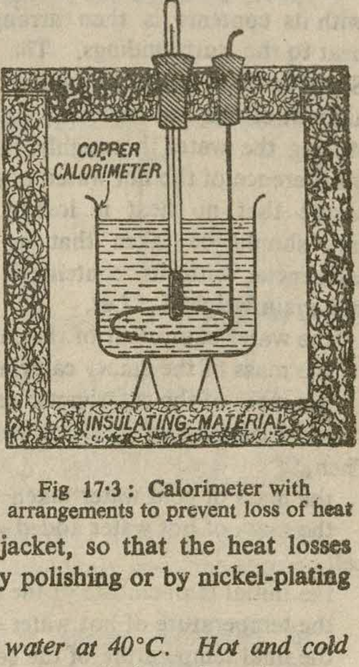


Fig 17·3 : Calorimeter with arrangements to prevent loss of heat

Example 17·6. *A bath contains 100 kg. of water at 40°C. Hot and cold taps are then turned on to deliver 30 kg. of water per minute each at temperatures of 50°C. respectively. How long will it be before the temperature in the bath has dropped to 30°C.? Assume complete mixing of the water and ignore heat losses.*

Solution : Suppose both the taps were turned for t min. for which mass of hot and cold water delivered into the bath was $3\times10^4\times t$ gm. each.

Now, heat lost by the water of the bath

$$=1\times10^5\times(40-30)=1\times10^6 \text{ cal.}$$

Also, heat lost by the water that enters in the bath from hot tap,

$$=3\times10^4\times t\times(50-30)=6\times10^5 t \text{ cal.}$$

Heat gained by water that enters in the bath from cold water tap,

$$=3\times10^4\times t\times(30-5)=7\cdot5\times10^5\times t \text{ cal.}$$

But since heat lost=heat gained, we have,

$$1\times 10^6+6\times 10^5\times t=7\cdot5\times 10^5\times t$$

$$\text{or,}\quad 1\cdot5\times 10^5\times t=1\cdot\times 10^6$$

$$\text{or,}\quad t=\frac{10}{1\cdot5}=6\cdot67 \text{ min.}$$

17·11. Calorimetric measurements.

The relation (17·5) serves as the basis for the experimental determination of various thermal quantities. We shall now devote ourselves to the detailed study of the determination of these quantities.

(1) **Determination of the water equivalent of the calorimeter :** The calorimeter with the stirrer is cleaned, dried and weighed. It is then about half-filled with water at room temperature and again weighed. The calorimeter with its contents is then arranged as shown in Fig. 17·3 to prevent transfer of heat to the surroundings. The temperature is noted from the thermometer. Some water is heated in another vessel and its temperature is recorded. Suitable amount of this hot water is quickly transferred to the calorimeter and after stirring the water thoroughly, the maximum temperature attained is noted. The transference of the hot water must be done very quickly ; this is necessary to ensure that no heat is lost by the water during transference. During stirring, care should be taken that no water splashes out of the calorimeter. The calorimeter with its contents is then brought back to the room temperature and again weighed. Let,

the water equivalent of the calorimeter with the stirrer $=W$ gm.
the mass of the empty calorimeter and stirrer $=m_1$ gm.
the mass of the calorimeter and cold water $=m_2$ gm.
the final mass of the calorimeter with its contents $=m_3$ gm.

Then,

the mass of cold water taken $=(m_2-m_1)$ gm. $=m$ gm. (say)
the mass of hot water added $=(m_3-m_2)$ gm. $=m'$ gm. (say)

Let,

the initial temperature of the calorimeter with its contents $=t_1$°C.
the temperature of hot water $=t_2$°C.
the final temperature of the calorimeter with its contents $=t$°C.

Then,

Heat lost by hot water $=m'(t_2-t)$ cal. [$\because$ Sp. heat of water $=1$]
Heat gained by cold water $=m(t-t_1)$ cal.
Heat gained by calorimeter and the stirrer $=W(t-t_1)$ cal.

$$\because\ \text{Heat lost}=\text{Heat gained}$$

$$\therefore\ m'(t_2-t)=m(t-t_1)+W(t-t_1)$$

$$\therefore\ W=\frac{m'(t_2-t)}{t-t_1}-m \qquad \dots \qquad \dots \qquad (17\cdot6)$$

Hence W can be calculated.

(2) **Determination of specific heat of a solid by the method of mixture :** Let a body of mass m gm., specific heat s and of temperature t_1°C. be dropped inside a calorimeter containing a liquid. Let m_1 gm. be the mass

of the calorimeter with the stirrer, both being of the same material of specific heat s_1 and also let m_2 gm. be the mass of the calorimetric liquid of specific heat s_2. Then if t_2°C. be the initial temperature of the calorimeter and its content before insertion of the body, and if t°C. be the final temperature of the mixture $(t_1 > t > t_2)$, then from the principle of calorimetry.

Heat lost by the body = Heat gained by the calorimeter and its content,

or, $ms(t_1-t)=(m_1s_1+m_2s_2)(t-t_2)$

$$\therefore \quad s=\frac{(m_1s_1+m_2s_2)(t-t_2)}{m(t_1-t)} \quad \ldots \quad \ldots \quad (17{\cdot}7)$$

If W be the water equivalent of the calorimeter with the stirrer, then $W=m_1s_1$

$$\therefore \quad s=\frac{(m_2s_2+W)(t-t_2)}{m(t_1-t)} \quad \ldots \quad \ldots \quad (17{\cdot}8)$$

If water be taken as the calorimetric liquid, then $s_2=1$; the equation (17·8) then reduces to

$$s=\frac{(m_2+W)(t-t_2)}{m(t_1-t)} \quad \ldots \quad \ldots \quad (17{\cdot}9)$$

The specific heat s of the solid can be determined from any of the above formulae. In the actual experiment Regnault's apparatus is used. It consists essentially of a calorimeter *C* and a steam heater *H* separated by a movable partition *P* [Fig. 17·4]. The calorimeter is a thin walled copper can *C* partly filled with a liquid. A stirrer *S* of same material as that of the calorimeter and a thermometer T_2 are introduced into the liquid. To reduce heat loss from the calorimeter, it is placed on spikes of cork or cardboard inside another metallic vessel *D*. The combination is again placed within a wooden box *W*, the intervening space between *D* and *W* being packed by dry or felt. The box is closed at the top by a slotted wooden cover (not shown in the figure).

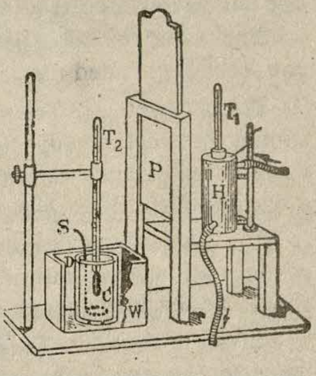


Fig. 17·4 : Regnault's apparatus

The steam heater *H*, shown separately in Fig. 17·5, consists of two concentric copper cylinders *M* and *J*. The annular space between them is closed at the top and the bottom. Steam is introduced within this space through the inlet tube *I* and it comes out of the exit tube *E*. The outside of the cylinder *J* is covered with felt or asbestos. The tube *M* is closed at the bottom by a removable shutter *L* and at the top by a cork *A* having two holes. Through one of them a thermometer T_1 is introduced, while through the other a string for suspending the solid is inserted. The experimental solid *B* is hung by this string just by the side

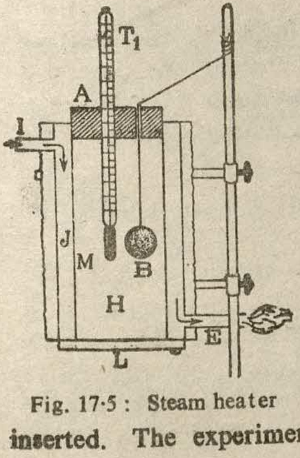


Fig. 17·5 : Steam heater

of the thermometer bulb. The steam heater enables the experimental solid to be heated without actual contact with the steam.

The mass (m) of the experimental solid is first determined with the help of a balance. It is then introduced in the steam heater. The calorimeter and the stirrer are cleaned and dried. Their mass m_1 is determined. Calorimetric liquid (usually paraffin, turpentine oil or water) just sufficient to immerse the experimental solid and the bulb of the thermometer T_2 is taken in the calorimeter and the mass (say, M) is measured. The mass m_2 of the liquid is then given by $m_2 = M - m_1$. The calorimeter is then introduced in the box W. The temperature (t_2°C.) is noted by the thermometer T_2. Meanwhile the solid is being heated and when its temperature becomes steady (t_1°C), it is noted from the thermometer T_1. By raising the partition P, the box W containing the calorimeter is slided to a position below the heater. Now by removing the shutter L, the string is snapped whence the solid drops inside the calorimeter. The calorimeter is then brought quickly back to its initial position and the partition is lowered. The top of the calorimeter is closed with the slotted lid. The thermometer T_2 is introduced into the liquid which is stirred briskly. The maximum temperature attained (t°C.) is noted.

The value of the specific heat of the solid, s, can then be calculated from eqn. (17·7), provided the values of s_1 and s_2 are known.

Precautions: (i) The highest temperature recorded by the thermometer T_1 should be kept steady for some time so that the experimental solid attains uniform temperature throughout its entire mass. This is very important if it be a bad conductor like glass, marble or rubber bung etc.

(ii) The accuracy of the result depends mainly on the accurate measurement of the rise in temperature of the calorimeter. Hence the thermometer used should be sensitive enough to be read upto $\frac{1}{5}$th or $\frac{1}{10}$th of a centigrade degree.

(iii) The amount of the calorimetric liquid taken should be just sufficient to immerse the experimental body and the bulb of the thermometer T_2. Excess liquid will cause the rise in temperature of the mixture too small to be read accurately.

(iv) The experimental solid should be lowered inside the calorimeter as quickly as possible to minimise the heat loss during this process. At the same time care should be taken so that no liquid splashes out of the calorimeter during such transfer. Similar care should also be taken during stirring of the mixture.

(v) The experimental solid should not dissolve in or react chemically with calorimetric liquid used.

(vi) Heat lost by radiation from the calorimeter and its content after mixing is not accounted for in the above working relation. The calorimeter is suitably insulated to minimise this heat loss as far as practicable. This difficulty may be avoided partially by suitably lowering the initial temperature of the liquid in the calorimeter (by mixing ice-cold water) so that the temperature of the surrounding equals the mean of the initial and final temperatures of the liquid. Thus, if a rise of 6°C. is expected and the room temperature is 30°C., the initial temperature of the liquid should be 27°C. The amount of heat lost to the surroundings by calorimeter is then compensated by the amount of heat gained by the calorimeter.

N. B. (i) If the calorimeter and the solid be made of the same substance, the water equivalent W of the calorimeter becomes equal to $m_1 s$. In that case the equation (17·9) reduces to

$$ms(t_1-t)=(m_2+m_1 s)(t-t_2) \qquad \dots \quad (17\cdot10)$$

Hence, knowing all other quantities, the value of s can be obtained. Thus without knowing the specific heat of any other substance, that of any particular substance say, copper, can be obtained.

(ii) The water equivalent W of the calorimeter can be determined by the above method. The experimental procedure is the same, but the specific heat of the hot body must be known, when W can be found out by using equation (17·9).

(3) **Determination of the specific heat of a liquid :** To carry out this determination, the specific heat of the heated solid and the water equivalent of the calorimeter must be known. The experimental procedure is identical. The specific heat s_2 of the liquid is then worked but by using equation (17·7) and is given by

$$s_2=\frac{ms(t_1-t)}{m_2(t-t_2)}-\frac{m_1 s_1}{m_2} \qquad \dots \quad \dots \quad (17\cdot11)$$

17·12. Choice of water as a Calorimetric substance.

Though water is widely used as a calorimetric substance, the choice is not an ideal one. The specific heat of water is much larger than that of any other liquid. Hence for given quantity of heat supplied, the rise in temperature of water will be much less than that of other liquids of the same mass. Also, the boiling point of water being 100°C., it becomes impossible to carry out measurements by heating the sample to a high temperature since some water might get vaporised.

Oils of known specific heat are sometimes used as calorimetric substances. The choice is better in many respects. Specific heat of an oil being almost half that of the water, rise in temperature of the oil is almost twice that for a corresponding mass of water. Moreover, oils are lighter than water. Thus, for the given volume of the calorimeter mass of oil contained will be less than that for water and hence, the rise in temperature will be correspondingly greater. However, the main difficulty with oil lies in the fact that the composition of an oil may vary from one sample to other. Hence the specific heat of an oil may vary from sample to sample.

Now a days *aniline is choosen as the best calorimetric substance.* It is a pure chemical with a low specific heat (0·62) and has a high boiling point (183·9°C).

Example 17·7. *100 gm. of copper are heated to 100°C. and put into 150 gm. of an oil (sp. heat 0·51) at 10°C., contained in a copper calorimeter weighing 40 gm. The temperature rises to 19·5°C. Find the specific heat of copper.*

Solution : Let s be the required sp. heat of copper.
Heat given out by copper $=100\times s\times(100-19{\cdot}5)=8050\ s$ cal.
Heat gained by the oil $=150\times0{\cdot}51\times(19{\cdot}5-10)=726{\cdot}75$ cal.
Heat gained by the calorimeter $=40\times s\times(19{\cdot}5-10)=380\ s$ cal.
$\because$ Heat lost = Heat gained, $\therefore$ $8050s=726{\cdot}75+380s$, or, $s=0{\cdot}095$.

Example 17·8. *A lead ball weighing 200 gm. at 150°C. is added to a calorimeter of water equivalent 50 gm. and containing 50 gm. of water at 30°C. Find the common temperature (sp. heat of lead 0·03).*

Solution : Heat lost by the lead ball $=200\times0{\cdot}03\times(150-t)$ cal.
where t is the common temperature.
Heat gained by the calorimeter $=50\times(t-30)$ calorie
" " " water $=50\times1\times(t-30)$ calorie
$\because$ Heat lost = Heat gained
$\therefore$ $200\times0{\cdot}03\times(150-t)=(t-30)(50+50)$
Solving we get $t=36{\cdot}8°C$.

Example 17·9. *An iron ball weighing 180 gm. is heated to 140°C. and dropped in a copper calorimeter containing glycerine, when a rise in temperature from 20°C. to 50°C. is observed. The mass of the calorimeter is 50 gm. and that of the glycerine is 100 gm. Calculate the specific heat of glycerine. Given, sp. heat of iron = 0·11, and of copper = 0·09.*

Solution : Heat lost by the iron ball $=180\times0{\cdot}11\times(140-50)$
$=1782$ cal.
Heat gained by the calorimeter $=50\times0{\cdot}09\times(50-20)=135$ cal.
Heat gained by glycerine $=100\times s\times(50-20)=3000s$ cal.
where s is the sp. heat of glycerine.
$\because$ Heat lost = Heat gained
$\therefore$ $1782=135+3000\ s$, or, $s=0{\cdot}55$.

Example 17·10. *5 gm. of carbon detaches from the tip of an electric arc and falls in 1 kg. of water at 25°C. The temperature of water is thereby raised to 29°C. What is the temperature of the electric arc? (specific heat of carbon = 0·2).*

Solution : Let t°C. be the temperature of the electric arc.
Heat lost by the piece of carbon $=5\times0{\cdot}2\times(t-29)$ cal.
Heat gained by water $=1000\times1\times(29-25)=4000$ cal.
$\therefore$ $5\times0{\cdot}2\times(t-29)=4000$ cal. or, $t=4029$°C.

Example 17·11. *A calorimeter contains 570 gm. of water at 30°C. 200 gm. of water at 50°C. is thoroughly mixed with it. If the resulting temperature is 35°C. find the water equivalent of the calorimeter.*

Solution : Let the water equivalent of the calorimeter be W gm.
Heat lost by warm water $=200\times(50-35)=3000$ cal.
Heat gained by cold water $=570\times(35-30)=2850$ cal.
Heat gained by the calorimeter $=W\times(35-30)=5W$ cal.,
$\therefore$ $3000=2850+5W$, or, $W=30$ gm.

Example 17·12. *A calorimeter of water equivalent 15 gm. contains 60 gm. of water at 40°C. Due to the loss of heat by radiation its temperature falls to 30°C. in 5 minute. If the same calorimeter is filled with 50 gm. of another liquid, the time in which the same drop of temperature takes place is found to be 2 minute. Calculate the specific heat of the liquid if the rate of heat loss in both the cases are assumed to be equal.*

Solution : The heat lost by the calorimeter and water

$=(60+15)(40-30)=750$ cal.

$\therefore$ Rate of loss of heat$=\frac{750}{5}=150$ cal./min. If the specific heat of the liquid be s, then the heat lost by the calorimeter and the liquid$=(50\times s+15)(40-30)=(500s+150)$ cal. $\therefore$ Rate of loss of heat$=(500s+150)/2$ cal./min. As the rate of heat loss in both the cases are equal, we have $(500s+150)/2=150$, or, $s=0{\cdot}3$.

● EXERCISE ●

[A] Essay type questions

1. (a) What are the factors on which the quantity of heat possessed by a body depend ? Explain your answer with reference to a suitable experiment.

(b) Define :—Calorie, mean calorie, B. Th. U., C. H. U., therm, Kilo-calorie, 15° caloric and pound calorie.

Find, the relation between C. G. S. and F. P. S. thermal units.

2. Define specific heat of a substance indicating its nature of dependance on the unit of mass and the scale of temperature. Describe briefly a method for determination of specific heat of a solid insoluble in water. [*Model Questions, H. S. Council, 1978*]

3. (i) Define thermal capacity and water equivalent of a body. Compare them.

(ii) In experiments by the method of mixture in a calorimeter, we assume that the heat lost by the warmer bodies is equal to the heat gained by the cooler bodies. In order that the relation may hold, no heat must be allowed to enter or leave the calorimeter or be developed or absorbed inside. Will then, the relation hold, if.

(a) the calorimeter contains water and the solid is sugar, (b) the solid and the liquid in the calorimeter react chemically, (c) the calorimeter is kept on a table and exposed to the air ? Explain your answer briefly. [*H. S. 1963*]

4. What is the 'fundamental principle of calorimetry' ? Show that in an ordinary calorimeter, the basic assumption behind this principle is not fulfilled accurately. What steps are generally taken to fulfil the assumption in a precision type calorimeter ?

5. Describe an ordinary calorimeter. What is measured by the aid of a calorimeter ? What precautions are generally taken to perform this experiment ?

6. (a) Explain different methods of defining specific heat of a substance and state the relevant units.

(b) Describe a simple experiment for demonstrating the difference in specific heats of different substances. [*Jt. Entrance '73*]

[B] Short answer type questions

1. What is the advantage of taking water as the hot substance in a hot water bottle ? [*H. S. '82*]

2. The specific heat of a substance is equal to its thermal capacity per unit mass. Explain,

3. What is the difference between specific heat and specific gravity ? *[Jt. Entrance '79]*

4. Specific heat of Copper is 0·09—Explain. *[H. S. '83]*

5. "Specific heat of a substance is generally defind in two ways"—Write the said two definitions. Is there any essential difference between them ? Justify your answer.

6. Two exactly similar kettles—one containing water and the other an equal mass of milk—are placed side by side on fire. The rise of temperature of milk is found to take place at a quicker rate than in the case of water. Explain. *[H. S. '60]*

7. Suppose that in the determination of specific heat of a solid you are not provided with specific heat of material of the calorimeter. Then how do you proceed to perform the experiment ?

8. Is water a good choice as calorimetric substance ? If not, state with proper reason your suggestion about a better one.

9. Suppose you were given a thermometer reading from 0°C. to 110°C. How could you measure the temperature of a solid using this thermometer, if it is insoluble in water and kept below 0°C ?

10. Two exactly similar vessels—one containing milk and the other water—are heated to a common higher temperature from the room temperature. In which case, will the heat required be comparatively smaller and why is it so ? *[Model Question, H. S. Council, '78]*

11. You are provided with a thermometer which can read from 0°C to 100°C. Explain how with the help of this thermometer you can measure the temperature of a furnace where temperature is 2000°C. *[H. S. '80]*

12. Why are calorimeters made of metals (generally copper) ?

[C] Simple Problems

1. 300 gm. of water at 15°C. are mixed with 500 gm. of water at 30°C. What is the resulting temperature ? [*Ans.* 24·38°C.]

2. Some hot water is added to four times its mass of water at 10°C. The resulting temperature was 25°C. What was the temperature of the hot water ? [*Ans.* 85°C.]

3. An electric heater provides 10^4 calorie per minute. How much cold water at 0°C. could be placed on the heater to raise its temperature to 100°C. in 1 minute ? Assume complete utilization of heat. [*Ans.* 100 gm.]

4. A water bath contains 120 kg. of water at 50°C. Hot and cold taps are turned on to deliver 20 kg. of water per minute each at temperatures of 60°C. and 10°C. respectively. How long will it be before the temperature in the bath has dropped to 40°C. Assume complete mixing of the water and ignore heat losses. [*Ans.* 6 min.]

5. Equal volumes of mercury and glass have the same capacity of heat. Calculate the specific heat of glass of density 2·6 gm./c.c., if the specific heat of mercury is 0·034 and density is 13·59 gm./c.c. [*Ans.* 0·178]

6. How much heat is required to raise the temperature of 7 gm. aluminium of specific heat 0·21 from 10°C. to 30°C. ? Find also the thermal capacity and water equivalent of the metal. [*Ans.* 29·4 cal, 1·47 cal., 1·47 gm.]

7. 80 gm. of iron at 0°C. cools some water contained in a calorimeter from 25°C. to 24°C. Calculate the thermal capacity of the calorimeter and its contents. [Specific heat of iron=0·11] [*Ans.* 211·2 cal.]

8. An iron saucepan contains 100 gm. of water of 25°C. 50 gm. of water at 60°C. are poured into it and the resultant temperature is found to be 35°C. Calculate the water equivalent of the pan, assuming no loss of heat by radiation or otherwise. If the mass of the pan be 238 gm., what is the specific heat of iron ? *[H. S. comp. '60]* [*Ans.* 25 gm. ; 0·105]

9. An aluminium container of mass 50 gm. contains 100 gm. of water at 25°C. Heat is added to the system at the rate of 50 cal./sec. What is the temperature of the system after 2 min ? Given that, the specific heat of aluminium=0·2. [*Ans.* 79·5°C.]

10. If 900 gm. of water at 25°C. contained in a calorimeter, are heated in 30 min. to 85°C. by an electric heater placed in the water, how many calories of heat are given to the water per second by the heater ? Neglect the water equivalent of the calorimeter. [*Ans.* 30 calorie]

11. A piece of iron 100 gm. in weight is placed for some time in an electric furnace and is then quickly immersed in 550 gm. of water at 30°C. contained in a non-conducting vessel. If the temperature of water rises to 40°C., calculate the temperature of the furnace. (Sp. heat of iron=0·11) [*Ans.* 540°C.]

12. A piece of copper weighing 100 gm. having its specific heat 0·095 is heated in a steam chamber which has a uniform temperature of 100°C. The piece is then quickly put into 150 gm. of an oil at 10°C. contained in a copper calorimeter weighing 40 gm. If the temperature rises to 19·5°C. calculate the specific heat of the oil. [*Ans.* 0·51]

13. A liquid of sp. ht. 0·6 at 50°C. is mixed with another liquid of sp. ht. 0·4 at 10°C. After mixing the temperature of the mixture becomes 20°C. In what proportion by weight are the liquids mixed ? [*Ans.* 2 : 9]

14. Two liquids *A* and *B* are at temperatures 70°C. and 30°C. respectively. Their masses are in the ratio 3 : 5 and their specific heats are in the ratio 4 : 5. Calculate the resultant temperature if the liquids *A* and *B* are mixed. Neglect water equivalent of the calorimeter. [*Ans.* 42·97°C.]

15. A button of charcoal weighing 2 gm. is heated uniformly in an electric arc and dropped into 50 gm. of water at 20°C. The temperature of water becomes 46°C. What is the temperature of the arc, given specific heat of charcoal is 0·2. [*Ans.* 3296°C.]

16. A metal forging of mass 600 gm. and specific heat 0·1 is heated to 700°C. and then quenched by plunging it into oil. What is the minimum mass of oil at 20°C. contained in a vessel of water equivalent 1 kg. that must be used so that its temperature does not rise above 30°C. Given specific heat of oil=0·45. [*Ans.* 6711 gm.]

17. In an industrial undertaking the flue gas at 360°C. is made to warm the water going into the boiler. In doing this the temperature of the flue gas falls to 200°C. Calculate how much water could be raised in temperature from 20°C. to its boiling point by 0·5 kg. of flue gas, assuming the specific heat of flue gas as 0·25. [*Ans.* 250 gm.]

18. Equal volumes of turpentine and water are allowed to cool from 60°C. to 40°C., the time taken being respectively 4 min. and 10 min. Calculate the specific heat of turpentine assuming that the rate of heat loss from the water and turpentine are the same. Given, the specific gravity of turpentine=0·88. [*Ans.* 0·45]

[D] Harder Problems

1. 50 gm. of boiling water at 100°C. is added into a calorimeter containing 200 gm. of water at 10°C. so that temperature rises to a maximum of 27°C. Now if a metal at 10°C. of unknown specific heat but of mass 100 gm. is dropped into the calorimeter, the temperature falls to 20°C. Calculate the specific heat of the metal. [*Ans.* 0·165]

2. A hot solid weighing 70 gm. is dropped into a calorimeter of water equivalent 10 gm. containing 116 gm. of water. If the fall in temperature of the solid is 15 times as great as the rise in temperature of the water, find out the specific heat of the solid.
(*Jt. Entrance* '67) [*Ans.* 0·12]

3. When a calorimeter of water equivalent 10 gm. containing 100 gm. of water at 90°C. is exposed to air, its temperature drops to 80°C. in 5 min. But when this calorimeter containing an oil of mass 50 gm. is exposed to air, it takes 2 min. for the same drop of temperature. Calculate the specific heat of the oil assuming that the rate of heat loss in both the cases are equal. [*Ans.* 0·68]

4. Three liquids *A*, *B* and *C* are at 40°C., 30°C. and 20°C. respectively. When equal parts (by weight) of *A* and *B* are mixed, the temperature of the mixture is 36°C. and when equal parts (by weight) of *A* and *C* are mixed the temperature is 35°C. Find the resulting temperature when equal parts of *B* and *C* are mixed. [*Ans*. 26·67°C.]

5. Exactly 4 gm. carbon was burnt to CO_2 in a copper calorimeter. The mass of the calorimeter is 1·6 kg., and the mass of water in the calorimeter is 3 kg. The initial temperature was 25°C. and the final temperature 35°C. Calculate the heat value of carbon in calorie per gm. assuming specific heat of copper to be 0·093. [*Ans*. 7872 cal./gm.]

6. One metric ton of water is heated from 10°C. to 70°C. using coal having a heat of combustion of $1{\cdot}5\times10^4$ calorie/gm. How many kgs. of coal are required if 60% of the heat is useful ? One metric ton of water weighs 10^3 kg. [*Ans*. 6·67 kg.]

7. The temperature of equal masses of three different liquids *A*, *B* and *C* are 12°, 19° and 28°C respectively. The temperature when *A* and *B* are mixed is 16°C ; and when *B* and *C* are mixed, it is 23°C. What will be the temperature when *A* and *C* are mixed. (*I. I. T. '76*) [*Ans*. 20·25°C]

8. 60 cu. ft. of water meant for bath is heated by a gas burner. The heat given out by burning 1 cu. ft. of the gas is 600 B. Th. U. and the cost of the gas is 95 paise per 1000 cu. ft. What will be the cost of heating the bath water from 55°F to 100°F assuming 70% of the heat given out by the burner is delivered to the water ? (*Jt. Entrance '81*) [*Ans*. 38 paise]

18
CHAPTER

CHANGE OF STATE : PROPERTIES OF VAPOURS : HYGROMETRY

18·1. Change of state : Latent heat.

A substance can exist in any of the three states—gaseous, liquid or solid. In the last chapter, we saw that the addition or removal of heat from a substance is accompanied by a change in its temperature. This is, however, not always true. It is found that during transition from one state to other, a substance absorbs or liberates heat without any corresponding change in its temperature. The absorbed or liberated heat goes to bring about the change of state. *This quantity of heat associated with a change of state is known as the* **latent heat.** It is so called because the heat does not manifest its presence by producing a change in temperature ; "latent" means hidden or concealed. On the other hand, when the heat produces a change in temperature of a substance, it is called **sensible heat.**

The latent heat of a substance corresponding to a particular change of state depends upon the initial and final states. In the succeeding articles, we shall, therefore, discuss the phenomenon specifying the initial and final states i.e., whether the change is from solid to liquid state or from liquid to gaseous state etc.

18·2. Fusion and solidification.

The change of a substance from solid to liquid state is known as **fusion** *or* **melting.** *The reverse process i.e., the change from liquid to solid state is called* **freezing** *or* **solidification.** Such change of state of a substance is always accompanied by a change in its volume.

When a solid is heated, its temperature rises till the body begins to melt. As soon as the process of melting starts, no further change in the temperature is observed ; more supply of heat goes to continue the process of melting. As soon as the solid is completely melted, with further addition of heat the temperature of the resulting liquid again begins to rise.

This behaviour of a substance can be easily demonstrated if we plot the temperature of the substance against time. The plot, known as temperature-time curve, is shown in the Fig. 18·1. The portion *AB* of the curve represents the increase in temperature of the solid with absorption of heat. At the point *B*, melting begins and the temperature becomes constant. This constancy of temperature throughout the process of melting is represented by the portion *BC*

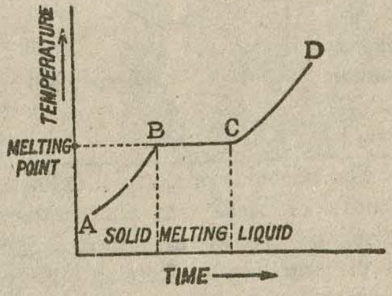


Fig. 18·1·: Temperature-time curve for fusion of the curve. At *C*, the melting is completed. The portion *CD* represents the consequent rise in temperature of the liquid with further supply of heat.

The phenomenon just gets reversed when the liquid is gradually cooled by withdrawing heat from it. Consequently, the temperature of the liquid slowly falls until it just begins to freeze. Further withdrawal of heat does not produce any change in temperature till the freezing is completed. The temperature at which freezing takes place is found to be the same as that at which the solid melts. When the last drop of the liquid has solidified, the temperature again begins to fall. The temperature-time curve in this case would have the same variation as *DCBA*, but the curve should be drawn the other way round [Fig. 18·2].

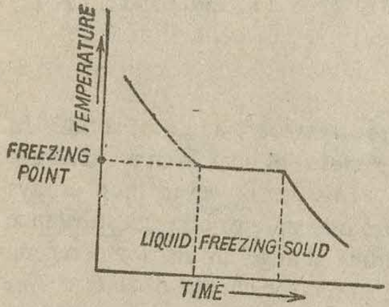


Fig. 18·2 : Temperature-time curve for freezing

The temperature at which a substance melts or freezes is known as the melting point or the freezing point of the substance. Obviously the freezing point is the same as the melting point. They are found to be different for different substances. For a particular substance, they vary with the pressure on the substance.

We thus define *the* **melting point of a solid** *substance as the definite temperature at which it melts under a given pressure.*

The **freezing point of liquid** *substance is the definite temperature at which the substance freezes under a given pressure.* Usually, the freezing point is the same as the melting point.

For a pressure of one standard atmosphere, the corresponding melting point or the freezing point is known as the *normal melting point* or *normal freezing point.* The normal melting points for some common substances are listed below :—

Substance	Melting point (°C)	Substance	Melting point (°C)
Ice	0	Copper	1038
Mercury	—39·5	Aluminium	657
Sulphur	115	Gold	1063
Cast Iron	1200 (approx.)	Silver	962
Zinc	418	Platinu	1755
Lead	327	Tungsten	3300 (approx.)

The existence of sharply defined melting point or freezing point and the identity of these two temperatures hold accurately only for pure crystalline substances. In the case of amorphous or plastic substances, such as glass, wax, fat etc. melting or freezing takes place over a certain range of temperature and hence they do not possess sharply defined melting points. Butter is found to melt between 28°C. and 33°C., but solidifies between 20°C. and 22°C. Impure substances and mixtures also do not have sharp melting points.

Obviously, during melting or freezing, heat supplied or given out remains hidden. We thus formally define *the* **latent heat of fusion** (*or* **melting**) *as the amount of heat required to convert unit mass of the substance from the solid* *to the liquid state without change of temperature.* The same amount of heat is released when unit mass of the liquid solidifies at the same temperature.

The latent heat of fusion is usually denoted by the letter L. It follows that if Q be the amount of heat required (or given out) to melt (or to freeze) a mass m of a substance, then

$$Q=m.L. \quad \ldots \quad \ldots \quad (18{\cdot}1)$$

Hence latent heat of fusion is expressed in calorie per gram or B. Th. U. per pound. For conversion,

$$L \text{ cal. per gm.} = L\times 453{\cdot}6 \text{ cal. per lb.} \quad (\because 1 \text{ lb.}=453{\cdot}6 \text{ gm.})$$

$$=\frac{L\times 453{\cdot}6}{252} \text{ B. Th. U. per lb.} \quad (\because 1 \text{ B. Th. U.}=252 \text{ calorie})$$

$$=L\times \tfrac{9}{5} \text{ B. Th. U. per lb.}$$

Note that the conversion factor $\frac{9}{5}$ is simply equal to the ratio of a degree C. to a degree F. The value of latent heat does not, therefore, depend on the unit of mass chosen, but does depend on the scale of temperature adopted.

For example, "the latent heat of fusion of ice is 80 cal. per gm.", means that in converting 1 gm. of ice at 0°C., to 1 gm. of water at 0°C., 80 calorie of heat must be supplied or in changing 1 gm. of water at 0°C to 1 gm. of ice at 0°C., 80 calorie of heat will be given out. The value of latent heat of fusion of ice in the F.P.S. system will be $80\times\frac{9}{5}=144$ B. Th. U. per lb.

18·3. Determination of latent heat of fusion of ice.

(i) *Method of mixture* : The latent heat of fusion of ice can simply and accurately be determined by the method of mixture. A clean and dry calorimeter C provided with a wire gauge stirrer S is taken and weighed when empty (M gm). It is filled up partly with water and again weighed. The difference of these two gives the mass of water taken (m_1 gm). To reduce heat loss from the calorimeter, it is placed on spikes of cork or card board inside another metallic vessel D [Fig. 18·3]. The combination is again placed within a wooden box W, the intervening space between D and W being packed by dry cotton or felt. The box is closed at the top by a wooden cover provided with holes to accomodate a thermometer T and the stem of the stirrer. The initial temperature (t_1 °C.) of the calorimeter with water is noted from the thermometer.

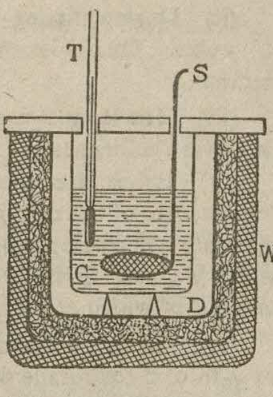


Fig. 18·3

One or two small pieces of ice, after being carefully dried with the help of a blotting paper, are quickly dropped into the calorimeter. Pressing the ice always below the surface of water by the stirrer, the mixture is continuously stirred. The final temperature (t°C.) is noted after all the ice has melted. The mass of the calorimeter with its contents is then again determined. The increase in mass gives the mass of ice (m_2 gm.) added to the calorimeter. If W be the water equivalent of the calorimeter, then,

the heat lost by the calorimeter$=W(t_1-t)$ calorie, and
the heat lost by the water contained in it$=m(t_1-t)$ calorie.

This heat converts the ice at 0°C. to water at 0°C., and raises the temperature of the resulting water from 0°C. to t°C. If L be the latent heat of fusion of fusion of ice, then

heat gained by ice in melting$=m_2L$ calorie, and the heat gained by the resulting water$=m_2t$ calorie.

Since, heat lost=heat gained,

$$\therefore \quad (W+m_1)(t_1-t)=m_2(L+t)$$

$$\text{or,} \quad L=\frac{(W+m_1)(t_1-t)}{m_2}-t \quad \cdots \quad (18{\cdot}2)$$

If s be the specific heat of the material of the calorimeter,

$$W=Ms$$

$$\therefore \quad L=\frac{(Ms+m_1)(t_1-t)}{m_2}-t$$

Hence the value of L can be found.

Precautions : (i) The chief difficulty in the experiment is to get completely dry ice. If the pieces of ice are not dried carefully, some water formed by melting will adhere to them and the observed value for the mass of ice will become greater than the actual value. The error introduced will lower the experimentally obtained value of L.

(ii) During stirring, the pieces of ice should always be kept under the surface of water. Otherwise some amount of ice may melt by gaining heat from outside.

(iii) The ice pieces should be dropped very gently so that no water splashes out of the calorimeter.

(iv) Excess ice should not be added to the calorimeter. In that case, temperature of the calorimeter may get lowered below the dew point. Hence moisture will condense on its outer surface. The latent heat of condensation which is given up in such deposition will be gained by the calorimeter, the amount of which can not be correctly ascertained.

(v) The thermometer must be sensitive one capable of reading upto $\frac{1}{8}$th or or $\frac{1}{10}$th of a centigrade degree.

(vi) Since the final temperature is less than room temperature, the calorimeter may acquire some quantity of heat from the surroundings. This can be prevented by suitably insulating the calorimeter as discussed before. An alternative way is to raise the initial temperature of the water in the calorimeter 4°C. or 5°C. above the room temperature. The amount of ice to be added is suitably adjusted, so that the final temperature of mixture lies below the room temperature by the same amount. The amount of heat gained from the surroundings by the calorimeter is then compensated by the amount of heat lost by calorimeter.

(ii) *Black's ice calorimeter* : It consisted simply of a block of pure ice, free from bubbles, in which a cavity has been scooped out [Fig. 18·4]. The upper surface of the block is made plane and a flat slab of ice is placed on it to cover the mouth of the cavity. A hollow space covered on all sides by ice at the melting point is thereby obtained.

A suitable piece of a substance of known specific heat is taken and is weighed. It is heated to a known temperature. The cavity is then carefully wiped dry with a piece of cotton wool or blotting paper. The hot body is then quickly placed inside it and immediately the upper slab, which has also been wipped dry, is placed in position, In a few minutes, the temperature of the body will fell to that of melting ice and some ice will melt. The slab is then removed from the block. The cavity as well as the surface of the body is wiped dry with a piece of cotton wool or blotting paper which had been previously weighed. It is then again weighed. The increase in weight gives the mass of ice melted.

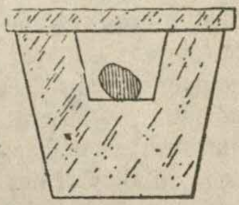


Fig. 18·4 : Black's ice calorimeter

Let, the mass of the body $=m$ gm.
its initial temperature $=t$°C.
its specific heat $=s$

Then, since its final temperature is 0°C.
heat lost by the body $=mst$ cal.

If the mass of ice melted be M gm. and L be the latent heat of fusion of ice, then

heat gained by ice in melting $=ML$ cal.

$$\therefore\quad ML=mst \quad \text{or,} \quad L=\frac{mst}{M} \quad \cdots \quad \cdots \quad (18{\cdot}3)$$

Hence L can be found..

The advantages of the method are (i) there is no loss of heat from the calorimeter by radiation and (ii) only the temperature of the hot body is to be measured. This may be accurately determined. There is no necessity of measuring the rise or fall of temperature in the calorimeter unlike other methods.

The main disadvantages of the method are : (i) it is very difficult to obtain large block of ice of sufficient purity and (ii) some water always gets evaporated from the cotton wool during the final weighings.

18·4. Effect of dissolved substances on melting point.

A solution has a lower freezing point than the pure solvent. As for example, the freezing point of a solution of common salt in water is much lower than 0°C., the freezing point of water. The actual freezing point which may be as low as −21°C., depends on the concentration of the salt solution. Sea-water, thus, has a freezing point below 0°C. Similarly, milk freezes at a temperature of −1·03°C.

When a solution is allowed to cool, crystals of pure solvent separate out from the solution. This leaves the solution with a higher percentage of dissolved substance making the freezing point still lower. So the temperature of the solution drops as time goes on until the solution freezes as a whole.

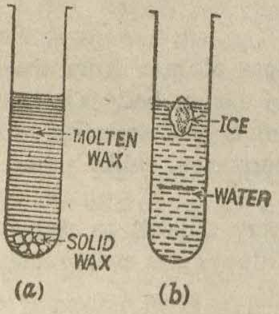
Alloys of different metals possess melting points usually lower than those of their constituents. For example, *Wood's metal* which is an alloy of lead, tin, cadmium and bismuth have a melting point of 60·5°C. although each of the constituent metal has a much higher melting point. The melting point of *Rose's metal*, an alloy of tin, lead and bismuth is only 94·5°C. Electrical fuses are usually made of these alloys.

Freezing mixture : When common salt is mixed with ice, the mixture is above its freezing point. Some of the ice, therefore, melts taking the necessary latent heat from the mixture. The temperature of the mixture falls and ultimately it becomes very cold (—21°C). A mixture of ice and salt (usually 3 parts of ice to 1 part of salt) is thus called a *freezing mixture*. An even colder one (temperature ; —52°C) may be prepared by mixing calcium chloride (3 parts) with ice (2 parts).

18·5. Change in volume during melting and freezing.

The transition from the liquid to the solid state is usually accompanied by a change in volume of the substance, although during transition temperature remains constant. Most substances increase in volume while they liquefy and contract during solidification. Hence their densities increase on freezing so that the solid form sinks in the liquid form.

There are, however, some important exceptions to this general rule. Ice, cast iron, antimony, type-metal (an alloy of copper, lead and antimony) etc., contract on melting and expand on solidification. Hence, their densities decrease on solidification. As a result, a solid of any of these substances floats on its liquid form. Thus ice floats on water.



(a) (b)

Fig. 18·5

Demonstration experiment : A test tube is half-filled with flakes of wax. On heating in a burner, the wax will melt. If two or three small pieces of wax are then dropped inside the test tube, they are found to sink below [Fig. 18·5 (*a*)]. Wax, therefore, contracts on solidification. If in another test tube half-filled with water, a small piece of ice is dropped, it will be found that ice floats on water [Fig. 18·5 (*b*)]. Hence water expands on solidification.

Practical applications : In the case of ice, 10 c.c. of water gives nearly 11 c.c. ice. Due to this fairly large expansion, a strong force is developed and precautions must be taken in cold countries to prevent damage to water containers. Freezing of water in the cooling system round the engine of a car may produce a burst in the radiator. So, 'anti-freeze' (e.g. glycerine, ethyl glycol) is added to give a low freezing point solution. Exposed water pipes are also found to burst during very cold weather due to the same reason. Rocks are split by the large force exerted when water which seeped into crevices, freezes during winter.

The phenomenon of expansion of a substance on solidification also serves useful purposes. The fact that ice floats in water helps to preserve marine life [vide Art. 15·10]. When molten type-metal is poured into a mould, it expands on freezing, so that a sharp impression is obtained. Due to the same reason, good castings of various designs are obtained with iron.

18·6. Effect of pressure on the melting point.

We have already mentioned that the melting point of a substance depends upon the pressure to which the substance is subjected. The nature of the dependence is as follows :

(*1*) *For substances which undergo expansion in volume on melting, an increase in pressure results in an elevation of the melting point.*

(*2*) *For substances which undergo contraction in volume on melting, the melting point is depressed with increase in pressure.*

For example, when paraffin melts, its volume increases and the melting point of paraffin increases by about 0·04°C. per one atmosphere increase in pressure. Again, the volume of ice diminishes when it melts and the melting point of ice is lowered by about 0·0073°C. per one atmosphere increase in pressure.

The above facts can be explained very simply. An increase in pressure always produces a diminution in the volume of a substance. Thus, for a substance which expands on melting, an increase in pressure opposes the process of melting. The melting point, therefore, gets elevated. On the other hand, for a substance which contracts on melting, an increase in pressure facilitates the process of melting. The melting point thus gets lowered.

Regelation : If two lumps of ice at 0°C. are squeezed together tightly and then released, it will be found that the pieces stick together. The applied pressure causes a lowering of the melting point of ice at the surface of contact. Hence, the ice at these points melts. When the pressure is released, the melting point again increases to 0°C., so that the water freezes and binds the pieces firmly together. This phenomenon of melting under pressure and freezing again after the pressure is released, is called *regelation* (refreezing).

Demonstration experiment : The phenomenon of regelation can be demonstrated strikingly by the following simple experiment originally performed by Bottomley. A thin copper wire carrying two equal heavy weights at the two ends is placed over the surface of an ice block supported at its ends [Fig. 18·6]. The wire is found to cut right through the block, but the block remains intact. Due to the heavy weights, the copper wire produces a high pressure on the ice. This pressure lowers the melting point of ice directly beneath the wire, so that ice melts and the resulting water flows up above the wire. The latent heat required for the

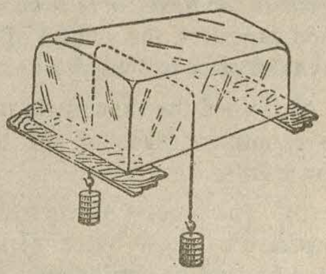


Fig. 18·6 : Bottomley's experiment

melting is supplied at the first instance by the surrounding atmosphere. As the pressure is then released, the water above the wire refreezes. The latent heat released in the process is conducted down through the wire to supply the necessary heat for further melting of the ice below the wire. This goes on until the wire passes through the ice block.

From the above discussion, it is obvious that the conduction of heat through the wire, from the water freezing above the wire to the ice beneath it, is necessary. Since iron has a lower thermal conductivity than copper, an iron wire is less effective for a quick demonstration. A cotton string of low conductivity is quite unsuitable for this demonstration.

18·7. Laws of Fusion.

The various facts discussed in connection with fusion can be summarized in the following laws, commonly known as the *laws of fusion* :—

(1) Every pure substance under a definite pressure melts at a particular temperature, called its melting point. This temperature remains constant throughout the process of melting. The melting point of a solid is the same as the freezing point of the corresponding liquid. Different substance melt at different temperatures.

(2) With increase in pressure, the melting points of the substances which contract on melting are lowered and the melting points of those which expand on melting are elevated.

(3) The freezing point of a solution is always lower than that of the pure solvent.

(4) During fusion, unit mass of the substance absorbs a certain constant amount of heat, which is called the latent heat of fusion. Unit mass of the corresponding liquid gives out the same amount of heat during solidification. The latent heat of fusion, though constant for a particular substance, varies from substance to substance.

18·8. Vaporisation and Condensation.

The conversion of a substance from liquid to the vapour or gaseous state is known as **vaporisation.** *The reverse process i.e., the transformation from the vapour to the liquid state is known as* **condensation** *or* **liquefaction.** When a liquid vaporises, the volume of the substance invariably increases. Vaporisation may take place in two ways viz., (i) *evaporation* and *boiling*.

(i) When the vaporisation of a liquid occurs *slowly* at any temperature from the exposed surface of the liquid, the process of vaporisation is known as **evaporation.**

(ii) The process of vaporisation of a liquid which occurs *rapidly* at a fixed temperature whose value depends upon the pressure on the liquid surface, is called **boiling** or **ebullition.** Boiling takes place throughout the whole volume of the liquid.

We shall now discuss these two phenomena in somewhat greater detail.

18·9. Evaporation.

It is a common experience that if a shallow dish filled up with water is left in the open, the amount of water contained in it diminishes gradually and ultimately the dish dries up completely. The obvious explanation is that the water evaporates slowly and continuously from the exposed surface. Evaporation is responsible for the gradual drying up of small pools, tanks etc. in summer. Wet garments hung up in the open become completely dry, because water escapes by evaporation.

Some liquids e.g. ether, alcohol etc. are found to evaporate very rapidly ; they are called *volatile liquids*. On the other hand, these liquids which do not evaporate rapidly are called *non-volatile liquid*, e.g., olive oil, coconut oil, glycerine etc.

When a liquid evaporates, heat must be supplied to bring about this change of state. *The amount of heat necessary for evaporation of a unit mass of a liquid is called its latent heat of evaporation*. Its value is found to depend upon the temperature at which evaporation takes place.

If the heat is not supplied from outside, the liquid itself supplies the latent heat necessary for evaporation, so that its temperature goes down. Thus *evaporation produces cooling*. If a few drops of ether or spirit be placed on our skin, we get the sensation of cold. As the drops evaporate quickly, the latent heat necessary for the purpose is obtained from the skin which thus cools. Due to the same reason, we feel cold when after perspiration we expose our body to a breeze or sit below a fan. Water is kept cool in summer time by using an unglazed earthen container. Water seeps out through the pores and evaporates producing cooling. Wind coming through a screen of *khas-khas* sprinkled with water gets cooled due to the evaporation of water.

Factors affecting evaporation.

(i) *Effect of temperature :* The rate of evaporation of a liquid increases with increase in temperature.

(ii) *Effect of area of the exposed surface :* The rate of evaporation increases as the area of the exposed surface of the liquid increases.

(iii) *Nature of liquid :* The rate of evaporation varies with the nature of the liquid itself. The lower the boiling point of a liquid, the greater is its rate of evaporation. Volatile liquids thus have low boiling points.

(iv) *Effect of superincumbent pressure :* The rate of evaporation increases with decrease in the superincumbent pressure i.e., the pressure on the liquid surface. So the rate of evaporation is maximum in vacuum.

(v) *Effect of the quantity of vapour in contact with the liquid :* The rate of evaporation decreases with the increase in the amount of vapour of the liquid present over the liquid surface. So in humid weather, water evaporates slowly. That is why clothes dry up more quickly in winter than in the rainy season.

(vi) *Effect of wind :* The rate of evaporation increases, with the flow of air in contact with the liquid surface. Essentially, this is due to the fact that the wind carries away the liquid vapour so that the space near the liquid becomes more or less free from its vapour and vaporisation takes place quickly. This explains why wet clothes dry rapidly on a windy day and the perspiration evaporates rapidly from our bodies when we sit below a fan.

18·10. Boiling.

If a liquid is gradually heated, the temperature of the liquid rises steadily. Meanwhile, the evaporation of the liquid from its surface continually takes place, becoming brisker, with rise in temperature. With further increase in temperature, a stage is reached when bubbles of vapour form throughout the liquid and rise to the surface producing a simmering sound. This stage is called the boiling of the liquid. The temperature then remains constant until all the liquid has been converted into vapour. The heat supplied during this process is thus utilized only in effecting the change of state from liquid to vapour. *This definite temperature at which a liquid boils is called the boiling point of the liquid.* It is found to be different for different liquids.

The boiling point of a liquid depends upon the superincumbent pressure. The *normal boiling point* of a liquid is defined as that temperature at which a liquid boils under normal atmospheric pressure. For water, the normal boiling point is 100°C.

Boiling point of a liquid is affected by the presence of dissolved substances in it. But the temperature of the vapour remains the same as in the case of the pure liquid boiling under the same pressure. For example, if common salt is dissolved in water, the solution boils at about 109°C., but the temperature of the steam issuing out is found to be 100°C.

Similar to the case of fusion, we can visualise the concept of the latent heat of vaporisation. *The quantity of heat that must be supplied to unit mass of a liquid at its boiling point to convert it completely to a gas at the same temperature, is called the* **latent heat of vaporisation** *of the liquid.*

On cooling a gas, its temperature decreases and at the same temperature at which boiling took place, the gas begins to condense. This temperature also remains constant till the change to the liquid form is completed. In this process, heat is given up and the heat so given up per unit mass, is called the *latent heat of condensation.* It equals the latent heat of vaporisation.

The latent heat of vaporisation, usually denoted by the symbol L, is expressed in calorie per gram, or B. Th. U. per pound. For water, the value is 539 cal./gm. or 970 B. Th. U./lb. As in the case of fusion, if Q be the amount of heat required (or given up) to vaporise (or to condense) a mass m of a substance, then

$$Q = mL \qquad \dots \qquad \dots \qquad (18\cdot4)$$

Effect of pressure on boiling point :

We have already mentioned that the boiling point of a liquid depends upon the superincumbent pressure. *With an increase in pressure on the liquid surface,* *the boiling point of the liquid rises while the boiling point gets lowered with a fall of pressure on the liquid surface.* The explanation is obvious if we remember that vaporisation always produces an increase in volume.

Demonstration Experiment.

The lowering of the boiling point of a liquid with decrease in the superincumbent pressure can be beautifully demonstrated by a simple experiment due to Franklin.

A round bottomed flask is half-filled with water and is heated by a Bunsen burner. When the water boils vigorously, a rubber stopper provided with a thermometer is tightly inserted into its mouth. The flask is then removed from the flame, wherefore the boiling ceases in a short time. Next the flask is turned upside down and clamped as shown in the Fig. 18·7. If some cold water is poured over the flask, it is found that the water inside the flask begins to boil again. The reading of the thermometer now indicates that the water boils at a temperature few degrees below 100°C.

The explanation of the phenomenon is as follows. The space above the water in the flask is filled up with water vapour. When the flask is cooled, some of the vapour condenses thereby reducing the superincumbent pressure. Due to this, the boiling point of the liquid is also lowered. The temperature of water which is already fairly high, now becomes greater than the reduced boiling point. Consequently, the water again starts boiling.

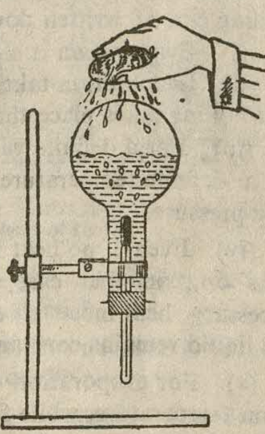


Fig. 18·7 : Franklin's experiment

Variation of boiling point with altitude : The atmospheric pressure decreases as we go upwards being greatest at the sea-level. Hence the boiling point of a liquid should decrease if it is taken to higher altitudes. It follows that in hill stations like Darjeeling, water will boil at a lower temperature than on the plains. This phenomenon renders the cooking of meat etc. more difficult at hill stations than in the plains.

It has been found that for an increase of about 300 metre in altitude, the boiling point of water decreases roughly by 1°C. On the peak of Mt. Everest (8·7 km. high), water boils at 70°C. approx.

Practical applications : The raising or lowering of boiling point with rise or fall of pressure has got wide industrial applications. Water is boiled at pressures above the atmospheric and steam at temperature higher than 100°C. is obtained. This is utilised for sterilisation work in hospitals or for sterilising canned food etc. Steam at about 200°C. is used in boilers of locomotive engines. A common household application is found in *pressure cookers.* In a pressure cooker, the water is taken in a closed vessel provided with a valve.

This is adjusted to open at some desired pressure above the atmosphere. When the water is heated, the pressure inside the cooker rises gradually till it becomes sufficient to open the valve. The water then boils steadily at the temperature to which it has risen. The pressure inside the cooker is kept constant at the higher value by the valve which allows steam to escape as rapidly as it is produced. Generally, the valve is so adjusted that within the cooker water boils at about 110°−115°C. Cooking becomes quicker due to such greater temperature and pressure.

18·11. Distinction between Evaporation and Boiling.

From the previous discussions, the distinction between evaporation and boiling can be written down as :

(i) Evaporation is a slow process while boiling takes place rapidly.

(ii) Evaporation takes place only from the exposed surface of the liquid, but boiling takes place throughout the entire volume of the liquid.

(iii) For a definite value of the superincumbent pressure, boiling takes place at a definite temperature, but evaporation takes place at any temperature under any pressure.

(iv) Even if no heat is supplied from outside, the evaporation of a liquid goes on ; in that case, its temperature is lowered. But for boiling, the necessary heat must always be supplied from outside and the temparature of the liquid remains constant throughout.

(v) For evaporation of a liquid, the corresponding latent heat depends upon temperature, while for boiling, the latent heat is a constant for a particular substance.

18·12. Laws of Boiling.

The various facts about boiling can be summarized in a set of laws known as the *laws of boiling*. The laws are :

(1) For a particular value of the superincumbent pressure, every liquid boils at a definite temperature, called its boiling point. Until all the liquid has vaporized, this temperature remains constant.

(2) The boiling point of a liquid gets elevated or lowered, according as the superincumbent pressure increases or decreases.

(3) On dissolving solids or gases in a liquid, its boiling point shifts. In general, solids dissolved in liquids raise the boiling point. Gases dissolved in liquids usually lower the boiling point.

(4) If two or more liquids are mixed, the mixture, in general, boils at a temperature lying between the boiling points of the pure liquids.

18·13. Determination of the latent heat of steam.

A clean and dry calorimeter is weighed together with the stirrer. It is then filled up with water upto about two-thirds and again weighed. The difference between these two weights gives the mass (m gm.) of the water taken. The initial temperature (t°C.) is noted with a sensitive thermometer.

Steam issuing out from a boiler B is then led by an exit tube E into the water in the calorimeter C [Fig. 18·8]. Care must be taken so that any water which might be produced by the condensation of steam during its passage from the boiler to the calorimeter, does not reach the calorimeter. A steam trap S is used to prevent this. This consists simply of a wide bore glass tube fitted with rubber stoppers. The tube E leads into this glass tube through the upper stopper as shown in the figure. Water formed due to condensation of steam is collected at the bottom and is drained out through the side tube T. Dry steam goes out of S through the tube D and enters the calorimeter. In order that any direct heat does not reach the calorimeter from the boiler, a wooden partition P is placed between them. To prevent the loss from the calorimeter to the surroundings by radiation, the calorimeter is properly insulated.

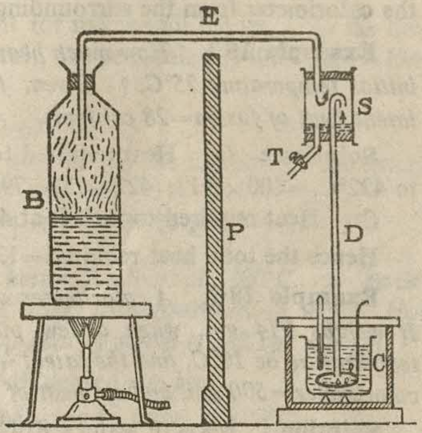


Fig. 18·8 : Determination of latent heat of steam

As the steam passes into the water in the calorimeter, it condenses giving up the latent heat so that the temperature of water will gradually rise.

The water is continually stirred and after some time, the supply of steam is cut off and the final temperatue (t°C.) is carefully noted from the thermometer. The calorimeter with its contents is then allowed to cool, and then again weighed. The difference of the last two weights gives the mass of the steam condensed (m' gm.).

If L be the latent heat of steam in calorie per gm., then heat lost by the steam at 100°C. in condensing into water at $100°\text{C}=m'L$ calorie.

Heat lost by this water in cooling to a temperature of $t°\text{C.}=m'(100-t)$ calorie.

If W be the water equivalent of the calorimeter, then heat gained by the calorimeter $= W(t-t_1)$ calorie.

Heat gained by the water contained in it $=m(t-t_1)$ calorie.

Since, heat lost = heat gained,

$$\therefore \quad m'L+m'(100-t)=(W+m)(t-t_1)$$

$$\text{or,} \quad L=\frac{(W+m)(t-t_1)}{m'}-(100-t) \qquad \ldots \qquad \ldots \qquad (18{\cdot}5)$$

Hence L can be found out.

In this experiment, error due to heat loss by radiation from the calorimeter may be minimised by an alternative method. The calorimeter with its contents is cooled so that the initial temperature lies a few degrees below the room temperature. At the instant the temperature of the calorimeter rises by the same amount above the room temperature, steam is shut off. The heat lost by the calorimeter to the surrounding is then compensated by the heat gained by the calorimeter from the surroundings.

Example 18·1. *How much heat is required to melt 200 gm. of zinc, having initial temperature 25°C. ? Given, for zinc, sp. heat=0·1, melting point=422°C. latent heat of fusion=28 cal./gm.*

Solution : (a) Heat required to raise the temperature of zinc from 25°C. to 422°C.$=200\times0{\cdot}1\times(422-25)=7940$ cal.

(b) Heat required to melt it at 422°C.$=200\times28=5600$ cal.

Hence the total heat required$=13{,}540$ cal.

Example 18·2. *A calorimeter with some water at 35°C. weighs 814 gm. It weighs 914 gm. when a few pieces of ice are dropped into it. If the final temperature be 10°C, find the latent heat of fusion of ice. Given mass of empty calorimeter=500 gm. and sp. heat of copper=0·09.*

Solution : Mass of water$=814-500=314$ gm.

Mass of ice$=914-814=100$ gm.

The heat given out by the calorimeter and water

$$=500\times0{\cdot}09\times(35-10)+314\times(35-10)=1125+7850=8975 \text{ cal.}$$

Let the latent heat of fusion of ice$=L$ cal./gm.

Heat required to melt 100 gm. of ice to water at 0°C.$=100\,L$ cal.

Heat required to raise 100 gm. of water at 0°C to 10°C$=100\times10=1000$ cal.

Hence from the principle of calorimetry, $100\,L+1000=8975$

$$\therefore\quad L=79{\cdot}75 \text{ cal./gm.}$$

Example 18·3. *A calorimeter at 20°C, contains 500 gm. of mustard oil. The water equivalent of the calorimeter is 50 gm. When 30 gm. of ice at 0°C. are added to it, the temperature falls to 10°C. Find the specific heat of the oil. Given, latent heat of fusion of ice=80 cal./gm.*

Solution : Heat gained by ice in melting$=30\times80=2400$ cal.

Heat gained by water, formed by melting, in being heated from 0°C. to 10°C.$=30\times(10-0)=300$ cal.

Heat lost by the calorimeter$=50\times(20-10)=500$ cal.

If s be the specific heat of the oil, then heat lost by the oil

$$=500\times s\times(20-10)=5000\times s \text{ cal.}$$

$\therefore$ From the principle of calorimetry,

$$5000\,s+500=2400+300 \qquad \text{or, } s=0{\cdot}44.$$

Example 18·4. *20 gm. of water at 70°C. is mixed with 20 gm. of ice at −10°C. Find the result. Given, sp. heat of ice=0·5, latent heat of fusion of ice=80 cal./gm.*

Solution : Heat required to raise the temperature of ice from −10°C. to 0°C.$=20\times0{\cdot}5\times\{0-(-10)\}=100$ cal.

Heat required to change the ice at 0°C. to water at 0°C.

$$=20\times80=1600 \text{ cal.}$$

Thus to convert the whole mass of ice to water at 0°C., total amount of heat required $=100+1600=1700$ cal. Now, heat given out by water when its temperature falls from 70°C. to 0°C. $=20\times(70-0)=1400$ cal.

This amount of heat is not sufficient for melting all the ice. As the heat required to bring the ice up to 0°C. is 100 cal., the remaining $(1400-100)=1300$ cal. is utilised for melting ice.

$\therefore$ amount of ice melted $=\frac{1300}{80}=16{\cdot}25$ gm.

Hence the result is (i) whole mixture is at 0°C. (ii) the amount of ice which remains unmelted $=(20-16{\cdot}25)=3{\cdot}75$ gm.

(iii) the amount of water at 0°C. $=(20+16{\cdot}25)=36{\cdot}25$ gm.

Example 18·5. *Dry steam from a kettle, boiling at 100°C., is passed directly in 4 kg. of water at 10°C. and raises its temperature to 40°C. How much steam has been condensed, if the latent heat of steam is 540 cal. per gm.* ?

Solution : Let the mass of the steam condensed $=m$ gm.

Heat given out by steam at 100°C. in changing to water at 100°C.

$=m\times540$ cal.

Heat given out by water at 100°C. in cooling to 40°C. $=m.(100-40)$ cal.

Heat gained by cold water $=4000(40-10)$ cal.

$\because$ Total heat lost by steam = heat gained by cold water

$\therefore$ $m\times540+m(100-40)=4000(40-10)$ whence, $m=200$ gm.

Example 18·6. *A calorimeter of water equivalent 100 gm. contains 500 gm. of water at 25°C. If 200 gm. of ice at 0°C. are added to it, then (a) how much ice will remain unmelted ? (b) How much steam at 100°C. must then be blown into the mixture to raise the temperature to 25°C. ? (Latent heat of fusion of ice=80 cal./gm. Latent heat of vaporisation of water=540 cal./gm.)*

Solution : (a) Let m gm. of ice be melted: The heat given out by the calorimeter and its contents $=(500+100)(25-0)$ cal. Heat taken in by m gm. of ice in melting at 0°C. $=m\times80$ cal.

$\therefore$ From the principle of calorimetry,

$(500+100)(25-0)=m\times80$; or, $m=187{\cdot}5$ gm.

Thus $(200-187{\cdot}5)$ or 12·5 gm. of ice will remain unmelted.

(b) Let w gm. of steam at 100°C. be blown into the mixture in order to raise its temperature to 25°C. $\therefore$ Heat lost by the steam

$=w\times540+w\times(100-25)=w\times615$ cal. ... ... (i)

The heat gained by the calorimeter and its contents = heat required to melt 12·5 gm. of ice at 0°C. + heat required to raise the temperature of the calorimeter with its contents from 0°C. to 25°C.

$=12{\cdot}5\times80+(500+100+200)(25-0)=21{,}000$ cal. ... ... (ii)

Equating (i) and (ii) we get $615w=21{,}000$; or, $w=34{\cdot}15$ gm.

Example 18·7. *A vessel contains a small quantity of water in it. Air is being rapidly pumped out from the vessel. As a result of intensive evaporation* *the water gradually freezes. What fraction of the initial quantity of water can be converted into ice in this process ? Given for water, latent heat of fusion=80 cal./gm. ; of vaporisation=540 cal./gm.*

Solution : Let m=mass of water before pumping began.

m_1=mass of water converted into ice.

and m_2=mass of water converted into vapour.

$\therefore \quad m=m_1+m_2$

The heat required to produce the vapour can be obtained only from the latent heat of fusion liberated during the freezing of the water.

Heat liberated during the freezing of m_1 gm. of water is $H=m_1L_1$ where L_1 is the latent heat of fusion of water.

Since m_1 gm. of vapour are produced with this heat, we have $m_2=\frac{m_1L_1}{L_2}$ where L_2 is the latent heat of vaporization of water.

$$\therefore \quad m=m_1+m_2=m_1+\frac{m_1L_1}{L_2}=m_1\left(1+\frac{L_1}{L_2}\right)$$

$$\text{or,} \quad \frac{m_1}{m}=\frac{L_2}{L_1+L_2}=\frac{540}{80+540}=\frac{540}{620}=\frac{27}{31}$$

Example 18·8. *A sealed metal container is immersed for a few minutes in liquid oxygen at −180°C. and then transferred rapidly to a copper calorimeter containing water and of total thermal capacity 100 cal. deg^{-1} C. The tempreature of the water is observed to fall from 22°C. to 20°C. When the experiment is repeated with the container partly filled with 5 gm. of water and re-sealed, the water in the calorimeter cools from 20°C to 10°C. Assuming no heat exchange between the calorimeter and its surroundings, obtain a value for the mean specific heat of ice between −180° to 0°C. [The latent heat of fusion of ice=80 cal. gm.$^{-1}$]*

(London)

Solution : *When metal container was empty.*

Let W be the thermal capacity of the empty metal container.

$\because$ Heat gained by the container=Heat lost by the calorimeter and the water in it,

$$\therefore \quad W(180+20)=100\times 2$$

$$\text{or} \quad W=1 \text{ cal./°C.}$$

When the metal-container contains water in it.

When dipped in liquid oxygen the water in the container immediately freezes into ice at −180°C.

$\because$ Heat gained by the container+contents=Heat lost by the calorimeter and the water in it,

$$\therefore \quad W(180+10)+5\times s\times 180+5\times 80=100\times 10$$

where s is the specific heat of ice between −180°C. to 0°C. Putting $W=1$ cal./°C., we get $s=0{\cdot}4$ cal./gm. °C.

PROPERTIES OF VAPOURS

18·14. Vapour exerts pressure.

When a liquid evaporates, the resulting vapour exerts a pressure, like gas, on any surface with which it comes into contact. This pressure is known as *vapour pressure*, existence of which can be demonstrated by the following simple experiment.

Demonstration Experiment.

A small glass bulb containing a volatile liquid, such as ether or alcohol, which floats on water is connected by rubber tubing to a flask partly filled with water [Fig. 18·9]. With the help of a pinch-cock attached to the rubber tubing, the inflow of the liquid into the flask can be regulated. The mouth of the flask is closed tightly by a rubber stopper through which a glass tube is inserted into the flask. One end of this tube dips under the surface of the water. If the pinch-cock is now opened, the liquid enters the flask and evaporates. A little later, the pressure exerted by the resulting vapour will become sufficient to drive the water up through the glass tube. A spray of water then issues out of it.

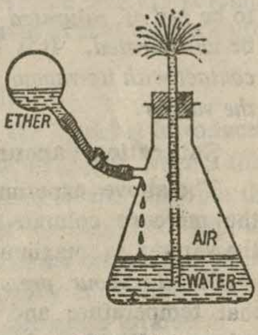


Fig. 18.9

18·15. Saturated and unsaturated vapour.

If a liquid is allowed to evaporate in closed space, it is found that after a small interval of time evaporation of the liquid ceases. Thus, *at a given temperature, there exists a maximum limit to the amount of vapour the space can hold*. The space is then said to be *saturated with the vapour*. Whenever this condition is fulfilled, the vapour is known as **saturated vapour**.

At a given temperature, if a space contains a smaller quantity of vapour than the maximum possible amount the space is then said to be *unsaturated*. The corresponding vapour is also called **saturated vapour**.

Demonstration Experiment.

Two barometric tubes, each about 1 metre long, are completely filled up with mercury and then inverted over a mercury trough [Fig. 18·10.] In each tube, mercury will stand to the same height corresponding to the atmospheric pressure. The space at the top of each tube represents the Torricellian vacuum. Now with the help of a dropper, small quantity of a liquid, say water or ether, is introduced into one of the tubes drop by drop. Being lighter than mercury, the liquid drop rises to the top of the mercury. As the pressure in this space is extremely low, the liquid immediately vaporises. The pressure exerted by the vapour so formed, depresses the mercury column

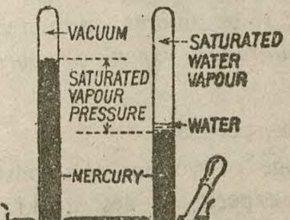


Fig. 18·10

slightly. As more liquid is introduced drop by drop, the depression of the mercury column continues indicating the increase in the pressure exerted by the vapour.

However, on continuing this process, a stage is soon reached when the evaporation of the liquid ceases and no further depression of the mercury column is observed. A thin layer of liquid then collects at the top of the mercury column. This demonstrates that a closed space, (in this case, the Torricellian vacuum) at a given temperature, has a maximum limit to its ability to accomodate a vapour. The space is then *saturated with the vapour* or said to be *full of saturated vapour*. Before this stage is reached, the space is said to be *unsaturated*. It is obvious that *in a closed space, the presence of a liquid in contact with its vapour provides a sure sign that the said space is saturated with the vapour*.

Saturation Vapour Pressure.

The above experiment shows that at the saturation stage the depression of the mercury column is the greatest, which means that the pressure exerted by the vapour is maximum at that temperature. This pressure is known as the *saturated vapour pressure* (abbreviated S. V. P) of the liquid corresponding to that temperature and is measured by the maximum depression of the mercury column. We thus formally define that the **saturation vapour pressure** *of a liquid at a particular temperature is the maximum pressure exerted by its vapour at that temperature.*

Keeping this in mind, the definitions of saturated and unsaturated vapour can also be given as :

(i) *When the pressure exerted by a vapour at a particular temperature equals the maximum value it can exert at that temperature, it is said to be* **saturated.**

(ii) *When the pressure exerted by a vapour at a particular temperature is less than the maximum value it can exert at that temperature, it is said to be* **unsaturated** ; *the corresponding vapour pressure is called the* **unsaturated vapour pressure.**

The saturation vapour pressure of water vapour is also known as *aqueous tension.*

If the above experiment is repeated using different liquids, the maximum depression of the mercury column will be found to be different in each case. This shows that *at a particular temperature the saturation vapour pressure is different for different liquids.*

18·16. Effect of change of volume at constant temperature.

(*a*) *Unsaturated vapour* : The effect of change of volume at a constant temperature can be easily demonstrated by the experiment described in the previous article. We have already noted that prior to any accumulation of liquid in the space above the mercury column, it is filled up with unsaturated vapour. The pressure exerted by this vapour is equal to the depression of the mercury column and its volume can be read off directly from graduations marked on the tube.

If the tube is now gradually raised, the lower end of the tube always remaining under mercury, the volume occupied by the vapour increases. It will be noticed that the length of the mercury column in the tube always increases ; this shows that the pressure of the vapour decreases. If for different positions of the tube, the pressure and volume of the vapour are noted, it will be found that their product remains practically constant.

Similar approximate constancy of the product of pressure and volume of unsaturated vapour is observed when the tube is pushed into the trough, thereby decreasing the volume occupied by the vapour. This holds till a part of the vapour condenses into liquid and collects above the mercury column. The vapour is then no longer unsaturated but is saturated at that temperature. Hence, we conclude that,

an unsaturated vapour obeys Boyle's law approximately.

(b) *Saturated vapour* : If additional amount of the liquid is introduced gradually into the tube, the space will ultimately become saturated, when a thin layer of liquid collects over the mercury column. Lowering of the tube into the mercury trough shall produce a decrease in the volume of the saturated vapour. But it is found that the height of the mercury column remains unchanged, while some vapour condenses, shown by the gradual thickening of the liquid layer. On raising the tube, some liquid evaporates to keep the space saturated. In this case also, the height of the mercury column remains unchanged. Thus,

at a given temperature, the saturation vapour pressure is independent of the volume occupied by the vapour. Hence, *Boyle's law is not obeyed by a saturated vapour.*

The above facts are represented graphically in the Fig. 18·11. For a given quantity of unsaturated vapour, reduction in volume produces a corresponding increase in pressure according to Boyle's law. This is represented by the portion *AB* of the curve. At *B*, the vapour gets saturated, when with reduction of volume its pressure remains the same. The saturated vapour gradually condenses in liquid. The portion *BC* of the curve represents this change. At *C* the condensation gets completed, when further increase in pressure produces negligible change in volume because a liquid is almost incompressible. The portion *CD* of the curve is, therefore, very nearly parallel to the pressure axis.

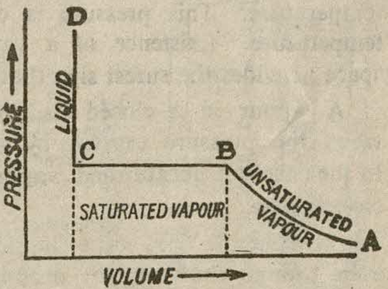


Fig. 18·11

18·17. Effect of change of temperature.

(a) *Unsaturated vapour* : The variation with temperature of the pressure exerted by a vapour can be simply studied by surrounding the tubes with a suitable bath.

For unsaturated vapour, the pressure at constant volume is found to increase with increase in temperature according to the Pressure law (vide Art. 16·5).

If on keeping the volume constant, the temperature is lowered, the pressure exerted by the unsaturated vapour decreases according to the same law till the space is saturated at the lower temperature. At this stage with further lowering of temperature, part of the vapour condenses and the pressure of the saturated vapour varies in a different manner, as will be discussed below.

(b) *Saturated vapour* : With increase in temperature, the saturation vapour pressure of a liquid generally increases. The increase, however, does not take place according to the Pressure law, but takes place much more rapidly. Thus, *saturated vapour does not obey Pressure law.*

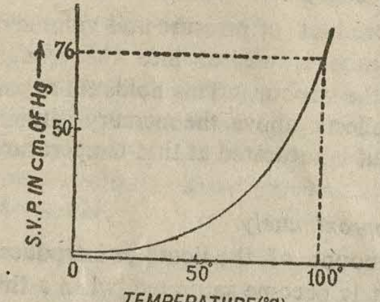


Fig. 18·12 : Saturation vapour pressure curve for water

Fig. 18·12 shows the variation of S.V.P. of water with temperature. From the graph, we see that the S.V.P. is not zero at 0°C. being equal to about 0·4 cm. of Hg. With increase in temperature, the S. V. P. at first increases gradually and then much more steeply. At 100°C. the S. V. P. is 76 cm. of Hg. For other pure liquids, the vapour pressure curve is of the same general shape.

No theoretical equation connecting the S. V. P. with temperature has yet been established ; several empirical relations have, however, been suggested.

18·18. Difference between unsaturated and saturated vapours.

(1) A vapour in a closed space at a given temperature is said to be saturated when the pressure exerted by it is the maximum corresponding to that temperature. This pressure is called the saturation vapour pressure at that temperature. Existence of a vapour in contact with its own liquid in a closed space provides the surest sign that it is saturated.

A vapour in a closed space at a given temperature is said to be unsaturated when the pressure exerted by it is less than its S. V. P. at that temperature. In the case of unsaturated vapour, no liquid can remain in contact with it in a closed space.

(2) The S. V. P. of a liquid remains constant at a particular temperature even though the volume occupied by the vapour may change. Saturated vapour, therefore, does not obey Boyle's law. With decrease in volume some vapour condenses while increase in volume produces more evaporation of the liquid if some liquid be present. If no liquid be present, then with increase in volume, the vapour becomes unsaturated.

In the case of unsaturated vapour, change in volume at a particular temperature produces a corresponding change in its pressure obeying Boyle's law approximately.

(3) The S. V. P. of a liquid increases with rise in temperature but the change is not linear even if the volume be kept constant. Hence, saturated vapour does not obey the Pressure law.

For unsaturated vapour, change in temperature at constant volume produces a change in pressure according to the Pressure law. The pressure-temperature curve for an unsaturated vapour is, therefore, linear.

18·19. Condition for boiling.

Boiling of a liquid takes place at a temperature when its saturation vapour pressure is equal to the superincumbent pressure i.e., the pressure on the exposed surface of the liquid.

We know that during boiling, bubbles of vapour are formed throughout the entire mass of the liquid. The pressure inside a bubble is due to the vapour contained in it and equals the S. V. P. of the liquid at the particular temperature. The external pressure on a bubble inside the liquid is the pressure on the liquid surface plus the pressure due to the liquid column above the bubble. Ordinarily this hydrostatic pressure exerted by the liquid itself is very small compared to the superincumbent pressure and can be neglected. If the pressure inside the bubble is less than the external pressure, it can not grow. A bubble can grow only when its internal pressure just exceeds the external pressure. With increase of temperature, the S. V. P. of the liquid increases and at the boiling point, it just exceeds the external pressure on the bubble. The bubble then grows and rises to the surface and the liquid is said to be boiling.

Demonstration Experiment.

A U-tube of glass with one end fitted with a stop-cock is filled up partly with mercury. By opening the stop-cock, some amount of well-boiled water is introduced at the top of the mercury level in that limb. The stop-cock is then closed. Some mercury is then removed from the open limb by introducing a thin solid rod in it so that the level of mercury in that limb stands below the level in the closed limb [Fig. 18·13(*a*)]. The closed limb is surrounded by a glass jacket through which steam is passed. Water inside the closed limb begins to evaporate and the vapour pressure pushes the mercury column in that limb downwards. Finally, the two mercury columns assume the same level [Fig. 18·13(*b*)]. It should be noted that water must be taken in a sufficient amount so that some water always remains present inside the closed limb during the experiment. This experiment shows that the S. V. P. of water at the temperature of steam i.e., at the boiling point equals the atmospheric pressure.

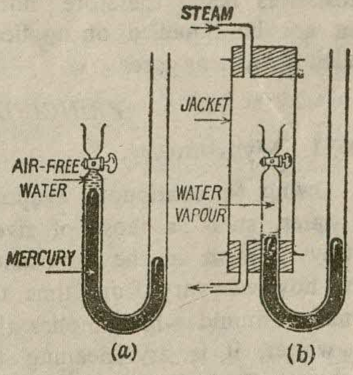


Fig. 18·13

Explanation of variation of boiling point with pressure : We know that the S. V. P. of a liquid increases with rise in temperature. It is obvious that if the superincumbent pressure rises, the S. V. P. of the liquid shall be equal to this enhanced value at a higher temperature ; from the condition for boiling, it follows that the boiling point of the liquid rises. Similarly we can explain the lowering of the boiling point with a fall in superincumbent pressure.

18·20. Distinction between gases and vapours.

We have already seen that an unsaturated vapour behaves like a gas. Both the terms 'vapour' and 'gas' are ordinarily used to designate a substance in the gaseous state. In the stricter sense, however, a distinction between the two is drawn.

It is found that every gaseous substance can be liquified by applying a suitable pressure if its temperature is below a certain value. If the temperature of the gaseous substance exceeds that particular value, it cannot be liquified by simply increasing the applied pressure. This limiting temperature is called **the critical temperature.** The value of the critical temperature depends on the nature of the substance. The pressure necessary to liquefy a substance at its critical temperature is called **critical pressure.**

When a gaseous substance is below its critical temperature, it is called a vapour, thereby implying that it can be condensed easily by pressure alone. *When the substance is above its critical temperature, it is called a gas* which can not be liquefied by pressure alone.

Critical temperature for water vapour is 361°C. ; hence, it can be condensed at ordinary temperatures by pressure. For the same reason, ammonia, carbon dioxide, sulphur dioxide etc. can be easily liquefied. We call the substances to be in the vapour state. On the other hand critical temperatures of oxygen, hydrogen, etc. are very low, viz. −119°C. and −240°C. respectively. These substances being therefore normally well above their critical temperatures, can not be liquefied on application of pressure alone. Hence these gases are called *permanent gases.*

PRINCIPLES OF HYGROMETRY

18·21. Hygrometry.

Owing to continuous evaporation at all temperatures from open surfaces of water, such as those of rivers, lakes etc., aqueous vapour more or less, is always present in the atmosphere. The amount of aqueous vapour present may however vary from time to time. Thus in rainy season, the air generally remains humid which implies the presence of large quantity of water vapour ; in winter, it is dry meaning that smaller quantities of water vapour are present. Evidence of water vapour in the atmosphere is also supplied by the formation of dew, cloud, fog etc.

The temperature of the human body is maintained constant at about 98·4°F. Any extra heat generated within our body cause us to perspire. The sweat generated evaporates and the required latent heat is supplied from our body, so that we get cooled. Presence of large amount of moisture in the air impedes this process of evaporation giving rise to a feeling of discomfort. Presence of very small amount of moisture in the atmosphere also produces a feeling of discomfort, since rapid evaporation from our body occurs. Human body, therefore, feels comfortable at a given temperature when the moisture content of air lies within a definite range. Changes in weather conditions are associated with a corresponding change in the moisture content of air and its observation thus helps us to forecast the weather. Also in some industrial operations, it is necessary to keep the moisture content of air within a definite range. Knowledge of the amount of water vapour present in air is, therefore, important. *The subject of* **hygrometry** *is concerned with the measurment of the water vapour content in the air.* That is, it is a science of measuring the *humidity* of the atmosphere.

18·22. The dew point.

Under ordinary circumstances, water vapour present in air is not sufficient to produce saturation. But if the temperature of a certain mass of air is gradually lowered, the air may become saturated by the same quantity of of vapour, because less vapour is required to produce saturation at lower temperature. The temperature at which this takes place is called the *dew-point.* With still further lowering of temperature, some amount of vapour gets condensed and is deposited as dew. Thus *the temperature at which a given mass of air gets just saturated with the quantity of water vapour contained in it is called the* **dew point**.

As the amount of water vapour present in air varies from day to day and place to place, the dew point also changes correspondingly. It should also be noted that since the cooling takes place at constant pressure, *the saturation vapour pressure at the dew point equals the pressure exerted by water vapour under the original conditions.*

Obviously, we can conceive the dew point simply as the temperature of air at which dew is deposited. In principle, the dew point can be experimentally determined by gradually cooling a bright metal surface in the air. At the instant a film of dew appears on it, the surface loses its brightness. The temperature of the surface is then noted. If the surface is now slowly warmed, the film again disappears and the temperature at which this takes place is again noted. The mean of these two temperatures gives the dew point. Hygrometers based on the above principle have been constructed. These will be discussed later on.

For example, let the temperature of air in a room be 30°C. Gradual cooling of a metal surface shows that dew just appears on its surface when its temperature is 14·9°C. On allowing the surface to warm up, dew disappears when its temperature is 15·1°C. We thus conclude that the dew point is the mean of 14·9°C. and 15·1°C. i.e., 15°C. and also that the pressure of water vapour in the room at 30°C. is equal to the S. V. P. of water at 15°C.

18·23. Humidity of air.

The *humidity* of air provides a measure of the water vapour content of air. It is expressed in two alternative ways, viz., (i) by absolute humidity and (ii) by the relative humidity.

***The* absolute humidity** *is defined as the mass of vapour present per unit volume of air.* It is usually expressed in grams per cubic metre of air.

We have already mentioned that our sense of comfort is largely controlled by the evaporation of perspiration. This evaporation depends on the question—how near to saturation is the water vapour present in the air. If the air is almost saturated, little evaporation is possible and hence our sense of discomfort grows. Conversely, if the amount of water vapour present in the air is not close to the saturation value, quick evaporation is possible. We feel cool and comfort.

It is now obvious that the evaporation of water from a surface depends on two factors, viz. (i) the actual amount of water vapour present in the air, and (ii) the amount of water vapour necessary to saturate the air under identical temperature. This consideration leads us to the concept of relative humidity. It is defined as :

$$\text{Relative humidity} = \frac{\textit{Mass of water vapour actually present in a certain volume of air}}{\textit{Mass of water vapour required to saturate the same volume at the same temperature}}$$

Since unsaturated vapour obeys Boyle's law approximately up to the saturation point, its density and hence the mass present in a certain volume will be proportional to its pressure. Making use of this in the case of water vapour present in the air, we get,

$$\text{Relative humidity} = \frac{\text{Pressure of water vapour actually present in the air}}{\text{S. V. P. of water at the temperature of the air}}$$

But the pressure of water vapour actually present in the air is identical with the S.V.P. of water at the dew point. Hence,

$$\text{Retative humidity} = \frac{\text{S.V.P. of water at the dew point}}{\text{S.V.P. of water at the air temperature}} \quad \cdots \quad (18{\cdot}6)$$

Generally, relative humidity is expressed as a percentage and can be obtained by multiplying any of these three expression by 100.

From the eqn. (18·6) it is obvious that relative humidity depends on two temperatures—dew point and air temperature. The moisture content of the air remaining constant, an increase in air temperature clearly causes a decrease in relative humidity. The reverse is also true ; relative humidity increases with decrease in air temperature.

Example 18·9. *100 litre of atmospheric air contains 1·5 gm. water vapour at room temperature. For complete saturation, the same volume of air requires 3·75 gm. of water vapour. Find the relative humidity.*

Solution : From the definition,

$$\text{the relative humidity} = \frac{1{\cdot}5}{3{\cdot}75} \times 100 = 40\%$$

Example 18·10. *Calculate what fraction of the mass of the water vapour in the air would condense if the temperature of the air fell from 25°C. to 5°C. and if originally at 25°C. the relative humidity was 80%. (Saturation vapour pressure at 25°C.=23·5 mm., at 5°C.=6·5 mm. of Hg.)*

Solution : Saturation vapour pressure at 25°C.

$$P=23{\cdot}5 \text{ mm.} ; R.H.=80\%.$$

Let actual vapour pressure at 25°C.$=p$. From the definition of relative humidity,

$$R.H.=\frac{p}{P}\times 100$$

$$\text{or, } 80=\frac{p}{23{\cdot}5}\times 100$$

$$\therefore \quad p=18{\cdot}8 \text{ mm.}$$

Since, saturation vapour pressure at 5°C.=6·5 mm.

∴ fall in vapour pressure due to condensation as a result of fall of temperature from 25°C. to 5°C.=18·8−6·5=12·3 mm.

Since the mass of water vapour present in air is directly proportional to the vapour pressure,

$$\therefore \text{ fraction of mass of water vapour condensed}=\frac{12{\cdot}3}{18{\cdot}8}=0{\cdot}654$$

Example 18·11. *The temperature of air on a certain day is 29·5°C. while the dew-point is 16°C. Calculate the relative humidity. Given S. V.P. of water at 16°C., 29°C. and 30°C. are 13·5, 29·8 and 31·4 mm. of mercury respectively.*

Solution : Since the S.V.P. of water at 29°C. is 29·8 mm. and at 30°C. is 31·4 mm., hence the increase in S.V.P. for 1°C. rise in temperature=(31·4−29·8) =1·6 mm. Thus for 0·5°C. rise in temperature, the S.V.P. will increase by 1·6/2=0·8 mm. Therefore, the S.V.P. of water at 29·5°C. is (29·8+0·8) =30·6 mm.

$$\text{From eqn. (18·6), relative humidity}=\frac{13{\cdot}5}{30{\cdot}6}\times 100=44{\cdot}1\%\text{(approx).}$$

Example 18·12. *The relative humidity in a closed room at 17°C. is 40 percent. If the temperature increases to 25°C., what will be the value of the relative humidity ? Given, S. V.P. of water at 17°C. and 25°C. are 14·4 and 23·5 mm. of mercury respectively.*

Solution : From the definition of relative humidity, the pressure of water vapour in the room at 17°C.

$$=\frac{40}{100}\times 14{\cdot}4=5{\cdot}76 \text{ mm.}=P_1 \text{ (say)}$$

If the pressure exerted by this water vapour at 25°C. be P_2, then since unsaturated vapour obeys Pressure law, we have

$$\frac{P_1}{P_2}=\frac{T_1}{T_2} \quad \text{or,} \quad \frac{5{\cdot}76}{P_2}=\frac{290}{298} \quad \text{or, } P_2=5{\cdot}92 \text{ mm.}$$

$\therefore$ Relative humidity at 25°C. $=\frac{5{\cdot}92}{23{\cdot}5}\times 100=25{\cdot}2\%$.

Example 18·13. *500 c.c. of hydrogen is collected over water at 17°C. when the barometer reads 74 cm. The surface of water in the collecting vessel is 3·4 cm. above the level of water in the trough. Find the volume of hydrogen at N.T.P. (Aqueous vapour pressure at 17°C.=14·4 mm. of Hg. and density of Hg=13·6 gm./c.c.).*

Solution : Pressure of hydrogen and aqueous vapour,

$$=\left(74-\frac{3{\cdot}4}{13{\cdot}6}\right)$$

$$=(74-0{\cdot}25)=73{\cdot}75 \text{ cm.}$$

From Dalton's Law of partial pressure, we get

pressure of dry hydrogen $=73{\cdot}75-1{\cdot}44=72{\cdot}31$ cm.

Now, $P_1=72{\cdot}31$ cm. ; $V_1=500$ c.c. ; $T_1=(273+17)=290$°K.

$P_2=76$ cm. $V_2=?$; $T_2=273$°K.

Applying eqn. (16·15),

$$V_2=\frac{P_1V_1}{T_1}\times\frac{T_2}{P_2}=\frac{72{\cdot}31\times 500\times 273}{290\times 76}=434{\cdot}4 \text{ c.c}$$

Example 18·14. *Outside air at 30°C. and 80% relative humidity is drawn by an air conditioning plant where it is cooled to 20°C. and the relative humidity is decreased to a comfortable value of 50%. How many gram of water must be condensed out from a cubic metre of outside air to accomplish this ? Given, the density of saturated water vapour in air at 20°C. is 17·3 gm./m.³, and at 30°C. is 30·4 gm./m³.*

Solution : Density of water vapour in outside air at 30°C.

$=0{\cdot}8\times 30{\cdot}4$ gm./m³. $=24{\cdot}32$ gm./m³.

Density of water vapour in cooled comfortable air

$=0{\cdot}5\times 17{\cdot}3$ gm./m³. $=8{\cdot}65$ gm./m³.

1 m³. of water vapour at 30°C. contracts to $\frac{293}{303}$ m³. at 20°C.

$\therefore$ Mass of water vapour in $\frac{293}{303}$ m³. at 20°C.

$=\frac{293}{303}\times 8{\cdot}65=8{\cdot}366$ gm.

$\therefore$ Mass of water to be condensed out of each m³. of outside air

$=24{\cdot}32-8{\cdot}366=15{\cdot}954$ gm.

18·24. Regnault's Hygrometer.

A hygrometer is an instrument with which we can measure the dew point and hence the relative humidity of the air. Regnault designed a hygrometer which determines the dew point accurately ; the relative humidity is then obtained from eqn. (18·6). The instrument is, therefore, also called Regnault's dew point hygrometer.

The hygrometer consists of two glass tubes *A* and *B* whose lower ends are fitted with highly polished silver cabs *P* and *Q* [Fig. 18·14]. The tube *B* is closed at its upper end by a stopper through which passes a thermometer T_2. The mouth of the tube *A* is also closed by a stopper through which a thermometer T_1 and an inlet tube *D* are introduced. The tube *A* is also provided with a side tube *E* to which an aspirator is connected. The tube *A* contains ether which fills the silver cap completely : the tube *B* is empty. By means of the aspirator, bubbles of air are drawn at a steady rate through the inlet tube dipping into the ether. This produces rapid evaporation of the ether and also stirs it. The latent heat required for the evaporation being taken from the liquid itself, the silver cap *P* is slowly cooled down. Ultimately a thin film of dew appears on *P*. The temperature at that instant is noted from the thermometer T_1. Since the liquid is being thoroughly stirred, this reading gives the temperature of the silver cap accurately. The other silver cap *Q* helps in detecting the first appearence of dew on *P* by comparison of the brightness of the two surfaces. The thermometer T_2 records the air temperature. The flow of air is then stopped, so that the temperature of *A* begins to rise slowly. The temperature at which dew just disappears is noted by the thermometer T_1. The dew point is then taken as the mean of these two readings.

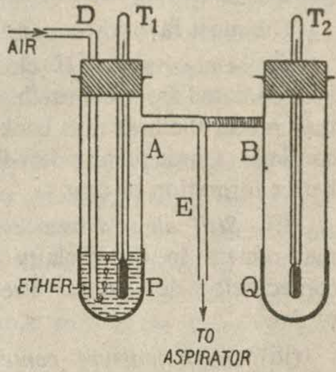


Fig. 18·14 : Regnault's hygrometer

From the standard chart containing S. V. P. values of water at different temperatures, S.V.P. of water at the dew point and also at the air temperature is obtained. Since the relative humidity is given as the ratio of the two, its value can easily be calculated.

During the experiment, necessary precautions must be taken so that the humidity and the temperature of the air surrounding the instrument are not disturbed by the breath of the observer or by the method of passing the air through ether. The observer should therefore stand as far as possible from the instrument and a glass plate is interposed between him and the instrument. A better method is to enclose the instrument in a glass box and observe from a distance with the help of a telescope.

18·25. Condensation of water vapour in nature.

Under suitable conditions, the water vapour present in the atmosphere gets condensed giving rise to the formation of cloud, mist, dew etc.

Dew : In day time, the various bodies on the earth's surface and the air become heated ; the amount of water vapour present in the air is quite insufficient to produce saturation at day temperatures. At night the bodies are cooled by losing heat due to radiation. In this process, those which are good radiators, sometimes cool below the temperature of the surrounding air. The air in their immediate vicinity gets cooled by them and may reach the saturated state. Further cooling causes some of the vapour to condense on the surface of the cold body, producing the phenomenon of dew. Copious deposition of dew occurs on grass and other vegetations, because plants give off water vapour, thereby increasing the humidity in their proximity.

The most favourable conditions for the formation of dew are :

(i) *A clear sky* : If clouds are present in the sky, they do not allow the heat radiated from the earth's surface to escape into space. On the contrary, they reflect the heat rays back to the earth, thereby retarding the process of cooling. Consequently dew formation is impeded. Clear sky therefore ensures better formation of dew.

(ii) *Still air* : Absence of wind ensures that a particular volume of air may remain in the vicinity of a cold object for a sufficient time so as to be cooled below dew point. Presence of wind, therefore, obstructs the formation of dew.

(iii) *High moisture content of the air* : If the relative humidity of the air be high, a little cooling is sufficient to produce saturation. Dew formation is thus facilitated.

Substances suitable for deposition of dew should have following characteristics :—

(i) They should be good radiators but bad conductors of heat, so that they cool rapidly and the loss of heat is not compensated by conduction from the surface of the earth.

(ii) They should lie near the surface of the earth. Otherwise the cooled air in their proximity being heavier will sink downwards and will be replaced by warm air from above ; as a result none of the air will be cooled below the dew point.

Fog and Mist : During a still night, a large region of the atmosphere near the earth's surface may be cooled slowly and ultimately the whole mass of air becomes saturated with water vapour. The atmosphere contains abundant number of dust particles and condensation takes place on each dust particle, giving rise to *fog or mist*. No fundamental difference exists between the two. A dense mist is called a fog. In large towns and industrial areas, very dense fogs, called *smogs*, are formed due to condensation of water vapour on smoke and soot particles. In mountainous areas, fogs are formed when the warm moist air from the valleys comes into contact with the cold air near the mountain.

In general, fogs disappear before noon. With rise in temperature water droplets evaporate causing disappearance of the fog. The atmosphere then again becomes unsaturated.

Cloud : Clouds are formed due to condensation of water vapour in the upper strata of the atmosphere. Continuous evaporation from the seas, lakes, rivers, ponds, moist surface of the earth, vegetations etc. causes the warm air in contact with the earth's surface to become moist. The warm, moist air being lighter than dry air rises upwards. As the pressure decreases upwards, the ascending air expands and gets cooled. Upper layers of the atmosphere being themselves colder, further cooling of the rising air is produced by contact with these layers. Finally the mass of moist air is cooled below the saturation stage, when water vapour condenses giving rise to clouds. The condensation takes place on particles of dust, common salt or other kinds of hygroscopic particles present in the atmosphere. A cloud is thus essentially a mist or fog, formed in the upper atmosphere.

As the tiny droplets of water in a cloud coalesce, they grow bigger and ultimately they fall downwards under the action of gravity, giving rise to the phenomenon of rain. Whether a drop may reach the ground depends on various factors, two important ones being (i) the downward velocity of the rain drops must be greater than the velocity of ascent of the moist air and (ii) the lower layers of the atmosphere must be sufficient moist so that the drops are not evaporated during their descent.

● EXERCISE ●

[A] Essay type Questions.

1. Define sensible heat and latent heat. When does a substance absorb or release latent heat ? State in general terms the effect of application of heat to ice, say at —8°C., until the temperature of 150°C. is reached.

How would you express freezing and condensation in terms of latent heat ?

2. Give a systematic account of finding the melting point of a solid from its time-temperature graph for fusion.

What is meant by the statement 'Latent heat of fusion of ice is 80 calorie per gm' ? What is its value in F.P.S. system ?

3. State laws of fusion. What is meant by 'latent heat of fusion' ?

Describe the experiment by which the latent heat of fusion of ice may be determined. Mention the sources of error in the experiment and the precautions.

4. Define 'melting point' of a solid. State why electrical fuses are made of alloys.

When can heat be supplied to a liquid without causing rise of temperature ? How can this heat be recovered ?

5. Enumerate how a solid generally changes in volume during melting and freezing. Is there any exception to this general rule? Give one example in support of your statement.

What is measured by 'Black's ice calorimeter' ? Describe it and explain its working principle.

6. Describe the experiment for determination of 'latent heat of steam.' How could you reduce the error in this experiment due to heat loss from the calorimeter ?

7. What is the effect of pressure on melting point ? How would you explain it ? What is meant by 'Regelation' ? Describe Bottomley's experiment to demonstrate it and explain the experimental results. State with reasons what will happen in this experiment if (i) the copper wire be replaced by a steel wire or (ii) by an ordinary thread and (iii) temperature of air goes below 0°C.

8. What is meant by 'evaporation' ? Enumerate the factors which influence the rate of evaporation of a liquid. Why does evaporation cause cooling ? Explain it with a few examples.

9. What is 'boiling'? What distinguishes boiling from evaporation? Water may be made to boil at various temperatures—describe an experiment to illustrate it.

10. What are the laws of boiling? State and explain the effect of pressure on boiling point. Describe one practical application of it.

11. What is meant by 'Latent heat of vaporisation'? What is its difference with 'Latent heat of evaporation'? State the condition of boiling and demonstrate it by an experiment.

12. State what characteristic features distinguish a 'saturated vapour' from an 'unsaturated vapour'. Define saturated vapour pressure and unsaturated vapour pressure. Discuss a simple experiment by which the vapour pressure of a liquid at the room temperature may be determined. Can the saturated vapour pressure be also determined by this experiment? If so, how?

13. How can you demonstrate that vapour exerts pressure? A liquid boils at a temperature when its saturation vapour pressure equals the superincumbent pressure. Give an experimental evidence in support of this statement.

14. Distinguish between a gas and a vapour. Why is oxygen called a permanent gas.

How would you show experimentally that unsaturated vapour pressure obey's Boyle's law, but saturated vapour pressure does not?

15. Discuss the effects of change in temperature at constant volume and change of volume at constant temperature on saturated and unsaturated vapours.

16. Define 'Absolute' and 'Relative humidity'. Which one of them is more important for weather forecasting? Why? Describe Regnault's hygrometer and explain how this instrument is used for the determination of relative humidity of the atmosphere.

17. Define 'Dew-point'. Is the dew point a fixed point in the sense that the melting point is? Describe a method of determining the dew-point. Why is its determination important?

What would be the condition of the atmosphere, if the room temperature is equal to the dew-point?

18. What is dew? How is it formed? State and explain under which conditions dew is formed. What are the characteristics of substances suitable for deposition of dew?

19. 'Cloud is essentially a mist or fog formed in the upper atmosphere'—explain. What is smog? Discuss two most important conditions that must be satisfied in order that the rain drops from the cloud reach the ground.

[B] Short-answer type questions.

1. (*a*) What is the advantage of taking water as the hot substance in a hot water bottle?

(*b*) A person gets a more severe burn by steam at 100°C. than by boiling water. Why? [*H. S.* '*82*]

2. A beaker of water is kept in a table in a room. Can this water be made to boil by passing steam through it at atmospheric pressure? Give reasons for your answer. [*H. S.* '*65*]

3. A small quantity of water is placed in a hole in a large block of melting ice. Will it freeze? Give reasons for your answer.

4. 'Exposed water pipes are found to burst during very cold weather'. Explain why this occurs.

5. A brass pitcher and a porous earthenware jug are filled with water and placed side by side in air. Would you notice any difference of temperature between the two after some time? If so, why?

6. Some astronaut on the surface of the moon took some water at about 20°C. out of his thermosflask and poured over into a glass beaker. Briefly explain what you would expect to happen to water. [*Jt. Entrance*, '*76*]

7. Why does warm moist air cause more discomfort than warmer dry air? [*H. S.* '*65*]

8. What is freezing mixture ? Why is it so called ?

9. What is meant by the statement—'Latent heat of steam is 540 cal./gm' ?

10. Will water ever freeze in a bottle immersed in a mixture of water and ice ? Can we get water to boil at room temperature ? [*Jt. Entrance*, '72]

11. If the temperature of a room be raised, explain what the effect will be on (i) the dew point, (ii) the relative humidity of the air in the room. [*H. S.* '60]

12. Can two cubes of ice when pressed together form one mass ? Explain your answer with reasons. [*Jt. Entrance* '77]

13. It is found that at ordinary temperatures some gases like ammonia can be liquified with suitable pressures, whereas other like oxygen cannot be so liquified whatever be the pressure applied. Explain why it is so. [*Jt. Entrance* ' 73]

14. Although the room temperature in the winter season is much less, wet clothes dry up much quicker in the winter than in the rainy season. Why ? [*H. S.* '80]

15. Is it possible for the dew point to be below 0°C ? State your reasons briefly.

16. If water is sprinkled inside a closed room, how will the dew-point and the relative humidity change ?

17. What is the saturated vapour pressure of water at 100°C ?

18. Why much longer time is required to boil away a quantity of water than that necessary to bring it to the boiling point, heat being supplied always at the same rate ?

19. Why the reading of a thermometer is lowered when its bulb is wrapped with a wet rag ? What difference will be observed when the rag is wetted with (i) ether, and (ii) water ? [*Jt. Entrance*, '74]

20. 'Lumps of ice are more effective in cooling a drink than iced water.' Explain.

21. Two metal spheres one of aluminium and the other of lead, each having the same weight, are suspended with threads. A vessel containing molten napthalene in equilibrium with solid napthalene is taken and the two spheres at room temperature are simultaneously plunged into the clear molten liquid napthalene at the top of the vessel, kept immersed for about a minute and then taken out. Each gets coated with some frozen napthalene and on weighing the napthalene coated aluminium sphere appears heavier than the napthalene coated lead sphere.

Briefly explain the reason for this difference and state your conclusion about the relevant properties. [*Jt. Entrance*, '74]

22. If a few drops of ether is poured on the bulb of a thermometer an immediate lowering of its reading is produced. But when the thermometer is dipped in a bottle of ether, no such lowering is observed. Explain.

23. A solid material is supplied with heat at a constant rate. The temperature of the material is changing with the heat input as shown in the graph. Study carefully and answer the following questions.

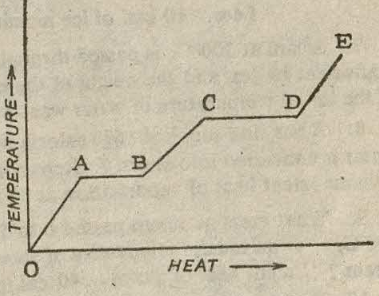
(i) What do the horizontal regions *AB* and *CD* represent ?

(ii) If *CD* is equal to 2*AB*, what do you infer ?

(iii) What does the slope of *DE* represent ?

(iv) The slope of *OA* is greater than slope of *BC* what does this indicate ?

[*I. I. T.* '80]



24. Explain the following statements with scientific reasons.

(i) Water boils at Darjeeling at much lower temperature that 100°C.

(ii) It is unwise to sit in a draught with wet clothes on.

(iii) A fan gives a feeling of comfort during hot weather.

(iv) When a *khus-khus* moistened with water is placed in the door, a cooling effect is produced in the room.

(v) Temperature of a boiling liquid cannot be increased further on application of heat,

(vi) Water can be boiled at temperatures lower than 100°C.

(vii) If a few drops of ether is poured on the hand, a cold sensation is produced.

(viii) When ice cooled water is kept within a glass vessel, dews are formed on the outer surface of the vessel.

(ix) Dew is formed at night more on the blades of grass than on the leaves of trees.

(x) Cloudless nights are better than cloudy nights for dew to deposit.

(xi) In cold countries when the temperature falls below 0°C, ethyl glycol should be added to the water in the radiators of automobiles.

(xii) Human body feels comfortable at a given temperature when humidity lies within a definite range.

(xiii) When pressure is increased, melting point of ice decreases but that of wax increases.

(xiv) We apply glycerine on the lips during winter.

[C] Simple Problems.

1. (a) How many calorie of heat are needed to melt 60 gm. of ice at 0°C. and raise the temperature of the melted ice to 80°C. ?

(b) A kilogram of water at 20°C. is placed in a refrigerator. How much heat must be extracted before all the water turns to ice ? Latent heat of ice=80 cal./gm.

[*Ans.* (a) 9600 cal. ; (b) 10^5 cal.]

2. A thermosflask whose water equivalent is 30 gm. contains 750 gm. of water at 20°C. If 300 gm. of ice at 0°C. are put into it, how much of the ice will remain unmelted ? Latent heat of ice=80 cal./gm.

[*Ans.* 105 gm]

3. After a few piece of dry ice had been added to 120 gm. of water at 46°C. the final temperature of the mixture was found to be 0°C. What is the mass of ice added ? Latent heat of ice=80 cal./gm.

[*Ans.* 69 gm.]

4. What is the final temperature when (a) 3 gm., (b) 2 gm. of steam at 100°C. are passed into 60 gm. of water at 20°C. ? Given, latent heat of steam=540 cal./gm.

[*Ans.* (a) 49·5°C. (b) 40°C.]

5. Steam is passed through a mixture of 2 kg. of ice and 1 kg. of water and the whole mixture is found to gain weight by 150 gm. How much ice will melt ? (Latent heat of fusion of ice=80 cal./gm. ; Latent heat of condensation of steam=540 cal./gm). [*Ans.* 1200 gm.]

6. If 64,800 calorie of heat are withdrawn from 100 gm. of steam at 100°C., what will be the resultant ? (Latent heat of steam=540 cal./gm., latent heat of fusion of ice=80 cal./gm.)

(*H.S.* '65)

[*Ans.* 10 gm. of ice remains mixed up with 90 gm. of water, both being at 0°C.]

7. Steam at 100°C. is passed through 200 gm. of water contained in a calorimeter of water equivalent 10 gm. and the weight of the calorimeter and its content thereby increases by 6 gm. If the initial temperature of water was 25°C., find the final temperature. [*Ans.* 42·1°C.]

8. A gas ring supplied 1620 calorie of heat to a kettle full of water at 100°C. How much water is converted into steam ? Repeat assuming the number of calorie supplied to be 2160. Assume latent heat of vaporisation as 540 calorie/gm.

[*Ans.* 3 gm. ; 4 gm.]

9. What mass of steam passed into 50 gm. of water at 10°C. will raise the temperature to 100°C. ? If an indefinite quantity of steam is available, will the 50 gm. of water be turned into steam ? Latent heat of steam=540 cal./gm.

[*Ans.* 8·33 gm. ; No.]

10. In an experiment 10 gm. of steam were passed into 285 gm. of water at 20°C. contained in a vessel of water equivalent 15 gm. The final temperature was 40°C. What value did this experiment give for the latent heat of vaporisation of water ?

[*Ans.* 540 cal./gm.]

11. What is the result of mixing 10 kg. of copper at 100°C. with 3 kg. of ice at 0°C. ? (sp. heat of copper=0·1, latent heat of fusion of ice=80 calorie/gm.)

[*Ans.* 1·25 kg. of ice melts, and 1·75 kg. of ice remains unmelted ; whole mixture is at 0°C.]

12. 10 gm. of iron (sp. ht.=0·1) are heated into a Bunsen flame and dropped into a cavity in a lump of ice. 20 gm. of ice are melted. What is the flame temperature ? Latent heat of ice=80 cal./gm.

[*Ans.* 1600°C.]

13. How much heat is required to convert 2 gm. of ice at −20°C. to steam at 100°C. ? (Given, sp. heat. of ice=0·5 ; latent heat of ice=80 cal./gm. ; latent heat of condensation of steam=540 cal./gm.)

[*Ans.* 1460 cal.]

14. Vapour from a liquid boiling at 85°C. and of a latent heat 200 calorie per gm. is led into 100 gm. of this liquid at 20°C. contained in a calorimeter weighing 50 gm. and of specific heat 0·08. If 4 gm. of vapour bring the final temperature to 35°C. find the specific heat of the liquid.

[*Ans.* 0·569]

15. A polar explorer wants to make tea but the only water he has is 1000 gm. of ice at −50°C. How many calorie of heat will be required to convert this ice into water at 100°C. ? The specific heat of ice is 0·5 cal./gm. °C. and the latent heat of ice=80 cal./gm.

[*Ans.* $2{\cdot}05\times10^5$ cal.]

16. A nickel cylinder weighing 100 gm. is cooled to the temperature of liquid air and then quickly transferred to water at 0°C. It is found that 12 gm. of ice are formed on the nickel. Calculate from these date the temperature of the liquid air. Assume sp. ht. of nickel as 0·086 and latent heat of fusion of ice=80 cal./gm.

[*Ans.* −111·6°C]

17. A copper calorimeter weighing 500 gm. contains 400 gm. of liquid paraffin at 25°C. When 50 gm. of ice is added to it, the steady temperature is found to be 10°C. Find the sp. heat of liquid paraffin. Given, sp. heat of copper=0·09, latent of fusion of ice=80 cal./gm.

[*Ans.* 0·64]

18. A calorimeter contains 500 gm. of water and 100 gm. of ice at an equilibrium temperature of 0°C. Then 200 gm. of steam at 100°C. are fed into it. Calculate the final temperature and composition of the mixture assuming that the calorimeter is of negligible water equivalent. Latent heat of ice=80 cal./gm. ; of steam=540 cal./gm.

[*Ans.* 74 gm. steam and 726 gm. water, both being at 100°C.]

19. Whilst ethane gas provides heat of combustion 373 K. cal. per mole, only 60% of this quantity is useful. Calculate how many litre of ethane gas measured at N.T.P. should be used in order to convert 50 kg. of water at 10°C. into steam at 100°C. Assume one mole of a gas occupies 22·4 litre at N.T.P. and also latent heat of vaporisation of water=540 cal./gm.

[*Ans.* 3153 litre]

20. When a piece of metal weighing 48·3 gm. at 10·7°C. was immersed in a current of steam at 100°C., 0·762 gm. of steam was found to condense. Calculate the specific heat of metal. Assume latent heat of vaporisation of water as 540 cal./gm.

(*I.I.T.* '*63*) [*Ans.* 0·095]

21. What mass of ether at 0°C. must be evaporated in order to freeze 10 gm. of water at 0°C. ? Assume latent heat of vaporisation of ether and latent heat of ice at 0°C. as 95 and 80 calorie per gm. respectively.

[*Ans.* 8·422 gm.]

22. What will be the result of mixing 110 gm. of ice at −10°C. with 40 gm. of water at 90°C. ? Assume sp. ht. of ice is 0·5 and latent heat of ice=80 cal./gm.

[*Ans.* 38·125 gm. will melt ; 71·875 gm. remain unmelted at 0°C.]

23. A lagged metal can contains a mixture of water and crushed ice at 0°C. It is weighed and immediately afterwards the mixture is well stirred while dry steam at 100°C. is passed through it. When all the ice has just melted, the steam supply is cut off and a second weighing of the can and its contents shows an increase in mass of 10 gm. Calculate the mass of ice originally present. Given, latent heat of ice=80 cal./gm., of steam=540 cal./gm.

[*Ans.* 80 gm.]

24. 20 gm. of ice at 0°C. are dropped into a mixture of oil and water at 30°C. and are just melted in cooling the mixture to 0°C. If the specific heat of oil is 0·6 and the total weight of the liquid at the end of the experiment is 85 gm., find the quantities of oil and water in the original mixture.

[*Ans.* 29·2 gm. oil ; 35·8 gm. water]

25. On a certain day, the temperature of air is 24·5°C. and the dew point is 13°C. Find the relative humidity. The S.V.P. of water at 24°C., 25°C., and 13°C are 22·2, 23·5 and 11·1 mm. of mercury respectively. [*Ans.* 48·57%]

26. Calculate the relative humidity when the air temperature on a day was 29°C. and the hygrometer condensation point was 10°C. Given, aqueous tension at 10°C. and 29°C. are 9·1 mm. and 29·9 mm. respectively. [*Ans.* 30·43%]

27. Find the dew point when the relative humidity is 70%, the temperature being 13°C. Given that, for pressures of 7, 9 and 11 mm. of mercury, the corresponding boiling points of water are 6°, 10° and 13°C, respectively. [*Ans.* 7·4°C.]

28. What is the pressure of water vapour in the air on a warm day when the temperature is 30°C. and the relative humidity is 80% ? (Pressure of saturated vapour at 30°C. is 31·7 mm. of Hg.) [*Ans.* 25·36 mm. of Hg.]

29. A closed room contains moist air at 20°C., the relative humidity being 40%. What would the relative humidity become if the room was cooled to 10°C. ? State any assumption you make in your calculation (S.V.P. of water at 20°C.=17·5 mm. at 10°C.=9·2 mm. of Hg.) [*Ans.* 73·5%]

30. A flask is completely filled with 1 gm. of saturated steam at 100°C. How much water will condense into water if the temperature of the flask in reduced to 25°C. ? (Aqueous tension at 25°C.=30 mm. of Hg). (*Jt. Entrance* '81) [*Ans.* 0·96 gm.]

31. In an industrial process 10 kg. of water per hour is to be heated from 20°C. to 80°C. To do this, steam at 150°C. is passed from a boiler into a copper coil immersed(in water. The steam condenses in the coil and is returned to the boiler as water at 90°C. How many kg of steam are required per hour ? (Specific heat of steam=1 cal./gm./°C. ; Latent heat of steam=540 cal./gm.) (*I.I.T.* '72) [*Ans.* 1 kg.]

32. How should 1 kg. of water at 5°C. be divided in two parts so that one part of it when turned into ice at 0°C. would by this change of state give out a quantity of heat that would be sufficient to vapourise the other part ? (Latent heat of steam=540 cals, Latent heat of ice=80 cals.) (*Jt. Entrance* '80) [*Ans.* 882 gm. into ice and 118 gm. into steam]

33. 100 gm. of ice cubes at 0°C. are dropped into a calorimeter containing 1 kg. of salt water at 0°C. It is observed that the temperature of the mixture drops to −2°C. Find the amount of ice melted (mass of the calorimeter may be neglected, the specific heat of salt water may be taken as 0·9, and latent heat of ice=80 cal./gm.). [*Ans.* 23·02 gm.]

34. An aluminium container of mass 100 gm. contains 200 gm. of ice at −20°C. Heat is added to the system at the rate of 100 cal./sec. What is the temperature of the system after 4 minutes ? Draw a rough sketch showing the variation of the temperature of the system as a function of time. (Specific heat of ice = 0·5 cal./gm./°C ; Specific heat of aluminium=0·2 cal./gm./°C ; Latent heat of fusion of ice=80 cal./gm.) (*I.I.T.* '73) [*Ans.* 25·5°C.]

[D] Harder Problems.

1. Specific gravity of ice is 0·92. 10 gm. of a metal at 100°C. is immersed in a mixture of ice and water and the volume of the mixture is found to be reduced by 0·1 c.c. without change of temperature. Find the specific heat of the metal assuming the specific gravity of water at 0°C. to be 1. (*I.I.T. 1964*) [*Ans.* 0·092]

2. A mixture of 250 gm. of water and 200 gm. of ice at 0°C. is kept in a calorimeter which has a water equivalent of 50 gm. If 200 gm. of steam at 100°C. is passed through this mixture, calculate the final temperature and weight of the content of the calorimeter. (*I.I.T.* '74) [*Ans.* 100°C. ; 572·2 gm.]

3. A test tube containing 3 gm. of ether is immersed in a beaker of water which is surrounded by melting ice. When the whole is at 0°C. a stream of air at 0°C. is blown through the ether until it has all evaporated. The cap of ice formed round the test tube has a mass of 3·14 gm. Find the latent heat of vaporisation of ether. Latent heat of fusion of ice=80·0 cal. gm^{-1}. (*Oxford*) [*Ans.* 83·73 cal./gm.]

4. 20 minutes were required to heat a certain quantity of water from 0°C. to the boiling point with an electric heater. A further 1 hour and 48 minutes were needed to turn all the water into steam under the same condition. Determine from these data the latent heat of vaporization of water. [*Ans.* 540 cal./gm.]

5. A copper calorimeter weighing 100 gm. contains 150 gm. of water at 30°C. Pieces of ice which have not been dried are dropped in and the final temperature after stirring is 5°C. The weight of the calorimeter and its contents is then 300 gm. How much water was put in with the pieces of ice ? (Take the latent heat of ice as 80 cal./gm. and the specific heat of copper as 0·1). (*Oxford*) [*Ans.* 3·125 gm.]

6. A piece of ice at 0°C. suspended in a large hall takes 10 hr. to melt completely into water at 0°C. and the water so formed takes 30 min. to rise to a temperature of 4°C. Calculate from these data an approximate value for the latent heat of ice. [*Ans.* 80 cal./gm.]

7. 7·5 gm. of copper at 27°C. were dropped into liquid oxygen at its boiling point (—183°C.), and the oxygen evaporated occupied 1·89 litre at 20°C. and 750 mm. pressure. Calculate the latent heat of vaporization of oxygen. Specific heat of copper=0·08, Density of oxygen at S.T.P.=1·429 gm. litre^{-1}. (*Cambridge*) [*Ans.* 50·72 cal./gm.]

8. Steam containing some water at 100°C. is passed into an empty 10 kg. vessel originally at 15°C. When the temperature has risen to 60°C., the water in the vessel is found to weigh 150 gm. What is the percentage of water in the mixture ? (The latent heat of steam is 540 cal./gm. ; the specific heat of the material of the vessel is 0·12) [*Ans.* 40·74%]

9. 15·0 gm. of ice at 0°C. are put into a copper calorimeter of mass 100 gm. containing 200 gm. of water, and the final temperature reached is observed. The experiment is then repeated with the same masses and calorimeter and with the water at the same initial temperature but with the ice initially cooled to —180°C. The final temperature is then 6·0 deg. C. lower than before. Neglecting heat exchange between the calorimeter and its surroundings, calculate a value for the mean specific heat of ice between —180°C. to 0°C. The specific heat of copper may be taken as 0·10 cal. gm.$^{-1}$ deg^{-1}C. (*London*) [*Ans.* 0·5 cal. gm.$^{-1}$ deg^{-1} C.]

10. The volume of a quantity of moist air, originally at a temperature of 17°C, and a pressure of 75·8 cm. of mercury is reduced to one half. Simultaneously the temperature is lowered to 10°C. and the air becomes saturated. Given that the relative humidity of air originally was 70 percent and that the pressure of saturated vapour at 17°C. and 10°C. are respectively 14 mm. and 9·14 mm. of mercury, calculate the final pressure. (*London*)
[*Ans.* 146·914 cm. of Hg.]

11. A vessel of water is put in a dry sealed room of volume 76 m^3. at a temperature of 17°C. The saturated vapour pressure of water at 17°C. is 15 mm. of Hg. How much water will evaporate before the water is in equlibrium with its vapour ? (*I.I.T.* '78) [*Ans.* 1·135 kg.]

(*Hint* : Density of water vapour at N.T.P. $=\frac{18}{22400}$(gm./c.c)

12. Air at 30°C. and 90% relative humidity is drawn into an air conditioning unit and cooled to 20°C., the relative humidity being reduced to 50%. How many gm. of water vapour must be removed by the air conditioner from a cublic metre of air ? Neglect the change of volume of air. (Given, density of saturated vapour in air at 30°C. is 30 gm./m.^{3} and at 20°C. is 17 gm./m.^{3}) (*Jt. Entrance* '72) [*Ans.* 18·79 gm.]

13. A jar contains a gas and a few drops of water at T°K. The pressure in the jar is 830 mm. of mercury. The temperature of the jar is reduced by 1%. The saturated vapour pressures of water at the two temperatures are 30 and 25 mm. of Hg. Calcutate the new pressure in the jar. (*I.I.T.* '81) [*Ans.* 817 mm. of Hg.]

14. Outside air at 5°C and 20% relative humidity is introduced into a heating and airconditioning plant where it is heated to 20°C and relative humidity increased to 50%. How many gms. of water must be evaporated into a cubic metre of air at 5°C to accomplish this ? The density of saturated water vapour at 5°C is 6·8 gm./m.^{3} and at 20°C is 17·3 gm./m.^{3}. The pressure being normal. [*Ans.* 7·76 gm.]

15. 10 gms. of a substance was taken in the solid state at —10°C. 64 calories were required to heat it to −2°C (still in the solid state) and 880 and 900 calories were required to heat it to the liquid state at 1°C and 3°C respectively. Assuming that the specific heat of the material in the solid and liquid state has values s_1 and s_2 ($s_1 \neq s_2$) respectively, find their values. Show that the latent heat of fusion L is related to the melting point temperature t_m by

$$L=79+0·2\,t_m$$

(*Jt. Entrance* '82) [*Ans.* 0·8 ; 1 0]

16. A cubic metre of air at 24°C was passed through an U tube containing fused calcium chloride, the weight of which was found to increase by 11·2 gm. find the relative humidity of air. (Aqueous tension at 24°C=22·2 mm of Hg.) [*Ans.* 51·91%]

17. Calculate the mass of 1 lt. of moist air at 27°C when the barometer reads 753·6 mm. of Hg and the dew point is 16·1°C. (Saturation vapour pressure at 16·1°C is equal to 13·6 mm. of Hg, the density of air at N.T.P. is equal to 0·001293 gm./c.c. density of saturated water vapour at N.T.P. is equal to 0·000808 gm./c.c.) (*I. I.T.* '77) [*Ans.* 1·159 gm.]

19

CHAPTER

MECHANICAL EQUIVALENT OF HEAT

19·1. Heat as a from of energy.

Upto the middle of the 19th century, the conception of heat as a material substance was prevalent among the contemporary scientists. Heat was considered to be an invisible weightless fluid, called *caloric*. This theory regarding the nature of heat was known as the caloric theory. A body gaining heat was said to have increased its caloric content, whereas loss of heat was interpreted as a decrease in its caloric content.

In the middle of the 19th century, the concept of heat as a material substance was finally abandoned and instead, the energy principle was extended to incorporate the thermal phenomena. This success was achieved by the persistent and painstaking efforts of Count Rumford, Mayer, Joule and many others.

In the year 1798, while superintending the boring of gun barrels at an arsenal in Munich, Count Rumford noticed that large amount of heat was produced during the boring. The supply of heat was found to be quite unlimited; the longer the boring took place, larger was the amount of heat produced. According to the caloric theory, this occurred due to the squeezing out of the caloric from the metal by the action of the drill. Rumford observed that the amount of metal chips obtained by boring is very small and he argued that it is quite impossible for such a small amount of metal shavings to store such large amount of heat (caloric). Use of a blunt borer, produced less metal shavings, but the heat produced was found to be greater.

It was then argued that the caloric might have been supplied by the surrounding air. To verify this, Rumford carried out the boring for some hours inside a tank of water which began to boil. Obviously in this case caloric could not have been supplied by the water since the water itself has gained caloric.

From these observations, Rumford rejected the caloric theory of heat and concluded that heat is a form of motion. He ascribed the heat produced to the mechanical energy spent by the borer.

At about the same time, Davy conducted an experiment in which two pieces of ice were rubbed together in vacuum at a temperature of −2°C. Precautions were taken so that no heat from outside could reach them. A little later, it was found that ice melted which showed that heat was produced by rubbing.

In spite of these experiments, the theory that heat is not a material substance but is a form of energy did not gain favour till 1847, when the results of a series of experiments conducted by James Joule was published. These experiments proved beyond all reasonable doubts that heat is a form of energy and also established the exact relation of equivalence between heat and mechanical energy. In a later article, we shall discuss one of these numerous experiments carried out by Joule.

The heat energy is possessed by a body by virtue of the state of motion of its molecules. Heat imparted to a substance goes towards increasing the energy, both kinetic and potential, of its molecules. Detailed discussions on this aspect will be carried out in the next chapter.

19·2. Mechanical equivalent of heat.

As was stated in the last article, Joule first found out the exact equivalent between the mechanical work done and the amount of heat produced. According to him (*i*) *work may be fully converted into heat and* (*ii*) *whenever some mechanical energy is fully transformed into heat energy, the heat so obtained, is directly proportional to the work done.*

Thus, if W and H represent respectively, the mechanical work done and the corresponding heat generated, we have

$$W \propto H$$

$$\text{i.e., } W=JH \text{ or, } J=\frac{W}{H} \quad \cdots \quad \cdots \quad (19{\cdot}1)$$

when, J is the constant of proportionality and is known as the *mechanical equivalent of heat*.

If $H=1$, then from the eqn. (19·1) $W=J$. We can, therefore, define the **mechanical equivalent of heat** *as the work done to produce unit quantity of heat.*

It is obvious that numerical value of J depends on the units in which W and H are expressed. Thus, if W is expressed in ergs and H in calorie, the value of J was found experimentally to be equal to $4{\cdot}186\times10^7$ erg per calorie. Hence, when $4{\cdot}186\times10^7$ erg of work are fully converted into heat, the heat so produced equals one calorie. Since 1 Joule$=10^7$ erg, the value of J, when W is expressed in joule and H in calorie, is 4·186 Joule per calorie. For ordinary work, the value of J may be taken approximately as $4{\cdot}2\times10^7$ erg per calorie, or 4·2 Joule per calorie.

If W is expressed in ft. lb; and H in B. Th. U. then since 1 ft. lb.$=1{\cdot}356$ Joule and 1 B. Th. U.$=252$ cal.

we have $$J=\frac{4{\cdot}186\times252}{1{\cdot}356}=778 \text{ ft. lb. per B. Th. U.}$$

19·3. Determination of Mechanical equivalent of heat.

The determination of the mechanical equivalent of heat was first carried out by Joule and the type of apparatus employed by him is illustrated in Fig. 19·1. It consists of a calorimeter C containing water with vanes V fixed to the walls of the calorimeter. Two equal weights M, M were placed at the same height above the ground by means of two strings of equal length attached to them. The strings passed over the pulleys P_1 and P_2 and were wound round a cylinder S. With the help of a pin P, the cylinder is connected to a paddle D inside the calorimeter. The paddle carries a set of vanes which pass through spaces within the fixed vanes. When the masses are allowed to descend, they cause the cylinder and in turn the paddle D to rotate. The paddle thus churns the water in the calorimeter, while the fixed vanes prevent the rotational motion of the water. Hence a thorough churning of water is produced. The cylinder S may be detached from the paddle D by removing the pin P and the masses can be brought back to the same height by wrapping the string over the cylinder with the help of a handle H. The paddle and the cylinder are then connected by inserting the pin, and the weights are again allowed to descend from the same height. In this way, the operation can be repeated several times and a considerable amount of heat can be generated in the calorimeter due to churning of the water. The temperature of the water and calorimeter thus rise and can be measured by a sensitive thermometer inserted through an aperture on the top of the calorimeter (not shown in the figure).

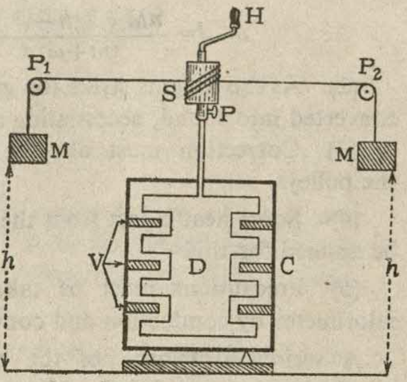


Fig. 19·1 : Joule's experiment

As the weights descend to the ground, they lose a certain amount of P.E., which is converted into heat. If the mass and the height of descent of each be M and h respectively, the loss in P.E. $=2\times Mgh$. If they are allowed to descend n times, then the total loss in P.E. $=2n\times Mgh$. Let the water equivalent of the calorimeter be w and the mass of water contained in it be m. If the resulting heat produces a rise in temperature $t°$ of the water and the calorimeter then the total heat developed, $H=(m+w)t$.

∴ From eqn. (19·1) we have,

$$J=\frac{W}{H}=\frac{2nMgh}{(m+w)t} \qquad \cdots \qquad \cdots \qquad (19{\cdot}2)$$

Since all quantities in the right hand side of the equation is known, J can be found out.

Corrections must be made for various sources of error in the experiment.

(1) As the weights descend, they touch the ground with a definite velocity, however small it might be. Hence their K. E. must be subtracted from the expression for the loss of their P. E. to get the net amount of mechanical energy converted into heat.

Let the velocity with which the weights touch the ground be v.

So, the total K. E. of the weights when they touch the ground

$$=2\times\tfrac{1}{2}Mv^2=Mv^2$$

∴ The mechanical energy converted into heat for each downward journey of the weights $=2Mgh-Mv^2$

As the weights are allowed to descend n times, hence the total amount of mechanical energy converted into heat $= n.M\,(2gh - v^2)$

$$\therefore\ J = \frac{nM\,(2gh - v^2)}{(m+\omega)\,t}$$

(2) As the weights strike the ground, some amount of mechanical energy is converted into sound, necessiating a correction on this account.

(3) Correction must also be made for the energy lost due to friction at the pulleys.

(4) Some heat is lost from the calorimeter by radiation. Correction should be applied for this.

(5) Precautions must be taken to minimise the loss of heat from the calorimeter by conduction and convection.

A serious drawback of the experiment was that a very small rise in temperature was produced. Yet the results obtained were surprisingly close to the present day accepted accurate value of J. More accurate experiments were later performed by Calender, Rowland and others. One such electrical method for finding out the value of J will be discussed later in this text in the "Current Electricity" section.

Example 19·1. *A waterfall discharges 60 litre of water per sec. from a vertical height of 200 m., and it falls in a pool below the fall. Find (a) the quantity of heat produced per second and (b) the rise in temperature of water, assuming that all the heat generated is absorbed by water.*

Solution : Here P.E. of the water is converted into heat. Since $m = 60 \times 1000$ gm. and $h = 200 \times 100$ cm., P. E. of the water $= mgh = 6 \times 10^4 \times 980 \times 2 \times 10^4 = 1\cdot176 \times 10^{12}$ erg.

(*a*) From eqn. (19·1), $W = JH$. Putting $J = 4\cdot2 \times 10^7$ erg/cal. the heat produced $H = \dfrac{W}{J} = \dfrac{1\cdot176 \times 10^{12}}{4\cdot2 \times 10^7} = 28 \times 10^3$ cal.

(*b*) If the rise of temperature be t°C., then the heat absorbed by water $= m.t$ cal.

$$\therefore\ t = \frac{H}{m} = \frac{28 \times 10^3}{6 \times 10^4} = 0\cdot47^\circ\text{C} \quad \text{(approx).}$$

Example 19·2. *What is the height from which a piece of lead must be dropped so that its temperature may rise through 100°C. due to heat generated at the impact. Assume that all the heat generated is absorbed by the lead piece and the specific heat of lead$=0\cdot03$.*

Solution : Let h cm. be the required height and m gm. be the mass of the lead piece.

$\therefore$ P. E. of the lead piece $= mgh$ erg.

This P. E. is converted into heat due to the impact.

$\therefore$ Heat produced $H = \dfrac{mgh}{J} = \dfrac{m \times 980 \times h}{4\cdot2 \times 10^7}$ cal.

Again, heat required to raise the lead piece through 100°C is $H_1 = mst = m \times 0{\cdot}03 \times 100$ cal.

Since all the heat generated is absorbed by the lead piece

$$H = H_1$$

$$\text{or,} \quad \frac{m \times 980 \times h}{4{\cdot}2 \times 10^7} = m \times 0{\cdot}03 \times 100$$

$$\text{or,} \quad h = 1{\cdot}286 \times 10^5 \text{ cm.} = 1286 \text{ m.}$$

Example 19·3. *A block of ice at 0°C. is dropped from a height and melts completely due to impact with the ground. Find the height from which it is dropped assuming that 60% of its energy is converted into heat. Given, latent heat of ice=80 cal./gm.*

Solution : Let h cm. be the required height and m gm. be the mass of the ice block.

∴ P. E. of the ice block$=mgh$ erg. Since 60% of this P. E. is converted into heat.

$$\therefore \quad \text{Heat produced} \quad H = \frac{mgh}{4{\cdot}2 \times 10^7} \times \frac{60}{100}$$

$$= \frac{6m \times 980 \times h}{4{\cdot}2 \times 10^8} \text{ cal.}$$

But heat required to melt m gm. of ice is

$$H_1 = m \times 80 \text{ cal.}$$

∴ By question $H = H_1$

$$\text{or,} \quad \frac{6m \times 980 \times h}{4{\cdot}2 \times 10^8} = m \times 80$$

$$\therefore \quad h = 5{\cdot}715 \times 10^6 \text{ cm.} = 57150 \text{ m.}$$

Example 19·4. *At what speed must a lead bullet at 15°C. strike a fixed target in order that the heat produced an impact be just sufficient to melt the bullet ? Given sp. heat of lead=0·032, its melting point=327°C. and its latent heat of fusion=5·4 cal./gm.*

Solution : The heat produced by the impact is due to the conversion of the K. E. of the bullet into heat energy. If m be the mass of the bullet, then heat produced $= m \times 0{\cdot}032 \times (327 - 15) + 5{\cdot}4 \times m = 15{\cdot}38m$. If v be the velocity of the bullet just before impact, then $\frac{1}{2} mv^2 = JH = 4{\cdot}2 \times 10^7 \times 15{\cdot}38m$ or, $v = 35{\cdot}95 \times 10^3$ cm./sec.

Example 19·5. *Two lead balls of masses 100 gm. and 400 gm. respectively are moving towards each other with speeds 100 cm./sec. and 10 cm./sec. respectively. They suffer a head-on collision and stick together and the combined mass moves with a velocity of 12 cm/sec. If the temperature of both the balls before collision be the same, find the increase in temperature of the combined mass assuming $J = 4{\cdot}2 \times 10^7$ erg/cal. and specific heat of lead=0·03,*

Solution : Total K. E. of the balls before collision

$E_1=\frac{1}{2}\times 100\times 100^2+\frac{1}{2}\times 400\times 10^2=5{\cdot}2\times 10^5$ erg.

The K. E. of the combined mass after collision

$E_2=\frac{1}{2}\ 100+400)\times 12^2=0{\cdot}36\times 10^5$ erg

$\therefore$ Loss of energy during collision $=(5{\cdot}2-0{\cdot}36)\times 10^5$

$=4{\cdot}84\times 10^5$ erg

$\therefore$ Amount of heat produced

$$H=\frac{4{\cdot}84\times 10^5}{4{\cdot}2\times 10^7}\text{ cal.}$$

If t°C. be the increase in temperature of the combined mass, we get

$$\frac{4{\cdot}84\times 10^5}{4{\cdot}2\times 10^7}=(100+400)\times 0{\cdot}03\times t$$

$$\therefore\quad t=7{\cdot}62\times 10^{-4}\,{}^\circ\text{C.}$$

Example 19·6. *A 18 kg. weight descends through a vertical height of 4·9 m., so that in doing so, it turns a paddle which stirs 400 gm. of water in a vessel of water-equivalent 20 gm. If the process is repeated 10 times, the temperature of the water rises by 5°C. Hence find out the mechanical equivalent of heat.*

Solution : As the weight descends, the loss in its P. E.

$=18\times 10^3\times 980\times 4{\cdot}9\times 10^2$ erg.

Hence the total loss in P. E. during the experiment is 10 times the above value. This is converted into heat.

$$\therefore\quad J=\frac{W}{H}=\frac{18\times 10^3\times 980\times 4{\cdot}9\times 10^2\times 10}{(400+20)\times 5}=4{\cdot}12\times 10^7\text{ erg/cal.}$$

19·4. First law of Thermodynamics.

Thermodynamics *is that branch of Physics which is concerned with the study of mutual conversion of heat and mechanical energy and their interrelation.* In general, however, the study of the relations of heat with all other forms of energy falls within the scope of thermodynamics.

We have just seen how the works of Rumford, Joule etc. proved that heat is a form of energy and established the equivalence of heat and mechanical energy. *This equivalence relationship is generally known as* **first law of thermodynamics** which can, therefore, be expressed in the form

$$W=JH$$

The law states that *whenever mechanical energy is converted into heat or vice versa, an exact equivalance exists between the two i.e., the ratio of work done to the amount of heat developed is always constant.*

It follows that the first law of thermodynamics is simply a particular case of the more general principle of conservation of energy applied to the phenomenon of mutual transformation of heat and mechanical energy.

As suggested by Clausius, the first law of thermodynamics can be expressed in a more general form. When heat is supplied to a substance, the energy so supplied is used up generally in two ways, viz., (i) to increase the internal energy of the substance, (ii) to perform external work.

By the internal energy of a substance, we mean the kinetic energy and the potential energy of the molecules of which the substance is composed (vide Art. 20·2). A part of the heat energy supplied, therefore, goes towards increasing the energy of the molecules. The increase in internal energy is manifested by a rise in temperature or a change in state. The actual amount of internal energy possessed by a body can never be measured. But this poses no problem, because in practice we are concerned only with the changes in internal energy ; this can always be measured with certainty.

The rest of the heat supplied is used for performing external work. For example, external work is performed when the substance, subjected to a definite pressure, expands. As the volume is increased, work is done against external pressure.

Thus, if Q be the amount of heat supplied to a substance so that its internal energy changes from U_1 to U_2 and W be the amount of external work done, then

$$Q=U_2-U_1+W \qquad \ldots \qquad \ldots \qquad (19{\cdot}3)$$

This is the most general mathematical statement of the first law of thermodynamics. In applying this expression, it must be borne in mind that (i) all quantities must be expressed in the same unit i.e., either in thermal units or in mechanical units, (ii) Q is positive when heat is absorbed by the substance, (iii) W is positive when the external work is done by the substance. In this form the law applies equally well for solids, liquids and gases.

If the amount of heat supplied to a substance be infinitesimally small viz. δQ, the corresponding increase in its internal energy and the external work done by it will be infinitesimally small. Thus if δU be the increase in its internal energy and δW the amount of external work done by it, the first law of thermodynamics can be written in the form

$$\delta Q=\delta U+\delta W$$

19·5. Internal energy of a gas.

The thermal state of a gas is defined by its temperature T, pressure p or the volume V. Evidently, it may be argued that internal energy of a gas depends on all these factors. This matter was investigated experimentally by Joule and the result is known as Joule's law or Mayer's hypothesis. It states that

The internal energy of a given amount of gas depends only upon its temperature and is independent of its pressure or volume.

The law implies that the change in internal energy of a given mass of a gas can be known provided its change in temperature is given, no matter what the pressure or volume may be. Evidently, if the temperature of a gas does not change, its internal energy remains the same, though the pressure and the volume of the gas might vary.

The hypothesis holds exactly only for an ideal gas. In practice, for real gases, the internal energy is found to depend on its volume. The effect is, however, very small so that the hypothesis holds extremely closely for all real gases. For our work, we shall disregard this small departure in the behaviour of real gas from that of an ideal gas.

19·6. External work done by an expanding gas.

As an example of the external work done by a substance, let us consider the expansion of a gas subjected to a constant pressure. For convenience, we assume the gas to be enclosed in a cylinder equipped with a movable piston [Fig. 19·2]. We imagine the piston to be frictionless.

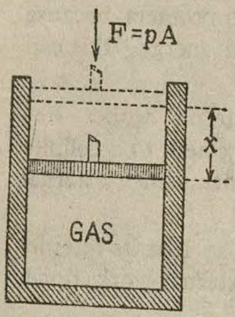


Fig. 19·2 : Work done by an expanding gas

If the pressure of the gas be p, then it exerts a force $F=pA$ on the piston, where A is the area of the piston. Hence, for equilibrium, an equal and opposite external force F must be applied on the piston.

If the gas is heated keeping the pressure constant, it expands and the piston is pushed back through a distance, say x. From the definition of work, the work done by the gas=force on piston $\times x = F \times x = pA \times x$.

But $(A \times x)$ represents the change in volume of the gas.

$\therefore$ Work done$=p\times$change in volume

or, $W=p(V_2-V_1)$ ... ... (19·4)

where V_1 and V_2 are the initial and final volumes of the gas respectively.

If the change in volume is infinitesimally small, we can write $V_2-V_1=dV$, so that the expression for work done becomes

$$\delta W=pdV \qquad \text{... ... (19·5)}$$

The work done is considered positive when the gas performs the work i.e., when it expands. When work is done on the gas i.e., when it is compressed, the work is taken to be negative.

Example 19·7. *Calculate the change in the internal energy and indicate whether there will be a rise or fall in temperature in each of the following processes: (a) A system absorbs 500 calorie of heat and performs 200 Joule of external work. (b) A system absorbs no heat while at the same time 500 Joule of work are done on it.*

Solution : (a) Here $Q=500$ cal., $W=\frac{200}{4 \cdot 2}=47{\cdot}6$ cal.

$\therefore$ From eqn. (19·3), the change in internal energy$=U_2-U_1=Q-W=500-47{\cdot}6=452{\cdot}4$ cal. As the internal energy increases, the temperature of the system will rise.

(b) Here $Q=0$, $W=-\frac{500}{4 \cdot 2}=-119$ cal. $\therefore$ From eqn. (19·3), the change in internal energy$=U_2-U_1=Q-W=0-(-199)$

$=119$ cal.

The increase in internal energy indicates that the temperature of the system will rise.

Example 19·8. *To a gas confined in a cylinder, 600 calorie of heat are supplied and at the same time the gas expands against the atmospheric pressure of 76 cm. of mercury. There is no change in its internal energy. Find its increase in volume.*

Solution : Here $Q=600$ cal.$=600\times4{\cdot}2\times10^7$ erg, and $U_2=U_1$.

$\therefore$ From eqn. (19·3), the external work performed by the gas

$$=W=Q=252\times10^8 \text{ erg}.$$

Since atmospheric pressure $p=76\times13{\cdot}6\times980=1{\cdot}013\times16^6$ dyne/cm^2., hence from eqn. (19·4), the increase in volume of the gas

$$V_2-V_1=\frac{W}{p}=\frac{252\times10^8}{1{\cdot}013\times10^6}=24{\cdot}9\times10^3 \text{ c.c.}$$

19·7. Specific heats of gases.

The specific heat of a substance (vide Art. 17·4) is defined as the amount of heat absorbed per unit mass per unit rise in temperature. There is no difficulty about this definition when applied to solids and liquids ; but special considerations are necessary to apply this definition in the case of gases, because with change in temperature both the volume and the pressure of a gas vary considerably. The amount of heat absorbed by a gas depends upon the conditions of its volume and pressure ; hence it is customary to define the specific heat of a gas by keeping either of the two factors, p and V, constant. Accordingly, two specific heats of a gas have came in vogue, viz. the specific heat at constant volume (c_v) and the specific heat at constant pressure (c_p).

The **specific heat of a gas at constant volume** (c_v) *is the amount of heat required to raise the temperature of unit mass of the gas through 1°, when its volume is kept constant throughout.* Thus, the heat required in calorie to raise the temperature of m gm. of a gas through t°C. at constant volume is given by

$$Q=mc_vt \qquad \ldots \qquad \ldots \qquad (19{\cdot}6)$$

The unit of c_v is calorie per gm. per°C. or B. Th. U. per lb per° F.

It is often convenient to deal with one gram-molecule of the gas, instead of one gram of the gas. The corresponding specific heat (C_v) is then called *gram-molecular specific heat* or simply **molar specific heat** of the gas at constant volume. If M be the molecular weight of the gas, then,

$$C_v=Mc_v$$

The **specific heat of a gas at constant pressure** (c_p) *is the amount of heat required to raise the temperature of unit mass of the gas through 1°, when its pressure is kept constant throughout.* So the heat required in calorie to raise the temperature of m gm. of a gas through t°C. at constant pressure is given by

$$Q'=mc_pt \qquad \ldots \qquad \ldots \qquad (19{\cdot}7)$$

c_p is expressed in the same unit as c_v.

As before, the molar specific heat of the gas at constant pressure is given by

$$C_p=Mc_p$$

19·8. The relation between the two specific heats of a gas.

From the first law of thermodynamics, we know that the heat supplied to a substance is equal to the increase in its internal energy plus the external work done.

When a gas is heated at constant volume, no external work is performed and hence the heat supplied is used up only in increasing the internal energy of the gas.

But if the gas is heated at constant pressure, then the heat supplied not only produces an increase in its internal energy but is also used up in doing some external work.

As the internal energy of a gas depends only upon its temperature, hence more heat will be required to produce a given change in temperature of a fixed mass of a gas at constant pressure than that required to produce the same rise in temperature at constant volume. Hence *for a gas* c_p *is greater than* c_v.

To find the relation between the two specific heats, let us consider one gm. molecule of a gas. If its temperature is raised at constant volume from T_1 to T_2, then the heat supplied is $C_v\ (T_2-T_1)$, where C_v is the molar specific heat at constant volume. Since no external work is done, first law of thermodynamics gives

$$C_v(T_2-T_1)=U_2-U_1 \qquad \cdots \quad \cdots \quad (19\cdot8)$$

If, however, the same amount of the gas is heated at constant pressure p, so that the same rise in temperature takes place, the heat supplied is $C_p\ (T_2-T_1)$, where C_p is the molar specific heat at constant pressure. If the volume of the gas changes from V_1 to V_2 the external work done by the gas during expansion is $p\ (V_2-V_1)$. Since the internal energy of the gas depends only on temperature, the change in the internal energy must be the same as in the previous case, Hence,

$$C_p(T_2-T_1)=U_2-U_1+p(V_2-V_1)=C_v(T_2-T_1)+p(V_2-V_1)$$

$$\text{or,}\quad (C_p-C_v)(T_2-T_1)=p(V_2-V_1) \qquad \cdots \quad \cdots \quad (19\cdot9)$$

But we know that $pV=RT$, where R is the gas constant for one gram-molecule of the gas. Hence, $pV_1=RT_1$ and $pV_2=RT_2$. Substituting these in eqn. (19·9), we get

$$C_p-C_v=R \qquad \cdots \quad \cdots \quad (19\cdot10)$$

Thus, *the difference in the molar specific heats of a gas at constant pressure and constant volume is equal to the universal gas constant.*

In the equation (19·10), R must be expressed in the same units as C_p and C_v, usually calorie per mole per°C. Since $R=8\cdot31$ Joule per mole per °C. and 4·19 Joule=1 calorie, we have, therefore, $R=8\cdot31/4\cdot19=1\cdot99$ cal. per mole per °C. or very nearly equal to 2 calorie per mole per °C.

This expression is valid accurately only for an ideal gas ; but for real gases at moderate pressure, the agreement with it is very close.

From eqn. (19·8), C_v provides the measure of the rate of change of internal energy with temperature and hence like internal energy, the value of C_v must be independent of the pressure or volume of the gas. Since from eqn. (19·10), the difference of C_p and C_v is constant, C_p must also be independent of pressure or volume of the gas.

It can be proved that the values of C_v and C_p depend upon the number of atoms in the molecules of a gas. For a monoatomic gas, e.g. helium, neon etc. C_v has the value $\frac{3}{2}R$; hence C_p is equal to $\frac{5}{2}R$. For a diatomic gas, e.g., hydrogen, oxygen etc., the value of C_v is $\frac{5}{2}R$ and that of C_p is $\frac{7}{2}R$.

Example 19·9. *Find out the molar specific heats C_p and C_v for an ideal gas for which $\gamma=1·67$.*

Solution : Since $\gamma=C_p/C_v=1·67$, therefore, $C_p=1·67\ C_v$. From eqn. (19·10), $C_p-C_v=1·99$; or, $(1·67-1)\ C_v=1·99$

$\therefore\ C_v=2·97$ and $C_p=4·96$.

Example 19·10. *The specific heat of oxygen at constant volume is 0·155 cal./gm./°C. Find its specific heat at constant pressure. (Molecular weight of O_2= 32 gm./mole. Universal gas constant=2 cal./mole/°C.)*

Solution : Molar specific heat of oxygen at constant volume $C_v=32\times0·155$. From eqn. (19·10),

$$C_p=C_v+R=32\times0·155+2=6·96 \text{ cal./mole/°C.}$$

Hence the specific heat at constant pressure,

$$c_p=C_p/M=6·96/32=0·218 \text{ cal./gm./°C.}$$

Example 19·11. *At N.T.P., one litre of hydrogen weighs 0·0896 gm. and its specific heat at constant volume is 2·47 cal./gm./°C. Calculate its specific heat at constant pressure. Given, J=4·2 Joule/cal. and density of mercury at 0°C.=13·6 gm./c.c*

Solution : At N.T.P. 1 gm. of hydrogen occupies the volume $V=1000/0·0896$ c.c. The pressure $p=76\times980\times13·6$ dyne/cm². and temperature $T=273°$K. Also if M be the molecular weight of hydrogen, then 1 gm. of hydrogen $=\frac{1}{M}$ mole. Hence from the relation $pV=nRT$ where n is the number of moles of the gas$=1/M$, we have $nR=\frac{R}{M}=\frac{pV}{T}=\frac{76\times980\times13·6\times1000}{273\times0·0896}$

$$c_p-c_v=\frac{R}{MJ}=\frac{76\times980\times13·3\times1000}{273\times0·0896\times4·2\times10^7}=0·986$$

$$c_p=c_v+0·986=2·47+0·986=3·456 \text{ cal./gm./°C.}$$

Example 19·12. *The temperature of 10 gm. of oxygen gas is raised from 20°C. to 180°C. Find the amount of heat supplied, the increase in internal energy and the external work performed by the gas, if the gas is heated (a) at constant volume, and (b) at constant pressure. Given for oxygen, $c_v=0·155$ cal./gm./°C, $c_p=0·218$ cal./gm./°C.*

Solution : (a) From eqn. (19·6), the heat supplied at constant volume $Q=mc_v(t_2-t_1)=10\times0·155\times(180-20)=248$ cal.

Since the volume remains constant, the external work done $W=0$. Hence from the first law of thermodynamics, the change in internal energy $U_2-U_1=Q=248$ cal.

(b) From eqn. (19·7), the heat supplied at constant pressure, $Q = mc_p(t_2 - t_1) = 10 \times 0{\cdot}218 \times (180 - 20) = 348{\cdot}8$ cal. Since the internal energy of the gas depends only on temperature, the change in the internal energy is the same as in the previous case, i.e., $U_2 - U_1 = 248$ cal. From the first law of thermodynamics, the external work done $W = Q - (U_2 - U_1) = 348{\cdot}8 - 248 = 100{\cdot}8$ cal.

19·9. Isothermal changes.

When the change of pressure and volume of a gas takes place without any change in temperature, it is called an **isothermal change.**

In order to explain how such a change can be brought about, let us consider a quantity of gas in a metal cylinder fitted with a frictionless movable piston. If the gas is now allowed to expand *very slowly*, some work is done by the gas, which will tend to produce a decrease in its internal energy. The temperature of the gas, therefore, falls slightly. But the metal cylinder being a good conductor of heat, heat flows from the surroundings through the cylinder to the gas and exactly compensates the loss in internal energy. The temperature of the gas, therefore, remains constant and the gas is said to have undergone an *isothermal expansion.*

Similarly, let us suppose that the gas is compressed *very slowly*. This time, the work is done on the gas, and the internal energy and hence the temperature of the gas tends to increase. The heat, so produced, escapes through the cylinder to the surroundings and the temperature of the gas is thereby maintained constant. This reverse process is called the *isothermal compression* of the gas.

From the above discussions, it follows that to allow the exchange of heat between the gas and the surroundings, isothermal changes must always be performed in a thin-walled, highly conducting vessel. But, in practice, there is no such conductor known through which conduction of heat takes place as rapidly as required to maintain the temperature of the system constant. A fairly close approximation to isothermal change may be obtained by causing the movement of the piston at a very slow rate, thus allowing sufficient time for transfer of heat between the gas and the surrounding by conduction through the walls of the enclosure. Hence, *isothermal change is essentially a very slow process.*

It is obvious that *a gas undergoing isothermal change obeys Boyle's law, pV = constant.* A plot of p against V will be a rectangular hyperbola [Fig. 19·3]. Similar shaped curves will be obtained at different constant temperature. These curves are called the *isothermals* of the gas, the lower ones corresponding to smaller values of temperature.

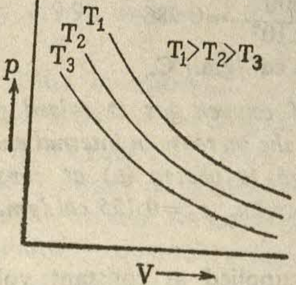


Fig. 19·3 : Isothermals of a gas

As the internal energy of a gas depends only upon its temperature, the internal energy of the gas remains unchanged during an isothermal process. From the first law of thermodynamics, since $U_2 = U_1$, $Q = W$. Hence *in an isothermal expansion, the heat supplied equals the external work done by the gas.*

19·10. Adiabatic change.

If the change of pressure and volume of a gas takes place under such a condition that no heat enters or leaves the gas, it is said to have undergone an **adiabatic change.** The heat content of the gas therefore, remains constant during such a change.

Now, if a gas in an enclosure is allowed to expand adiabatically then $Q=0$. Evidently, the first law of thermodynamics gives,

$$U_2-U_1=-W \qquad \ldots \qquad \ldots \qquad (19\cdot11)$$

$\therefore \quad U_2<U_1$. Thus in doing external work, the internal energy content of the gas is lowered. From Mayer's hypothesis, the temperature of the gas also falls by a certain amount. It is a common experience that when the compressed air in a bicycle tire is released, the air on rushing out becomes colder.

Similarly, if a gas is compressed adiabatically, then $Q=0$ and since W is negative when work is done on the gas, we have

$$U_2-U_1=W \qquad \ldots \qquad \ldots \qquad (19\cdot12)$$

In this case $U_2>U_1$. Hence the internal energy of the gas increases, and a consequent rise in temperature is produced. Thus, when air is pumped into a bicycle tire, the air becomes hotter which can be easily felt.

A gas undergoing an adiabatic change must be kept in an enclosure surrounded by a thick layer of heat insulating material, so that no heat enters or leaves the enclosure. The process should be performed quickly. The flow of heat is a slow process so that if the changes are brought about rapidly, no appreciable exchange of heat takes place. *Adiabatic change is, therefore, a quick one.*

Unlike isothermal changes, Boyle's law does not hold for adiabatic changes. In this case, the pressure-volume changes of the gas obeys the law,

$$pV^{\gamma}=\text{constant} \qquad \ldots \qquad \ldots \qquad (19\cdot13)$$

where, γ is the ratio of the specific heat of the gas at constant pressure to the specific heat at constant volume, i. e., $\gamma=\frac{C_p}{C_v}$. Since $C_p>C_v$, γ is always greater than unity. For monoatomic gases like argon, helium etc., $\gamma=5/3$; for diatomic gases and air, $\gamma=7/5$. The proof of the eqn. (19·13) is beyond the scope of this book.

The equation $pV=RT$, which is always true under all possible conditions, holds for adiabatic changes also. Eliminating either p or V from the eqn. (19·13) with the help of this expression, we get the following relations for an adiabatic change,

$$TV^{\gamma-1}=\text{constant} \qquad \ldots \qquad \ldots \qquad (19\cdot14)$$

$$\frac{p^{\gamma-1}}{T^{\gamma}}=\text{constant} \qquad \ldots \qquad \ldots \qquad (19\cdot15)$$

The plot of p vs. V as shown in Fig. 19·4 is called an *adiabatic curve.* A complete family of adiabatics for a fixed mass of a gas is obtained for different initial values of the pressure and volume of the gas. The figure also illustrates the isothermals for the same mass of the gas. From the inspection of the curves, it follows that the adiabatics are steeper than the isothermals. The slope of the adiabatic through any point is γ times the slope of the isothermal through the same point. This can be proved simply by using calculus. The slope of the curve is given by $\frac{dp}{dV}$. For isothermals, $pV=$ constant. Differentiation gives $pdV+Vdp=0$,

$$\frac{dp}{dV}=-\frac{p}{V}$$

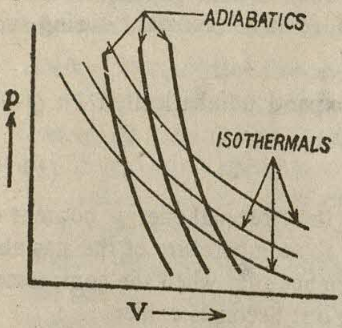


Fig. 19·4 : Adiabatics and isothermals of a gas

For adiabetics, $pV^{\gamma}=$constant. Differentiating we get,

$$\gamma pV^{\gamma-1}dV+V^{\gamma}dp=0, \quad \text{or,} \quad \frac{dp}{dV}=-\gamma\frac{p}{V}$$

The adiabatic expansion has many important applications, as cooling is produced by the sudden adiabatic expansion of compressed gases. An example for such cooling effect is the solidification of carbon dioxide gas, called dry ice. If a cylinder containing compressed carbon dioxide gas is suddenly opened and a piece of wet cloth is held before it, the issuing gas is deposited in the form of solid carbon dioxide. Oxygen was first liquefied by utilising this principle. In some refrigerators, the required cooling is produced by the adiabatic expansion of compressed air.

Example 19·13. *A certain volume of a gas at the pressure of 1 atmosphere and 0°C. is compressed to 1/5th of its original volume. If the compression is adiabatic, find (a) the final pressure, and (b) the final temperature. (c) What would be the final pressure if the compression is isothermal? Given γ of the gas $=1{\cdot}4$.*

Solution : (a) For adiabatic compression, $p_1V_1^{\gamma}=p_2V_2^{\gamma}$

$\therefore \quad 1\times(V_1)^{1\cdot4}=p_2(V_1/5)^{1\cdot4}$

or, $p_2=(5)^{1\cdot4}=9{\cdot}52$ atm.

(b) $T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$

or, $273\,(V_1)^{0\cdot4}=T_2(V_1/5)^{0\cdot4}$

or, $T_2=273\times(5)^{0\cdot4}=520$°K

$\therefore \quad t_2=247$°C.

(c) For isothermal compression, $p_1V_1=p_2V$.

$\therefore \quad 1\times V_1=p_2\times V_1/5$

or, $p_2=5$ atm.

● EXERCISE ●

[A] Essay-Type Questions.

1. Heat is a form of energy'—Justify the statement. What in meant by 'Mechanical equivalent of heat' ? State its value in C.G.S. and F.P.S. units.

2. State the 'First law of thermodynamics.' Describe the general form of the law in terms of internal energy, external work done and heat supplied.

3. What in mechanical equivalent of heat ? Describe a laboratory method for the determination of mechanical equivalent of heat. Indicate the sources of error in the experiment.

4. Give two examples to show the conversion of mechanical work into heat. What is meant by the 'internal energy' of a gas ? State and explain 'Mayer's hypothesis'. Does it remain valid for real gases ? Obtain an expression for the external work done by an expanding gas.

5. Define the two 'specific heats' of a gas. Show that the specific heat at constant preasure (c_p) is always greater than the specific heat at constant volume (c_v).

6. What is meant by gram-molecular specific heat ? Prove that $c_p - c_v = R$ where c_p and c_v express molar specific heats at constant pressure and constant volume, and R is the universal gas constant.

7. What do you mean by an isothermal change ? How is it realised in practice ? Show that in an isothermal expansion, the heat supplied equals the external work done by the gas.

8. What is meant by adiabetic change ? How do you account for the decrease (or rise) in temperature of a gas when it expands (or contracts) adiabatically ? Does Boyle's law hold for it ? If not, state the relation connecting the pressure and volume of a gas undergoing adiabatic change.

9. What are adiabatics and isothermals of a gas ? Show that the adiabatics are steeper than the isothermals.

[B] Short-answer type Questions.

1. What is the relation between 'calorie' and 'erg' ?

2. What do you mean by the statement that mechanical equivalent of heat is 4·2 joule/cal ?

3. Explain why a gas has two specific heats while solids and liquids have only one specific heat.

4. 'Isothermal change is essentially a very slow process'—Explain.

5. 'Adiabatic change is essentially a very fast process.'—Explain.

6. If a cylinder containing compressed carbon dioxide gas is suddenly opened and a piece of wet cloth is held before it, the issuing gas is deposited in the form of dry ice. Why ?

7. Explain with reasons the following statements :
   (a) Palms get heated when these are rubbed together.
   (b) A bicycle pump gets heated when air is pumped into a tyre.
   (c) When the valve of a bicycle tyre or of a car tyre is opened, the on rushing air becomes colder.
   (d) On hammering a nail, it gets heated.
   (e) There is a difference between the temperature of water at the top and at the bottom of a waterfall.
   (f) Fire sparks are emitted when a scissor or a knife is held against the edge of a rotating sharpener.

[C] Simple Problems.

1. How much work is done in supplying heat necessary to convert 15 gm. of ice at −10°C. into steam at 100°C ? Assume specific heat of ice=0·5 and latent heat of ice=80 cal./gm. and latent heat of steam=536 cal./gm. [*Ans.* 45423 Joule]

2. A steel ball of specific heat 0·12 has its temperature increased by 0·5°C. on falling through a certain height. If $J=4{\cdot}2\times10^7$ erg./cal., calculate the height. [*Ans.* 25·71 m.]

3. A piece of lead of specific heat 0·03 falls from a height of 40 metre. If on reaching the ground 75% of its energy is converted into heat, calculate the rise in temperature.

[*Ans.* 2·33°C.]

4. A lead bullet of mass 200 gm. and specific heat 0·03 moving with a speed of 50 m./sec. is suddenly stopped by a non-conducting material. Assuming that all the heat developed is absorbed by the bullet and no heat is lost by conduction or radiation, calculate the rise in temperature of the bullet. [*Ans.* 9·92°C]

5. The height of a waterfall is 40 m. Calculate the difference between the temperature of water at the top and the bottom of the fall. Given that $J=4{\cdot}2\times10^7$ erg/cal. [*Ans.* 0·093°C.]

6. A locomotive pulls a train at a steady speed for a distance of 3 miles. If the locomotive exerts a total pulling force of 5 ton wt., calculate the total heat generated in the 3 mile run. Given 1 ton=2240 lb. and $J=776$ ft. lb./B. Th. U. [*Ans.* $2{\cdot}29\times10^5$ B. Th. U.]

7. Some quantity of lead shot is placed in a cardboard cylinder of length 50 cm. The shots are then allowed to fall backwards and forwards from one end of the tube to the other 100 times in succession, so that the temperature of them is found to have risen by 3·7°C. Find the mechanical equivalent of heat. (Sp. heat of lead=0·03) [*Ans.* $4{\cdot}41\times10^7$ erg/cal.]

8. A lead bullet strikes a target with a velocity of 225 m./sec. After the impact its temperature rises by 100°C. Calculate the value of J, assuming that half the kinetic energy is utilised to raise the temperature of the bullet and that the specific heat of lead is 0·03.

[*Ans.* $J=4{\cdot}22\times10^7$ erg./cal.]

9. A metal bullet travelling with a speed of 100 metre/sec. is brought to rest after hitting a target. Assuming that 80% of the kinetic energy is converted into heat in the bullet itself, find the rise in temperature of the bullet after impact; given sp. ht. of metal=0·1.

[*Ans.* 9·52°C.]

10. A lead bullet moving horizontally enters a traget and is brought to rest, being just melted by the heat produced. If the initial temperature of the bullet was 17°C. find the speed at which the bullet enters the target. Given specific heat of lead=0·03; melting point of lead =327°C. and latent heat of fusion of lead=5 cal./gm. [*Ans.* $3{\cdot}47\times10^4$ cm./sec.]

11. How many metre can a 70 kg. man climb by using the energy from a slice of bread which produces 10^5 calorie of heat? Assume that human body works at an efficiency of 25%.

[*Ans.* 153·1 m.]

12. Two bodies A and B of masses 200 gm. and 500 gm. respectively are moving towards each other with speeds 120 cm./sec. and 20 cm./sec. respectively. They suffer a head on collision and stick together and the combined mass moves with a velocity 20 cm./sec. Calculate the heat developed in calorie due to this collision. [*Ans.* $3{\cdot}33\times10^{-2}$ cal.]

13. A drill rated 0·56 h.p. is used to bore a hole in a piece of iron of mass 150 gm. Assuming that 70% of the energy heats the iron, calculate the rise in temperature which occurs in 30 sec. Assume that $4{\cdot}2\times10^7$ erg=1 cal., 1 h.p.=746 watt; specific heat of iron=0·11 cal. gm^{-1} deg.$^{-1}$C. (*London*) [*Ans.* 126·6 deg C.]

14. In an experiment for the determination of J; the mass of each weight was 14 kg. They were allowed to fall through 2 metre, 50 times in succession. The water equivalent of the calorimeter and its content was 6 kg., and the rise in temperature was found to be 1·1°C. Find the value of J. [*Ans.* 4·16 Joule/cal.]

15. A car weighing one metric tonne and moving with a speed of 54 km./hr. is brought to rest by the application of brakes equally on its four wheels. The brake bands are of steel of specific heat 0·12 and 75% of heat developed is absorbed by them. Calculate the mass of each brake band if its temperature rises by 40°C. ($J=4{\cdot}2\times10^7$ erg per cal.) [*Ans.* 1·047 kg.]

16. Calculate the change in the internal energy when 10 gm. of air are heated from 0°C. to 1°C. at constant volume. Given, for air $c_v=0{\cdot}17$ cal. gm.$^{-1}$ °C^{-1}. *Ans.* 7·14 Joule]

17. The temperature of 6 gm. of nitrogen gas is increased from 15°C. to 115°C. at constant pressure. Calculate the amount of heat supplied, the increase in internal energy and the external work done by the gas. If the same process is accomplished at constant volume calculate the heat supplied. Given, for nitrogen c_v=0·177 and c_p=0·248 cal. gm. $^{-1}$ °C $^{-1}$.

[*Ans.* 148·8 cal., 106·2 cal., 42·6 cal., 106·2 cal.]

18. Given that 1 litre of air at N.T.P. weighs 1·293 gm. and c_v=0·169 cal. gm. $^{-1}$ °C $^{-1}$ find c_p. Density of mercury=13·6 gm./c.c. at 0°C. $J=4{\cdot}2\times10^7$ erg/cal.

[*Ans.* 0·2373 cal. gm. $^{-1}$ °C. $^{-1}$]

19. The specific heat of argon at constant pressure is 0·12 and the specific heat at constant volume is 0·08. Find the density of argon at N.T.P. Given, $J=4{\cdot}2\times10^7$ erg/cal. and normal atmospheric pressure=$1{\cdot}01\times10^6$ dyne/cm^2. [*Ans.* $2{\cdot}2\times10^{-3}$ gm./c.c.]

20. Some quantity of air at 0°C. is compressed (i) slowly, (ii) suddenly, to $\frac{1}{5}$th of its original volume ; calculate the rise in temperature in each case. Given that γ for air=1·4.

[*Ans.* (i) No rise of temperature ; (ii) 246·7°C.]

21. The tyre of an automobile car is pumped up to a pressure of three atmospheres at 25°C. when it suddenly bursts. Find the resulting drop in temperature of the air. Given that γ for air=1·4. [*Ans.* 80·3°C.]

22. A body of mass 2 kg. is being dragged with a uniform velocity of 2 m./sec. on a rough horizontal plane. The coefficient of friction between the body and the surface is 0·20. Calculate the amount of heat generated in 5 sec. [J=4·2 Joule/cal. and g=9·8 m./sec^2.]

(*I.I.T.* '80] [*Ans.* 9·33 cal.]

23. A lead bullet just melts when stopped by an obstacle. Assuming that 25 per cent of heat is absorbed by the obstacle, find the velocity of the bullet if its initial temperature is 27°C. [M.P. of lead=327°C, specific heat=0·03 cal./gm. °C, latent heat of lead=6 cal./gm., J=4·2 Joule./cal.] (*I.I.T* '*81*) [*Ans.* 410 m./sec.]

24. A gas in compressed from a volume of 20 litre to 10 litre by a constant pressure of 10^6 dynes/cm.^{2} Calculate the heat developed. (*Jt. Entrance* '82) [*Ans.* 238·1 cal.]

[D] Harder Problems.

1. A 10 kg. mass falls through 25 m. on to the ground and bounces to a height of 0·50 m. Assuming that all the potential energy lost is used in heating up the mass, calculate the temperature rise. Given specific heat of the material is 0·06 and the acceleration due to gravity is 980 cm./sec^2. [*Ans.* 0·95°C.]

2. A bullet strikes against a steel armour plate with a velocity of 400 metre/sec. If the bullet falls dead after the impact, find the rise in temperature, assuming that the heat produced is equally shared between it and the target. Given the sp. ht. of the material of the bullet=0·03 and $J=4{\cdot}2\times10^7$ erg./cal. [*Ans.* 317·5°C.]

3. A fire hose with a nozzle diameter of 5·00 cm. delivers a cubic metre of water in 10·0 sec. Assuming all the kinetic energy to be converted into heat and retained in the water, calculate the rise in temperature of the water when the jet strikes a rigid wall. 4·2 Joule=1 cal. (*London*) [*Ans.* 0·31 deg C.]

4. A copper calorimeter of mass 100 gm. contains 1 kg. of an oil which is stirred well by a rotating paddle by applying a torque of 10^8 dyne-cm. If the temperature of the liquid is found to have increased by 5°C. after 220 revolutions. Calculate the mechanical equivalent of heat. Given that the specific heat of copper=0·09 and specific heat of oil=0·64.

[*Ans.* $4{\cdot}26\times10^7$ erg./cal.]

5. A steel drill making 180 revolutions a minute is used to drill a hole in a block of steel. The mass of the steel block and drill is 180 gram. If the entire mechanical work is used up in producing heat and the rate of rise in temperature of the block is 0·5°C. per second, find (a) the rate of working of the drill in watt and (b) the couple required to drive the drill. (sp. ht. of steel=0·10 cal./gm./°C.) (*I.I.T.* 1969)

[*Ans.* (a) 37·8 watt, (b) 2 Newton-metre]

6. A block of ice of mass 3 kg. is dragged over a horizontal sheet of ice for a distance of 50 m. all the ice being at 0°C. Calculate the mass or ice melted, taking the coefficient of friction between the ice surfaces as 0·03 and the latent heat of fusion of ice as 80 cal./gm.

[*Ans.* 0·131 gm.]

7. A petrol engine consumes 20 kg. of petrol per hr. The calorific value of the fuel is 1×10^7 cal. per kg. The power of the engine is 84 kilowatt. Calculate the efficiency of the engine.

[*Ans.* 36%]

8. A steel tool of mass 250 gm. held against the edge of a grindstone of radius 8·4 cm. spinning at 4 rev. per sec., exerts a tangential force of 500 gm. wt. on the gridstone. If 90% of the mechanical work done is converted into heat, which is absorbed uniformily by the tool, find the rise in temperature of the tool at the end of one minute. Take the value of *J* to be $4{\cdot}2\times10^7$ erg/cal. and the sp. ht. of steel to be 0·12.

[*Ans.* 4·43°C.]

9. A vessel containing liquid at its boiling point and suitably jacketed to prevent heat losses, is stirred by a motor driven stirrer using 75 watt. If 250 gm. of the liquid are evaporated in 1 hour, what is the latent heat of vaporisation of the liquid? Given $J=4{\cdot}2$ Joule/cal.

[*Ans.* 257·1 cal./gm.]

10. An engine doing work at the rate of 10 kW uses 3 0 kg. of fuel per hour, the calorific value of the fuel being 12,000 cal. gm.^{-1}. One-quarter of the heat produced in the combustion escapes with the exhaust gases. Calculate (a) the percentage of heat produced in the combustion of the fuel which is converted into work, (b) the initial rate of rise of temperature of the engine, if its total water equivalent is 8·0 kg.

(*Cambridge*)

[*Ans.* (a) 23·81% (b) 0·64 deg C./sec.]

11. Calculate the heat equivalent of the internal and external work done in the process of vaporization when 1 gm. water (1 c.c. water boiled at 100°C and 76 cm. of pressure) becomes 1801 c.c. of steam, assuming latent heat of vaporisation at this temperature being 536 cal./gm.

[*Ans.* 492·59 cal.; 43·41 cal.]

12. One mole of oxygen is heated at constant pressure from 0°C. What quantity of heat must be added to the gas for its volume to be doubled? Specific heat of oxygen under these conditions is 0·2 and mol. wt. of oxygen=32.

[*Ans.* 1748 cal.]

13. A vertical cylinder containing a gas is closed on top by a frictionless piston weighing 25 kg. wt. The area of the bottom of the cylinder is 15 cm^2. The initial volume of the gas in the cylinder is 12 litre at a temperature of 0°C. What quantity of heat is necessary to raise the temperature of the gas by 15 degree under these conditions if it is known that the heat required to raise the temperature of the same mass through 15 degree at constant volume with the piston in its initial position is 60 cal.? Neglect the atmospheric pressure.

[*Ans.* 85·64 cal.]

14. Calculate the number of calorie used in the performance of external work when 1 gm. of water at its normal boiling point is converted into saturated vapour at the same temperature, assuming the densities of water and its saturated vapour at the normal boiling point may be taken as 0·958 and 0·000606 gm. cm.^{-3} respectively.

[*Ans.* 39·77 cal.]

15. The density of hydrogen gas at N.T.P. is $8{\cdot}96\times10^{-5}$ gm./c.c. and its specific heat at constant volume is 2·41 and the ratio of specific heat at constant pressure to that at constant volume is 1·4. Find mechanical equivalent of heat.

[*Ans.* $4{\cdot}296\times10^7$ erg./cal.]

16. The anode of a wireless valve is bombarded by a stream of electrons each of mass 9×10^{-23} gm. moving with a velocity of 10^9 cm./sec. If the mass of the anode is 0·5., its specific heat is 0·1 and 3×10^{12} electrons hit it per second, calculate the rate at which its temperature rises on the assumption that all the energy of the electrons is converted into heat at the anode.

(*Cambridge*) [*Ans.* 64·3°C./sec.]

17. The volume of a certain gas, having a density of 0·00125 gm./c.c. under a pressure of 1 atmosphere and at a temperature of 0°C., is 8 litres. 30 calories of heat are required to raise the temperature of this gas to 15°C at constant pressure. Find the sp. ht. of the gas at constant pressure and also at constant volume. ($R=2$ cal./°C. mole). (*Jt. Entrance '81*)

[*Ans.* 5·6 and 3·6 cal./°C. mole]

18. A glass tube 1 metre long, closed at the ends, contains 25·0 gms of lead shot, the remainder of the tube being filled with 1 litre of water. The tube is held vertically and then inverted, causing the shot to fall the entire length of the tube. How many times the action is to be repeated to produce a rise of temperature of 1°C ? Neglect the heat supplied to the glass. (sp. ht. of lead$=0{\cdot}02$, $J=4{\cdot}2$ joules/cal.) *(Jt. Entrance '81)* [*Ans.* 17,151]

19. A heating coil is embedded in a copper cylinder which also carries a thermocouple. The whole is thermally equivalent to 30 gm. of copper. The cylinder is suspended in liquid nitrogen until the thermocouple reading is constant. The cylinder is taken out and rapidly transferred into a beaker of water at 0°C. A coating of ice is formed on the cylinder and when its temperature in again constant it is taken out of the water and suspended in a space maintained at 0°C. The heating coil is switched on which supplies energy at a steady rate of 25 watts. After 1 min. 19 sec. the whole of the ice has just melted. Find the temperature of the liquid nitrogen. Specific heat of copper$=0{\cdot}08$. [*Ans.* $-195{\cdot}9$°C.]

20. A lead bullet at 27°C strikes a steel plate with a velocity of 700 m./sec. and due to the heat produced by impact, the bullet just melts. Assuming that heat generated has been shared by the bullet and the plate, what percentage of heat has been absorbed by the steel target. Specific heat of lead$=0{\cdot}03$, its melting point$=327$°C., its latent heat of fusion$=5{\cdot}4$ cal./gm. *(Jt. Entrance '84)* [*Ans.* 75·3%]

20

CHAPTER

KINETIC THEORY OF GASES

20·1. Molecular theory of matter.

At the beginning of the last century, the concept of the molecular structure of matter was developed by Dalton in a coherent form. We now know that every substance whether solid, liquid or gas is composed of a very large number of tiny particles, called molecules. The molecules of a substance are all identical with one another and possess all the characteristic properties of the substance.

It has also been established that the molecules exert forces on one another. From a study of the observed differences in the physical behaviours of solids, liquids and gases, the nature of the molecular forces can be surmised.

In solids, the molecules are placed very close to each other. Since a large force is required to pull a solid apart, the inter-molecular forces must be attractive in nature and should be of fairly large magnitude.

In liquids, the distance between the molecules is usually larger than that in solids. Since much smaller forces are required to separate parts of liquids, the forces of attraction between the molecules must be smaller.

In gases, the separation of the molecules is still larger. No external force is required to separate parts of gases, so that a gas automatically fills up the entire space available to it. This shows that inter-molecular attraction in a gas may be considered to be negligible.

The above facts indicate that in general, *with increase in the separation, the force of attraction between the molecules decreases rapidly.*

20·2. Heat as motion of molecules.

In the last chapter, we saw that heat is a form of energy. The question that naturally arises is—energy of what ? The answer to this question forms the basis of the kinetic theory of matter. According to this theory, the molecules of a substance are in a state of incessant motion. They are always vibrating or moving and this motion is correlated with the heat energy. When a substance is heated, the molecules move faster so that an increase in the kinetic energy of the molecules takes place. Heat is, therefore, identified with the motion of the molecules.

In solids, the inter-molecular attraction is so large that the molecules cannot move freely but remain in definite positions. As a result, the molecules can execute only vibratory motion about their mean positions. Solids, therefore, possess definite shape and volume.

In the case of liquids, attractive forces between molecules are relatively weak. The molecules of a liquid, therefore, vibrate and also move randomly through the liquid. Liquids can, therefore, change their shape and flow from one place to another.

In gases, the attractive force between molecules is almost negligible. The gas molecules, therefore, move freely in a random manner in every possible direction over the entire space of the containing vessel. In doing so, they collide with each other and also with the walls of the container. So a gas does not possess either a definite shape or a definite volume.

We shall devote ourselves mainly to the discussion of the kinetic theory of gases. The kinetic theory of solids and liquids is much more complex and a detailed discussion is beyond our scope. The kinetic theory of gases was fully developed in the middle of the 19th century. The theory was placed on a firm mathematical basis by such celebrated scientists like Bernoulli, Clausius, Maxwell, Boltzmann, Jeans, Van der Waals and many others.

20·3. Evidences in favour of the kinetic theory.

The motion of the molecules cannot be directly seen as the molecules are too small to be visible. Various important evidences are, however, available in favour of the kinetic theory.

(1) *Diffusion* : If a cylinder filled with hydrogen is inverted over another containing a heavier gas, say carbon dioxide, it is found that the two gases get mixed up against gravity. With lapse of time, a uniform mixture is obtained. This can be easily explained by the kinetic theory. As the molecules of the two gases move randomly in all directions over the entire space of the container, they intermingle with one another and soon a uniform mixture is produced. This phenomenon in which a substance mixes with another because of molecular motion is called diffusion. Liquids and solids are also found to diffuse. For example, if a few copper sulphate crystals are dropped in a beaker of water, the water at the bottom will first become coloured with dissolution of the crystals. Though the copper sulphate solution is heavier than water, it will be observed that it slowly diffuses upwards in spite of gravity. After the lapse of sufficient time, all the water becomes coloured. Even the diffusion of one solid into another has been observed. Diffusion of gold into lead has been demonstrated by Rebert Austen.

(2) *Evaporation and vapour pressure* : Simple explanations of these phenomena are provided by the kinetic theory (vide Art. 20·5).

(3) *Agreement with results obtained from other branches of Physics* : It is possible to deduce the gas laws viz. Charle's law and Boyle's law, from the kinetic theory. Expressions for the specific heat, Avogadro's number, molecular diameter etc. are also obtained from this theory. Values of the coefficient of viscosity and the conductivity of a gas are also derivable from this theory. All these results agree quite well with those obtained from experimental investigations or obtained theoretically from other branches of Physics.

(4) *Expansibility of gases :* A gas completely fills up a container of any shape and volume. This property clearly indicates that gas molecules are free to move randomly over the entire space available. This also explains the fact that a gas can be compressed easily. Compression of a gas is simply equivalent to lessening the space available to the moving molecules.

(5) *Brownian motion :* This phenomenon was first observed by the British botanist Robert Brown in 1827. It provides one of the most direct piece of visual evidence of molecular motion. We shall discuss it in detail in the next article.

20·4. Brownian motion.

Robert Brown discovered that when fine pollen grains suspended in water are viewed under a high power microscope, they are seen to be dancing wildly, as if they are alive. They are, however, found to execute the same kind of motion even after they had been boiled. Later it was found that the phenomenon is a general one. Any kind of fine particles suspended in both liquids and gases exhibit such motion. The particles move continually in a completely random manner describing a succession of short displacments rapidly from one way to other. Such motions are called *Brownian motions,* after their discoverer. The path of a single particle is very much like that shown in the Fig. 20·1. In water or air, Brownian motion is exhibited by those suspended particles whose diameters are of the order of 10^{-3} to 10^{-5} cm.

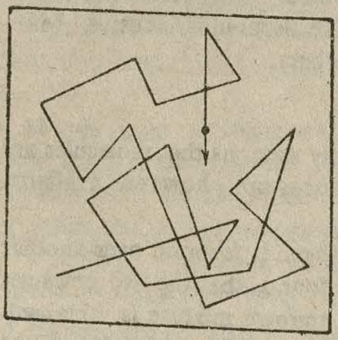


Fig. 20·1 : Brownian motioo

The *characteristics of Brownian motion are :—*

(i) This motion is spontaneous, perpetual and completely random. Even in the same locality, no two particles exhibit the same kind of motion.

(ii) The motion does not depend upon the chemical nature of the suspended particles. At the same temperature, different particles of identical sizes are found to be equally agitated.

(iii) With rise in temparature, the motion becomes more vigorous.

(iv) With particles of smaller dimensions, the motion becomes more rapid.

(v) More vigorous motion is witnessed when the particles are suspended in a fluid of smaller viscosity. Hence the motion is most pronounced in the case of gases.

(vi) The motion is independent of the state of motion of the vessel containing the fluid.

The *cause of the Brownian motion* cannot be attributed to any convection or eddy current in the fluid, because the motion is completely random in nature. The correct explanation is provided by the kinetic theory. The motion of a suspended particle is due to the bombardment from different sides by the molecules of the surrounding gas or liquid, which themselves are in continual motion. Very small particles are essential. If the particle is fairly large, the forces due to the molecular impart will cancel out and there will be no average resultant force on the particle. Which therefore remains at rest. For example, a piece of paper or a watch-stick floating in water does not exhibit Brownian motion. Only if the particle is very small, there is little chance that the effects of these forces will exactly cancel out at any instant. Since the impacts are quite random, the resulting motion of the particle is completely erratic. Obviously smaller particles will exhibit more vigorous motion. With rise in temperature, the intensity of the thermal motion of the molecules increases. Hence the magnitude of the unbalanced force acting on the particles becomes greater producing in consequence greater agitation in them.

After about three quarters of a century of the discovery of the effect, a theoritical explanation was given by Einstein on the basis of the kinetic theory. He showed that the average of the squares of the displacements of a Brownian particle should be proportional to the elapsed time. This was confirmed experitally by Perrin, R.A. Millikan and others. The experiments led to a determination of Avogadro's number according to Einstein's theoretical formula. The value, so obtained, was found to be in close agreement with the results obtained, by other observations of the most diverse kinds. Brownian motion thus furnishes the most direct and most convincing evidence of the validity of the kinetic theory.

Demonstration Experiment : A little amount of smoke is drawn into a small glass cell *G* [Fig. 20·2]. The small amount of smoke can be conveniently obtained from a burning cigarette. The glass cell is illuminated strongly from one side by converging the light from a bright source *S* with a lens *L*. The illuminated smoke particles are viewed from above with a high power microscope *M* through the window *W*. The tiny smoke particles are seen as bright specks of light dancing to and fro in a completely random manner. Brownian motion in a liquid can be similarly demonstrated by observing the small particles in colloidal solution.

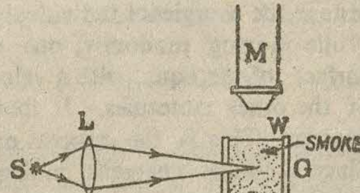


Fig. 20·2 : Demonstration of Brownian motion

20·5. Explanation of different thermal phenomena.

Different thermal phenomena are explained in an elegant manner by the kinetic theory. Some of these are discussed below :

(i) *Thermal expansion* ; When a solid is heated, the kinetic energy of the constituent molecules increases so that they now vibrate with a greater amplitude. Hence larger space is required to accommodate these increased amplitudes of vibration of the molecules. The molecules, therefore, move apart and an expansion in the linear dimensions of the solid is said to have taken place.

A liquid possesses no definite shape so that only an expansion in volume takes place due to the same cause.

(ii) *Change of state ; Latent heat* : We have already mentioned that the molecules in a solid are arranged in definite positions and execute vibration about these mean positions. With rise in temperature, the amplitude of the vibration increases. This goes on until the melting point is reached, when the vibrations become so voilent that the attractive forces are no longer able to bound the molecules to fixed positions. The ordered arrangement is then destroyed and the molecules become comparatively free to move, giving rise to the liquid. The latent heat of fusion represents the energy that must be supplied to perform the necessary work to free the molecules in this way. Therefore, during the change of state, the heat energy supplied is spent not in increasing the K.E. of the molecules, but in breaking the bonds of intermolecular attractive force i.e., in increasing the P.E. of the molecules. Hence, no rise in temperature occurs during such change.

Similarly, the latent heat of vaporization can be explained. In a gas, the molecules are much farther apart than in a liquid, and hence energy must be supplied to bring about such increased separation. Since the change of state from liquid to gas is always accompanied by an increase in volume, a part of the latent heat is spent in performing the necessary external work.

(iii) *Evaporation ; Vapour pressure* : In a liquid, the molecules are bound by attractive forces within a fixed volume and move randomly inside the liquid. At any particular temperature, the *average* K.E. of the molecules remains constant, but this does not mean that all the molecules have the same K.E. At any instant, the energies of individual molecules are distributed about this mean value. While moving randomly, one of the faster moving molecules may reach the surface of the liquid with a velocity sufficient to overcome the attractive forces of the other molecules. It then escapes from the liquid and passes into the vapour. This is the process of evaporation which takes place continuously. Since the most energetic molecules escape from the liquid in this way, the average energy of the remaining molecules in the liquid is reduced i.e., a lowering of temperature of the liquid is produced. Evaporation, therefore, always gives rise to cooling.

When a liquid is confined in a closed vessel, the molecules leaving the surface gather in the space above the liquid. They move randomly in all directions and this process collide with one another and bombard the walls of the vessel. Some of them also strike the surface of the liquid, when they again get absorbed. Bombardment of the walls of the vessel produces a pressure on these walls, which is called the vapour pressure. If the number of molecules escaping from the liquid in a given time is greater than the number returning to the liquid, a net loss of the liquid or evaporation takes place. The vapour is said to be unsaturated, the corresponding pressure being called the unsaturated vapour pressure.

As time goes on, the number of molecules in the closed space increases and correspondingly the number returning to the liquid in a given time also increases. Ultimately a condition is reached, when as many molecules return to the liquid in a given time as escape from it in the same interval. This is called a state of dynamic equilibrium. The number of vapour molecules in the closed space then attains the maximum value under the given conditions of volume and temperature and remains steady. The vapour is then said to be saturated. The pressure exerted by it is called saturation vapour pressure, which is obviously the maximum possible pressure at the particular temperature since the vapour then possesses the maximum possible density at that temperature. Increase in the temperature results in an increase of the speed of the molecules inside the liquid. Hence in a given time, more molecules are now able to escape from the liquid so that when dynamic equilibrium is again established, greater number of moleclues are present in the space above the liquid. So an increase in the density of the vapour molecules takes place, producing a corresponding increase in the saturation vapour pressure.

If the volume of the space above the liquid decreases (or increases), then the rate at which the vapour molecules enter the liquid increases (or decreases). If the temperature remains constant, then the rate of escape of the molecules from the liquid remains the same. It obviously means that some vapour is condensed (or some liquid is evaporated) till the dynamical equilibrium is again reached. So the density of the vapour and hence the saturation vapour pressure remains unchanged by such change in volume.

20·6. Basic assumption of the kinetic theory of ideal gases.

In order to develop the kinetic theory of gases on a quantitative basis, the following simple assumptions are made :

(i) A gas consists of a very large number of molecules which are exactly alike in all respects.

(ii) The molecules are continually in motion and this motion is completely random i.e., they are moving in all possible directions with all possible velocities ranging from zero to infinity.

(iii) In such motion, the molecules collide with one another and with the walls of the container. At each collision their velocities are altered both in magnitude and direction. But in the steady state, the molecular density of the gas is not affected by these collisions, the number of molecules in any small element of volume remains the same regardless of the position of the element.

(iv) The collisions are assumed to be perfectly elastic i.e., both momentum and kinetic energy are conserved in such collisions.

(v) Between two consecutive collisions a molecule travels in a straight line with uniform velocity according to Newton's first law of motion. The distance a molecule moves between two successive collisions is known as its free path.

(vi) The duration through which a collision lasts is negligible compared with the interval between two collisions.

(vii) Except when they collide, the molecules do not exert any force of attraction or repulsion on one another or on the walls of the container.

(viii) The actual volume occupied by the molecules is negligible compared with the volume occupied by the gas i.e., we can regard the molecules as mere mass-points.

A gas for which all the above assumptions hold ture is called a *perfect gas* or an *ideal gas*. In reality, no such gas exists for which the above assumptions are strictly valid. Hence the behaviour of a real gas deviates from that of an ideal gas.

20·7. Mean velocity, Mean square velocity and R.M.S. velocity.

Let us consider an enclosed volume of a gas containing a number of molecules, say N. If the velocities of the molecules be $c_1, c_2 \cdots c_N$ respectively, then the **mean velocity** c is given by the relation

$$\bar{c} \quad \frac{c_1+c_2+\cdots+c_N}{N} \qquad \cdots \qquad \cdots \qquad (20{\cdot}1)$$

The **mean square velocity** *is defined as the mean of the square of all the molecular velocities.* Thus mean square velocity

$$\overline{c^2}=\frac{c_1^2+c_2^2+\cdots+c_N^2}{N} \qquad \cdots \qquad \cdots \qquad (20{\cdot}2)$$

The square root of the mean square velocity is known as the **root mean square** *or* **R.M.S. velocity.** Thus if C denotes the R. M. S. velocity, then

$$C=\sqrt{\overline{c^2}}=\sqrt{\frac{c_1^2+c_2^2+\cdots+c_N^2}{N}} \qquad \cdots \qquad \cdots \qquad (20{\cdot}3)$$

$$\text{or,} \quad C^2=\overline{c^2}=\frac{c_1^2+c_2^2+\ldots c_N^2}{N} \qquad \cdots \qquad \cdots \qquad (20{\cdot}4)$$

So the *square of the R. M. S. velocity equals the average of the squares of the velocities of the molecules.*

The R.M.S. velocity is not identical with the mean velocity. For example, suppose, we have only 3 molecules with velocities of 20, 26 and 50 m./sec. respectively. Then the mean velocity for the molecules of this "gas" is $c=\frac{20+26+50}{3}$ $=32$ m./sec., whereas the R.M.S. velocity is $C=\sqrt{\frac{20^2+26^2+50^2}{3}}=34{\cdot}5$ m./sec. In general, the mean velocity is somewhat less than the R.M.S. velocity is of practical importance, not the mean velocity.

20·8. Pressure of an ideal gas.

On the basis of the foregoing assumptions, Joule, in 1848, obtained an expression for the pressure of an ideal gas in terms of the number, velocities and mass of the molecules. In course of their incessant random motions, the molecules of a gas collide with the walls of the container (Fig. 20·3), These continuous blows on the walls give rise to a force tending to push the walls outwards, which is felt as the pressure exerted by the gas. As a molecule collides with the wall then since the collision is elastic, it rebounds with the same velocity. Hence, a change in its momentum takes place. This change in the momentum of a molecule per collision is multiplied by the rate at which the molecules strike the wall, whence the total rate of change of momentum for all the molecules is obtained. From Newton's second law, this gives the force exerted by the wall on the gas molecules. From Newton's third law, the gas molecules exert the same force on the wall, which when divided by the area of the wall gives the pressure exerted by the gas. Calculations carried out on this basis yield the following expression for the pressure p of an ideal gas

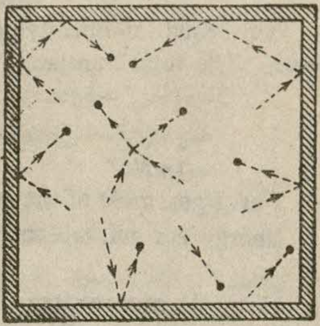


Fig. 20·3 Explanation of pressure of gas

$$p=\tfrac{1}{3}mn\overline{c^2} \qquad \ldots \qquad \ldots \qquad (20{\cdot}5)$$

where m is the mass of a molecule, n is the number of the molecules per unit volume and $\overline{c^2}$ is the mean square velocity. Now mn is the mass per unit volume and is therefore, equal to the density ρ of the gas. If C be the R.M.S. velocity, we get

$$p=\tfrac{1}{3}mn\overline{c^2}=\tfrac{1}{3}\rho C^2 \qquad \ldots \qquad \ldots \qquad (20{\cdot}6)$$

If N be the total number of molecules in a gas of volume V and mass M, then $\rho=M/V=mN/V$. Therefore,

$$p=\tfrac{1}{3}\frac{M}{V}C^2=\tfrac{1}{3}\frac{mN}{V}C^2 \qquad \ldots \qquad \ldots \qquad (20{\cdot}7)$$

Rearranging the eqn. (20·6), we get

$$C=\sqrt{\frac{3p}{\rho}} \qquad \ldots \qquad \ldots \qquad (20{\cdot}8)$$

This equation enables us to calculate the R.M.S. velocity of the molecules of the gas from the directly measurable values of the pressure and the density of the gas.

20·9. Interpretation of temperature from kinetic theory.

So far in our discussions, we have not considered the change in temperature of the gas. To introduce the temperature into the kinetic theory, we make use of the equation of state of an ideal gas obtained experimentally viz., $pV=RT$, where R is the universal gas constant. For simplicity, we consider 1 mole of the gas so that in eqn. (20·7) M becomes the molecular weight of gas and N is the number of molecules in a gram-molecule i. e., the Avogadro number. Hence we have,

$$\tfrac{1}{3}mNC^2=RT \qquad \dots \qquad \dots \qquad (20{\cdot}9)$$

This equation directly provides the kinetic theory interpretation of temperature. The total translation K. E. of the molecules of the gas is given by,

$$\begin{aligned} E&=\tfrac{1}{2}mc_1^2+\tfrac{1}{2}mc_2^2+\dots\dots+\tfrac{1}{2}mc_N^2 \\ &=\tfrac{1}{2}m(c_1^2+c_2^2+\dots\dots+c_N^2)=\tfrac{1}{2}mN\overline{c^2} \\ &=\tfrac{1}{2}mNC^2 \qquad \dots \qquad \dots \qquad (20{\cdot}10) \end{aligned}$$

For 1 gm. mole of the gas, we can make use of eqn (20·9) and get

Energy per gm. mol. of a gas, $E=\frac{3}{2}\cdot\frac{1}{3}\,mNC^2$

$$=\tfrac{3}{2}\,RT \qquad \dots \qquad \dots \qquad (20{\cdot}11)$$

Now, the average translational K.E. per molecule of the gas is

$$\bar{E}=\frac{E}{N}=\tfrac{1}{2}mC^2=\tfrac{3}{2}\,\frac{RT}{N}=\tfrac{3}{2}\;kT \qquad \dots \qquad \dots \qquad (20{\cdot}12)$$

where $k=R/N$ is a universal constant, known as the Boltzmann's constant. This equation shows that *the average translational K.E. of a molecule in an ideal gas is directly proportional to the absolute temperature and is independent of the kind of gas.*

This gives the meaning of temperature in the kinetic theory, viz. that *temperature is a measure of the average molecular K. E. of translation.* This equation also confirms the concept of heat as a form of energy.

It also follows from this equation that at $T=0$, $\bar{E}=0$; hence at the absolute zero of temperature, the molecules of an ideal gas do not possess any K.E. of translation. In other words, all molecular motion ceases at the absolute zero of temperature. This, however, is an ideal picture not exactly true in practice since all substances cease to be in the gaseous state before the absolute zero of temperature is reached.

An expression for the R.M.S. velocity can be obtained from the eqn. (20·12) We have,

$$\bar{E}=\tfrac{1}{2}mC^2=\tfrac{3}{2}kT$$

$$\therefore\quad C^2=\frac{3kT}{m}=\frac{2RT}{M}$$

$$\text{or,}\quad C=\sqrt{\frac{3RT}{M}} \qquad \dots \qquad \dots \qquad (20{\cdot}13)$$

$$\therefore\quad C\propto\sqrt{T}$$

Lastly, we note that from eqns. (20·7) and (20·10), we get

$$pV=\tfrac{1}{3}mNC^2=\tfrac{2}{3}\times\tfrac{1}{2}mNC^2=\tfrac{2}{3}E \qquad \dots \qquad \dots \qquad (20{\cdot}14)$$

Hence *the pressure exerted by unit volume of an ideal gas is equal to two-third of the total K.E. of the molecules of the gas.*

Example 20·1. *A 500 c.c. flask contains $5{\cdot}5\times10^{24}$ molecules of hydrogen each of mass $3{\cdot}32\times10^{-24}$ gm. and having R.M.S. velocity 3×10^4 cm./sec. Calculate the pressure in the flask and express it in atmosphere.*

Solution : Here $V=500$ c.c., $m=3\cdot32\times10^{-24}$ gm., $N=5\cdot5\times10^{24}$ $C=3\times10^{4}$ cm./sec. Hence from eqn. (20·7),

$$p=\tfrac{1}{3}\frac{mN}{V}C^2=\frac{3\cdot32\times10^{-24}\times5\cdot5\times10^{24}\times9\times10^{8}}{3\times500}=1\cdot096\times10^{7}\text{ dyne/cm.}^2$$

Since one atmospheric pressure$=1\cdot013\times10^{6}$ dyne/cm.^{2}

$$\therefore\quad p=\frac{1\cdot096\times10^{7}}{1\cdot013\times10^{6}}=10\cdot8\text{ atmosphere.}$$

Example 20·2. *Calculate the R.M.S. velocity of oxygen at N.T.P. (one litre of oxygen weighs 1·429 gm. at N.T.P.) and also at 1365°C.*

Solution : Hence the pressure $p=76\times13\cdot6\times981$ dyne/cm.^{2} ; density $\rho=1\cdot429\times10^{-3}$ gm./c.c. Hence from eqn. (20·8), for oxygen the R.M.S. velocity is given by,

$$C=\sqrt{\frac{3p}{\rho}}=\sqrt{\frac{3\times76\times13\cdot6\times981}{0\cdot001429}}=4\cdot613\times10^{4}\text{ cm./sec.}$$

Let the R.M.S. velocity be C' when the temperature is 1365°C. i.e., $(1365+273)=1638°\text{K}=T'$ (say).

From eqn. (20·13), we have, $C\propto\sqrt{T}$

$$\text{or,}\quad \frac{C'}{C}=\sqrt{\frac{T'}{273}}=\sqrt{\frac{1638}{273}}=\sqrt{6}$$

$$\therefore\quad C'=\sqrt{6}\times4\cdot613\times10^{4}=1\cdot13\times10^{5}\text{ cm./sec.}$$

Example 20·3. *At what temperature is the R.M.S. velocity of hydrogen molecules equal to that of oxygen molecules at 47°C ? An oxygen molecule is about 16 times as massive as a hydrogen molecule.*

Solution : Let the required temperature be T°K. Then from eqn. (20·13), we have

$RT=\tfrac{1}{3}MC^2$ and $R\,(273+47)=\tfrac{1}{3}\,16MC^2$.

$$\text{Dividing, we get}\quad \frac{T}{320}=\tfrac{1}{16},\quad T=20°\text{K}$$

Example 20·4. *If the average distance between collisions of nitrogen molecules at 1 atm. pressure and 0°C. is $9\cdot44\times10^{-6}$ cm., what is the time between collisions ? Given $R=8\cdot31\times10^{7}$ erg./°C./mole.*

Solution : Molecular weight of nitrogen$=28$ gm./mol. So from eqn. (20·13), the R.M.S velocity at 0°C. is given by

$$C=\sqrt{\frac{3RT}{M}}=\sqrt{\frac{3\times8\cdot31\times10^{7}\times273}{28}}=4\cdot93\times10^{4}\text{ cm./sec.}$$

$$\therefore\quad\text{Time between collisions}=\frac{\text{average distance}}{\text{R.M.S. velocity}}=\frac{9\cdot44\times10^{-6}}{4\cdot93\times10^{4}}=1\cdot9\times10^{-10}\text{ sec.}$$

Example 20·5. *What is the K.E. of translation of the molecules in 20 gm. of CO_2 gas at 27°C. ? Given R=8·31 Joule/°C./mole.*

Solution : Molecular weight of $CO_2=44$ gm./mol. Hence the number of mole present $=\frac{20}{44}=x$ (say). From eqn. (20·12), the K.E. of translation of the molecules in 1 mole of a gas $=\bar{E}N=\frac{3}{2}RT$

Hence for x moles, the K.E. of translation of the molecules

$$\frac{3}{2}xRT=\frac{3\times 20\times 8{\cdot}31\times(273+27)}{2\times 44}=1699 \text{ Joule.}$$

20·10. Deduction of gas laws.

(1) *Boyle's law* : From eqn. (20·7), we have $pV=\frac{1}{3}\ MC^2$. If temperature is maintained constant, the R.M.S. velocity C remains constant. We, therefore, get that pV is constant for any fixed mass M of the gas at constant temperature, which is Boyle's law.

(2) *Avogadro's hypothesis* : This hypothesis states that *at the same temperature and pressure, equal volumes of all gases contain the same number of molecules.*

From eqn. (20·7), we have for equal volumes of two different gases 1 and 2 at the same temperature and pressure.

$$pV=\tfrac{1}{3}m_1N_1C_1^2=\tfrac{1}{3}m_2N_2C_2^2$$

Now, since the average K.E. of translation of a gas molecule depends only on temperature, the average molecular K.E. of translation for the two gases must be the same because both are at the same temperature. Hence, $\frac{1}{2}m_1C_1^2 =\frac{1}{2}m_2C_2^2$.

So, we obtain $N_1=N_2$, i.e. the number of molecules in both the gases are the same.

(3) *Dalton's law of partial pressure* : The law states that the pressure exerted by a mixture of gases is equal to the sum of the pressures exerted by each of the constituent gases if it alone occupied same volume at the same temperature.

Let a number of gases having densites ρ_1, ρ_2, ρ_3......and R.M.S velocities C_1, C_2, C_3......etc. be mixed together in the same volume. From the assumption that the molecules move quite independently of one another, the resultant pressure p will be due to all these different types of molecules. Hence

$$p=\tfrac{1}{3}\rho_1C_1^2+\tfrac{1}{3}\rho_2C_2^2+\tfrac{1}{3}\rho_3C_3^2+.\ ...$$
$$=p_1+p_2+p_3+......$$

where $p_1, p_2, p_3\cdots$ are the pressure exerted by each gas if it alone occupied the same volume. Hence Dalton's law is proved.

20·11. Limitations of ideal gas laws.

The equation of state $pV=RT$ holds accurately only for an ideal gas. We have seen that this equation is obtained from the kinetic theory with a set of ideal assumptions. Boyle himself found that the law is obeyed by real gases only at high temperature and low pressure. Even under ordinary conditions, deviations of behaviour of real gases from this law were noticed. The deviations become more pronounced at low temperature and high pressure.

In developing the kinetic theory, we assumed that the volumes of the molecules are negligible and that they do not exert any force on each other. Deviations in the behaviour of real gases from the ideal gas equation are traced to the inaccuracy of these assumptions. Obviously a molecule possesses finite size, however small it might be. At high pressures when the gas is compressed to a very small volume, the volume of the molecules can not be entirely neglected. Hence to get the effective volume, a correction term b which is related to the total volume of the gas molecules must be subtracted from the measured volume V. Hence in place of V in the ideal gas equation $pV=RT$, we must write $(V-b)$.

The molecules also exert attractive forces on each other. When a molecule approaches the wall of the container, it is attracted from inside by the molecules in its neighbourhood. This is similar to the phenomenon of surface tension in liquids. Hence the pressure p experienced by the wall is less than the pressure that would be exerted in the absence of inter-molecular attraction. At low temperatures when the velocities of the molecules become very small, these inter-molecule attractive forces play a prominent role leading to the deviation from the ideal gas equation $pV=RT$. It is obvious now that to account for this factor, a correction term must be added to the observed value of pressure p. This correction term was shown by Van der Waals to be equal to a/V^2, where a is a constant for a given mass of gas.

Hence for real gases, the equation of state takes the form

$$\left(p+\frac{a}{V^2}\right)\left(V-b\right)=RT$$

This is known as Van der Waal's equation.

● EXERCISE ●

[A] Essay type questions.

1. How is the motion of the molecules in a gas different from that of the molecules in a liquid and in a solid ? Mention very briefly the experimental evidences, if any, in support of the kinetic theory.

2. Give a short account of the kinetic theory of matter, indicating how this theory explains (i) Thermal expansion, (ii) Change of state and (iii) Evaporation and vapour pressure.

3. State the basic assumptions of the kinetic theory of ideal gases. Explain what are meant by mean velocity and root-mean square velocity. Are the two velocities identical ? Which one is more important in kinetic theory ?

4. (a) When heat is supplied to a body what physical process goes on inside it ? [*Jt. Entrance '79*]

(b) Account for the diffusion phenomenon from the kinetic theory of gases.

5. What do you mean by Brownian motion ? What are its characteristics ? How do you account for such motion ?

Describe how can you demonstate the phenomenon.

6. How the pressure exerted by a gas on the wall of its container is explained from kinetic theory ? Write down an expression for it in terms of the density and r.m.s. velocity of molecules of the gas.

Prove that for a gas of molecular weight M and temperature T°K, the R.M.S. velocity of its molecules is

$$C=\sqrt{\frac{3RT}{M}}.$$

7. How can Boyle's Law, Avogadro's hypothesis and Dalton's law of partial pressure be accounted for by the kinetic theory of gas ?

What is the interpretation of temperature on this theory ?

8. State and explain the limitations of the ideal gas equation $PV=RT$. Write down the equation which holds for a real gas.

Explain how this equation is established.

9. (i) Prove that the average kinetic energy of a molecule is the same for all gases at the same temperature.

(ii) Prove that the pressure exerted by unit volume of an ideal gas is equal to two-third of the total kinetic energy of the molecules of the gas.

[B] Short answer type questions.

1. What is an ideal gas ? What is the difference between an ideal gas and a real gas ?

2. 'At same temperature molecules of different gases have the same R. M. S. velocity'. Is it true ? Justify your answer. [*I. I. T.* '*81*]

3. How does the R. M. S. velocity of the molecules of an ideal gas change, when (i) temperature is increased, and (ii) density is increased.

4. In Brownian motion, do we observe the motion of the molecules of a liquid or a gas or of some other particles ?

[C] Simple Problems.

1. Find the increase in average translational kinetic energy per molecule of a gas for 5°C. rise in temperature. Given that $R=8\cdot31$ Joule/°C/mole and $N=6\cdot023\times10^{23}$.

[*Ans.* $1\cdot035\times10^{-22}$ Joule]

2. Find the R.M.S. velocity of molecules of CO_2 at a pressure of 76 cm. of Hg. Given that the density of mercury $=13\cdot56$ gm./c.c., $g=981$ cm./sec.^{2} and density of $CO_2=1\cdot997\times10^{-3}$ gm./c.c.

[*Ans.* $3\cdot896\times10^{4}$ cm./sec.]

3. A gas is kept at 0°C. If one has to double the R.M.S. speed of the molecules, to what temperature in degree centigrade must the gas be raised ? [*Ans.* 819°C.]

4. At what temperature is the R.M.S. velocity of hydrogen molecules equal to that of carbon dioxide molecules as 57°C. A carbon dioxide molecule is about 22 times heavier than a hydrogen molecule.

[*Ans.* 15°K]

5., Nitrogen gas of molecular weight 28 gm./mole is at 1 atmos. pressure at 0°C. Find the average distance between collisions of nitrogen molecules if time between collisions is only 2×10^{-10} sec. Given $R=8\cdot31\times10^{7}$ erg/°C/mole.

[*Ans.* $9\cdot86\times10^{-6}$ cm.]

6. Calculate the R.M.S. velocity of oxygen at N.T.P. and also at 1400°C. Assume one litre of oxygen weighs $1\cdot429$ gm. at N.T.P. [*Ans.* $4\cdot613\times10^{4}$ cm./sec., $1\cdot142\times10^{5}$ cm./sec.]

7. An one litre flask contains $2\cdot6\times10^{23}$ molecules of nitrogen each of mass $4\cdot65\times10^{-23}$ gm. and having R. M. S. velocity 5×10^{4} cm./sec. Calculate the pressure in the flask and express it in atmosphere.

[*Ans.* $9\cdot946$ atmos.]

8. What is the kinetic energy of translation of the molecules in 15 gm. of ammonia gas at 37°C ? Given molecular weight of ammonia is $17\cdot03$ gm./mole. and $R=8\cdot31\times10^{7}$ erg/mole/°C.

[*Ans.* 3403 Joule]

9. Calculate the total molecular kinetic energy of a mole of a gas at N.T.P.

[*Ans.* 3407 Joule]

10. The R.M.S. velocity of helium molecules under normal pressure is $1\cdot33\times10^{3}$ m./sec. Calculate the density of helium under such condition. [*Ans.* $1\cdot718\times10^{-4}$ gm./c. c.]

11. Calculate the density of an ideal gas at S.T.P. if the R.M.S. velocity of the molecules be 0·5 km./sec. What will be density of the gas at 21°C, the pressure remaining the same ? (Assume 1 standard atmosphere$=10^5$ N/m^2). (*Jt. Entrance* '*83*)

[*Ans.* 1·2 kg./m^3 ; 1·114 kg./m^3.]

12. The r.m.s speed of hydrogen molecules is $2{\cdot}0\times10^5$ cm./sec. at a certain temperature. Calculate the r.m.s. speed of oxygen molecules at the same temperature. (Their molecular weights are 2 and 32 respectively). [*Ans.* 5×10^4 cm./sec.]

13. Smoke particles in the air have masses of the order of 10^{-16} kg. The Brownian motion of these particles resulting from collisions with air molecules can be observed with a microscope. Find the r.m.s. speed of Brownian motion for such a particle in air at 27°C. (Boltzmann's constant $k=1{\cdot}38\times10^{-16}$ erg/molecule °K). [*Ans.* 1·11 cm./sec.]

[D] Harder Problems

1. Methane gas of molecular weight 16 is at 20°C. and 5 atmos. Calculate its density, and hence find the R.M.S. velocity of the gas. Given $R=8{\cdot}31\times10^7$ erg./°C/mole. and 1 atmos.$=$ $1{\cdot}013\times10^6$ dyne/cm^2. [*Ans.* 3·328 gm./lt., $6{\cdot}757\times10^4$ cm./sec.]

2. Calculate the number of molecules in 1 c. c. of a perfect gas at 27°C and at a pressure of 76 cm. of Hg. Given that the mean kinetic energy of a molecule at 27°C$=4\times10^{-4}$ erg. and density of mercury$=$13·6 gm./c.c. [*Ans.* $3{\cdot}798\times10^9$]

3. At 18°C. and 765 mm. of mercury, 1·29 litre of an ideal gas weighs 2·71 gm. Compute the molecular weight of the gas. Hence, find the R.M.S. velocity of the gas, assuming $R=8{\cdot}31\times10^7$ erg./°C/mole. [*Ans.* 49·83 gm./mole. ; $3{\cdot}816\times10^4$ cm./sec.]

4. Calculate the R.M.S. velocity of a molecule of hydrogen at 30°C. and 70 cm. of mercury pressure. Molecular weight of hydrogen is 2·016 gm./mole. [*Ans.* 1936 m./sec.]

5. Two vessels contain hydrogen and oxygen at the same temperature. If the R.M.S. velocity of oxygen molecules be 400 m./sec., calculate that of hydrogen molecules. Given that molecular weights of hydrogen and oxygen are 2 and 32 respectively. [*Ans.* 1600 m./sec.]

6. One gm. mole of helium at 60°C is mixed with one gm. mole of argon at 30°C. What is the temperature of the mixture ? [*Ans.* 45°C]

21 CHAPTER

TRANSMISSION OF HEAT

21·1. Method of heat propagation.

We know that temperature is that thermal state of a body which determines the direction of flow of heat. Whenever there exists a difference in temperature between two bodies or between adjoining portions in a single body, heat flows from one at higher temperature to the other at lower temperature, till the said difference is equalised. This transference of heat is effected by one or more of three distinct methods, known as *conduction*, *convection* and *radiation*. In the subsequent sections we shall confine ourselves to an introductory discussion of these three fundamental methods of heat propagation.

21·2. Thermal conduction.

If one end of a metal rod is held by hand and its other end is heated, it is our experience that the part of the rod one is holding becomes hotter and hotter, though this part has not been directly heated. By some means heat has passed from the hotter to the colder end. It is obivous that during this process, no transference of the particles of the rod from the hotter end to the colder end takes place. Such *transfer of heat through a substance without any detectable motion of the particles of the substance is called* **conduction**.

A simple explanation of the phenomenon can be put forward from the kinetic theory of matter. When a particular region of a substance is heated, the molecules in that region vibrate with increased violence without changing their mean positions. As they collide with the more slowly vibrating molecules in the adjacent regions, some of their energy of motion is passed on to these neighbours, which, therefore, begin to vibrate more vigorously about their mean positions. These molecules in turn excite greater vibrations in their next neighbours. In this way the energy of thermal motion (heat) is transferred through the substance from one molecules to the next until the remote end is reached. Thus, in conduction through a substance, thermal energy is passed from molecule to neighbouring molecule, no bodily movement of the molecules themselves being involved.

The ability to conduct heat varies from substance o substance. Those substances through which heat can conduct easily from one region to other are known as *good conductors* ; on the other hand, those through which heat cannot conduct easily from one region to other are known as *bad conductors* (or *good insulators*).

In general, metals are very good conductors of heat. Silver is the best conductor of heat, the next in order of its ability being copper. Metals are also known to be good conductors of electricity, which is ascribed to the presence of a large number of so called "free electrons" inside them. These electrons are free in the sense that they have become separated from their parent atoms; inside the metal, they move about randomly like molecules of a perfect gas. Along with molecules, the free electrons also take part in the process of conduction of heat. This is the reason why metals are such good conductors of heat.

Substances such as cork, wood, cotton, wool, felt etc., are bad conductors of heat. Consequently these are used as good insulators. Liquids, in general, are bad conductors ; mercury is an exception. Gases also are very poor conductors of heat.

Demonstration experiments.

(1) To show that copper is a good conductor of heat, we can device an experiment based on the action of copper gauge on flames. When a wire gauge is held just above a Bunsen flame, the flame does not pass through the gauge but becomes confined in the portion below the gauge [Fig. 21·1 (*a*)]. This effect can be attributed to the fact that copper is a very good conductor. The inflammable gas burns only when its temperature reaches a certain value, known as its '*ignition temperature*.' The copper wire of the gauge being good conductor, the heat received by it from the flame is quickly conducted through it, so that the temperature of the gas above the gauge remains below the ignition temperature.

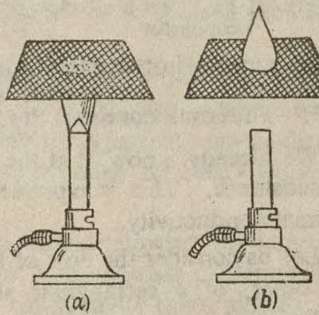


Fig. 21·1 : Copper is a good conductor of heat

The flame is now put off and the gauge is held about 3 cm. above the burner. The gas above the gauge is then lighted when it is found that the flame continues to burn only above the gauge [Fig. 21·1 (*b*)]. This time the wire-gauge being a good conductor prevents the temperature of the gas below the gauge from reaching the ignition temperature. These experiments may also be performed with an iron gauge, although not so satisfactorily.

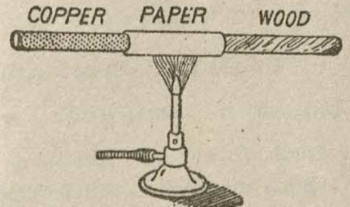


Fig. 21·2 : Different substances conduct different amount of heat

(2) The difference in the abilities of two substances to conduct heat can be illustrated by the following experiment. A thin pieec of paper is wrapped once around a composite rod, one half of which consists of wood (a bad conductor) and the other half of copper (a good conductor), as shown in the Fig. 21·2. When held over the flame of a burner, the portion of the paper would over the wooden half begins to burn, while the other portion which is in contact with the copper remains quite uneffected. Copper being a good conductor, transmits

the heat received by the paper very quickly into the interior of the metal, while wood, being a poor conductor cannot conduct heat in the same manner. Consequently, in the latter case the paper is heated to a high temperature and begins to burn.

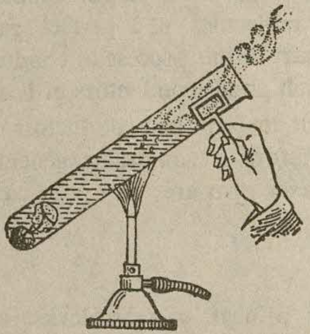


Fig. 21·3 : Water is a bad conductor

(3) The following simple experiment nicely demonstrates that water is a bad conductor of heat. A test tube is filled up partly with ice-cold water. A piece of ice tied to a piece of iron is introduced into the test tube, so that the combination sinks to the bottom [Fig. 21·3]. Now water at the top of the test tube is heated by a bunsen flame. It will be found that the water at the top boils vigorously, while ice at the bottom remains more or less unmelted. This shows clearly that very small amount of heat can conduct through water from the hotter upper portion to the lower colder portion.

21·3. Thermal conductivity.

We already know that the ability to conduct heat differs from substance to substance. This is expressed quantitatively by introducing the concept of thermal conductivity.

Let us consider the flow of heat from one face to the other, of a rectangular slab of a substance of thickness d and cross-sectional area A [Fig. 21·4]. Let us suppose that the whole of the left side of the slab is at a temperature θ_2 and the whole of the right side is at a lower temperature θ_1. Under such conditions, heat will be conducted *steadily* from left to right through the slab. Experiments show that the quantity of heat Q which flows through the slab perpendicular to its face is

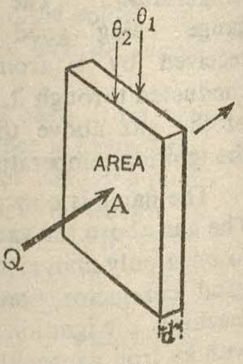


Fig. 21·4

(i) proportional to the area A,
(ii) proportional to the temperature difference $(\theta_2-\theta_1)$,
(iii) inversely proportional to the thickness d,
(iv) proportional to the time of flow t.

$$\text{Thus, } Q \propto A\left(\frac{\theta_2-\theta_1}{d}\right)t,$$

$$\therefore\quad Q=KA\left(\frac{\theta_2-\theta_1}{d}\right)t \qquad \ldots \qquad \ldots \qquad (21\cdot1)$$

where K is a constant depending on the nature of the material of the slab, and is known as the *thermal conductivity*. $\left(\frac{\theta_2-\theta_1}{d}\right)$ denotes the space rate of change of temperature from one point to another within the slab and is known as the *temperature gradient.*

This equation for the conduction of heat through a substance holds *provided no heat is lost from its sides.*

If $A=1$, $d=1$, $(\theta_2-\theta_1)=1$, $t=1$, we have from eqn. (21·1), $Q=K$.

Thus *the* **Thermal conductivity** *of the material of a substance is numerically equal to the quantity of heat that conducts in one second normally through a slab of unit length and unit area, the difference of temperature between its end faces being one degree.*

In C.G.S. system, the unit of thermal conductivity, as obtained from eqn. (21·1), is cal. cm. $^{-1}$ sec.$^{-1}$ degree C^{-1}, while in the F.P.S. system, the corresponding unit is B. Th. U. ft.$^{-1}$ sec^{-1} degree F^{-1}.

The statement that the *thermal conductivity of silver is 0·97 C.G.S. unit,* means that siver allows a flow of heat of 0·97 calorie per sec., from one face to the opposite face of a unit cube, when the difference of temperature between the two faces is 1°C.

It is obvious from the above discussion that a material for which K is large, is a good conductor of heat ; on the other hand, if K is small, the material is a poor conductor. For a perfect conductor $K=\infty$, and for a perfect non-conductor $K=0$. No material is, however, found to possess either of these two extreme values of conductivity.

Thermal conductivity of a substance is found to change slowly with temperature ; for solids and liquids, conductivity decreases with increase in temperature, while for gases an increase in the value of the conductivity takes places.

Thermal resistance : From eqn. (21·1), the amount of heat flow due to conduction per second may be written as

$$\theta/t=(\theta_2-\theta_1)/\left(\frac{1}{K}.\frac{d}{A}\right)$$

This can be compared with the expression for current flow as obtained from Ohm's law, viz.

$$i=\frac{V_2-V_1}{R}=\frac{V_2-V_1}{\rho l/A}$$

where, (V_2-V_1) is the potential difference between the ends of a conductor of resistance R, length l and area of cross-section A. ρ is the specific resistance of the material of the conductor. So from this analogy we can call $\frac{1}{K}\cdot\frac{d}{A}$ as the thermal resistance of the slab.

21·4. Comparison of thermal conductivity.

A simple experiment developed by Ingenhausz is useful in comparing the conductivities of different solids. The apparatus (Fig. 21·5) consists of a trough, one face of which is provided with a number of holes. A number of rods of different materials uniformly polished and having the same length and diameter are introduced into the trough through corks fitted in the holes. The portion of rods projecting outside the trough are coated uniformly with wax. The trough is filled with water maintained at the boiling point. After some time it is noticed that the wax has melted to different distances along the rods showing the differences in their thermal conductivities.

In can be established theoretically that in the steady state the thermal conductivities of the different rods are proportional to the squares of their respective lengths upto which wax has been melted. Thus, if K_1, K_2, $K_3 \cdots$, etc. represent the conductivities of the rods and l_1, l_2, l_3, etc., are the lengths of the melted wax, we have $K_1 : K_2 : K_3 \cdots\cdots = l_1^2 : l_2^2 : l_3^2 \cdots\cdots$. Hence by measuring the lengths to which wax has melted, conductivities of different substances may be compared.

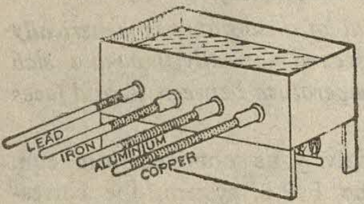


Fig. 21·5 : Ingenhausz's experiment

21·5. The variable and the steady state : Diffusivity

Let us consider a metal bar, one end of which is heated by introducing it in a constant temperature bath as shown in the Fig. 21·6. Evidently, heat will flow through the adjacent layers A, B and C along the length of the bar. Heat received by the layer B from its preceding layer A is spent up in three ways viz., (i) a portion is absorbed by the layer B producing a rise in its temperature, (ii) a protion is lost by radiation from the surface of the layer B and by convection of the surrounding gas, (iii) the remaining portion is transferred to the next layer C. This is true for every layer of the bar. Such situation persists for some time and the bar then said to be in *variable state*.

But with the passage of time, a stage is reached when it is found that the temperature of each section of the bar becomes constant i.e. steady ; the temperature of a section is found to be diminishing with its distance from the hottest end. The bar is then said to be in *steady state*. The constancy of the temperature of each section of the bar in such state indicates that no heat is being absorbed by the section.

From the definition of the thermal conductivity, it follows that the amount of heat which reaches a section of the bar depends on the value of the conductivity of its material : in the variable state, a fraction of this amount of heat is absorbed by the section and corresponding rise in its temperature depends on the value of the specific heat of the material. Hence, in the variable state, the rate of rise of temperature depends upon both the thermal conductivity and the specific heat of the material of the bar. In the steady state, no more absorption of heat takes place so that the flow of heat along the bar is determined solely by its thermal conductivity. Due to this reason, all measurements of thermal conductivity are carried out only after attainment of the steady state.

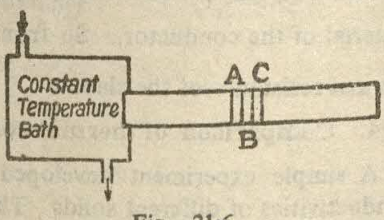


Fig. 21·6

If K be the conductivity, s the specific heat and ρ the density of a substance then it can be shown that the rate at which temperature rises in the initial variable state is determined by the ratio

$$\frac{K}{\rho s}=\frac{\text{thermal conductivity}}{\text{thermal capacity per unit volume}}$$

This ratio is called the *diffusivity* or the *thermometric conductivity* of the substance.

For example, for iron $\rho=7{\cdot}8$ gm. per c.c., $s=0{\cdot}11$ and so $\rho s=0{\cdot}86$; since $K=0{\cdot}16$ C.G.S. unit, the diffusivity for iron$=0{\cdot}19$ C.G.S. unit. For lead, $\rho=11{\cdot}4$ gm. per c.c., $s=0{\cdot}03$ and hence $\rho s=0{\cdot}34$; as $K=0{\cdot}08$ C.G.S. unit, the diffusivity for lead$=0{\cdot}24$ C.G.S. unit. Thus although the conductivity of lead is less than that of iron, its diffusivity is greater and consequently under the identical conditions of heat flow, the temperature of different sections in a lead bar rises more rapidly than in an iron bar. This can be easily demonstrated by the Ingenhausz's, experiment described before. It is found that at the beginning of the experiment i. e., before the steady state is reached, wax melts at a faster rate along a rod of lead than along a rod of iron. But finally at the steady state, wax melts through a greater distance on the iron rod due to its larger thermal conductivity.

21·6. Measurement of the thermal conductivity of a metal by Searle's method.

A bar AB of the metal under investigation is taken and polished. The end A of the bar is surrounded by a chamber through which steam is passed [Fig. 21·7]. Near the other end B is wound a spiral of copper tubing through which a stream of water is passed at a constant rate. The temperatures of the incoming and outgoing water are recorded by the thermometers T_4 and T_3 respectively. Two small holes at a distance say, d, apart are drilled in the metal bar. Thermometers T_1 and T_2 are introduced in the holes to measure the temperature gradient. Little mercury is poured in the holes to ensure good thermal contact with the bulbs of the thermometers and the metal bar. To prevent loss of heat from the sides of the bar, it is surrounded by dry felt and enclosed in a wooden box not shown in the figure.

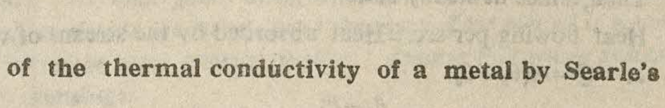


Fig. 21·7 : Searle's apparatus

When the end A is heated by passing steam through the chamber, heat is conducted along the length of the bar. The readings of the thermometers T_1 T_2 will rise. The heat reaching the end B is carried away by the stream of water. The consequent rise in temperature of the outgoing water will be indicated by the thermometer T_3. Soon the steady state will be reached when all the thermometers will indicate steady readings. The temperatures are all recorded. The mass of water flowing per second is also measured.

Let θ_1 and θ_2 be the steady temperatures recorded by the thermometers T_1 T_2. Since they are placed at a distance d apart,

$$\text{the temperature gradient}=\frac{\theta_1-\theta_2}{d}$$

If Q be the heat flowing per second across any cross-section of the bar, then

$$Q=KA\frac{\theta_1-\theta_2}{d}$$

where A is the cross-section of the bar which is measured and K is thermal conductivity of the material of the bar.

Let m be the mass of water flowing per second and θ_3, θ_4 be the temperatures of the outgoing and incoming water respectively.

Then, since at steady state,

Heat flowing per sec. = Heat absorbed by the stream of water per sec.

$$\therefore \quad Q=m(\theta_3-\theta_4)$$

$$\text{or,} \quad KA\frac{\theta_1-\theta_2}{d}=m(\theta_3-\theta_4)$$

$$\therefore \quad K=\frac{m(\theta_3-\theta_4)d}{A(\theta_1-\theta_2)}$$

Hence the value of K can be found. It should be noted that the method is not suitable for bad conductors.

Example 21·1. *An iron plate 2·5 cm. thick has a cross-sectional area of 1000 sq. cm. One side of the plate is maintained at 100°C., while the other side is at 80°C. How much heat is transmitted per second? (Thermal conductivity of iron=0·16 cal. sec.$^{-1}$ degree C.$^{-1}$ cm.$^{-1}$).*

Solution : Since $K=0{\cdot}16$ unit, $A=1000$ sq. cm., $(\theta_2-\theta_1)=(100-80)=$ 20°C., $d=2{\cdot}5$ cm., we have from eqn. (21·1),

$$Q=\frac{0{\cdot}16\times1000\times20}{2{\cdot}5}=1280 \text{ calorie per sec.}$$

Example 21·2. *A plate of nickel 4 cm. thick has a temperature difference of 32°C. between its faces. It transmits 110 calorie per sec. through an area of 100 sq. cm. Calculate the thermal conductivity of nickel.*

Solution : We have $Q=110$ calorie, $A=100$ sq. cm., $(\theta_2-\theta_1)=32$C°., $d=$ 4 cm. and $t=1$ sec. We get, therefore, from eqn. (21·1)

$$K=\frac{Q/t}{A\times(\theta_2-\theta_1)/d}=\frac{110\times4}{100\times32}=0{\cdot}137 \text{ cal. sec.}^{-1} \text{ degree C.}^{-1} \text{ cm.}^{-1}.$$

Example 21·3. *A glass tank 30 cm. long and 15 cm. wide contains ice at 0°C. Find the mass of the ice which melts per minute, when the tank is immersed to a depth of 10 cm. in a water bath at 30°C. The thickness of glass is 4 mm. and the thermal conductivity of glass is 0·0025 c.g.s. unit. Latent heat of ice=80 cal/gm.*

Solution : Area of the tank in contact with 30°C. water

$$A=2(30+15)\times10+30\times15$$
$$=1350 \text{ cm.}^2$$

Temperature difference $(\theta_2-\theta_1)=30°\text{C}$.

Thickness of glass $d=0·4$ cm.

Time $t=1$ min.$=60$ sec.

Conductivity$=0·0025$ c.g.s. unit

Let m gm. of ice melts in 1 min.

$$\therefore\quad m\times80=\frac{0·0025\times1350\times30\times60}{0·4}$$

whence, $m=189·9$ gm.

Example 21·4. *One end of a copper rod of uniform cross-section and of length 2 metre is in contact with ice and the other end is introduced into a bath maintained at 100°C. At what point along its length should a temperature of 200°C. be maintained, so that in the steady state the mass of ice melted is equal to that of steam produced in the same interval of time. Assume that whole system is insulated from the surroundings.*

Solution : In an interval of t sec., let the mass of ice melted be m gm. The heat necessary for this$=m\times80$ cal. Since the mass of steam produced is also m gm., the required heat$=m\times540$ cal.

Let the distance between the end of the rod in contact with ice and the point where the temperature is maintained at 200°C. be x metre. If K be the conductivity of the material of the rod and A its cross-section, then

$$m\times80=\frac{KA(200-0)t}{x}\quad\text{and}\quad m\times540=\frac{KA(200-100)t}{2-x}$$

Dividing, we get $\dfrac{540}{80}=\dfrac{x\times100}{(2-x)\times200}$; $\therefore\ x=1·86$ metre.

Example 21·5. *In an Ingenhauz's experiment, wax melted over 15 cm. of copper rod and 6 cm. of iron rod. What is the conductivity of iron, when the conductivity of copper=0·90 c.g.s. unit ?*

Solution : Length of the wax melted over copper rod $l_1=15$ cm.

Length of the wax melted over iron rod $l_2=6$ cm.

Conductivity of copper $K_1=0·9$ c.g.s. unit.

Let K_2 be the conductivity of iron.

$$\because\quad \frac{K_1}{K_2}=\frac{l_1^2}{l_2^2},\quad\therefore\quad K_2=\frac{l_2^2}{l_1^2}\cdot K_1$$

$$=\frac{6^2}{15^2}\times0·9=0·144\ \text{c.g.s. unit.}$$

21·7. Practical applications of thermal conduction.

Good conductors have their wide uses. (*a*) Copper and aluminium pans are much used for cooking utensils, since heat is conducted most rapidly through these metals. Boilers are, for this reason, often made with copper.

(*b*) Davy's safety lamp (Fig. 21·8) used widely in mines is an example in which the high conductivity of metal has been utilised. It consists of a simple oil lamp completely surrounded by a cylinder of wire gauze. Any combustible gas, if present inside the mine penetrates the gauze and comes in contact with the naked flame. The gas thus burns inside the wire gauze. But due to its high conductivity the gauze conducts away the heat developed so quickly that the temperature of any portion of it and hence of the gas outside the lamp never reaches ignition point of gas. Thus the outer gas does not burn and the safety of the miners is ensured. Moreover, the presence of any combustible gas within the mine can be detected by the colour and the nature of the flame.

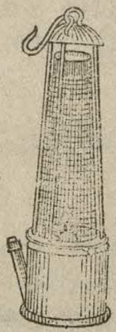


Fig. 21·8 : Davy's safety lamp

Like good conductors bad conductors are also made very useful in our everyday life. *These are used as heat insulators.* (i) Fibrous materials such as felt or asbestos are very poor conductors while they are dry. These are often used in heat engines to lag the pipes carrying the steam or hot gases. (ii) Saw dust being a poor conductor is used to cover ice clocks and prevents them from melting quickly. (iii) Ice boxes, ice-cream boxes and refrigerators have inner linings of glass-wool which is a very poor conductor. Foamed rubber or polystyrene are often used as pakaging materials for the instruments which may get damaged if heated above moderate temperature during transit. Besides the poor conductivity of their materials, these substances contain innumerable holes which serve as air-traps. Air being a very bad conductor, these substances act as very efficient heat insulators. (iv) Woollen clothes, although themselves are not warm, are comfortable in winter season. The texture of these clothes encloses innumerable air pockets wich prevent the body from losing heat to the colder surroundings and thus help the body to remain warm. (v) Wearing two cotton shirts, one above another a person can feel more warm than wearing a single shirt, even though the thickness of the cloth of the single shirt is equal to the total thickness of that of the two shirts. The reason is quite obvious ; for two shirts, a layer of air being trapped in between them, the flow of heat by conduction from the body to the surroundings is prevented to a greater extent and the person feels more warm. (vi) A very practical but interesting phenomenon is observed with a piece of wood (bad conductor) and a piece of iron (good conductor) both lying exposed to the sun and hence remaining at the same temperature. If one now touches them alternatively, the iron is felt warmer than the wood. Iron being a good conductor conveys heat very quickly from it to the observer's hand ; thus a sensation of warmth is experienced by him. But wood being a poor conductor is felt comparatively cold for it conveys less heat to his hand. Just the reverse happens if the two pieces are kept outside a room in cold night. In that case, their common temperature is less than that of the hand. But the iron conducts heat more efficiently than the wood. Hence the iron piece is felt colder than the wood piece.

21·8. Convection ; Convection current.

In a preceding article, we demonstrated that water is a poor conductor of heat. But our common experience seems to contravene this fact when we see that if a kettle filled with water is placed on a stove, water gets quickly heated. In this case, however, heat is transmitted through water not by conduction but by the second method of heat transmission, known as *convection* Water in the bottom of the kettle becomes hot before any other part, and its density, therefore, diminishes compared to the water above it. Consequently, warmer and hence lighter water rises to the top and the cooler water sinks to replace it. This, in turn, is heated and rises. Ultimately, the whole mass of water gets heated quickly. Thus in this process, the motion of water itself is responsible in transmitting heat through it. This process is primarily responsible for the transference of heat through liquids and gases although majority of them are bad conductors. Hence,

the process of transmission of heat from one place to other by the actual mass movement of the heated particles of a liquid or gas is known as convection.

Convection can not take place in a solid due to the reason that the constituent particles are not free to move.

It follows that in the process of convection, hotter portion of the fluid being lighter ascends while the colder portion being denser descends. This circulatory movement of the fluid is called *convection current*. Obviously, by this process heat can be transferred through a fluid only in the upward direction not in the downward or lateral direction. It should also be noted that in a place where force of gravity is zero, this process of heat transfer would cease.

Demonstration Experiment.

(1) In liquids, the convection current can be illustrated by dropping some crystals of potassium permanganate gently in a flask containing water. The flask is then heated from beneath by a burner. Convection current will be set up in water as shown in the Fig. 21·9; the current is rendered visible by the colour imparted by permanganate to the water.

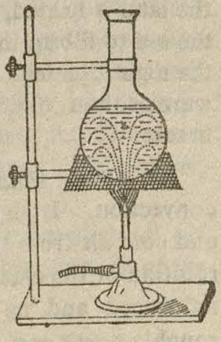


Fig. 21·9

(2) In gases, the convection current may be demonstrated by the following simple experiment. The apparatusconsists of two wide glass tubes fitted at the top of a rectangular wooden box with a removable glass front

[Fig. 21·10]. A candle is lit inside the box beneath one of the tubes. Air in the vicinity of the candle flame is heated, gets lighter and leaves the box through the tube above the canule, while cold air enters the box through the other tube. By holding a piece of smouldering paper at the top of the inlet tube, the direction of this convection current will be rendered visible by the path followed by smoke through the box.

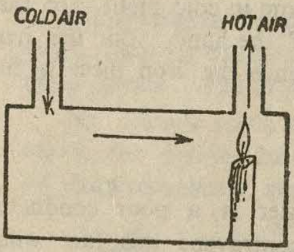


Fig. 21·10

21·9. Practical examples of convection.

There are many examples of natural convection in everyday life. Some of them are discussed below :

(1) The whole plan of ordinary ventilation in a room depends upon convection. The air which we exhale is warmer and consequently lighter. This lighter air moves up and escapes through the ventilators while cool and fresh air from outside rushes into the room through windows or doors.

(2) *Land and sea breeze* : At places on the coast, cool air from over the sea generally blows towards the land as *sea breeze* during the day, while at night the cool air, as a *land breeze*, blows from the land to the sea [Fig. 21·11]. These are local convection currents in the atmosphere.

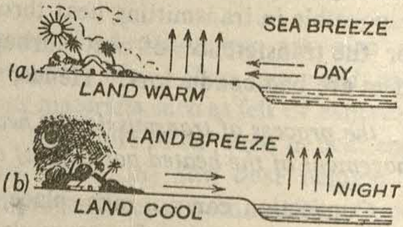


Fig. 21·11 : Sea breeze and land beeeze

During the day, land is heated by the sun to a higher temperature than the sea water. This is because water has a higher specific heat than earth and also it is a poor absorber of radiant heat compared to earth. As a result, sea water gets heated at a slower rate than the earth. Therefore, during the day, air over the land is heated, expands and rises while comparatively cold air flows in from the sea to fill up the gap. Since good absorbers are also good radiators, during the night, the land cools more quickly than does the sea. Thus sea is now warmer than the land and the effect is just reversed giving rise to the land breeze.

(3) *Trade winds* : The wind currents over the earth is attributed mainly to convection. It is produced when warm air in the equatorial region rises and cold air from the polar regions moves towards the equator. Owing to the rotation of the earth from west to east, the direction of the wind flow is influenced and as a result, in the northern hemisphere, the wind moves roughly north-east and in the southern hemisphere, the wind moves roughly south east. These winds are called the *north-east trade wind* and the *south-east tread wind* respectively.

(4) The domestic hot water supply system widely used in cold countries provides an application of convection current. On being heated, water in the boiler *B* [Fig. 21·12] rises through a pipe P_1, to the upper part of a hot water storage tank *C*, while the colder and denser water from *C* flows through the pipe P_2 to the bottom of *B* when it is heated up in turn. Thus, a circulation of water is set up and it continues until the water in *C* is fully heated. When hot water is drawn off from the tap *T*, an equal volume of water from the cold water reservoir *R* flows into the hot water storage tank *C* at the bottom through the pipe *D*. The water level in the reservoir is maintained by a supply from the mains through an automatic stopcock *S*. Thus a constant supply of hot water can be obtained from this arrangement.

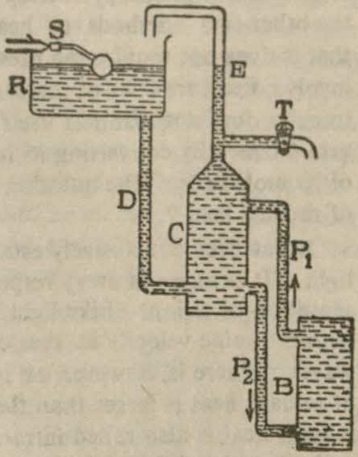


Fig. 21·12 : Domestic hot water supply system

Dissolved air comes out of water on heating and may cause air locks in the pipes. The system must be so designed that the flow of water is not impeded by such air locks. Consequently, an expansion pipe *E* is provided for allowing the air to escape. It also serves the purpose of safety precaution. If due to overheating the water boils, steam and hot water are discharged safely through it to the cold water tank.

21·10. Radiation.

When we sit besides an oven we feel warm. In this case heat reaches us not by conduction or convection. Air being a bad conductor of heat, allows little heat to pass through it and at the same time little convection takes place in the lateral direction as explained earlier. Here the process by which heat is transferred is called *radiation*.

Actually the transfer of heat by radiation does not require any material medium at all. In vacuum electric lamps, the heat emitted by the filament traverses the space between the filament and the glass, though there is vacuum inside the bulb. The heat from the sun which reaches the earth is transmitted across 149 million kilometre of space most of which is empty. Moreover, radiant heat from the sun, while passing through the earth's atmosphere does not heat it ; otherwise the atmospheric temperature should increase with altitude, which is not the case. Hence

Radiation *is the name given to the transfer of heat from a hot body to cold one without the help of any medium and without appreciable heating of the intervening medium, if any.*

21·11. Radiation as a form of energy.

We know that heat is a form of energy and is identified with the motion of molecules. Transference of heat by radiation, therefore, implies transference of energy from one body to another. Actually the word 'radiation' is used synonymously to the process of transfer of heat in this particular way, as well as to the stream of energy flowing out from the emitter. Radiation differs from the other two methods of heat transfer, viz. convection and conduction, in that it does not require the presence of any material medium and hence does not involve direct transfer of molecular motion. Radiant energy in the process of transfer does not manifest itself until it falls on a body which absorbs it and gets warmed by converting it into heat with a resulting increase in the motion of its molecules. The question that naturally arises is—what is then the nature of radiant heat ?

It has been conclusively established that radiant heat is of the same nature as light. It behaves in every respect as light except that it does not produce the sensation of vision. Like light, radiant heat is a transverse wave and travels with the same velocity as that of light. These waves are called electromagnetic waves. There is, however, an important point of difference. The wavelength of radiant heat is larger than the red part of the visible spectrum. Hence the radiant heat is also called infra-red radiation. The wavelength of infra-red rays varies between $7{\cdot}5\times10^{-5}$ cm. to about 3×10^{-2} cm., while the wavelength of visible light lies in the range of $3{\cdot}8\times10^{-5}$ cm. to about $7{\cdot}5\times10^{-5}$ cm.

It is now known that radio waves, ultraviolet rays, X-rays and γ-rays are all electromagnetic waves, one differing from the other only in wavelength. Radiowaves have larger wavelengths than infra-red rays ; the wavelengths of ultraviolet rays are smaller than those of violet part of the visible light ; those of X-rays and γ-rays are still smaller, decreasing in that order. In general, the term 'radiation' includes this entire family of electromagnetic waves. However, as the infra-red rays falling on our skin gives the sensation of warmth, radiations in this range are called thermal radiation or heat radiation.

21·12. Detection of radiant heat

Various instruments are used to detect and measure radiant heat. Of those only two are described below which are used chiefly for demonstration purposes. Description of accurate quantitative instruments is beyond the scope of this book.

(1) **Thermopile :** This instrument renders radiant heat measurable by converting it into electrical energy. It is constructed by connecting a number of thermocouples in series. A thermocouple is the name given to two unlike metal wires, say A and B, joined together at their ends [Fig. 21·13 (a)]. When one

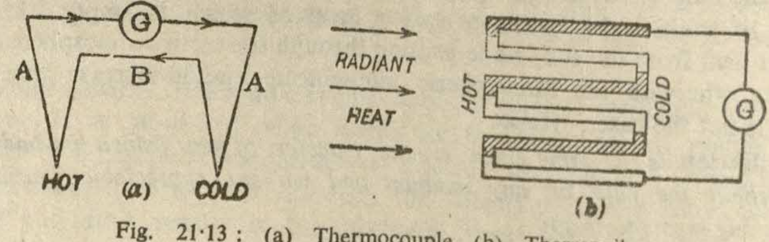


Fig. 21·13 : (a) Thermocouple (b) Thermopile junction is kept at higher temperature than the other, a small electric current flows in it which is detected by inserting a sensitive galvanometer G in the circuit. Usually antimony and bismuth are used in a thermocouple for their ability to show the effect to a marked degree. The use of a large number of thermocouples in a thermopile is just to magnify the effect. In a thermopile a number of antimony and bismuth rods, insulated from one another by mica foils, are soldered in pairs at their ends [Fig. 21·31 (*b*)]. The whole is mounted within a brass cylinder and a sensitive galvanometer is connected to the free ends. The 'hot' junctions are blackened and exposed to radiation, while the 'cold' junction are covered with a highly polished metal cap. The galvanometer registers the electric current flowing through the circuit. The blackened face is provided with a conical shield to protect it from stray radiations.

(2) **Bolometer :** Electrical resistance of a platinum wire changes markedly with the change of temperature. In a *bolometer* two similar thin strips of platinum [Fig. 21·14] are connected to the opposite arms of a Wheatstone Bridge. One of them is blackened and exposed to radiation. The other is shielded from radiation and designed to compensate for fluctuation in the resistance due to change in room temperature. The amount of radiation incident on the blackened strip is estimated from the measurement of the change in its resistance by means of the Wheatstone Bridge.

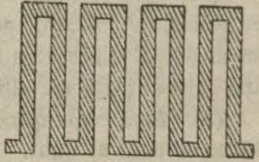


Fig. 21·14 : Platinum strip

21·13 Properties of thermal radiation.

As stated earlier, the radiant heat has many properties in common with light. These are summarised as follows :

(1) *Radiant heat travels through vacuum with the speed of light.* The following phenomenon provides a simple evidence of this fact. The radiant heat and light from the sun are cut off simultaneously by the moon during a solar eclipse. They also reappear at the same instant when the eclipse is over.

(2) *Radiant heat travels in straight lines.* When an object which is opaque to radiant heat is placed in front of a hot body of small size, it casts a sharply defined shadow. The boundary of the shadow can be traced with a thermopile. This is the reason why less heat is felt in the shadow of a tree ; the tree cuts off light as well as radiant heat from the sun.

(3) *Radiant heat obeys inverse square law.* The intensity of radiation at a point is the amount of radiant energy falling per second on unit area surrounding the point. The *inverse square law* states that the intensity of radiant energy at a point varies inversely as the square of the distance of the point from the source. That radiant heat obeys this law can be demonstrated simply with a thermopile and an ordinary electric bulb. When the distance between the thermopile and the bulb is doubled, the deflection of the galvanometer is reduced to a quarter of its previous value. As the galvanometer deflection is proportional to the intensity of the radiant heat incident on the thermopile, the conclusion follows immediately.

(4) *Reflection of thermal radiation.* Radiant heat is reflected from polished metal surfaces, whether plane or curved, obeying the laws of reflection like light. This may be demonstrated by the following simple experiment. Two concave mirrors are placed as shown in the Fig. 21·15, with their reflecting surfaces facing each other. A burning candle is placed at the focus of one so that the heat rays are reflected by it into a parallel beam. On reaching the second mirro these rays are reflected and brought to focus at *T*. When a thermopile i placed at *T* the galvanometer shows maximum deflection. On removing th candle, the deflection is reduced to zero.

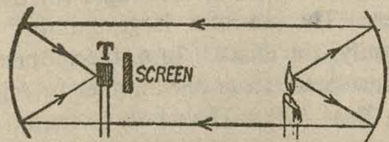


Fig. 21·15 : Reflection of thermal radiation

If a concave mirror is held facing the sun, the incident heat says converge t its focus. A piece of paper held at the focus begins to burn in a few second In medieval times, this method was utilised to set the enemy fleet ablaze wi the help of large metallic concave mirrors.

(5) *Refraction of thermal radiation.* Radiant heat may be refracted in the same manner as light. It is evident by the fact that an ordinary lens refracts the sun rays and brings them to a point very close to its focus and that a piece of paper placed at this point is ignited by the condensed heat rays. A more pronounced effect is obtained if a convex lens of rock-salt is used, because rock-salt is much more transparent to the radiant heat than glass.

Other phenomena associated with light viz., interference, diffraction, polarisation, photoelectric effect, photographic action, scattering etc., are also exhibited by radiant heat. All these lead to the conclusion that radiant heat is of same nature as light.

21·14. Emission and absorption of radiation.

The rate at which a body radiates heat depends on its temperature and on the nature of its surface. The rate of radiation is found to be smaller at lower temperature and increases rapidly with rise in temperature. For the same temperature, blackened surfaces are found to be most efficient radiators polished surfaces being least efficient.

At lower temperatures it is observed that the radiation from a body consists mainly of waves of longer wavelengths. As the temperature is increased, the radiation becomes richer in waves of shorter wavelengths. Thus, when a body is heated, we find that it begins to radiate heat energy i.e., infra-red waves and no visible glow is observed. As its temperature is gradually increased to higher and higher values, it begins to glow becoming, at first, red hot and then white hot. If the temperature can be raised still further, the body appears blue hot. This explains the reason for which the stars Sirius and Vega appear blue.

When radiant heat falls on a body, one part is reflected, another part is absorbed, while the rest is transmitted through the body if it is transparent to the incident radiation, If a body transmits all the incident radiant heat, it is said to be **diathermanous** i.e. transparent to the radiant heat. On the other hand if a body does not transmit any incident radiant heat, it is **adiathermanous** i.e., opaque to radiant heat. For example, water is adiathermanous while rock-salt and quartz are diathermanous. Ordinary glass transmits radiant heat of shorter wavelengths but is opaque to the rest.

If a body absorbs larger fraction of the incident radiant heat, then it is obvious that smaller fraction of the incident radiation will be reflected. Thus *a good absorber is a poor reflector of radiant heat.* From similar reasoning, it follows that the converse is also true. It can also be shown that *good radiator are also good absorbers* and vice-versa. Hence a good radiator is a poor reflector. Similarly a poor radiator is a good reflector. So it is found that blackened surfaces are best absorber and worst reflector of heat. The reverse is true for a polished surface. If a metal surface is made rough then it becomes a bad reflector ; hence it is a good absorber and radiator of heat.

Demonstration Experiments : (*a*) The above facts can be demonstrated simply with a Leslie's cube which is a hollow metallic cube filled with boiling water. Two vertical sides of the cube are coated with lamp black and white paint respectively. The third side is left untreated and the fourth is polished brightly. A thermopile *T* connected to a sensitive galvanometer *G* is placed at a suitable distance from the Leslie's cube *C* [Fig. 21·16]. By turning the cube, each side may be presented to the thermopile. The deflection of the galvanometer gives a measure of the radiation emitted from the surface. It is found that the lamp black surface produces the largest and the polished surface produces the smallest deflection. The deflection corresponding to the dull untreated surface and the white surface lie in between, that produced by the former being larger. This shows that *the lamp black surface is the best radiator, next in order being the dull surface, the white surface and the polished surface. Polished metallic surface is, therefore, the poorest radiator.*

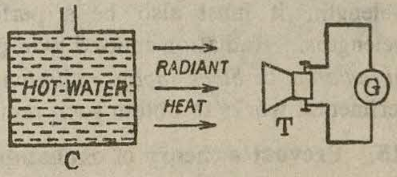


Fig. 21·16

(*b*) To show that good radiators are good absorbers and vice versa, two copper plates P_1 and P_2 are placed at equal distance on each side of a Leslie's cube as shown in the Fig. 21·17. Each face of the cube *C* is painted white. The copper plate P_1 is covered with lamp black, while P_2 is brightly polished. The temperature of each plate can be measured with a copper-constantan thermocouple. It is found that the temperature of P_1 is greater than that of P_2. This shows that *lamp black surface is a better absorber of radiant heat than a polished surface.*

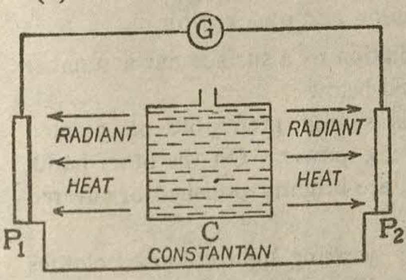


Fig. 21·17

Perfectly black body : *A perfectly black body is one which absorbs all the incident radiations without reflecting or transmitting any part of it.* Lamp black and platinum black provide nearest approach to such a body, absorbing respectively about 96% and 98% of the incident radiation.

A perfectly black body can be realized in practice by a small hole in the wall of a hollow body which is at uniform themperature [Fig. 21·18]. Any radiation entering the hollow body through the hole suffers a number of reflections and ultimately gets completely absorbed. This is facilitated by coating the interior surface with black, so that about 96% of the radiation is absorbed at each reflection. The portion of the interior surface opposite to the hole is made conical shaped to prevent any radiation incident along the axis of the hole from going out of the enclosure after reflection. Thus, the hole behaves like a perfectly black body. This is known as Fery's black body.

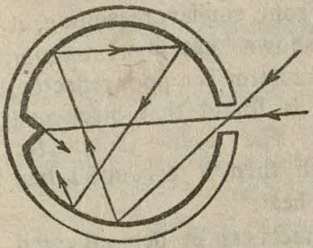


Fig. 21·18 : Black body

Since a black body serves as a perfect absorber of radiations of every wavelength, it must also be a perfect emitter of radiations of all possible wavelengths. Radiation emitted by a perfectly black body is, therefore, called *total radiation* or *black body radiation.* Fery's black body is widely used in experimental works to obtain such radiation.

21·15. Prevost's theory of exchange.

According to the idea of Pre'vost all objects, whether hot or cold, are continually emitting thermal radiation. At the same time they are also receiving radiation from their surroundings. The amount of heat radiated by a body increases with its temperature and is not affected by the presence of neighbouring bodies. When a body emits more radiation than what it receives from the surroundings, it cools. Conversely, it warms up when it receives more radiation than that it emits. At equilibrium, when rate of emission equals rate of absorption, the temperature of the body remains constant. Thus, *even at equilibrium, an exchange of radiation continues.*

21·16. Some practical applications of emission and absorption.

The nature of emission or absorption of radiation by a surface has a number of practical applications, some of which are cited below :

(i) For rapid heating, the cooking vessels should preferably be painted dull black outside so as to increase its absorbing power. On the other hand, the outer surfaces of tea pots, calorimeters etc. are brightly polished or silvered to minimise radiation loss.

(ii) In hot countries people keep cool by wearing light-coloured clothes and living in white painted house. This reduces the amount of heat absorbed, because white is a good reflector and therefore a poor absorber of radiant heat. On the other hand, black clothes and dark coloured houses are preferred in the cold countries.

(iii) Space crafts have highly polished surface to reflect away the radiated heat from the sun. The firemen wear garments with highly polished surfaces ; the heat from the fire is reflected and not absorbed. This enables the firemen to keep relatively cool and to approach the fire.

(iv) A cloudy night is found to be warmer than a clear night. This is due to the fact that cloud being practically opaque does not allow radiant heat from the earth's surface to escape into space.

(v) In colder countries, glass-walled house are constructed to preserve the vegetations during the winter season. These are called *greenhouses*, which serve as effective heat-traps. Radiations from the sun being stronger in shorter wavelengths are transmitted by glass so that the temperature of the objects inside the greenhouse is raised. As this temperature is not high, the radiations emitted by them is richer in longer wavelengths which are not transmitted by glass. Hence heat from the inside of the greenhouse cannot escape by radiation so that the temperature inside remains greater than temperature outside.

21·17. Stefan's Law.

The law regarding the relation between the radiant heat emitted by a hot body and its temperature was first guessed by Joseph Stefan in 1879, following a study of some experimental results. It asserts that *the total radiation E of all wavelengths emitted per unit time per unit area of a perfectly black body is proportional to the fourth power of its absolute temperature T.* This is known as the *Stefan's law*. Thus

$$E \propto T^4$$

$$\text{and hence } E = \sigma T^4 \qquad \ldots \qquad \ldots \qquad \ldots \qquad (21\cdot2)$$

where the constant of proportionality σ is known as the *Stefan's constant*. The value of σ is about $5{\cdot}7 \times 10^{-5}$ erg/cm.^{2}/sec./°K^4.

Stefan's law was later theoretically proved by Ludwing Boltzman in 1884. The law is now generally known as the *Stefan Boltzman law*.

If a black body is in equilibrium with its surroundings at a temperature T_0, it will emit energy at the rate of σT_0^4 per unit area. Hence it must be absorbing heat from the surroundings at the same rate. If the temerature of the black body is now raised to T, while the temperature of the surroundings remain at T_0, the rate of absorption of heat per unit area will still be σT_0^4 but the rate of emission now becomes σT^4 per unit area. Hence the net rate of loss of heat per unit area by radiation is

$$E = \sigma(T^4 - T_0^4) \qquad \ldots \qquad \ldots \qquad \ldots \qquad (21\cdot3)$$

This also holds when the black body being at a lower temperature gains heat from the surroundings. This is another formulation of Stefan's law. Obviously eqn. (21·3) holds when a body is gaining or losing heat by radiation only, the other methods of heat transference being absent.

Approximation from Stefan's law : If the temperature T of a black body radiator is slightly higher than the temperature T_0 of the surroundings, then putting $T = T_0 + x$, where x is assumed to be very small, we get,

$$\begin{aligned}E&=\sigma(T^4-T_0^4)=\sigma[(T_0+x)^4-T^4_0]\\&=\sigma(T_0^4+4T_0^3x+6T_0^2x^2+4T_0x^3+x^4-T_0^4)\\&=4\sigma T_0^3x \qquad\text{(neglecting higher powers of } x)\\&=Kx=K(T-T_0)\end{aligned}$$

where $K=4\sigma T_0^3$ is a constant.

Thus, *for small difference of temperature, heat lost per second by a black body due to radiation is proportional to the difference in temperature of the body with the surroundings.* This is known as *Newton's law af cooling.*

21·18. Applications of Stefan's law.

(1) *Measurement of high temperature* : Stefan's law is widely used for measurement of very temperature with the help of instruments called *radiation pyrometers.* Such pyrometers need not be placed in contact with the body whose temperature is being measured ; hence there is no upper limit to the range of temperature measureable with such pyrometers. However, in such measurements, we assume the body to be perfectly black which is not always so. Thus, the measured temperature will be less than the actual one.

In Fery's radiation pyrometer [Fig. 21·19] the radiation from the source entering from the right is focussed by a concave mirror M through the diaphragm D on a blackened plate P, known as *receiver*. The position of the mirror can be adjusted to achieve accurate focussing, which is judged by viewing the image of the source through an eyepiece E placed behind a hole in the mirror M. Ths receiver is attached to one junction (hot) of a thermocouple. The other junction (cold) is shielded from the radiation by T. A millivoltmeter mV is connected to the thermocouple. The incident radiation being absorbed by the receiver heats it up, whence the millivoltmeter shows a deflection.

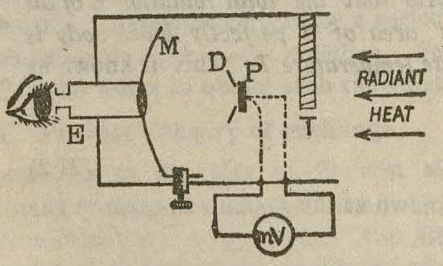


Fig. 21·19 : Fery's radiation pyrometer

As the e.m.f. E in the thermocouple is proportional to the intensity of the incident radiation, we have from Stefan's law,

$$E=a(T_1^b-T_2^b)$$

where T_1 and T_2 are respectively the temperature of the source and the receiver and a is a constant. The value of b varies between 3·8 and 4·2, being constant for a particular instrument. Hence, this pyrometer should be calibrated in comparison with a standardised thermocouple before use.

(2) Temperature of the sun.

Solar constant : The solar constant is the average amount of energy which will fall in one minute on one square cm. of a black surface held normal to the sun's rays and placed at mean distance of the earth from the sun provided there were no absorption in the earth's atmosphere. This constant can be measured with an instrument known as pyroheliometer and its value is found to be nearly 1·937 cal./cm.^{2}/min.

Temperature of the sun : Assuming the sun to be a perfectly black body of radius r and temperature T, then from Stefan's law, heat radiated by it per sec. is given by $Q=4\pi r^2\sigma T^4$. This radiant heat is distributed uniformly over the entire surface area $4\pi R^2$ of an imaginary sphere of radius R concentric with the sun, where R is the mean distance of the earth from the sun. Hence from definition, the solar constant S is given by,

$$S=\frac{4\pi r^2\sigma T^4}{4\pi R^2}\times 60$$

$$\text{or,}\quad T^4=\left(\frac{R}{r}\right)^2\frac{S}{\sigma}\times\frac{1}{60}$$

Substitution of known values of R, r, S, and σ gives $T=5723°K$.

Example 21·6. *If the temperature of the sun's surface be 5450°C., calculate the rate at which energy is radiated from each sq. cm. of its surface. Assume the temperature of the surrounding space to be 0°K. Given, Stefan's constant* $=5{\cdot}7\times10^{-5}$ *erg/sec./cm.^{2}/deg. K^4.*

Solution : The temperature of the sun's surface
$=5450+273=5723°K$.

From Stefan's law, $E=\sigma(T^4-T_0^4)$

$$\therefore\ \text{Energy radiated/sec./sq. cm.}=5{\cdot}7\times10^{-5}\times(5723^4-0^4)\ \text{erg.}$$
$$=5{\cdot}7\times10^{-5}\times5723^4/4{\cdot}2\times10^7\ \text{calorie.}$$
$$=1{\cdot}455\times10^3\ \text{calorie.}$$

● EXERCISE ●

[A] Essay-type questions.

1. Discuss the different methods of heat transfer with suitable examples. Indicate the fundamental differences among them.

2. Define thermal conductivity of a substance. Explain whether the establishment of steady state is essential for this definition to be applicable. [*Model Questions, H. S. Council*]
What is the unit of thermal conductivity in C. G. S. system ?

3. How is heat actually conducted through a body ? Describe an experiment to show that conductivity varies from substance to substance.

4. Describe briefly, with a diagram, one experiment in each case to show : (a) that copper is a good conductor of heat, (b) that water is a bad conductor of heat, (c) that copper is a good conductor while wood is a bad conductor of heat.

5. What do you understand by the steady and the variable state of heat flow ? What is diffusivity ? What is its difference with thermal conductivity ?

6. Describe an experiment for the determination of the thermal conductivity of a metal.

7. Explain the process of heat propagation by convection. What is convection current ? Explain how it originates.

8. Describe the experiments for the demonstration of convection current in liquids and gases. Discuss one useful application of convection current.

9. Write short notes on :

(a) Davy's safety lamp, (b) Sea breeze and Land breeze, (c) Trade winds, (d) Fery's black body, (e) Diathermanous and adiathermanous bodies.

10. What is radiation ? State the properties of thermal radiation. Describe one simple apparatus by which radiant heat may be detected.

What do you mean by a 'perfectly black body ? How is it realised in practice ?

11. Name the different factors on which the rate at which a body radiates heat depends. Describe one experiment in each case to prove the following statements :—

(a) A black surface is a better radiator than a bright polished surface.

(b) Good radiators are also good absorbers and vice-versa.

[B] Short answer type questions.

1. Explain what is meant by the statement, 'thermal conductivity of iron is 0·2 C.G.S unit'.

2. If you touch a piece of iron and a piece of wood lying exposed to the heat of the sun, which one feels hotter and why ?

3. Two metal bars *A* and *B*, of the same size, but of different materials are coated with equal thickness of wax and placed each with one end in a hot bath. It is noted that at first the wax on *A* melts at a greater rate than that on *B*, but that when a steady state has been reached, a greater length of wax has been melted on *B* than on *A*. Explain. [*C. U. '41*]

4. At what temperature wood a block of wood and a block of metal feel equally cold or equally hot when touched ? [*I. I. T. '76*]

5. Two rods *A* and *B* are of equal length. Each rod has its ends at temperatures T_1 and T_2. What is the condition that will ensure equal rates of flow of heat through the rods *A* and *B* ? [*I. I. T. '76*]

6. Two garments of equal thickness keep the body warmer than a single garment of double the thickness. Why ?

7. Explain why felt rather than air is used for thermal insulation even though the thermal conductivity of air is less than that of felt. [*I. I. T. '78*]

8. A piece of paper wrapped tightly on a wooden rod is found to get charred quickly when held over a flame, compared to a similar piece of paper when wrapped on a brass rod. Explain why. [*I. I. T. '74*]

9. Stainless steel cooking pans are preferred with extra copper bottoms. Why ? [*I. I. T. '77*]

10. Why the reading of a thermometer is lowered when its bulb is wrapped with a rag ? What difference will be observed when the rag is wetted with (i) ether (ii) water ? [*Jt. Entrance, '74*]

11. A sphere, a cube and a thin circular plate, all made of the same material and having the same mass are initially heated to a temperature of 200°C. Which of these will cool fastest and which one slowest when left in air at room temperature ? Give reasons. [*I. I. T. '72*]

12. Two thermometers are constracted in the same way except that one has a spherical bulb and the other an elongated cylindrical bulb. Which one will respend quickly to temperature changes ? [*I. I. T. '73*]

13. By supplying heat at a constant rate below a metallic pot, the water in the pot can be boiled quickly when its bottom is made black and rough but not so when the bottom is highly polished. [*Jt. Entrance '74*]

14. Heat is generated continuously in an electric heater but its temperature becomes constant after some time. Why ? [*I. I. T. '75*]

15. Two persons ordered tea in a restaurant and waited for a friend to arrive. One of them poured tea in his cup and mixed cold milk in it, and the other pored his tea and mixed the milk after the friend arrived. Whose cup of tea was hotter ? Give reasons for your answer. [*I. I. T.* '67]

16. Explain with proper reasoning :—

(a) The outside of a cooking vessel should preferably be rough and painted dull black whereas the outside of a calorimeter should be bright and polished.

(b) In hot countries people prefer to wear white clothes but in cold countries dark coloured clothes are preferred.

(c) A cloudy night is warmer than a clear night.

(d) The temperature inside a greenhouse remains greater than that at the outside.

(e) In a desert, it is too hot during day time and too cold during night.

(f) For cooking, a black is preferable then a bright vessel.

(g) The birds puff their feathers out on a cold day.

(h) Wooden clothes are more comfortable than cotton clothes in winter.

(i) The handle of a kettle is wrapped with cane.

(j) Heat can not be transmitted by convection through a solid.

17. You are given two ordinary clear glass thermometers and the bulbs of one of them is coated with lamp black. Compare their readings when exposed (a) or a damp cloudy night (b) on a clear dry night in the cold weather, and (c) in the sun.

[C] Simple Problems.

1. If the conductivity of a metal is 0·2 C.G.S. unit, what is its value in F.P.S. system ? [*Ans.* $1{\cdot}344\times10^{-2}$ F.P.S. unit]

2. Heat flows into one face of a 1 cm. cube and out of the opposite face at the rate of 10 cal./sec. No heat passes through the other four faces. If the temperature difference between the two faces is 1°C, what is the thermal conductivity of the material of the cube ? [*Ans.* 10 cal. cm.^{-1} sec.^{-1} deg.^{-}C.]

3. Two sides of an iron plate of thickness 1 cm. and area 2 sq m. are maintained at 100°C. and 50°C. respectively. Find the total amount of heat which flows through it from one side to the other in 1 min. Given, for iron $K=0{\cdot}16$ C.G. S. unit. [*Ans.* $9{\cdot}6\times10^{6}$ cal.]

4. It is found that 3×10^{6} cal. of heat are transmitted per minute across a sheet of a metal 10 cm. square and 1 mm. thick with a difference of temperature 100°C. between its faces. Find the co-efficient of thermal conductivity of the metal. [*Ans.* 0·5 cal. cm.^{1-} sec.^{-1} deg.^{-1} C.]

5. One end of a 20 cm. long bar of cross-section 1 sq. cm. is heated in a steam chamber, while its other end is introduced into melting ice. Calculate the amount of ice melted in 5 min. Given, K of the material of the rod$=0{\cdot}25$ C. G. S. unit. [*Ans.* 4·6875 gm.]

6. A spherical vessel of external radius 10 cm. is made of copper sheet 2 mm. thick. It is filled with melting ice and immersed in water at 100°C. Calculate the rate at which the ice melts. Given K for copper$=0{\cdot}72$ C.G.S. unit and latent heat of fusion of ice=80 cal./gm. [*Ans.* 5·652 kg./sec.]

7. A boiler with plate 2·5 cm. thick exposes 3 sq. metre of its surface to the furnace. If the temperature of the outside face is 110°C., find the amount of steam produced in one hour. Given, K for iron$=0{\cdot}16$ C.G.S. unit and latent heat of steam=540 cal./gm. [*Ans.* 128 kg.]

8. The ice on a pond is 5 cm. thick and the temperature of its surface in contact with the air is −10°C. At what rate heat is conducted through each square metre of ice ? Given, conductivity of ice=0·005 cal. cm.^{-1} sec.^{-1} deg.^{-1} C. [*Ans.* 100 cal./sec.]

9. In a copper boiler, the plates are 12 mm. thick and 30 kg. of water are evaporated per minute per square metre of the heating surface. Calculate the difference of temperature between the sides of the plate. Given, latent heat of steam=540 cal./gm. and thermal conductivity of copper=0·1 c.g.s. unit. [*Ans.* 324°C]

10. The walls of a hot water tank are 0·80 cm. thick and are made of material of thermal conductivity $1{\cdot}2\times10^{-1}$ cal. cm.$^{-1}$ sec.$^{-1}$ deg.$^{-1}$ C. The tank is protected by lagging of thickness 2·0 cm. and thermal conductivity $9{\cdot}6\times10^{-3}$ cal. cm.$^{-1}$ sec.$^{-1}$ deg^{-1} C. When conditions are steady the temperature of water in the tank is 70°C. and the temperature of the outer surface of the lagging is 40°C. Find (a) the temperature of the interface between the tank and lagging (b) the loss of heat per square metre per second. (*Oxford*) [*Ans.* 69·07°C ; 1395 cal.]

11. An ice box is built of wood 1·5 cm. thick with an inner lining of cork 3·0 cm. thick. It is filled with melting ice at 0°C. If the temperature of the outer surface of wood is 30°C. what is the temperature of cork-wood interface ? Given, the conductivity of wood is $6{\cdot}0\times10^{-4}$ and of cork $1{\cdot}2\times10^{-4}$ c.g.s. unit. [*Ans.* 27·3°C]

12. How much coal must be burned to make up for the loss of heat due to conduction for one day through two glass windows each 4 mm. thick and having area 2 m.^{2}, if the room temperature is 25°C and the outside air is at -10°C. Given, heat of combustion of coal used is 8000 cal./gm. and K for glass $=2{\cdot}4\times10^{-3}$ c.g.s. unit. [*Ans.* 90·72 kg.]

13. A box made up of an insulating material contains an electric heater. If a power input of 120 watt to the heater is required to keep the interior of the box 60°C. above the outside temperature, find the thermal conductivity of the material of the box. Given, the effective total area of the box is 2·5 m.^{2} and the wall thickness is 4 cm. [*Ans.* $7{\cdot}62\times10^{-5}$ c.g.s. unit]

14. The walls of a refrigerator are 10 cm. thick having a surface area of 10 m.^{2}. The temperature outside the refrigerator is 30°C. How much power in watt must the refrigerator motor supply in order to maintain a temperature of 0°C. inside the refrigerator ? Given, the thermal conductivity of walls=0·001 c.g.s. unit. [*Ans.* 1·26 kw.]

15. A vessel with walls 2·5 cm. thick, of effective area 2000 cm.^{2}, and conductivity 5×10^{-5} cal. sec.$^{-1}$ cm.$^{-1}$ deg.$^{-1}$ C., contains liquid oxygen at -188°C., evaporation being possible through an opening at the top of the vessel. Taking the latent heat of vaporization of oxygen to be 60 cal./gm. find the rate at which the liquid evaporates when the surroundings are at 12°C. (*Oxford*) [*Ans.* 8 gm./min.]

16. Apply Stefan's law to calculate the ratio of the rate of heat loss from a metal ball at 727°C. to that from the same ball at 427°C. if the temperature of the surrounding be 27°C. [*Ans.* 4·276 : 1]

17. A small hole of area 0·5 cm.^{2} is made in the wall of a furnace heated to 1527°C. Assuming the hole to be a perfectly black body, calculate the heat radiated per sec. from it. Given, Stefan's constant $\sigma=5{\cdot}7\times10^{-5}$ erg/cm.^{2}/sec./°K^4. [*Ans.* 7·123 cal./sec.]

18. Two rods A and B are of equal length. Each rod has its ends at temperatures T_1 and T_2. What is the condition that will ensure equal rates of flow of heat through the rods A and B ? [*I. I. T. '76*]

[*Ans.* $\frac{r_1}{r_2}=\sqrt{\frac{K_2}{K_1}}$]

[D] Harder Problems.

1. Assume that the thermal conductivity of copper is 4 times that of brass. Two rods of copper and brass having the same length and cross-section are joined end to end. The free end of the copper rod is kept at 0°C. and the free end of the brass rod is kept at 100°C. Calculate the temperature of the junction of the two rods at equilibrium. Neglect radiation losses. (*I. I. T. '70*) [*Ans.* 20°C.]

2. (a) One end of an insulated copper rod is held at 0°C. and the end at 100°C. The rod is 50 cm. long and has cross-sectional area of 4·0 cm.^{2} Calculate the heat current in the rod in watt. The thermal conductivity of copper is 0·90 cal./(sec. cm. °C).

(b) If the copper rod of part (a) is replaced by aluminium rod with the same cross-section but only 27 cm. long, the same heat current results. The two rods are then welded such that a compound rod 77 cm. in length is formed. What is the ratio of the heat current in the compound rod to that in the copper rod alone for the same temperature difference between opposite ends ? (*London*) [*Ans.* (a) 30·2 watt., (b) 1/2]

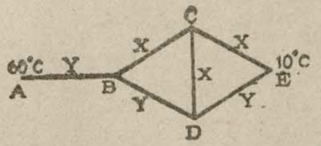
3. A slab consists of three parallel layers of different materials 5 cm., 10 cm. and 15 cm. thick and of thermal conductivity of 0·7, 0·4 and 0·2 c.g.s. unit respectively. If the opposite face of the slab are at 100°C. and 0°C. calculate the temperature of the interfaces and the quantity of heat passing by conduction through each square metre of the slab per second. [*Ans.* 93·3°C. ; 70°C ; 9333 cal.]

4. A layer of ice 14·9 cm. thick has formed on the surface of a deep pond. If the temperature of the upper surface of the ice is constant and equal to that of the air, which is at −1°C., determine approximately the time it will take for the thickness of ice to increase by 2 mm. Given, latent heat of fusion of ice=80 cal/gm., density of ice=0·9 gm./c.c., thermal conductivity of ice=0·006 c g.s. unit. [*Ans.* 10 hr.]

5. A copper block containing a resistance coil is suspended in vacuum by two wires 20 cm. long and 1·0 mm. in diameter. To maintain the temperature of the block at 40°C. above the other end of the suspension, a power of 0·084 watt must be disspated in the coil. Calculate the co-efficient of thermal conductivity of the material of the suspension wires. [Neglect the energy transferred by radiation] (*Cambridge*) [*Ans.* 0·6366 c.g.s. unit]

6. A glass disc 2 mm. thick is placed between and with its flat faces in good thermal contact with the ends of two identical coaxial solid copper cylinders, each 10 cm. long and having the same diameter as the disc. The outer ends of the cylinders are maintained at 100°C and 20°C. respectively, and the sides of the disc and the cylinders are perfectly lagged. Calculate the temperatures attained by the copper-glass interfaces when the steady state is reached. The thermal conductivities of copper and glass may be taken as 0·92 and 0·0024 cal. cm.$^{-1}$, sec^{-1}, deg^{-1}C. respectively. (*London*) [*Ans.* 91·725°C. ; 28·275°C.]

7. Three rods of material *X* and three rods of material *Y* are connected as shown in the figure. All the rods are of identical length and cross-sectional area. If the end *A* is maintained at 60°C and the junction *E* at 10°C, calculate the temperatures of the *B*, *C* and *D*. The thermal conductivity of *X* is 0·92 cal./sec. cm °C and that of *Y* is 0·46 cal./sec. cm. °C.



(*I. I. T.* '78) [*Ans.* 30°C, 20°C, 20°C]

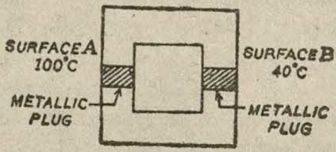
8. Estimate the surface temperature of the earth, assuming it to be in thermal equilibrium with the sun. Assume the surface temperature of the sun to be 5700°C, its radius 7×10^{10} cm. and its distance from the earth $1{\cdot}5\times10^{3}$ cm. (*Jt. Entrance* '83) [*Ans.* 137·9°C]

9. A copper calorimeter with a copper heating element fitted inside weighs 300 gm. It is filled to a certain mark with 200 gm. of a liquid. With 41 watt electrical input to the heater, the temperature rises from 20°C to 45°C in 10 minutes. When 140 gm. of the liquid is replaced by 1250 gm. of copper rivets such that the liquid level remains the same and the calorimeter heater is supplied with the same electrical input as before, the temperature rises from 20°C to 45°C in 9 mins 5 secs. It is found steady temperature of 45°C can be maintained in either case with an electrical input of 2 watt. The room temperature remaining constant at 20°C throughout the experiments. Calculate the specific heats of (i) copper and (ii) the liquid. (*Jt. Entrance* '80) [*Ans.* 0·095 ; 1·00]

10. The temperature on the inside and outside of a cylindrical tube of a bad conductor of heat whose coefficient of thermal conductivity is *k* are kept at θ_1 and θ_2. If the inside and outside diameters are 2*a* and 2*b* respectively, show that the heat flowing across unit length of the tube per sec. is $2\pi k\ (\theta_1-\theta_2)/(\log_e{}^{b}-\log_e{}^{a})$. (*Jt. Entrance* '79)

[Hints : Use calculus]

11. A closed cubical box made of perfectly insulating material has walls of thickness 8 cm. and the only way for heat to enter or leave the box is through two solid, cylindrical, metallic plugs, each of cross-sectional area 12 cm^2 and length 8 cm. fixed in the opposite walls of the box (see Fig.). The outer surface A of one plug is kept at 100°C while the outer surface B of the other plug is maintained at 4°C. The thermal conductivity of the material of the plug is 0·5 cal $deg^{-1}C\ cm^{-1}\ sec^{-1}$. A source of energy generating 36 cal/sec. is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is the same at all points on the inner surface.



(*I. I. T.* '72) [*Ans.* 76°C]

4

VIBRATIONS & WAVES

22
CHAPTER
VIBRATIONS

22·1. Introduction.

In the earlier section of mechanics, we considered, in detail, the linear and circular motion of a particle. In this section, we shall study another common type of motion, called *periodic* motion. *Any motion that repeats itself at regular intervals of time is known as* **periodic motion**. Many familiar phenomena can be cited as examples : the motion of a pendulum, of the balance wheel of a watch, of a planet round the sun, of a mass suspended by a spring, of the strings and air columns of musical instruments, etc. In the following discussions, we shall treat, for simplicity, a moving body as a particle. We shall also discuss, in detail, a particular type of periodic motion called vibratory or oscillatory motion.

22·2. Oscillation and its characteristics.

A particle in periodic motion is said to be executing **vibratory** *or* **oscillatory** *motion, when it continues to move back and forth repeatedly over the same path.* Previously we considered linear motion in which accelerations were constant both in magnitude and in direction, and circular motion in which the accelerations (centripetal) were constant in magnitude but not in direction. In oscillatory motion, however, as we shall see later on that the accelerations, like displacements and velocities, change periodically in both magnitude and direction. In such motion, one particular position of the particle can be found where no resultant force acts on the particle. This is known as the *equilibrium position* of the particle. On being slightly displaced from this position, a restoring force acts on the particle, tending to move it back towards the equilibrium position. As the particle returns back to this position, it overshoots due to its inertia of motion and moves on to the other side. Its motion, therefore, goes on repeating itself.

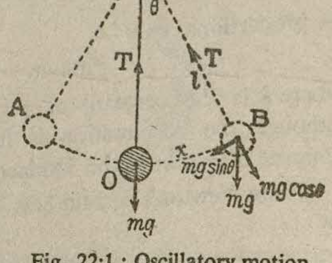


Fig. 22·1 : Oscillatory motion of a simple pendulum

We consider as an example, the motion of a simple pendulum [Fig. 22·1]. O is the equilibrium position ; when displaced from this position, the bob oscillates along the path AOB. Let θ be the angular displacement of the bob at any particular instant. The forces acting on the bob are its weight mg acting vertically downwards and the tension T in the string. At the position O, these two cancel each other, so that no net force acts on the bob.

At the displaced position, say B, the component $mg \cos \theta$ of the weight of the bob is balanced by the tension T in the string, while the component $mg \sin \theta$ always tries to bring the bob back to its equilibrium position O. This is, therefore, the restoring force acting on the bob of the pendulum.

We now define some important quantities in relation to oscillatory motion. *At any instant, the* **displacement** *of the particle is defined as the distance of the particle at that instant from its equilibrium position. The maximum value of the displacement is called the* **amplitude** *of its motion.* Thus, for the simple pendulum, OA or OB gives the amplitude of oscillation.

When the oscillating particle passing through any position returns to the same direction, a **complete vibration** *or* **oscillation** *is said to occur.* In the example of the simple pendulum, the trip from A to B and back to A, or the trip from O to B, from B to A and back to O represents a complete vibration or oscillation.

The time required for a complete oscillation is called the periodic time or time period or simply the **period** *of the oscillatory motion.* It is usually denoted by T.

The **frequency** (*symbol* n) *of the oscillatory motion is the number of oscillations performed per unit time.* Thus, frequency is the reciprocal of period.

Hence,
$$n=\frac{1}{T} \qquad \dots \qquad \dots \qquad (22\cdot1)$$

Obviously, the unit of frequency is per second. This unit is called Hertz, its abbreviated form being Hz.

22·2. Simple harmonic motion.

A particular type of oscillatory motion which is of great importance in all branches of physics is known as *simple harmonic motion* (often abbreviated as S.H.M.). It is the simplest type of oscillatory motion. Its importance lies in the fact that any complex oscillatory motion can be expressed as a combination of a number of S. H. Ms. *If the acceleration of an oscillating particle is directly proportional to its distance from the equilibrium position and is always directed towards that position, its motion is said to be* **simple harmonic**.

Fig. 22·2 represents a particle N oscillating in a straight line between the points A and B, about its equilibrium position O. At any instant, let its displacement measured from the equilibrium position, be x. For the motion to be simple harmonic, its acceleration f at that instant should be directed towards O and must be proportional to x i.e.

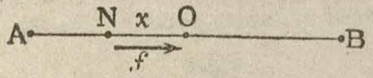


Fig. 22·2 : Simple harmonic motion

$$f \propto -x \qquad \text{or, } f=-kx \qquad \dots \qquad \dots \qquad (22\cdot2)$$

where k is the constant of proportionality. The negative sign indicates that although the acceleration is larger for larger displacement, it is always in the opposite direction to the displacement.

From Newton's second law, the force acting on the particle of mass m is given by

$$P=mf=-kmx=-Kx \qquad \dots \qquad (22\cdot3)$$

where $K=km$ is a constant. This equation indicates that the force on the particle executing S.H.M. is proportional to its displacement and is always directed towards the equilibrium position. The force is thus restoring in nature.

22·4. Relation of S.H.M. with uniform circular motion.

Let us consider a point P moving with a uniform angular velocity ω round a circle of radius a and centre O. Let PN be the perpendicular drawn from P on a fixed diameter XX' of the circle [Fig. 22·3]. Then as P revolves, the foot of the perpendicular N describes a to and fro motion along the diameter XX', with O, the centre of the circle, as its mean position. If the point P starts from X, then as it completes each revolution, the point N moves from X to X', through O, reverses its direction and returns again to X through O. It will be shown in the next article that this oscillatory motion of N is S.H.M. about O. Point P is known as the *reference point*, while the circle in which it moves with constant speed is called the *reference circle* or *auxiliary circle*. We may thus define **simple harmonic motion** alternatively *as the projection of a uniform circular motion on any diameter of the circle*.

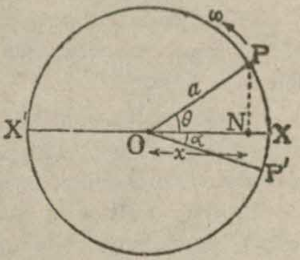


Fig. 22·3 : Circle of reference for analysing S.H.M.

The time spent in making one complete vibration is defined as the *Period* (T) of the particle executing S.H.M. It is obvious that this must be equal to the time spent by the reference point in making a complete revolution round the reference circle. Since the angular velocity of the reference point is ω, we have,

$$T=\frac{2\pi}{\omega} \qquad \dots \qquad \dots \qquad (22\cdot4)$$

If the frequency of the S.H.M. be n, then

$$n=\frac{1}{T}=\frac{\omega}{2\pi} \qquad \text{or,} \qquad \omega=2\pi n \qquad \dots \qquad (22\cdot5)$$

Demonstration experiment : A wheel fitted with a short handle is arranged to turn uniformly in a horizontal plane [Fig. 22·4]. A parallel beam of light casts

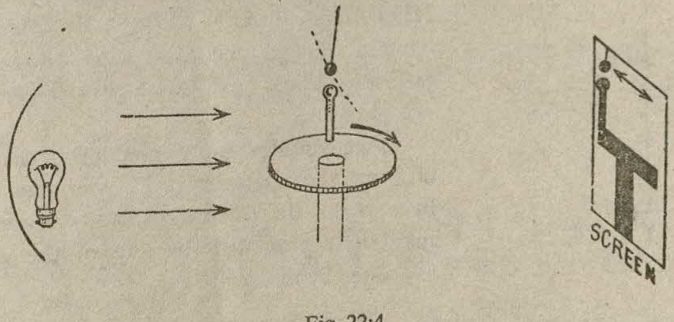


Fig. 22·4

its shadow on a vertical screen or on the wall of the room. It is found that as the handle executes a uniform circular motion, its shadow describes a to and fro motion on the screen. That this motion is S.H.M. can be shown by simultaneously projecting on the screen the shadow of the bob of an oscillating simple pendulum. The bob is suspended just above the centre of the wheel. Proper adjustment will produce identical motions of the shadows of the bob and of the handle.

22·5. Mathematical representation of S.H.M.

On the basis of the concept of reference circle, the mathematical representation of the various physical quantities associated with S.H.M. can be obtained in the following way :

(*a*) **Displacement and amplitude** : The displacement at any instant of the point N executing S.H.M. is given by its distance $ON=x$, measured from the mean position O at that instant (Fig. 22·3).

But $ON=OP\cos\theta$. $\therefore$ $x=a\cos\theta$ ... ... (22·6)

where θ is the angle which OP makes with OX at that instant. The radius a of the reference circle obviously gives the maximum value of displacement of the vibrating point N ; a thus represents the *amplitude* of the S.H.M. performed by the point N.

Let us assume that the time is counted from the instant when P passes through the point X i.e., when OP coincides with OX. If the time required by P in moving from X to P along the circle be t then $\theta=\omega t$, where ω is the uniform angular velocity of P. Thus,

$$x=a\cos\omega t \qquad \ldots \quad \ldots \quad (22\cdot7)$$

This equation connects the displacement of a particle executing S.H.M. with its amplitude and the angular velocity of the corresponding reference point and is called the **equation of S.H.M.**

Now, since $\omega=\frac{2\pi}{T}=2\pi n$, the equation of S.H.M. can also be written as

$$x=a\cos\frac{2\pi t}{T}=a\cos 2\pi nt \qquad \ldots \quad \ldots \quad (22\cdot8)$$

Alternatively, we may consider the reference point to be starting from Y and moving in a reverse direction, where YY' is another diameter perpendicular to XX' [Fig. 22·5]. Thus, if the $\angle YOP=\theta$ be described by the said point in time t, we have $\theta=\omega t$. In that case $x=OP\cos\angle PON=a\cos(90-\theta)=a\sin\theta$ $=a\sin\omega t$.

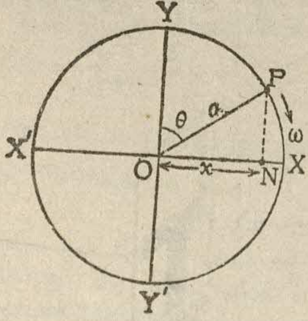


Fig. 22·5

If we consider the projection of the motion of P on the diameter YY', we may similarly have $y=a\sin\omega t$ or $a\cos\omega t$. Thus a S.H.M. may be represented by any of the equations expressed symbolically as,

$$\left.\begin{matrix} x \\ y \end{matrix}\right\} = a \left\{\begin{matrix} \cos\omega t \\ \sin\omega t \end{matrix}\right. \qquad \ldots \quad \ldots \quad (22\cdot9)$$

The displacement of a particle executing S.H.M., therefore, varies with time in *a sinusoidal manner*. If the displacement of any particle can be expressed in a form similar to equation (22·9), we can assert straightway that its motion is simple harmonic.

If the time is measured from the instant at which the reference point P passes through any other point P', such that $\angle P'OX = \alpha$, then $\angle P'OP = \omega t$ [Fig. 22·3]. Then the displacement becomes

$$x = a \cos \angle POX = a \cos \angle (P'OP - P'OX)$$
$$= a \cos (\omega t - \alpha) \qquad \ldots \qquad \ldots \qquad (22{\cdot}10)$$

Similarly, with proper choice of the instant from which time is counted, the expression for displacement can be written in the form

$$x = a \cos (\omega t + \alpha) \qquad \ldots \qquad \ldots \qquad (22{\cdot}11)$$

Equations (22·10) and (22·11) *represent the most general form of the equation of S.H.M.*

(*b*) **Velocity** : The linear velocity v_T of the reference point P at every instant is tangential to the circle and is given by $v_T = a\omega$. Now, since the motion of N arises out of the motion of P, the velocity v of N at any instant is given by the component of the velocity of P along XX' [Fig. 22·6]. It will be seen from the figure, that the velocity of N is given by

$$v = -v_T \cos (\pi/2 - \theta) = -a\omega \sin \theta$$
$$= -a\omega \sin \omega t \qquad \ldots \qquad \ldots \qquad (22{\cdot}12)$$

The negative sign is introduced because the velocity of N is directed towards left i.e., the negative direction of x. When P is below the diameter XX', the velocity of N is towards right, but since $\sin \theta$ is negative then, the negative sign is still required.

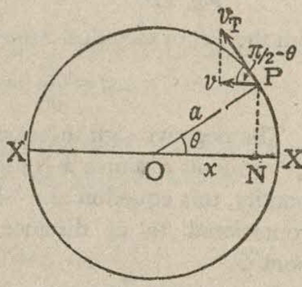


Fig. 22·6 : Velocity of a particle executing S.H.M.

The expression for the velocity can be obtained alternatively by applying differential calculus. We have for the displacement of the particle $x = a \cos \omega t$.

Since velocity is the rate of displacement we get,

$$v = \frac{dx}{dt} = \frac{d}{dt}(a \cos \omega t) = -a\omega \sin \omega t$$

We can also express the velocity of N in terms of its displacement x. Thus,

$$v = -a\omega \sin \theta = \pm a\omega\sqrt{1 - \cos^2\theta}$$
$$= \pm a\omega\sqrt{1 - \frac{x^2}{a^2}} \qquad [\because \; x = a \cos \theta]$$
$$= \pm \omega\sqrt{a^2 - x^2} \qquad \ldots \qquad \ldots \qquad (22{\cdot}13)$$

The symbol $\pm$ arises due to the fact that for a particular value of the displacement, the point N may be moving either towards left or towards right i.e., its velocity directions may be opposite.

From equation (22 13), the velocity of N is maximum when $x = 0$ i.e., when N is at O ; $v_{max} = \pm \omega a$, the same as that of P. The velocity of N is minimum when $x = a$ i.e., when N is at X or X' ; $v_{min} = 0$.

Thus, we conclude that *the velocity of a particle executing S.H.M. is different at different points along its path; the velocity is maximum, when it passes through the mean position and is minimum when it is at either of the extreme points of its path.* At intermediate points, the velocity of the particle lies within these two limits i.e., ωa and zero and may be directed either towards or away from the mean position O.

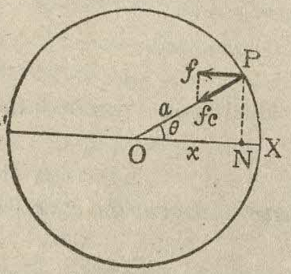


Fig. 22·7

(c) **Acceleration** : As the motion of N is due to that of P, so the acceleration of N is equal to the component of the acceleration of P parallel to XX'. The acceleration, f_c of P is $\omega^2 a$ directed towards the centre of the circle i.e., along PO [Fig. 22·7]. The component of this parallel to XX' is $\omega^2 a \cos\theta$. Hence, the acceleration f of N is

$$f = -\omega^2 a \cos\theta = -\omega^2 x \qquad \cdots \qquad (22{\cdot}14)$$

This expression can be alternatively obtained with the aid of calculus. Since, acceleration is the rate of change of velocity, we have,

$$f = \frac{dv}{dt} = \frac{d}{dt}(-a\omega \sin \omega t) = -a\omega^2 \cos \omega t = -\omega^2 x.$$

The negative sign indicates that the acceleration of N is always directed towards O, while its distance x is measured away from O. Since ω^2 is a positive constant quantity, this equation also shows that the acceleration of N towards O is directly proportional to its distance from O. Hence the motion of N is a S. H. M. about O.

The acceleration is maximum being equal to $a\omega^2$, when the displacement is maximum i.e., at the two extreme points of the path. The acceleration is minimum i.e., zero, when the displacement is also zero i.e., when the particle is at the mean position of its path. Thus, the acceleration of the particle varies both in magnitude and direction.

(d) **Time-period and Frequency** : From the expression (22·14) for the acceleration, the period of the particle executing S. H. M. can be put in a more useful form. We have (ignoring sign), $\omega^2 = f/x$.

$$\therefore \text{ from eqn. (22·4), } T = 2\pi\sqrt{\frac{1}{f/x}} \qquad \cdots \qquad \cdots \qquad (22{\cdot}15)$$

$$\text{i.e., Period} = 2\pi\sqrt{\frac{1}{\text{Acceleration at unit displacement}}}$$

Futher, if we multiply the numerator and denominator inside the root sign in eqn. (22·15) by m, the mass of the particle, we get

$$T = 2\pi\sqrt{\frac{m}{mf/x}}$$

But $mf = P$, the restoring force on the particle. Thus,

$$T = 2\pi\sqrt{\frac{m}{P/x}} \qquad \cdots \qquad \cdots \qquad (22{\cdot}16)$$

$$\text{or, Period} = 2\pi\sqrt{\frac{\text{Mass}}{\text{Restoring force at unit displacement}}}$$

From these two expressions, the period of a particle executing S. H. M. can be obtained in a simple manner. These expressions show that the *period is independent of the amplitude of vibration.*

If the frequency be n, then

$$n=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}=\frac{1}{2\pi}\sqrt{\text{Acceleration at unit displacement}}$$

(e) **Phase** : The term **'phase'** *of a particle describing S. H. M. at any instant of time, means its state of displacement and motion at that instant.* Thus, any quantity, which gives us the displacement and the direction of motion at any instant of an oscillating particle, may be taken as a measure of the phase of the particle at that instant.

If we look at the expressions for displacement, velocity and acceleration of a particle executing S.H.M., we shall see that if the factor ωt is known, then the values of these quantities at any instant t are determined. Hence, phase of a vibrating particle at an instant t is given by the angle ωt.

So, phase of a particle executing S.H.M. at any instant of time is measured by the angle described by the reference point. Since $\omega=2\pi/T$, hence the fraction t/T, also provides a measure of the phase.

Epoch *is the phase of the particle at the initial instant* i. e, at $t=0$. From the general equation for S.H.M., we find that when $t=0$, the phase is $-\alpha$ or $+\alpha$, which is the epoch.

In the equation $x=a \cos \omega t$, if we replace t by $t+T$ i.e., $t+\frac{2\pi}{\omega}$, we have $x=a \cos (2\pi+\omega t)=a \cos \omega t$. Hence the displacement and direction of motion of the particle become exactly the same as the previous values. Thus, *the phases of a vibrating particle become identical after an interval of a complete period i.e.,* when they differ by 2π. Similarly, if we replace t by $t+\frac{T}{2}$ i. e., $t+\frac{\pi}{\omega}$ we have $x=a \cos (\pi+\omega t)=-a \cos \omega t$. Hence the displacement of the particle is now equal and opposite to the previous value ; its direction of motion is also opposite. The particle is now said to be in opposite phase. Thus, *the phases of a vibrating particle become opposite after an interval of half a complete period i.e., when they differ by π.*

Phase difference exists when we consider S.H.Ms. which are not in step with each other. This indicates by how much angle or by how much time, given as a fraction of the period, one is ahead of the other. Thus, if the equations of the two S.H.Ms. of the same frequency are given as

$$x_1=a_1 \cos \omega t, \qquad x_2=a_2 \cos (\omega t+\phi)$$

ϕ is then called the phase difference between the two motions.

If at every instant, the states of vibration of two S.H.Ms. be identical, they are said to be in the *same phase* [$\phi=0$]. This holds true irrespective of whether their amplitudes and velocities are equal or not. For example, if the two vibrating particles pass through their mean positions in the same direction at the same instant, they are in the same phase ; they would also attain their maximum displacements in the same direction simultaneously. If for two S. H. Ms. the vibrating particles cross the mean positions simultaneously in opposite directions, they are said to be in the *opposite phase* [$\phi=\pi$]. Obviously they would simultaneously attain their maximum displacements in opposite directions.

22·6. Energy in simple harmonic motion.

A particle performing S. H. M. possesses kinetic energy by virtue of its motion. Also since a restoring force always acts on the particle in a direction opposite to its displacement, work is done during displacement. The particle thus also possesses potential energy. If no dissipative force is present, the total mechanical energy of the particle remains constant.

If the mass of the particle be m and if at any instant of its motion, its displacement and velocity be x and v respectively, then its K.E. at that instant is given by,

$$E_k=\frac{1}{2}mv^2=\frac{1}{2}m\omega^2(a^2-x^2) \qquad \cdots \qquad \cdots \qquad (22\cdot17)$$

where $\omega=\frac{2\pi}{T}$ and a is the amplitude. The K.E. is, therefore, maximum when $x=0$ i.e., at the mean position and is zero, when $x=a$ i.e., at the extreme positions.

Now, the acceleration of the particle is ω^2x (ignoring sign), so that the restoring force on the particle at that position is $m\omega^2x$. The force at the mean position is zero. Hence, the mean force acting on the particle during this displacement is $\frac{1}{2}m\omega^2x$. The work done in moving from the mean position to this displaced position is obtained by multiplying the mean force by the displacement, which gives the P.E. (E_p) of the particle at the displaced position.

$$\therefore \quad E_p=\frac{1}{2}m\omega^2x\times x=\frac{1}{2}m\omega^2x^2 \qquad \cdots \qquad \cdots \qquad (22\cdot18)$$

So the P.E. is maximum when $x=a$ i.e., at the extreme positions and is zero, when $x=0$ i.e., at the mean position.

Alternative calculation.

This expression for the P.E. of the particle can be obtained alternatively in a more accurate way by using calculus. The force acting on the particle when it is displaced through x is $m\omega^2x$. If the particle now gets displaced by an infinitesimally short distance dx over which the force practically remains constant, the work done on the particle is given by $m\omega^2xdx$. Hence the work done on the particle in moving through x i.e., its P.E. at that position is given by,

$$E_p=\int_0^x m\omega^2xdx\frac{1}{2}m\omega^2x^2$$

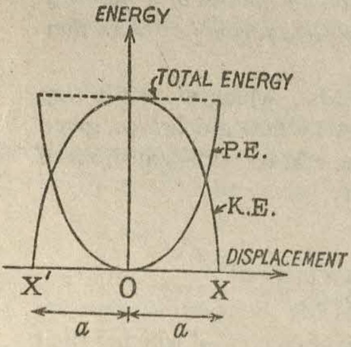


Fig. 22·8

The total mechanical energy of the vibrating particle is given by $E=E_k+E_p$

$$=\frac{1}{2}m\omega^2(a^2-x^2)+\frac{1}{2}m\omega^2x^2$$
$$=\frac{1}{2}m\omega^2a^2 \qquad \cdots \qquad \cdots \qquad (22\cdot19)$$

This is independent of the displacement x of the particle. The total mechanical energy thus remains constant during oscillation. Hence *the principle of* *conservation of mechanical energy holds good for a particle executing S.H.M.* Fig. 22·8 shows the variation of K.E., P. E. and total energy with displacement.

22·7. Examples of simple harmonic motion.

For small displacements, the motion of a simple pendulum and that of a mass hanging from a spiral spring provide two common examples of S.H.M. If a U-tube is partially filled up with a liquid and the levels are then altered by blowing into the tube, the liquid performs S.H.M. If a tuning fork is struck gently, the prongs of the tuning fork describe S.H.M. The motion of a steel ball or marble rolling back and forth inside a shallow spherical bowl is also simple harmonic. The oscillations of a magnetic needle displaced from its equilibrium position, the torsional oscillations performed by a body suspended by a twisted wire etc., provide some more familiar examples of S.H.M. We shall consider now in greater detail the first three examples cited above.

(i) **Simple pendulum** : From Fig. 22·1, the restoring force acting on the bob of the pendulum is $mg \sin\theta$. The resulting acceleration of the bob is given

$$f=-\frac{mg \sin\theta}{m}=-g \sin\theta.$$

If the angular displacement θ be very small, then $\sin\theta=\theta$, θ being measured in radians. Hence, $f=-g\theta$.

Now, if the displacement of the bob from its equilibrium position be x and if l be the length of the pendulum then $\theta=\frac{x}{l}$

$$\therefore \quad \text{acceleration} \quad f=-\frac{g}{l}x=-kx$$

where $k=g/l$ is a constant. This equation is of the same nature as eqn. (22·2). Hence, the motion of the pendulum is simple harmonic. This holds true only when θ is very small. Since the acceleration at unit displacement is equal to g/l, the period of a simple pendulum is given by

$$T=2\pi\sqrt{l/g}.$$

(ii) **An object hanging from a spiral spring** : An object, of mass m is suspended from the end of a spiral spring so that the spring gets elongated by, say, l [Fig. 22·9 (*a*) & (*b*)]. If the object is now depressed through a distance x from its equilibrium position and released [Fig. (c)], it will execute vertical oscillations. As the object is pulled down, the spring exerts on the body a restoring force P trying to bring it back to its initial position. From Hooke's law (vide Art. 9·2), this restoring force is proportional to the displacement x of the object but is oppositely directed, being given by,

$$P=-Kx$$

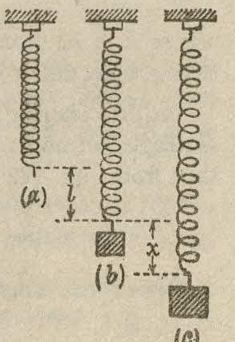


Fig. 22·9

where K is the force constant of the spring. When the object is released, it moves with an acceleration f due to the action of the force. This is given by

$$f=\frac{P}{m}=-\frac{K}{m}x=-kx$$

where m is the mass of the object. Since $k=\frac{K}{m}$ is also constant, the motion executed by the object is simple harmonic. As the acceleration at unit displacement is K/m, the period of the oscillating object is given by

$$T=2\pi\sqrt{m/K}.$$

Now, the weight mg of the object produced an elongation l of the spring.

$$\text{Hence } mg=Kl \quad \text{or, } \frac{m}{K}=\frac{l}{g} \quad \therefore \quad T=2\pi\sqrt{\frac{l}{g}}.$$

(iii) Liquid in a U-tube :

Let us consider a U-tube of uniform bore partially filled up with a liquid upto a level AB indicated in the figure by the dotted line [Fig. 22·10]. The level of liquid in the right hand arm is depressed by some means through a distance say, x. As a liquid is practically incompressible and as the cross-section of the tube is uniform, the liquid in the left hand arm will also rise through a distance x. Hence a difference in height of $2x$ is established between the levels of the liquid in the two arms. If the agent producing the depression of the liquid level in the right hand arm is removed, the weight of the liquid column of height $2x$ tries to restore the liquid levels in both arms to the initial positions. The liquid levels will be found to be executing oscillatory motion. Now

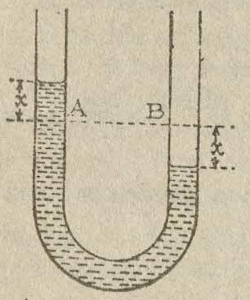


Fig. 22·10

the restoring force $=$Wt. of the liquid column of height $2x$

$$=-2x\,A\rho g$$

where A is the cross-section of the tube, ρ is the density of the liquid and g, the acceleration due to gravity.

As the restoring force is proportional to the displacement x, hence the motion is simple harmonic. If m be the mass of the entire liquid column in the U-tube, then from eqn. (22·16), the period of the oscillating liquid column is given by

$$T=2\pi\sqrt{\frac{m}{2A\rho g}}$$

Now if the length of the complete liquid column inside the U-tube be L, then

$$m=LA\rho$$

$$\therefore \quad T=2\pi\sqrt{\frac{L}{2g}}.$$

Example. 22·1. *The equation of motion of a point is given as* $x=4A\cos\left(100\,\pi t+\frac{4\pi}{7}\right)$. *Find its (a) amplitude, (b) period, (c) frequency and (d) epoch.*

Solution : Comparing with the equation of simple harmonic motion $x=a\cos(\omega t+\alpha)$ we get,

(a) Amplitude $a=4A$

(b) Angular velocity $\omega=100\pi$ rad./sec.

$\therefore$ period $T=\frac{2\pi}{\omega}=\frac{2\pi}{100\pi}=\frac{1}{50}$ sec.

(c) Frequency $n=\frac{1}{T}=50$ c.p.s.

(d) Epoch $\alpha=\frac{4\pi}{7}$ rad.

Example 22·2. *Write down the equation of a simple harmonic motion with an amplitude of 10 cm., a period of 4 sec. and an initial phase equal to 30°.*

Solution : Here $a=10$ cm.

$$T=\frac{2\pi}{\omega}=4 \text{ sec.}. \qquad \therefore \quad \omega=\frac{\pi}{2} \text{ rad./sec.}$$

$$\text{and} \quad \alpha=30^\circ=\frac{\pi}{180^\circ}\times 30^\circ=\frac{\pi}{6} \text{ rad.}$$

$\therefore$ The equation is

$$x=10\cos\left(\frac{\pi t}{2}+\frac{\pi}{6}\right).$$

Example 22·3. *A body weighing 100 gm. vibrates in S. H. M. with a period of 5 sec. and an amplitude of 1·5 cm. Find its velocity, acceleration, K. E. and P. E. (a) at the mean position, (b) at the end of the path and (c) at a point 1 cm. from the mean position.*

Solution : Since $T=5$ sec., $\omega=2\pi/T=2\pi/5=1{\cdot}26$ rad./sec.

(a) At the mean position, displacement $x=0$. $\therefore$ From eqn. (22·13), we have (ignoring sign) $v=a\omega=1{\cdot}5\times1{\cdot}26=1{\cdot}89$ cm./sec. From eqn. (22·14), when $x=0$, $f=\omega^2x=0$. Also, from eqns. (22·17) and (22·18), we have K.E. $=\frac{1}{2}mv^2=\frac{1}{2}\times100\times1{\cdot}89^2=178{\cdot}7$ erg., P.E. $=\frac{1}{2}m\omega^2x^2=0$.

(b) At the end of the path, $x=a$. Hence $v=0$, and $f=\omega^2a=1{\cdot}5\times1{\cdot}26^2=2{\cdot}38$ cm/sec.^{2} K.E. $=0$ and P.E. $=$ K.E. of body at the mean position $=178{\cdot}7$ erg.

(c) When, $x=1$ cm., then $v=\omega\sqrt{a^2-x^2}=1{\cdot}26\sqrt{1{\cdot}5^2-1^2}=1{\cdot}41$ cm./sec. and $f=\omega^2x=1{\cdot}26^2\times1=1{\cdot}58$ cm./sec.^{2}. K.E. $=\frac{1}{2}\times100\times1{\cdot}41^2=99{\cdot}2$ erg. P.E. may be found from the fact that the sum of P.E. and K.E. remains constant.

$\therefore$ P.E. $=178{\cdot}7-99{\cdot}2=79{\cdot}5$ erg.

Example 22·4. *A ball weighing 200 gm. is attached to one end of a vertical spring, the other end of which is clamped. The ball is pulled down through 5 cm.*

by applying a force of 50 gm. wt. vertically downwards and then released. Find the force constant of the spring and the period of the resulting S.H.M.

Solution : Force constant $K=\frac{P}{x}=\frac{50\times980}{5}=9800$ dyne/cm.

$$\text{Period } T=2\pi\sqrt{\frac{m}{K}}=2\times3{\cdot}14\times\sqrt{\frac{200}{9800}}=0{\cdot}9 \text{ sec. (nearly)}$$

Example 22·5. *A particle is executing S.H.M. in a line 6 cm. long. Its velocity when passing through the centre of the line is 12 cm./sec. Find its period.*

Solution : Amplitude of the partcle $a=\frac{6}{2}=3$ cm. From eqn. (22·13), its velocity at the mean position ($x=0$) is

$$v=a\omega \quad \text{(ignoring sign)}$$

$$\text{or,} \quad 12=3\times\omega \qquad \therefore \quad \omega=4 \text{ rad./sec.}$$

$$\text{Hence, period } T=\frac{2\pi}{\omega}=\frac{2\pi}{4}=1{\cdot}57 \text{ sec.}$$

Example 22·6. *A simple pendulum of length 80 cm. is oscillating with an amplitude of 4 cm. Calculate the velocity of the bob as it passes through the mid.point of its oscillation.*

Solution : The period $T=2\pi\sqrt{l/g}$, $\therefore$ $\omega=2\pi/T=\sqrt{g/l}$. Putting $g=980$ cm./sec.^{2} and $l=80$ cm., we get $\omega=7/2$ rad/sec. At the midpoint of oscillation, the displacement of the bob is zero. Hence, its velocity is $v=a\omega=4\times\frac{7}{2}=14$ cm./sec.

Example 22·7. *A body of mass 5 gm. vibrates in S.H.M. with a frequency of 36 vib./min. and an amplitude of 25 cm. Find the restoring force at an extremity.*

Solution : We have $n=36$ vib./min.$=3/5$ Hz

$$\therefore \quad \omega=2\pi n=6\pi/5 \text{ rad./sec.}$$

Now, acceleration at the extreme point$=\omega^2 a$

$$=\frac{36\pi^2}{25}\times25=36\pi^2 \text{ cm./sec.}^2$$

$$\therefore \quad \text{the restoring force}=5\times36\pi^2=1774 \text{ dyne.}$$

Example 22·8. *A particle of mass 4 gm. performs S.H.M. with a period of 2 sec. and ampltiude of 10 cm. (i) Calculate its displacement, velocity and acceleration when it is one-sixth period from one extremity. Find also the force acting on it at this position. (ii) Find the time that elapses when the particle is 7 cm. from the centre of oscillation.*

Solution : (i) Here $T=2$ sec., $a=10$ cm., $m=4$ gm.

Since the time is counted from one extreme point of the oscillation.

$$\therefore \quad \text{displacement } x=a\cos\omega t=10\cos\frac{2\pi}{T}\cdot\frac{T}{6}$$

$$=10\cos\frac{\pi}{3}=5 \text{ cm.}$$

$$\text{velocity } v=a\omega \sin \omega t=10.\frac{2\pi}{2}.\ \sin\frac{\pi}{3}$$

$$=10\pi\times 0{\cdot}866=27{\cdot}2 \text{ cm./sec.}$$

acceleration $f=\omega^2x=\pi^2\times 5=49{\cdot}3$ cm./sec^2.

Force $=P=mf=4\times 49{\cdot}3=197{\cdot}2$ dyne towards the centre of oscillation.

(ii) In this case, $7=10 \cos \omega t=10 \cos \pi t$

$\therefore \cos \pi t=0{\cdot}7=\cos 45{\cdot}57°$

$$\text{But } 45{\cdot}57°=\frac{45{\cdot}57\times\pi}{180}\text{rad.}$$

$$\therefore \quad \pi t=\frac{45{\cdot}57\times\pi}{180} \text{ or, } t=\frac{45{\cdot}57}{180}=0{\cdot}253 \text{ sec.}$$

Example 22·9. *A body having a mass 5 gm. executes S.H.M. When the displacement of the body is 25 cm., the force acting on it is 1280 dyne. Find its period.*

Solution : Here $m=5$ gm.

$$\text{Restoring force/unit displacement}=\frac{1280}{25}=\frac{256}{5}\text{dyne/cm.}$$

$\therefore$ From eqn. (22·16),

$$T=2\pi\sqrt{\frac{5}{256/5}}=2\pi\times\frac{5}{16}=1{\cdot}96 \text{ sec.}$$

Example 22·10. *5 kg. of mercury (density=13·6 gm./c.c.) is poured into a glass U-tube with uniform bore of 1·6 cm. diameter. When slightly disturbed, the mercury oscillates up and down freely about the position of equilibrium. Find the period.*

Solution : Cross-section of the tube $=\frac{\pi}{4}(1{\cdot}6)^2=0{\cdot}64\pi$ cm^2.

If the total length of the mercury column be L, then

$$L\times 0{\cdot}64\pi\times 13{\cdot}6=5\times 10^3$$

$$\text{Period } T=2\pi\sqrt{\frac{L}{2g}}=2\pi\sqrt{\frac{5\times 10^3}{0{\cdot}64\pi\times 13{\cdot}6\times 2\times 980}}=3{\cdot}68 \text{ sec.}$$

Example 22·11. *Some sand is sprinkled on a horizontal membrane which can be made to vibrate vertically with S.H.M. When the amplitude is 0·5 cm., the sand just fails to make continuous contact with the membrane. Explain the phenomenon and calculate the frequency of vibration.*

Solution : If the sand is not on the membrane, it would fall downwards with acceleration $g=980$ cm./sec^2. For the sand to remain in contact with the vibrating membrane, the maximum downward acceleration of the membrane must then be less than or just equal to g. Now, as the amplitude of vibration of the membrane increases, the maximum acceleration $(=\omega^2a)$ also increases. At the instant the maximum acceleration of the membrane becomes just greater than g, the sands will loose their contact with the membrane, Hence, the required frequency is given by

$$\omega^2 a = g \qquad \text{or, } \omega^2 = g/a$$

Since, $\omega = 2\pi n$, we have $n = \frac{1}{2\pi}\sqrt{\frac{g}{a}}$

$$= \frac{1}{2\pi}\sqrt{\frac{980}{0{\cdot}5}} \doteq 7 \text{ cycles per sec. (approx.)}$$

22·8. Graphical representation of S.H.M.

Let us consider the equation of S.H.M., $x = a\cos\omega t$. The velocity and acceleration of the particle are given respectively by $v = -a\omega\sin\omega t$ and $f = -a\omega^2\cos\omega t$. Since all these quantities change with time we can represent their variations graphically by plotting these quantities against time. Now,

(i) When $t = 0$, $\omega t = 0$ and $\cos\omega t = 1$, $\sin\omega t = 0$

Therefore, $x = a$, $v = 0$ and $f = -a\omega^2$

So the displacement and acceleration of the particle are maximum while its velocity is minimum.

(ii) When $t = \frac{T}{4}$, $\omega t = \frac{2\pi}{T}\cdot\frac{T}{4} = \frac{\pi}{2}$ and $\cos\omega t = 0$, $\sin\omega t = 1$. Hence, $x = 0$. $v = -a\omega$ and $f = 0$.

Thus, the displacement and acceleration of the particle are minimum but its velocity is maximum.

(iii) When $t = \frac{T}{2}$, $\omega t = \frac{2\pi}{T}\cdot\frac{T}{2} = \pi$ and $\cos\omega t = -1$, $\sin\omega t = 0$. So, $x = -a$, $v = 0$ and $f = a\omega^2$.

The displacement and acceleration of the particle are now again maximum but in opposite sense and its velocity is again minimum.

(vi) When $t = \frac{3T}{4}$, $\omega t = \frac{2\pi}{T}\cdot\frac{3T}{4} = \frac{3\pi}{2}$ and $\cos\omega t = 0$, $\sin\omega t = -1$.

Therefore, $x = 0$, $v = a\omega$, and $f = 0$.

So, the displacement and acceleration have again become minimum while the velocity is again maximum but is oppositely directed.

(v) When $t = T$, $\omega t = \frac{2\pi}{T}\cdot T = 2\pi$ and $\cos\omega t = 1$, $\sin\omega t = 0$.

Hence, $x = a$, $v = 0$ and $f = -a\omega^2$

The quantities, therefore, have again attained the initial values.

With further increase in time, the above variations are repeated over and over again. For convenience, the schemes of variation are shown in tabular form below :

Time	0	$T/4$	$T/2$	$3T/4$	T
Displacement	a	0	$-a$	0	a
Velocity	0	$-a\omega$	0	$a\omega$	0
Acceleration	$-a\omega^2$	0	$a\omega^2$	0	$-a\omega^2$

If now the quantities are plotted against time, curves shown in Fig. 22·11 are obtained. The curves are obviously sinusoidal. The displacement curve [Fig. 22·11 (*a*)] and the acceleration curve [Fig. 22·11 (*c*)] are cosine curves. It should be noted that they are similar but reciprocal in form ; this represents the fact that acceleration is always directed opposite to the displacement. The velocity curve [Fig. 22·11 (*b*)] is a sine curve.

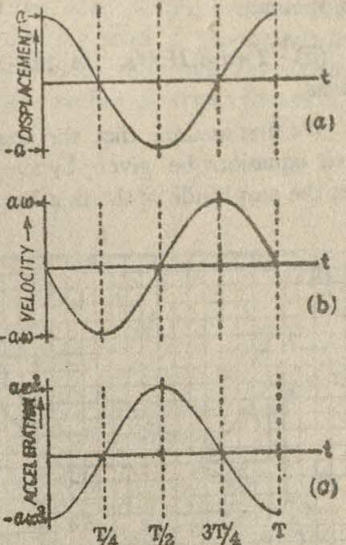


Fig. 22·11 : Graphs of displacement, velocity and acceleration of a particle executing S.H.M.

22·9. Superposition of two S.H.Ms. along the same line.

Let us suppose that two forces are simultaneously impressed on a particle. The nature of the forces is such that if impressed separately, each would cause the particle to describe a separate S. H. M., the two S.H.Ms. thus produced being in the same line. The motion of the particle will then be the resultant of the two simultaneous S. H. Ms. impressed upon it. Since the displacements produced by these two S.H.Ms. are in the same line the resultant displacement at any instant is simply the algebraic sum of their values at that instant. Thus the ordinate of the resultant displacement curve at each point is the algebraic sum of the ordinates of the component vibrations at the same point. We shall consider only the following two simple cases :

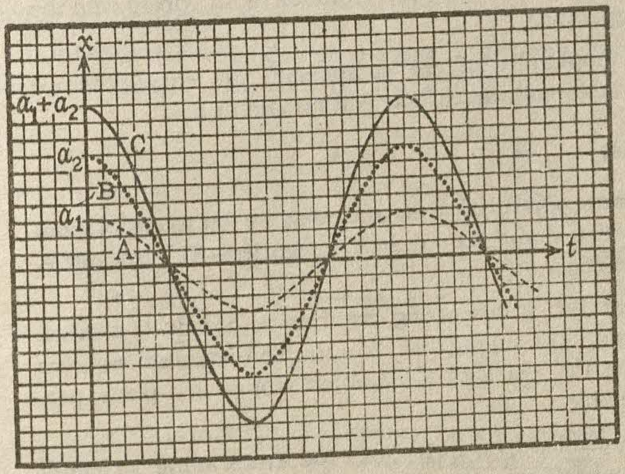


Fig. 22·12

(i) *Two S.H.Ms. in the same line having the same period, same phase but different amplitudes.*

Let the equations of the two S.H.Ms. be given by $x_1=a_1 \cos \omega t$ and $x_2=a_2 \cos \omega t$. The displacement curves of them are drawn as shown in Fig. 22·12. The curves *A* and *B* represent the first and the second component respectively. The resultant curve *C* is then obtained by simple algebraic addition of the ordinates at each point of the two components. So the resultant displacement at an instant is given by $x=x_1+x_2=(a_1+a_2) \cos \omega t$. It is obvious that the resultant motion of the particle is also S.H.M. with amplitude equal to the sum of the amplitudes of the components and of same period and same phase as those of the components.

(ii) *Two S.H.Ms. in the same line having the same period but opposite phase.*

We first assume that the amplitudes of the two S.H.Ms. are different and let their equations be given by $x_1=a_1 \cos \omega t$ and $x_2=a_2 \cos (\pi+\omega t)=-a_2 \cos \omega t$. Let the amplitude of the first be greater than that of the second i.e., $a_1>a_2$. As before, the displacement curves *A* and *B* of the two components are drawn [Fig. 22.13] and the resultant curve *C* is obtained by simple algebraic addition of the ordinates of the components at each point. In this case, the resultant displacement at any instant is given by $x=x_1+x_2=(a_1-a_2) \cos \omega t$. So the resultant motion of the particle is also S.H.M. having the same period as the components, but the amplitude being equal to the difference of their amplitudes. Also the resultant vibration is in phase with the component having greater amplitude.

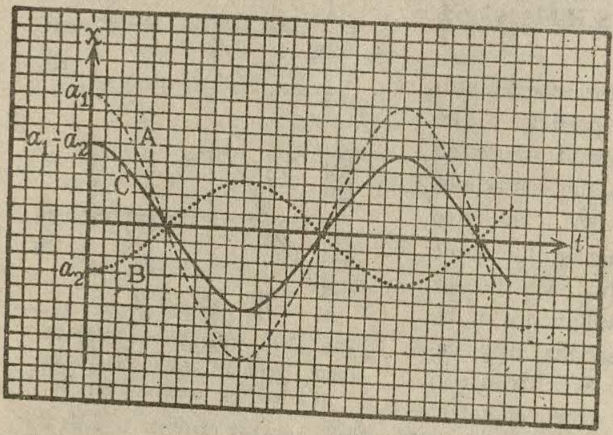


Fig. 22·13

It is obvious that if the amplitudes of the components are equal, the two S.H.Ms. will cancel out each other and the particle will remain at rest.

22·10. Nature of vibrations.

When properly excited any object of *finite size* may be set into vibration. Such vibrations may be classified into three distinct categories—(i) *transverse,* (ii) *longitudinal* and (iii) *torsional.*

The vibrations of a plucked or struck string fixed at both ends, of a struck metal strip clamped at one end, of a tuning fork, of a simple pendulum etc., are transverse. In such vibrations, the particles of the object vibrate in a plane perpendicular to the length of the object.

The vibrations of a mass suspended by a spring, of a rod struck along its length of air columns in musical instruments etc., are longitudinal. The particles of the object then vibrate along the length of the object.

Example of torsional vibration is provided by the motion of a loaded wire when it is released after twisting.

22·11. Free and Damped vibrations.

Whenever a body is set into vibrations, then in the absence of any externally impressed force upon it, the body will go on vibrating with a characteristic frequency of its own. The frequency depends upon the mass, shape and the elastic properties of the vibrating body. The vibration is then said to be **free vibration.** The corresponding period is termed the *free* or *natural period* and the frequency, the *natural frequency* of the vibrating body. Obviously such vibrations should continue indefinitely with the same amplitude. The displacement curve for such vibration is shown in the Fig. 22·14.

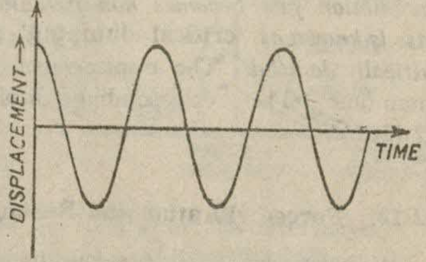


Fig. 22·14 : Displacement curve for free vibration

However, *a free vibration can not exist in practice because frictional forces opposing the motion of the vibrating body are always present. The energy of the vibrating body, therefore, gets continually dissipated. Its amplitude of vibration decreases regularly and the body finally comes to rest. Such vibrations are called* **resisted** or **damped vibrations.** For example the oscillations of a simple pendulum gradually die away due to the resistance offered by air to its motion.

In most cases, the frictional forces are of the viscous type and to work out the theory of such damped vibrations, the frictional force is assumed to be proportional to the velocity of the body. We have already seen that this is true for bodies moving with a small velocity through a fluid, e.g., air (vide Art. 12·3). The restoring force is taken to be proportional to the displacement i. e., this motion is a damped S.H.M.

It is found that when damping is small, the body oscillates with an amplitude which diminishes with time [Fig. 22·15 (*a*)]. *The frequency of such damped*

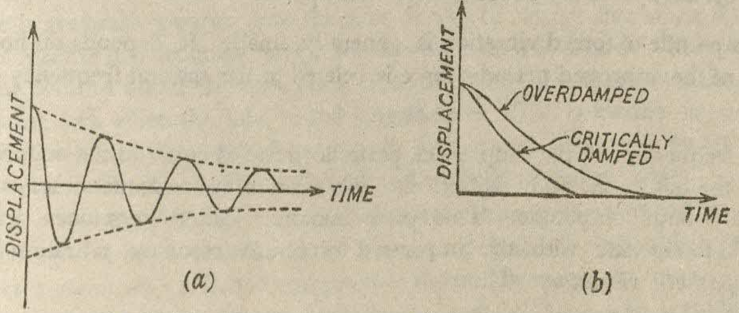


Fig. 22·15 : Displacement curve for (a) small damping
(b) heavy and critical damping

scillation is slightly less than the natural frequency of the vibrating body. Such diminution in frequency being usually very small, it may be neglected and consequently the frequency of the damped vibration executed by the body may be taken to be equal to its natural frequency.

When the motion of the body is heavily damped, it does not oscillate but its displacement gradually dies down to zero. The corresponding displacement curve is shown in Fig. 22·15 (*b*). Such motion is called **over damped, deadbeat** or **aperiodic motion**.

In between these two cases *there is a particular value of damping for which the motion just becomes non-oscillatory. The damping for this transition case is known as* '**critical damping**' and the motion of the body is said to be *critically damped.* The displacement of the body then becomes zero in a minimum time. The corresponding displacement curve is also shown in Fig. 22·15 (*b*).

22·12. Forced vibration and Resonance.

We have seen in the previous discussions that when a vibrating body is left to itself, then its vibrations gradually die away due to dissipation of energy by friction. Hence, in order to maintain its vibration, an external force must be applied so that the energy dissipated may be recouped. Familiar examples are provided by the maintenance of the vibrations of the pendulum of a clock by the coiled spring attached to it or of the vibrations of violin strings by a bow.

A particularly interesting case of considerable importance is when the externally impressed force is periodic. A force is said to be periodic when its magnitude and direction vary with time but become the same after a definite interval of time. The said interval is called the period of the force.

When an external periodic force is applied on a body, the period of the driving force being different from the natural period of the body, the body at first tries to vibrate with its natural frequency but very soon such vibrations completely die away and the body is then made to vibrate with the frequency of the driving force. *Such vibrations of the body with frequency equal to that of the externally impressed periodic force are called* **forced vibrations**.

The amplitude of forced vibration is generally small. It depends on how the frequency of the impressed periodic force is related to the natural frequency of the body.

If the frequency of the impressed periodic force is equal to the natural frequency of the body, the body is set into vibration almost spontaneously with a considerably large amplitude. This phenomenon is called ***resonance*** and the body is said to ***resonate*** with the impressed force. A resonant vibration is also called *sympathetic vibration*. Thus,

Resonant vibration *is a particular case of forced vibration when the frequency of the driving force equals the natural frequency of the body.* Even a very weak driving force can, in time, produce a large amplitude of motion when this condition is fulfilled.

The amplitude of forced vibration also depends on the amount of damping present. This is illustrated in the Fig. 22·16 which shows the resonance curves of a vibrating body for small, moderate and heavy damping. It is found that in each case, the amplitude of vibration is maximum when the driving frequency is equal to a particular value called the *resonant frequency*. The amplitude falls off on either side of this value. *For small damping the resonant frequency equals the natural frequency of the body. For large damping, it is slightly lower than the natural frequency.* It is also seen from the curves that greater response is obtained i. e., the resonance becomes sharper when the damping is smaller. It can be shown mathematically that in the ideal case when damping is completely absent, the amplitude of resonant vibration becomes infinitely large.

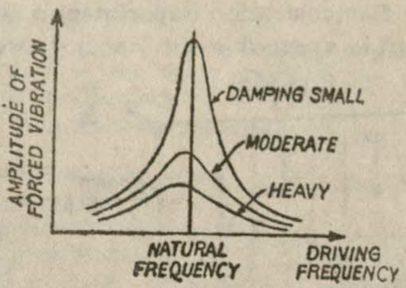


Fig. 22·16 : Resonance curves of a vibrating body

Practical Examples : When a vibrating tuning fork is held in hand, the sound emitted is almost inaudible. But when its stem is pressed on a table, a much louder sound is heard. This occurs because the vibration of the fork induces a forced vibration of the table with the same frequency. This large vibrating surface sets the neighbouring large volume of air into vibration so that the sound becomes louder.

For the same reason, stringed musical instruments are provided with a wooden box. The vibration of the string throws the box and the air inside it into a state of forced vibration. The sound emitted is, therefore, intensified. In some musical instruments like sitar, esraj etc., in addition to the main string, several side strings tuned to different notes are provided. When a note is played on the main string, the side string tuned to that particular note is thrown into resonance. Both the intensity and the sweetness of the note are thereby enhanced.

If a vibrating tunning fork is held over an empty cylindrical tube and then water is gradually poured into the tube, it will be found that when the surface of water reaches a particular level, an intense sound is heard. The reason is that the air column inside the tube above the water level is set into resonance with the vibrating fork when the loud sound is produced. This principle is utilised in the production of musical sounds in wind instruments where resonance takes place between the vibrations of air columns and of small vibrating reeds.

The most familiar example of mechanical resonance is provided by the motion of a swing. A large amplitude of swing is developed only when the series of pushes is given with a frequency equal to that of the swing.

Every structure can vibrate with its own natural frequency. While marching over a bridge, soldiers break steps, for otherwise, an accidental agreement of the frequency of their regular footsteps with the natural frequency of the bridge may take place. This would set in resonance and consequently the bridge vibrating with a large amplitude may collapse. The Tacoma Narrows Bridge in the State of Washington of U.S.A. collapsed due to resonance generated by the force of wind. Due to this reason, in designing a structure any periodic force that may act on it is always taken into account. It is so designed that its natural frequency should never become equal to that of the periodic force.

Demonstration Experiment : This experiment was devised by Barton. A cord is stretched a bit loosely between two rigid supports [Fig. 22·17]. A number of light pendulums made with small paper cones and strings of different lengths varying from 25 to 75 cm. are suspended from the cord. Due to the small mass, the motion of such a pendulum is heavily damped. Damping can be reduced by loading each paper cone with a small wire ring. A pendulum with a heavy bob and a length of 50 cm. is suspended from the same string. This acts as the driver pendulum.

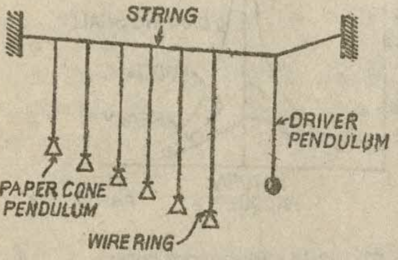


Fig. 22·17 : Barton's Pendulums

When the driver pendulum is set into vibration in a plane perpendicular to the plane in which all the pendulums are hanging, its motion is transmitted by the cord to the paper-cone pendulums. They also begin to vibrate. At first these vibrations are somewhat irregular ; ultimately all the pendulums vibrate with the same frequency as the driver but with different amplitudes. The pendulums are then executing forced vibrations.

Out of these pendulums, one whose length equals that of the driver pendulum, vibrates with the largest amplitude. This is so because their lengths being identical, the natural frequency of that paper-cone pendulum is equal to the frequency of the driver pendulum. Hence, resonance occurs between the two.

If the wire rings are now removed thereby increasing the damping, the pendulums vibrate with much reduced amplitudes and the phenomenon of resonance is found to be less pronounced.

● EXERCISE ●

[A] Essay type questions

1. Distinguish between a 'periodic motion' and a 'simple harmonic motion'. Explain with examples.

2. What do you understand by simple harmonic motion ? Define and explain the terms : (a) period, (b) frequency, (c) phase, (d) epoch and (e) amplitude of a S. H. M.

3. Deduce the equation for the simple harmonic motion of a particle. Show that the motion of a simple pendulum is simple harmonic when its amplitude of oscillation is small. Hence obtain an expression for its periodic time.

4. What are the characteristics of a simple harmonic motion ? Show that total mechanical energy possessed by a body executing S. H. M. is independent of the displacement of the body and the principle of conservation of energy holds good for the body.

5. Show that the projection of a uniform circular motion on any diameter of the circle represents simple harmonic motion. Describe an experiment to demonstrate this.

6. Derive expressions for the displacement, velocity and acceleration of a particle execu-ing S. H. M. Show that if an object hanging from a light spring is depressed slightly and then eleased, it executes S. H. M.

7. Give a precise definition of simple harmonic motion and discuss how far the motion of he simple pendulum is simple harmonic.

[*Jt. Entrance '76*]

8. A U-tube is partially filled with a liquid. When the liquid on one side is slightly depressed blowing gently down that side, show that the liquid levels in both the sides begin to execute S. H. M. Hence obtain an expression for its periodic time.

9. Show how can a S. H. M. be represented graphically.

A particle is subjected simultaneously to two simple harmonic vibrations of the same period and phase but of different amplitudes acting along the same line. Show graphically that the resultant motion is simple harmonic and find its amplitude and phase.

10. Obtain the resultant of superposition of two S. H. Ms. in the same line having the same period but opposite phase and different amplitudes.

11. Describe graphically what are meant by free and damped vibration. Explain when the vibration of a body is said to be (a) critically damped and (b) over-damped.

12. What are meant by forced vibration and resonance ? Explain with suitable examples. Describe one experiment for demonstrating the phenomenon of resonance.

13. What is resonance ? Explain resonance with a suitable example. Discuss the condition under which the forced vibration produces resonance. [*Jt. Entrance '80*]

14. Explain what you understand by the term phase of a particle executing a periodic motion and how it is expressed. Discuss whether the period of a simple pendulum should increase or decrease when the amplitude of vibration is appreciably increased (above 4°). [*Jt. Entrance '79*]

15. An external periodic force is applied on a particle executing damped S. H. M. Discuss graphically the relation of the resultant amplitude with the frequency of the applied force and hence obtain the condition for resonance.

16. If the earth were a homogeneous sphere and a straight hole were bored in it through its centre, show that a body dropped into the hole will execute a S. H. M. and calculate the time period of its vibration.

17. In the Question 16, even if the hole does not pass through the centre of the earth, prove that a body dropped in the hole will execute a S. H. M. and the time period of its vibration will remain the same.

18. If the period of revolution of the earth about its axis is the same as the period of oscillation of the body in the Question 16 or 17, show that a man at the equator feels himself weightless.

[B] Short answer type questions.

1. When the base of a vibrating tuning fork is pressed against a table the intensity of the sound is very much increased. Explain how this extra energy is obtained and show that the principle of conservation of energy is not violated. [*Jt. Entrance '75*]

2. 'All simple harmonic motions are periodic, but all periodic motions are not simple harmonic.' Explain the statement.

3. A wrist watch and a pendulum clock keep same time on the earth. If they are taken to the moon, will they keep same time ? Why ? What change in the length of the pendulum would be necessary so that both of them keep the same time ?

4. Give one example of a transverse, a longitudinal or a torsional vibration.

5 You have a light spring, a metre scale and a known mass. How will you find the time period of oscillation of the mass attached to the spring without the use of a clock ? [*I.I.T. '74*]

6. A carrom striker hits the edge of a smooth (frictionless) board perpendicularly, bounces back and strikes the opposite edge, bounces off it, and continues back to the opposite edge, and so forth. Is the striker moving with simple harmonic motion ? Justify your answer.

7. A point mass m is suspended at the end of a massless wire of length l and cross section A. If Y is Young's modulus for the wire, obtain the frequency of oscillation for the simple harmonic motion along the vertical line. [*I. I. T. '78*]

8. Explain with reasons the following :—

   (i) While marching over a bridge soldiers break steps.

   (ii) In sitar or esraj, in addition to the main string, several side strings are provided.

   (iii) Stringed musical instruments are provided with a hollow box.

[C] Simple Problems.

1. The equation of a simple harmonic motion is $x=15\sin(120\pi t-0{\cdot}2\pi)$ cm. What are the (a) amplitude, (b) period, (c) frequency and (d) initial phase of the motion ?

[*Ans.* (a) 15 cm. (b) $\frac{1}{60}$ sec. (c) 60 c. p. s. (d) $-0{\cdot}2\pi$ rad.]

2. Write the equation of a simple harmonic motion with an amplitude of 5 cm., if 300 oscillations are performed during one minute and the initial phase is 60°.

[*Ans.* $x=5\cos(10\pi t+\pi/3)$]

3. In what time after its motion begins will a point oscillating according to the equation $x=12\sin 0{\cdot}5\pi t$ move from the position of equilibrium to the maximum displacement ?

[*Ans.* 1 sec.]

4. A 2 kg. body vibrates in S. H. M. with an amplitude of 10 cm. with a period of 3·14 sec. Find the acceleration and the velocity (a) at the midpoint, (b) at the end of the path and (c) at a point 8 cm. from the midpoint.

[*Ans.* (a) 0, ±20 cm./scc. ; (b) 40 cm./sec^2., 0 ; (c) 32 cm./sec^2., ±12 cm./sec.]

5. A body weighing 40 gm. vibrates in S. H. M. with a period 4.sec. and an amplitude of 5 cm. Find its velocity, acceleration, K. E. and P. E., (a) at the mean position, (b) at the position of maximum displacement and (c) at a point 3 cm. from the mean position.

[*Ans.* (a) 7·85 cm./sec. ; 0 ; 1232 erg. ; 0 ; (b) 0 ; 12·32 cm./sec.^{2} ; 0 ; 1232 erg. (c) 6·28 cm./sec. ; 7·395 cm./sec^2. ; 788·8 erg. ; 443·2 erg.]

6. A particle executing S. H. M. in a straight line has velocities v_1, v_2 at distances l_1, l_2 from the centre of its path. Show that if T be the period of its motion,

$$T=2\pi\sqrt{\frac{l_1^2-l_2^2}{v_2^2-v_1^2}}$$

7. The equation of motion of a point is given by $x=6\sin\left(\frac{\pi}{2}t+\frac{\pi}{4}\right)$ cm. Find (a) the period, (b) the maximum velocity and (c) the maximum accleeration of the motion.

[*Ans.* (a) 4 sec. (b) 9·42 cm./sec. (c) 14·79 cm./sec^2.]

8. A particle of mass 250 gm. executing S. H. M. has velocities 150 cm./sec. and 120 cm./sec., respectively, when it is at a distance 75 cm. and 90 cm. from the centre of its path. Calculate its period and amplitude.

[*Ans.* 3·47 sec. ; 111·8 cm.]

9. A body of mass 20 gm. oscillates according to the equation $x=10\cos\left(\frac{\pi t}{5}+\frac{\pi}{6}\right)$ cm. Find the maximum force acting on the body and the total energy of the body.

[*Ans.* 78·88 dyne. ; 394·4 erg.]

10. If a seconds pendulum is oscillating with an amplitude 5 cm., find the velocity of the bob as it passes through the mid-point of its oscillation.

[*Ans.* 15·7 cm./sec.]

11. A simple pendulum of length 2 m. oscillates with an angular amplitude of 4°. Find the velocity of its bob when it passes through the position of equilibrium. [*Ans.* 30·9 cm./sec.]

12. What is the ratio between the K. E. of a simple harmonically oscillating point and its P. E. for the moments when the displacement of the point from the position of equilibrium is (i) $x=\frac{a}{4}$ (ii) $x=\frac{a}{2}$ and (iii) $x=a$ where a is the amplitude of oscillations.

[*Ans.* (i) 15 : 1, (ii) 3 : 1, (iii) 0]

13. What is the ratio between the K. E. of a simple harmonically oscillating point and its P. E. for the moments of time counted from an extreme point (i) $t=\frac{T}{12}$ (ii) $t=\frac{T}{8}$ and (iii) $t=\frac{T}{6}$ where T is the period of oscillation. The initial phase is zero.

[*Ans.* (i) 1 : 3 (ii) 1 : 1 (iii) 3 : 1]

14. A body weighing 8 kg. is suspended by a long and light spiral spring. An added force of 1 kg. stretches the spring 10 cm. Calculate (a) the force constant of the spring and (b) the period of vibration of the body if pulled down a little and then released.

[*Ans.* $9{\cdot}8\times10^4$ dyne/cm. ; 1·8 sec.]

15. A body having a mass of 10 gm. executes S. H. M. When the displacement of the body is 4 cm., the force acting on it is 8 gm. wt. Calculate the period. If the maximum velocity is 280 cm. per sec., find the amplitude and the maximum acceleration.
[*Ans.* 0·45 sec., 20 cm., 3920 cm./sec.^{2}]

16. A simple pendulum, 80 cm. long, swings to and fro through a distance 4 cm. Calculate its velocity at its lowest point, its acceleration at its highest point and the period.
[*Ans.* 7 cm./sec., 24·5 cm./sec.^{2}., 1·8 sec.]

17. A 40 gm. body hung on a spring causes it to elongate 1 cm. When another body is hung on the spring and set into oscillation, its period is 0·628 sec. What is the mass of the second body ? [*Ans.* 392 gm.]

18. Determine the maximum speed of the prong of a tuning fork of frequency 250 when vibrating with an amplitude of 2 mm. What fraction of the energy of vibration is lost when the amplitude has decreased to 1 mm. ? [*Ans.* 314 cm./sec., 0·75]

19. A body of mass 3 gm. performs S. H. M. with a period of 4 sec. and an amplitude of 5 cm. Determine the maximum velocity and maximum acceleration. Calculate also the velocity and acceleration when it is one-twelfth period from one extremity. Find the net force on it at this position.
[*Ans.* 7·85 cm./sec., 12·32 cm./sec.^{2}, 3·925 cm./sec., 10·67 cm./sec^2., 32 dyne]

20. A 49 gm. body vibrates in S. H. M. with a frequency of 24 vib./mim. and an amplitude of 12 cm. Calculate the restoring force at the instant its displacement is 5 cm. [*Ans.* 1546 dyne]

21. A ball of mass 10 gm. oscillates with a frequency of 4 c. p s. when suspended from a spiral spring of negligible mass. When the ball is hanging at rest, how much downward pull is needed to stretch the spring an additional 2 cm. ? [*Ans.* $1\cdot262\times10^4$ dyne]

22. A mass of 200 gm. when hung on a long and light spiral spring stretches it 8 cm. Calculate the period of vibration if pulled down a little and then released. [*Ans.* 0·57 sec.]

23. Equation of motion of a particle executing S. H. M. is $3f+12x=0$. Calculate the angular velocity of motion. [*Ans.* 2 rad./sec.]

24. A heavy brass sphere is hung from a spiral spring and executes vertical oscillations with period *T*. The sphere is now immersed in a ***non viscous*** liquid with a density one-tenth that of brass. What will be its period when it is set into vertical vibrations with the ball remaining inside the liquid all the time ? [*Ans.* *T*]

25. A wooden block of uniform cross-section 10 sq. cm. floats upright in water. The volume of the immersed portion of the block is 200 cc. If the block is slightly depressed and then released, find the frequency of its oscillation. ($g=980$ cm./sec.^{2}). (***Jt. Entrance '83***)
$\left[\textit{Ans.}\ \frac{7}{2\pi} \right]$

26. A particle is dropped down in a deep hole which extends to the centre of the earth. Calculate its velocity at a depth of one kilometre from the surface of the earth. Assume $g=$ 10 m./sec^2 and radius of the earth=6400 km. [*Ans.* 141·4 m./sec.]

27. A helical spring elongates 10 cm. when subjected to a tension of 5×10^5 dynes. Find the mass which should be attached to the bottom of the spring so that when pulled down and released the mass will vibrate exactly twice per second. Find also its maximum velocity when the amplitude of vibration is 1 cm. (*Jt. Entrance '79*) [*Ans.* 316·5 gm ; 12·56 cm./sec.]

[D] Harder Problems.

1. A body of mass 10 gm. executes S. H. M. with amplitude 5 cm. and total energy 320 erg. Write its equation of motion if its initial phase is 45°. [*Ans.* $x=5\cos(1\cdot6t+0\cdot25\pi)$]

2. A body executing S. H. M. has an amplitude of 12 cm. and period 2 sec. Calculate the time taken by the body to travel a distance of 6 cm. from its rest position. [*Ans.* 0·167 sec.]

3. If a particle executing S. H. M. has amplitude 20 cm., and time period 25 sec. what are

the phase and displacement of the particle at a time 30 sec. after a passage of the particle through its extreme point ? [*Ans.* $12\pi/5$, 6·18 cm.]

4. The initial phase of a S. H. M. of a particle is zero. If the particle starts from an extreme point then after the elapse of what fraction of the period will the velocity of the point be equal to half its maximum velocity. [*Ans.* 1/12]

5. Write down the eqn. of S. H. M. of a point, if the maximum acceleration of the point is 98·7 cm./sec.^{2}, the period of oscillation 2 sec. and the displacement of the point from equilibrium at the time when counting began is 5 cm. [*Ans.* $x=10\cos(\pi t\pm\pi/3)$]

6. The total energy of a body executing S. H. M. is 600 erg. and the maximum force acting on it is 150 dyne. If the period is 4 sec. and the initial phase is 60°, write down the equation of motion of the body. Calculate also its mass.

$$\left[\textit{Ans.}\quad x=8\cos\left(\frac{\pi t}{2}+\frac{\pi}{3}\right)\text{ cm. ; 7·61 gm.}\right]$$

7. A lead sphere suspended from a spring executes vertical oscillations. How will the period of oscillation change if an aluminium sphere of same size is attached to the spring instead of the lead sphere. Given, sp. gr. of lead=11·3, of aluminium=2·6.
[*Ans.* 0·48 of the period with lead sphere.]

8. A test tube of external diameter 2 cm. contains some lead shots. The total weight of the loaded test tube is 40 gm. The test tube floats vertically in a liquid. If it is now slightly pushed into the liquid and then released, it oscillates with a period of 1 sec. What is the density of the liquid ? Assume the oscillation of the test tube be undamped. [*Ans.* 0·513 gm./c.c.]

9. A spring elongates 2 cm. for a load of 80 gm. A body of mass 600 gm. is attached to the spring and displaced 8 cm. from the equilibrium position. Calculate the potential energy of the system in this position. From consideration of energy, determine the speed of the body when it is 4 cm. from the equilibrium position. [*Ans.* $125{\cdot}4\times10^4$ erg ; 56 cm./sec.]

10. A body of mass 6 kg. executes S. H. M. with an amplitude of 15 cm. The restoring force is 50 newton for a displacement of 12 cm. Calculate (i) its period (ii) maximum velocity (iii) the acceleration and its K. E. and P. E. when the displacement is 9 cm.
[*Ans.* 0·754 sec., 1·25 m./sec., 6·25 m./sec.^{2}, 3 joule, 1·7 joule]

11. A horizontal platform moves up and down in S. H. M. with an amplitude of 1 cm. What is its smallest period for which articles placed upon it will remain steadily in contact with it ? [*Ans.* 0·2 sec.]

12. If a particle P moves to and fro in a straight line between two fixed points A and B in such a way that its K. E. is always directly proportional to the product $AP.\ PB$, show that its motion is simple harmonic.

13. A particle describes S. H. M. with an amplitude 10 cm. and period 6 sec. Calculate the time that elapses as the particle passes through the points which are distant 8 cm. and 4 cm from the centre of oscillation and are on the same side of it. [*Ans.* 0·492 sec.]

14. A particle is performing a simple harmonic motion of a period T about a centre O and it passes through a point P with velocity v in the direction OP ; prove that the time which elapses before its return to P is $(T/\pi)\tan^{-1}(vT/2\pi.\ OP)$. (*London Univ.*)

15. An ideal gas is enclosed in a vertical cylindrical container and supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. Atmospheric pressure is p_0 and when the piston is in equilibrium the volume of the gas is V_0. The piston is now displaced slightly from the equilibrium position. Assuming the system is completely isolated from its surroundings, show that the piston executes simple harmonic motion and find the frequency of oscillation. (*I. I. T. '81*)

$$\left[\textit{Ans.}\quad \frac{A}{2\pi}\sqrt{\frac{\gamma p_0}{MV_0}}\right]$$

23 CHAPTER

WAVES

23·1. Progressive Waves.

It is a common experience that if a stone is thrown into a pool of still water, a series of waves spread out in gradually widening circles from the point where the stone entered the water. A floating piece of wood is found to move up and down as the waves sweep past it. This indicates that in this process, some energy has been imparted to it. If the oscillatory motion of the wood is carefully observed, it will be found that it does not move along with the waves but executes up and down motion at the same place.

These observations indicate that a disturbance is created at the point where the stone enters the water and the particles of water at that point are set into up and down oscillatory motion. This motion is then imparted to the neighbouring particles because of cohesive forces. So these particles also begin to execute the same type of motion. They in turn set into motion their immediately neighbouring particles and so on. In this way the disturbance proceeds along the surface of water, and this is called a *'progressive wave'*. *Only the form of the disturbance moves forward*; the particles of water vibrate up and down about their mean positions of rest but they do not move forward along with the waves. If this were not so, the floating piece of wood would have been found to be advancing with the waves. It is obvious that as the water particles are set into motion, some energy is communicated to each of them. We thus conclude that energy imparted by the stone to the water particles is transmitted from one point to other by means of waves without any actual flow of water in the direction of waves. The above discussion are quite general and holds for all kinds of waves. We thus define *a* **progressive wave** *as a disturbance that moves through a medium transferring energy from one point to another without any physical transport of the material between the points.*

As a wave propagates through a medium, the particles of the medium are thrown into identical type of oscillatory motion about their mean positions of rest. But such motions are not performed simultaneously. Each particle of the medium begins to vibrate a little later than its immediate predecessor.

From the above discussion we can also conclude that a wave propagates through a medium due to the action of the successive particles of a medium on each other. The force exerted by a particle on its immediate neighbour comes into play due to the elastic property of the medium. In other words, **such a wave can only pass through an elastic medium. It is, therefore, called an** ***elastic*** *wave*. Actually, if the particles of a medium can vibrate independently of each other, no wave would propagate.

23·2. Types of waves.

Several types of waves are possible. One type is distinguished from the other according to the nature of the motions of the individual particles of the medium. *If the motion of the particles be simple harmonic, then the corresponding wave is called a* **simple harmonic wave** or a **sinusoidal wave**. A simple harmonic wave may be of two types viz. (*i*) *transverse* and (*ii*) *longitudinal*.

Transverse wave : *When the motions of the particles of the medium are perpendicular to the direction of propagation of the wave, the wave is called a transverse wave*. The water waves discussed above provide an example of such transverse waves. If one end of a long cord is fastened to a support and the other end is moved to and fro perpendicular to its length, transverse waves will be generated and will proceed towards the fixed end. Transverse waves are also produced when a stretched string is struck or plucked. Light, radiant heat and in fact, all electromagnetic waves are transverse in nature.

Fig. 23·1 represents the successive stages during the propagation of a transverse wave through a medium. *A*, *B*, *C*, *D*., etc. represent particles which

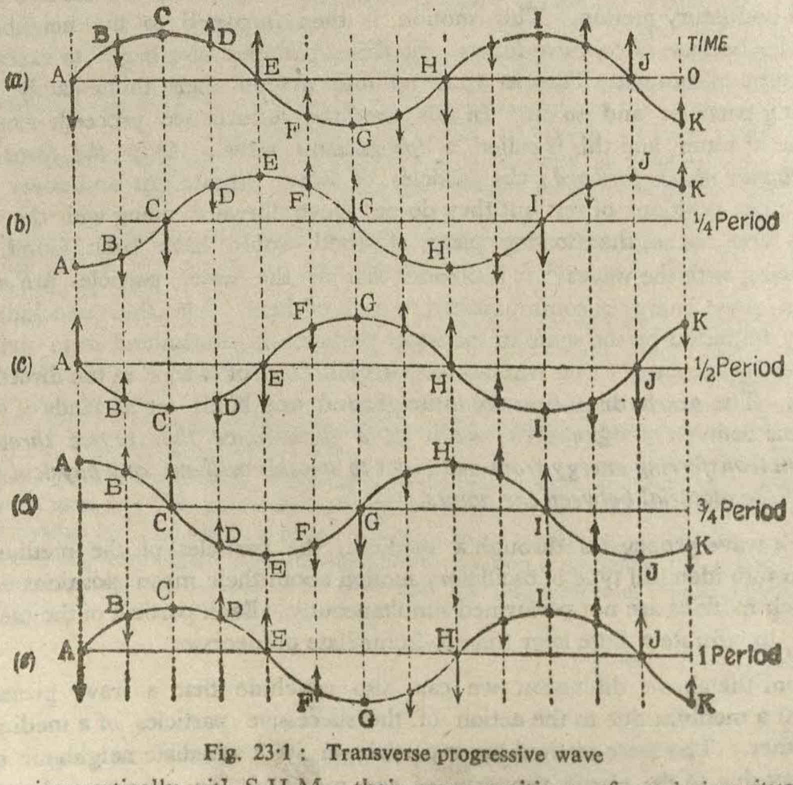


Fig. 23·1 : Transverse progressive wave

are moving vertically with S.H.M., their mean positions of rest being situated on the straight horizontal line. Each particle is vibrating with the same period and amplitude. The arrows indicate the magnitude and direction of the velocity of each particle. *At any instant, the curved line joining all the vibrating particles gives the form of the wave and is also known as the* **wave curve.** Inspection of Fig. 23·1 (*a*) shows that *it is a sine curve.* This figure represents the situation at a particular instant when the particle *A* is passing through the mean position of rest in a downward direction with the maximum velocity. The particle *B* is also moving downwards but with a somewhat smaller velocity while the particle *C* is at the end of its path and is momentarily at rest. The next three particles *D,E,F* are moving upwards, while the fourth one *G* is momentarily at rest and so on. From analogy with waves on the surface of water, the topmost point *C* is called a **crest** and the lowest point *G* is called a **trough.** The Fig. 23·1 (*b*) shows the situation quarter of a period later. The particle *A* has reached the downward extreme point of its path and is at rest at that instant. The particle *D* has already been at the top and is now moving downward. The particle *E* is now at the topmost position and is momentarily at rest and so on. It is obvious that the crest of the wave which was previously at *C* is now at *E*. Similarly, the wave curves can be constructed after $\frac{1}{2}$, $\frac{3}{4}$ and 1 period from the start [Figs. (*c*), (*d*) and (*e*)]. Study of these figures shows that as the motion of the particles continues, the wave travels more and more to the right and during one period a crest which was at *C* [Fig. (*a*)], has moved to *I* [Fig. (*e*)]. It should also be noticed that though each particle is performing exactly the same motion, every one of them is lagging behind its predecessor to the left. Because of this, a small phase difference exists between any two consecutive particles [in the figure, the value has been chosen to be 45° or $\frac{\pi}{4}$ radian]. Greater the distance separating any two particles, larger will be the phase difference between the two.

It should be noted that wave curve gives displacements at a particular instant of all the particles lying long the line of propagation of the wave. It is, therefore, called the **space-displacement curve.** But each particle is executing S.H.M. about the mean position of rest. Hence if we plot the displacement of any single particle against time, then also a sine curve results. This is called the **time displacement curve** of the particle. Thus *the time-displacement curve of a particle of the medium in which the wave is propagating is exactly identical with the spacedisplacement curve.*

In a transverse wave, the different layers of the medium move tangentially with respect to each other and hence the force exerted by one on its immediate neighbour is a shearing force. These forces can exist in solids and to a smaller extent in liquids, but not in gases. Hence *a transverse wave does not travel through a gas.*

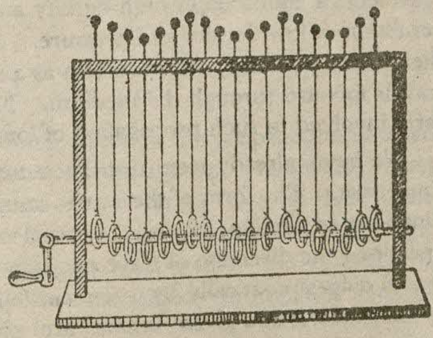


Fig. 23·2 : Wave machine

Demonstration Experiment : Transverse waves can be demonstrated with the help of an instrument known as wave machine [Fig. 23·2]. At one end of each of a number of straight vertical rods, a small ball is attached, while the other end of each rests on a wheel. The wheels are fitted eccentrically to a common spindle which can be rotated by a handle. When the handle is turned, the wheels rotate and each ball executes up and down motion but waves appear to move towards right or left, depending upon the direction of rotation of the wheels.

Longitudinal wave : *When the motions of the particles of the medium are parallel to the direction of propagation of the wave, the wave is called a longitudinal wave.*

Fig. 23·3 illustrates the propagation of a longitudinal wave through a medium. In Fig. (*a*) the vertical lines represent the undistributed positions of the layers of the medium, equidistant from each other. Fig (*b*) shows their positions at a given instant when a longitudinal wave is passing through the medium from left to right. The dots represent their undistributed positions. In this case since each particle of the medium vibrates about its position of rest parallel to the direction of propagation of the wave, no crests or troughs are produced. But in course of their movements, the particles come closer together in some places than their normal seperation in the medium and a *compression* is formed. In other places the particles move farther apart than their normal seperation and a *rarefaction* is formed. Obviously a region of compression represents a condition of high density and pressure while rarefaction represents a condition of low density and pressure. As the particles go on vibrating about their mean positions, the waveform as a succession of compressions and rarefactions travels forward through the medium. No bodily transfer of the medium is, therefore, involved in such propagation of longitudinal waves.

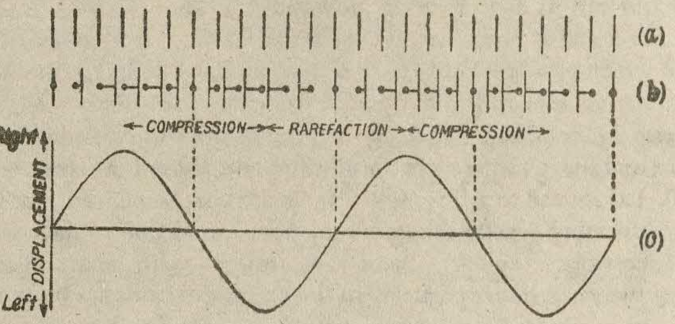


Fig. 23·3

We have already seen that the actual shape of a transverse wave is given by a sine curve. The form of the wave cannot be so obtained in the case of a longitudinal one as the line joining the actual positions of the particles will be a straight line along the direction of wave propagation. However, such wave may be represented diagrammatically by a sine curve in the following way. The displacements of the particles at a given instant are plotted in the ordinate against their mean positions shown in the abscissa. Such a curve is known as the *displacement curve* for the wave. This is shown in Fig. (*c*) where the displacements have been plotted on a magnified scale. In drawing the curve the displacements of the particles to the right of their mean positions are taken as positive and those to the left as negative. The figure shows that it is a sine curve and resembles the actual waveform of a transverse wave exactly. It is obvious that each particle excutes exactly the same periodic motion but there is a small phase difference between successive particles. The figure also shows that each compression extends from a crest to the next trough to the right of the crest and each rarefaction extends from a trough to the next crest to the right. Thus *points of maximum displacement represent the boundaries between consecutive compressions and rarefactions.* It should also be noted from the curve that *the points of no displacement are at the centres of the compressions and rarefactions where the effect is most pronounced.*

Sound waves are longitudinal waves (for a detailed discussion see Art. 24·1). Longitudinal waves are produced when one end of a long spring is made to vibrate to and fro along its length or when a long metal rod is stroke by a resined cloth.

Longitudinal waves can propagate in any medium because all substances are compressible to some extent. Thus solids, liquids and gases are all suitable for the propagation of such waves.

Demonstration Experiment.

One end of a long spring is attached to a small steel ball fixed at the end of a hacksaw blade [Fig. 23·4]. The other end of the spring is connected to a rigid support. The hacksaw blade is clamped tightly. If the steel ball is displaced slightly towards left and released, it will begin to execute S.H.M. which will be imparted to the spring. The disturbance will proceed along the spring as a longitudinal wave consisting of alternate compressions and rarefactions as shown in the lower figure.

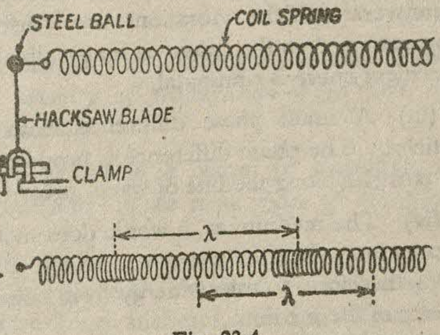


Fig. 23·4

23·3. Nature of the medium.

The nature of the medium has a profound effect on the propagation of waves through it. We have mentioned earlier that for propagation of transverse waves, the particles of the medium should be able to exert shearing force on their immediate neighbours. For this reason a transverse wave can not travel through a gas. On the other hand longitudinal waves are compressional waves. As all materials are compressible, so longitudinal waves may be transmitted through any material medium.

The speed with which a wave propagates is determined completely by the elastic property and inertia of the medium. In any particular example, the expression for the speed may be deduced by using the laws of mechanics. In general, expressions for wave speed are of the form

$$\text{wave speed} = \sqrt{\frac{\text{elastic force factor}}{\text{inertia factor}}}$$

A few expressions for the wave speed in some particular cases have been discussed in subsequent articles [vide Art. 23·12, 24·3].

Light, radiant heat, radio waves etc. are transverse waves. Here the disturbance is not mechanical but is electric and magnetic. These waves are, therefore, not elastic waves but of a separate type, called electromagnetic waves. For the propagation of electromagnetic ,waves, the presence of a material medium is not absolutely necessary ; they can propagate through vacuum. However, the speed of electromagnetic waves depends on the nature of the medium, the speed being greatest in vacuum.

23·4. Characteristics of progressive waves.

The characteristics of progressive waves may now be summarised as follows :

(i) Progressive waves are produced by the periodic vibrations of the particles of the medium. Such waves may be transverse or longitudinal depending upon the motions of the particles.

(ii) Each particle performs exactly the same vibration about their mean positions with identical frequency and amplitude. If the vibrations are performed in a direction at right angles to the direction of advance of the wave, then the wave is transverse. If the vibrations are performed parallel to the direction of wave propagation, then the wave is longitudinal. For both types of waves, the displacement curve is sinusoidal.

(iii) A small phase difference exists between the motions of successive particles. The phase difference is proportional to the distance of separation of the two particles along the line of wave propagation.

(iv) The medium as a whole does not proceed along the direction of wave propagation. It is the waveform which advances in that direction. Progressive waves, therefore, transfer energy from one place to another without any physical transfer of the medium.

(v) The velocity with which the wave propagates is different from the particle velocity. The velocity of waves is a constant for a particular medium and is determined by the elasticity and density of the medium. The velocities of the particles change continually being maximum at the mean positions and zero at their extreme positions.

23·5. Some terms and definitions.

(i) **Amplitude** : *The maximum displacement suffered by a particle from its* ***mean*** *(or equilibrium) position when a wave passes through a medium is called the* ***amplitude*** *(a) of the wave.* This is shown in Fig. 23·5.

(ii) **Wavelength** : *The shortest distance between any two particles which are in the same phase at any instant is called the wavelength* (λ). Thus in Fig. 23·5, *A* and *E* are in same phase ; so are *B* and *F* or *D* and *H* or *C* and *G* and so on. The wavelength is the distance between any of these pairs of particles.

It is obvious that *in the case of transverse waves, the distance between any two consecutive crests or consecutive troughs equals the wavelength*. Comparison with Fig. 23·3 will show that *in the case of longitudinal waves, wavelength is the distance between successive compressions or successive rarefactions*. See also Fig. 23·4.

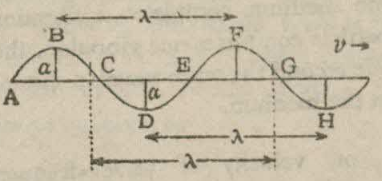


Fig. 23·5

Another important point should be mentioned. Closer examinanation of Fig. 23·1 will reveal that during the time a particle performs one complete vibration, the wave travels through a distance equal to its wavelength.

(iii) **Period** : *The time necessary for a particle in the path of the wave to execute one complete vibration is called the period* (*T*) *of the wave*. Obviously this is the same as the time required for one complete wave to pass by a given point.

(iv) **Frequency** : *The number of complete vibrations performed in one second by a particle in the path of the wave is called the frequency* (*n*) *of the wave*. Since during one complete vibration the wave moves a distance of one wavelength, frequency is also defined as the number of waves passing by any given point per second. It follows that $n=1/T$.

(v) **Velocity of propagation** : *In a medium, the distance travelled by the waveform in one second is defined as the velocity of propagation of the wave or simply the velocity of the wave* (*v*) *in that medium*.

(vi) **Wave front** : *In a medium through which a wave is travelling, a continuous surface passing through points which are in the same phase of vibration is called a wave front*.

In a homogeneous medium a wave travels with the same velocity in all directions. So for a point source of wave in a medium, the wave fronts are concentric spheres with the source at the centre. Such waves are called *spherical waves*. If the waves are confined to move on a plane, the wave fronts will be concentric circles. It is a common experience that when a stone is thrown in a pond, the waves spread out as concentric circles with the disturbed point at the centre. If the wave fronts are plane surfaces then the wave is called a *plane wave*. At large distances from the source, a small portion of the spherical wave fronts may be treated practically as a plane wave front.

(vii) **Ray** : *The direction of propagation of a wave is known as a ray*. Hence rays are represented by straight lines with an arrow. In a homogeneous medium, rays are normal to the wave front.

23·6. Relation between wavelength, frequency and velocity.

If n be the frequency and λ the wavelength, then by definition a particle of the medium completes n vibrations in one second. Since during the time a particle completes one vibration, the wave advances through a distance λ hence in one second the wave travels a distance $n\lambda$. But this is the velocity v of the wave in the medium.

$$\therefore \quad v=n\lambda \qquad \ldots \qquad \ldots \qquad (23{\cdot}1)$$

or, velocity of wave=frequency × wavelength. If T be the period, then $n=1/T$.

$$\therefore \quad \lambda=vT \qquad \ldots \qquad \ldots \qquad (23{\cdot}2)$$

Example 23·1. *A wave of frequency 100 vibrations per sec. has a wavelength of 320 cm. Find its velocity of propagation.*

Solution : $v=n\lambda=100\times320=32000$ cm./sec. $=320$ m./sec.

Example 23·2. *A body vibrating with a particular frequency generates waves having wavelength 20 cm. in a medium A travelling with velocity 160 m./sec. Calculate the wavelength of the wave it will generate in another medium B propagating with velocity 240 m./sec.*

Solution : Let n be the frequency of the wave common to both media. Then $v_A=n\times\lambda_A$ and $v_B=n\times\lambda_B$

$$\therefore \quad \frac{v_B}{v_A}=\frac{\lambda_B}{\lambda_A} \quad \text{or,} \quad \lambda_B=\lambda_A\frac{v_B}{v_A}=20\times\frac{240\times10^2}{160\times10^2}=30 \text{ cm.}$$

23·7. Equation of progressive wave.

As a wave advances through a medium, its particles execute S.H.M. about their mean positions. At any instant t, the displacement of a particle is then given by

$$y=a \sin \omega t \qquad \ldots \qquad \ldots \qquad (23{\cdot}3)$$

where a is the amplitude of vibration and $\omega=2\pi n=2\pi/T$, n being the frequency and T the period of the vibrating particle.

Now although every particle of the medium is executing identical vibrations, they are not all in the same phase. If the wave is assumed to be travelling in the positive direction of the x-axis i.e., from left to right, each particle lags behind its predecessor to the left. So at the same instant t, the displacement of another particle at a distance x to the right of the above particle is given by

$$y=a \sin (\omega-t\theta) \qquad \ldots \qquad \ldots \qquad (23{\cdot}4)$$

where θ gives the lag in phase. Now the phase difference between two particles separated by a distance of one wavelength i.e., λ is 2π.

$$\text{Hence,} \qquad \theta=\frac{2\pi}{\lambda}x$$

Substitution in the eqn. (23·4) gives

$$y=a \sin\left(\omega t-\frac{2\pi x}{\lambda}\right)=a \sin\left(\frac{2\pi t}{T}-\frac{2\pi x}{\lambda}\right)$$

$$=a \sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right) \qquad \ldots \qquad \ldots \qquad (23{\cdot}5)$$

This gives the equation of the progressive wave. If x is assigned a particular value, we get the equation of motion of the particle at that point. Again for a particular value of t; the equation gives the shape of the whole wave at that instant. It is also evident from eqn. (23·5), that both y vs. x (i.e., space-displacement curve) and y vs. t (i.e., time-displacement curve) plots are sine curves.

Since $v=n\lambda=\lambda/T$, we can write the above equation as

$$y=a\sin\frac{2\pi}{\lambda}\left(vt-x\right) \qquad \ldots \qquad (23{\cdot}6)$$

$$=a\sin 2\pi n\left(t-\frac{x}{v}\right) \qquad \ldots \qquad (23{\cdot}7)$$

Instead of a sine function, we can as well represent a S.H.M. by a cosine function. In that case exactly similar equations for a wave will be obtained ; only the sine function has to be replaced by the cosine function.

Example 23·3. *The equation of a progressive wave is given by $y=8\sin(140\pi t-0{\cdot}08\pi x)$ where x and y are in cm. and t in sec. Find the amplitude, frequency, speed and wavelength of the wave.*

Solution : Comparing with the eqn. $y=a\sin\left(\frac{2\pi t}{T}-\frac{2\pi x}{\lambda}\right)$ of a progressive wave, we get,

(a) Amplitude $a=8$ cm.

(b) $\frac{2\pi}{T}=140\pi$ $\quad\therefore\quad T=\frac{1}{70}$ sec.

$\therefore$ frequency $n=\frac{1}{T}=70$ c.p.s.

(c) $\frac{2\pi}{\lambda}=0{\cdot}08\pi$

$\therefore$ wavelength $\lambda=\frac{2}{0{\cdot}08}=25$ cm.

(d) speed $v=n\lambda=70\times 25=1750$ cm./sec.

$=17{\cdot}5$ m./sec.

Example 23·4. *Find the phase difference of two points 1 m. apart lying on a ray if the wavelength is 2 m.*

Solution : Phase difference $\theta=\frac{2\pi x}{\lambda}=\frac{2\pi\times 1}{2}=\pi$ rad.

Example 23·5. *Find the displacement from the position of equilibrium of a point lying at a distance $x=\lambda/6$ from the source of oscillations for the moment $t=T/3$. The amplitude of oscillation is 20 cm.*

Solution : Displacement $y=a\sin\left(\frac{2\pi t}{T}-\frac{2\pi x}{\lambda}\right)$

$$=20\sin\left(\frac{2\pi T}{3\times T}-\frac{2\pi\lambda}{6\times\lambda}\right)$$

$$=20\sin\pi/3$$

$$=17{\cdot}32\text{ cm.}$$

Example 23·6. *The equation of simple harmonic oscillations of a source is* $y=15 \sin 100 \pi t$ *cm. (a) Find the equation of the wave emitted, if it propagates with a velocity of 300 m./sec. (b) Write the equation of oscillation of a point at a distance of 150 m. from the source. (c) Write the equation of oscillation of the points of the wave at a moment=0·04 sec. after the oscillation begins.*

Solution : From the given eqn. we get, $\omega=100\pi$

$$\therefore \quad T=\frac{2\pi}{\omega}=\frac{2\pi}{100\pi}=\frac{1}{50} \text{ sec.}$$

$$\text{Hence } \lambda=vT=300\times\frac{1}{50}=6 \text{ m.}=600 \text{ cm.}$$

(a) The eqn. of the wave is

$$y=15 \sin\left(100\pi t-\frac{2\pi x}{600}\right) \text{cm.}$$

$$=15 \sin\left(100\pi t-\frac{\pi x}{300}\right) \text{cm.}$$

(b) Here $x=150$ m.$=150\times10^2$ cm.

$\therefore$ The eqn. of oscillation of the point is

$$y=15 \sin\left(100\pi t-\frac{\pi\times150\times10^2}{300}\right) \text{cm.}$$

$$=15 \sin(100\pi t-50\pi) \text{ cm.}$$

(c) The eqn. is

$$y=15 \sin\left(100\pi\times0{\cdot}04-\frac{\pi x}{300}\right) \text{cm.}$$

$$=15 \sin\left(4\pi-\frac{\pi x}{300}\right) \text{cm.}$$

Example 23·7. *The equation of simple harmonic oscillations of a source is* $y=10 \sin 20 \pi t$ *cm. Find the displacement from the position of equilibrium, the velocity and the acceleration of a point 10 m. away from the source of oscillation for a moment* $t=3$ *sec. after the oscillations begin. The oscillations propagate with a velocity of 200 m./sec.*

Solution.: From the given eqn. we get, $\omega=20\pi$

$$\therefore \quad T=\frac{2\pi}{\omega}=\frac{1}{10} \text{ sec.}$$

$$\text{Hence } \lambda=vT=200\times\frac{1}{10}=20 \text{ m.}=2000 \text{ cm.}$$

For the given point $x=10$ m.$=1000$ cm.

$\therefore$ The eqn. of oscillation of the point is

$$y=10 \sin\left(20\pi t-\frac{2\pi\times1000}{2000}\right) \text{cm.}$$

$$=10 \sin(20\pi t-\pi) \text{ cm.}$$

At $t=3$ sec.

(a) the displacement of the point is

$$y=10\sin(20\pi\times 3-\pi)=10\sin 59\pi=0$$

(b) the velocity $v=\omega\sqrt{a^2-y^2}$

$$=20\pi\times 10 \qquad [\because\ a=10 \text{ cm.}]$$

$$=628 \text{ cm./sec.}=6{\cdot}28 \text{ m./sec.}$$

(c) the acceleration $f=\omega^2 y=0$.

23·8. Reflection and Refraction of waves.

When energy in the form of wave is incident on the boundary separating two homogeneous media, (i) a part is reflected back into the first medium without any change in speed or wavelength and (ii) the rest is transmitted into the second medium.

In discussing the reflection of waves, it is convenient to make use of the concept of rays. In Fig. 23·6, *RR′* represents the reflecting surface on which a wave travelling in the direction *AB* is incident obliquely. Thus *AB* is the *incident ray*. After reflection, the wave travels in the direction *BC*, so that *BC* is the *reflected ray*. We draw *BN* normal to the reflecting surface at the point of incidence *B*. The angle between the incident ray and the normal is called the *angle of incidence* *i* and the angle between the reflected ray and the normal is called the *angle of reflection r*. It is found that waves are reflected according to two laws as stated below :

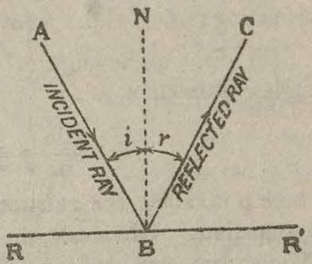


Fig. 23·6. Reflection of a wave

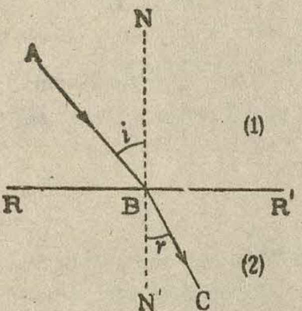
(i) The angle of incidence and the angle of reflection are equal ($i=r$), and

(ii) The incident ray *AB*, the reflected ray *BC* and the normal *BN* to the reflecting surface at the point of incidence lie on the same plane, called the plane of incidence. These laws are known as the **laws of reflection**.

Students are already familiar with these laws in the case of light. These laws are, however, true for all types of waves. For example, water waves on the surface of a pond get reflected from its edge obeying the same law. Sound waves are also reflected in accordance with the above laws giving rise to the phenomena of echo, reverberation etc. This property of sound waves is utilised in the construction of stethoscopes, speaking tubes, horns etc.

When a wave is incident on the boundary of two homogeneous media, the part of it that is transmitted into the second medium travels with a new velocity, a new wavelength and except in the special case when the incidence is normal, in a new direction. This change of direction is called **refraction**. The frequency of the wave remains unchanged after refraction.

As in the case of reflection we shall also discuss the phenomenon of refraction of waves in terms of rays. In Fig. 23·7, RR' represents the surface of separation of the two media, (1) and (2), and is called the *refracting surface*. A wave travelling in the first medium in the direction AB is incident obliquely on RR' at B. Hence AB is the incident ray. After refraction, the wave travels in the direction BC through the second medium. Thus BC represents the refracted ray. NN' is drawn normal to RR' through the point of incidence B. The angle between the incident ray and the normal is called the angle of incidence i and the angle between the refracted ray and the normal is called the angle of refraction r. It was found that the waves are refracted obeying the following two laws :



·Fig. 23·7 : Refraction of waves

(i) The incident ray AB, the refracted ray BC and the normal NN' to the refracting surface at the point of incidence lie on the same plane.

(ii) Sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction, i.e.,

$$\frac{\sin i}{\sin r} = \text{constant} = \mu \text{ (say)}$$

where μ is called the **refractive index** of the second medium with respect to the first medium. Obviously, the value of μ depends on the nature of the two media. The two laws, stated above are known **as laws of refraction**, of which the second one is particularly termed as **Snell's law** after the name of Snell, who first discovered it. It can be proved that the constant μ is equal to v_1/v_2 where v_1 and v_2 are the velocities of the wave in the first and the second medium respectively. Thus, Snell's law takes the form

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} \qquad \dots \qquad \dots \qquad (23{\cdot}7)$$

Now (a) if $v_1 > v_2$, i. e., if velocity of the wave in the first medium is greater than that in the second medium, then from eqn. (23·7),

$$\sin i > \sin r \qquad \text{or,} \quad i > r$$

i.e., the wave bends towards the normal after refraction.

(b) If $v_1 < v_2$, then from eqn. (23·7)

$$\sin i < \sin r \qquad \text{or,} \quad i < r$$

i.e., the wave bends away from the normal.

As in the case of reflection, we think of light when the term refraction is mentioned. In fact, all types of waves exhibit the phenomenon obeying the same laws. Thus refraction of sound waves by prisms and lenses of suitable materials can be demonstrated. [for a detailed discussion see Art. 24·8].

23·9. Superposition of waves.

In the foregoing articles we have considered the passage of a single wave through a medium. We shall now consider the effect when two or more waves proceed simultaneously through the same medium.

A simple law, known as the **principle of superposition** was put forward by Huygens which states that *when two or more waves are passing simultaneously through the same medium, each wave proceeds independently of the other. In the regions in which they overlap, the resulting displacement of any particle of the medium is the algebraic sum of the displacements of the particle due to the individual waves.*

Let us consider the simultaneous propagation of the waves through a medium. If one of the waves alone would have produced a displacement y_1 of any particle and if the other wave alone would have produced a displacement y_2 of the same particle, then according to this principle, the resultant displacement of the particle is given by

$$y=y_1+y_2 \qquad \ldots \qquad \ldots \qquad (23{\cdot}8)$$

The independence of the propagation of different waves is borne out by our experiences. If two stones are dropped into a pond, each set of ripples passes through the other without any change in its shape, direction or speed. The sounds of several independent conversations can be heard without any distortion. A clear photograph is obtained with a camera which shows that in their passage through the shutter opening, the light waves from different objects do not affect one another.

It should be mentioned that for any wave in a material medium, like water waves, sound waves etc., the principle holds provided the amplitudes of the waves are small. For light waves and other kinds of electromagnetic waves, the principle holds rigorously.

The superposition principle is a most general one and holds good regardless of the frequency or the direction of propagation of the individual waves. But we are mainly interested in the cases where they have equal or nearly equal frequencies. In the following articles we shall discuss three such cases, namely, stationary or standing waves, interference and beats.

23·10. Stationary waves.

When two identical progressive waves travelling in the same medium along the same straight line with the velocity but in opposite directions are superposed, they give rise to a system of waves alternately waxing and waning (expanding and shrinking), but not proceeding in either direction. Such waves are called **stationary waves** *or* **standing waves**. These waves are so called because they do not advance through the medium but remain fixed in their position.

Stationary waves may be produced by both tranverse and longitudinal waves. For example, when a string fixed at both ends is plucked, the resulting transverse waves travel along the string to both ends and are reflected. These reflected waves progress along the string in opposite direction with the same velocity and being identical in all respects with the incident wave give rise to transverse stationary waves. Similarly in an organ pipe, the longitudinal waves travelling from one end are reflected from the other. Superposition of these direct and reflected waves which are identical in all respects but oppositely moving gives rise to longitudinal stationary waves. Such waves are also produced when longitudinal vibrations are excited in a spring fixed at both ends.

In the following discussions we shall restrict ourselves to the consideration of transverse stationary waves for the sake of clarity of understanding. It should be remembered that all these discussions are equally applicable to longitudinal stationary waves, the only difference being that in the latter case the displacements and the velocities of the particles are along the direction of the wave.

In contrast to a progressive wave, the different particles in a stationary wave do not execute identical motions but each executes a S.H.M. with a particular amplitude which varies from zero at some points to a maximum at other points. *The points at which the amplitude is always zero i.e., where the particle are permanently at rest are called* **nodes.** The distance between consecutive nodes is always the same and is equal to half the wave length of the component progressive waves which produce the stationary wave. From one node to the next, the particles vibrate with amplitudes differing from point to point. The amplitude gradually increases from zero, being maximum at the points lying halfway between the nodes and then gradually decreases to zero. *These points at which the particles vibrate with the maximum amplitude are called* **antinodes.** Obviously, the distance between adjacent antinodes is also equal to half the wavelength. This maximum amplitude is called the amplitude of the stationary wave and is equal to the sum of the amplitudes of the component progressive waves. These discussions are illustrated in Fig. 23·8 where the points *N* denote the nodes and *A*, the antinodes. The arrows indicate the displacements of different particles in either direction from their mean positions.

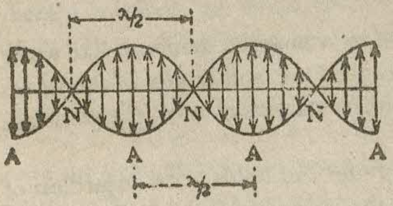


Fig. 23·8 : A stationary wave pattern

In between two nodes, all the particles vibrate in phase with one another i.e,, they achieve their maximum and zero displacements at the same time moving in the same direction. This region between two consecutive nodes is called a *loop*. In the next loop, the particles also vibrate in unison but differ in phase by 180° with those in the former i.e., the particles in adjacent loops reach their positions of maximum and minimum displacements at the same time but moving in opposite directions. This is illustrated in Fig. 23·9. The arrows indicate the magnitude and direction of velocities of the particles at the instant when they all possess zero displacements. In any loops, the arrows differ in length but are all in the same direction ; within the next loop, they

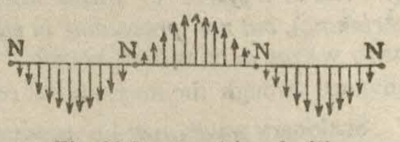


Fig. 23·9 : Particle velocities in a stationary wave are directed opposite. In the case of longitudinal stationary waves, it is found that maximum variation of pressure occurs at the nodes while at the antinodes no change of pressure takes place.

Formation of stationary waves can be explained by either of the following methods viz., (i) Graphical and (ii) Analytical.

(i) *Graphical Method.*

Fig. 23·10 shows two identical progressive waves travelling along the same line in opposite directions. One wave is shown in a continuous line and the other in a broken line. Let us consider the time $t=0$ at the instant when the two waves are in phase. The two waveforms then coincide and from the principle of superposition, the resultant waveform, drawn on a bold line, is obtained by simple addition of their ordinates at every point. This is shown in Fig. 23·10 (*a*).

One quarter of a period later, at $t=T/4$, one wave has moved through a distance $\lambda/4$ to the right, while the other has moved through the same distance to the left. At every point, the displacements corrresponding to the two waves are equal and opposite so that there is no resultant displacement. This is shown in Fig. 23·10 (*b*). Similarly the resultant waveforms at intervals of $T/4$ can be obtained and are shown in the figures 23·10. (*c*), (*d*) and (*e*).

Study of these figures shows that the resulting waveform is quite different from that of a progressive wave. It does not advance but remains confined in space. The points marked *N* remain always at rest. These are the nodes. Other points vibrate about their mean positions. Of these, the points marked *A* attain the maximum value of displacements. These are the antinodes. The figures also show that amplitude of the stationary wave is twice the amplitude of either of the progressive waves, while its wavelength is equal to that of the progressive waves. The distance between two adjacent nodes or adjacent antinodes is half the wavelength. The phase difference of π between the vibrations on the two sides of a node is also obvious from the figure.

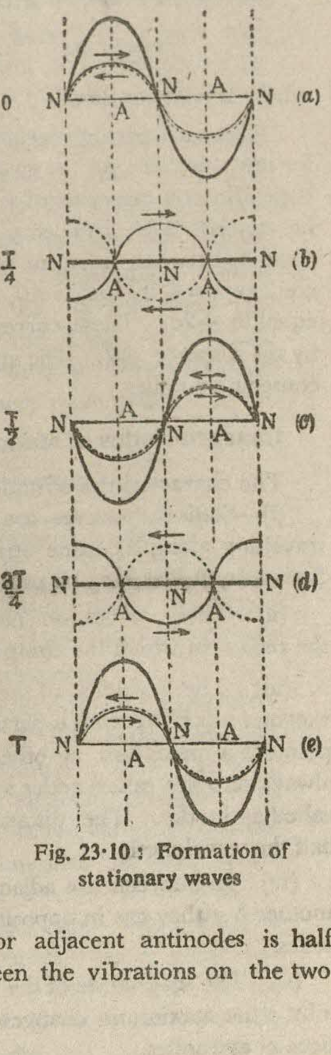


Fig. 23·10 : Formation of stationary waves

(ii) *Analytical method.*

The equation of a progressive wave travelling in the positive direction of x is given by

$$y_1=a \sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)$$

The equation of an identical progressive wave travelling in the negative direction of x is given by,

$$y_2=a \sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$$

From the principle of superposition, the resultant displacement y is given by

$$y=y_1+y_2$$

Since $\sin(A+B)=\sin A\cos B+\cos A\sin B$

and $\sin(A-B)=\sin A\cos B-\cos A\sin B$, we get

$$y=2a\cos\frac{2\pi x}{\lambda}\sin 2\pi\frac{t}{T}=A\sin 2\pi\frac{t}{T} \quad \ldots \quad \ldots \quad (23{\cdot}9)$$

where $A=2a\cos 2\pi x/\lambda$.

Thus each particle executes a S.H.M. with an amplitude A which is constant for any particle at a given point but which varies from particle to particle depending on the value of x. It follows that at the points for which $\cos 2\pi x/\lambda=0$, i.e. $x=\lambda/4, 3\lambda/4, 5\lambda/4$ etc., the amplitude is zero. These are the nodes, the distance between adjacent ones being obviously $\lambda/2$. At the points for which $\cos 2\pi x/\lambda=\pm 1$, i.e., $x=0, \lambda/2, \lambda, 3\lambda/2$ etc., the amplitude is maximum being equal to $\pm 2a$. These correspond to the antinodes which are obviously separated by the distance $\lambda/2$. The antinodes are also obviously situated midway between consecutive nodes.

Characteristics of stationary waves.

The characteristics of stationary waves may be summarised as follows :—

(i) Stationary waves are produced when two exactly alike progressive waves travelling along the same straight line but in opposite directions are superposed. Stationary waves may be produced by both transverse and longitudinal waves.

(ii) These waves do not advance through the medium but remain fixed in the region in which the component progressive waves overlap.

(iii) The particles of the medium do not execute identical motions but each executes a S.H.M. with a particular amplitude which varies from zero at some points to a maximum at other points. The points at which the amplitude is always zero are called nodes and those at which the amplitude is maximum are called antinodes. The distance between adjacent nodes or antinodes is equal to half the wavelength.

(iv) In between two adjacent nodes, all the particles vibrate in phase with one another but they are in opposite phase with the particles between the next pair of nodes.

(v) The magnitude of the change of pressure and density varies from point to point ; the maximum change occurs at nodes while the minimum change takes place at antinodes.

Demonstration Experiments.

(i) **Transverse** stationary waves can be demonstrated by the following simple experiment. One end of a string passes over a pulley and is attached to a scale pan [Fig. 23·11]. The other end is fixed to one prong of a tuning fork which vibrates perpendicular to the length of the string. The string is kept in tension by placing suitable weights on the scale pan. When the tuning fork is set into vibration, a progressive wave travels along the string and is reflected back from the other end. The incident and the reflected waves superpose and by suitable adjustment of the weights on the scale pan, stationary wave pattern with distinct nodes and antinodes can be obtained. The demonstration becomes more effective if an electrically maintained tuning fork is used.

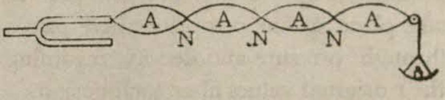


Fig. 23·11

(ii) For demonstrating longitudinal stationary waves, a long spiral of thick wire is attached to a hacksaw blade, while its other end is fixed to a rigid support [Fig. 23·12]. The hacksaw blade is clamped and set into transverse vibration. Longitudinal waves proceed along the spring and get reflected from the fixed end. On adjusting the frequency of the vibrating blade by changing the point of clamping, a stationary wave pattern can be obtained in the spring.

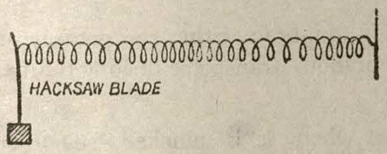


Fig. 23·12

23·11. Distinction between progressive and stationary waves.

Progressive waves	*Stationary waves*
(i) These are produced by the periodic vibrations of the portion of the medium.	(i) These are formed when two identical progressive waves travelling along the same line but in opposite direction are superposed.
(ii) Each particle of the medium executes identical periodic motion about its mean position of rest.	(ii) All the particles of the medium except those at the nodes execute periodic motions with varying amplitudes but identical periods. The amplitude of vibration varies from maximum at the antinodes to zero at the nodes.
(iii) At any instant, the phases of the particles along the line of wave propagation changes continually. The phase difference between any two particles is proportional to their distance of separation.	(iii) Between any two adjacent nodes all the particles are in phase with one another, but they are in opposite phase with the particles between the next pair of nodes.
(iv) As the particles are set into vibration one after another, a waveform proceeds through the medium.	(iv) The waveform does not proceed through the medium but remains confined within the region in which the component progressive waves overlap.

Progressive waves	*Stationary waves*
(v) At any point of the medium similar changes of pressure and density take place as one complete wave passes through pressure and density, regaining their original values after each period.	(v) The magnitude of the change of pressure and density varies from point to point. During each period, maximum change of pressure and density take place at nodes while minimum change take place at antinodes.
(vi) During a complete vibration, the particles never pass simultaneously through their mean positions of rest.	(vi) Twice in each complete vibration all the particles reach their positions of rest simultaneously.

VIBRATIONS OF STRINGS

23·12. Waves in stretched string.

A string stretched under tension can be made to vibrate either transversely or longitudinally. Depending upon the kind of vibration, transverse or longitudinal waves are set up in the string.

Transverse waves are generated in a string when it is plucked, struck or bowed. In all these methods of excitation, a certain portion of the string is made to vibrate laterally, whence transverse waves are produced. These waves travel along the length of the string with a definite velocity v. It can be proved that this velocity is given by

$$v=\sqrt{\frac{T}{m}}$$

where T is the tension and m the mass per unit length of the string.

On reaching the fixed ends of the string, these waves are reflected back in the opposite direction and are superposed on the incident wave. Superposition of these two identical progressive waves travelling along the string in opposite directions gives rise to **transverse stationery waves**. The spring, therefore, vibrates transversely with the formation of nodes and antinodes at definite points along its length.

In a similar fashion, longitudinal stationary waves may be generated in a stretched string by rubbing it along its length with a piece of resined chamois leather or wet flannel.

In stringed musical instruments, however, only the transverse vibrations of strings are excited. In the following sections, we shall, therefore, discuss only the transverse vibrations of a string.

23·13. Frequency of transverse vibration of a string.

When a stretched string fixed at both ends is set into transverse vibration, stationary wave patterns are formed due to the superposition of incident and reflected waves travelling up and down the string. Evidently the two fixed ends of the string remain permanently at rest and as such these are nodes. In the simplest or *fundamental mode of vibration* of the string, a single antinode is formed at its middle [Fig. 23·13]. The string then vibrates up and down as a whole. Since the distance between two consecutive nodes is equal to half the length of a wave, we have

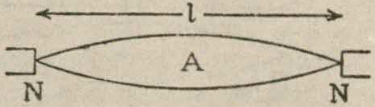


Fig. 23·13

$$l=\lambda/2$$

where l is the length of the string. If n be the frequency of vibration of the string, then

$$v=n\lambda$$

where v is the velocity of the transverse wave along the string. But $v=\sqrt{T/m}$; therefore,

$$n=\frac{v}{\lambda}=\frac{1}{\lambda}\sqrt{\frac{T}{m}}$$

$$\text{or,}\quad n=\frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \cdots \quad \cdots \qquad (23{\cdot}10)$$

This expression gives the frequency of the simplest or fundamental mode of transverse vibration of a string.

If the radius of the string be r and ρ be the density of its material, then $m=\pi r^2\rho$. Hence

$$n=\frac{1}{2l}\sqrt{\frac{T}{\pi r^2\rho}}=\frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}} \qquad \cdots \quad \cdots \qquad (23{\cdot}11)$$

23·14. Laws of transverse vibrations of strings.

The laws of transverse vibrations of strings follow immediately from the eqn. (23·10). These were first given by the French mathematician Mersenne in the year 1636 and as such are also called *Mersenne's laws.*

(i) **Law of length :** *Tension remaining constant, the frequency of transverse vibration of a particular string varies inversely as its length.*

Symbolically, $n \propto 1/l$, when T and m are constants.

(ii) **Law of tension** : *For a given length of a particular string, the frequency of transverse vibration varies directly as the square root of tension, i.e.,* $n \propto \sqrt{T}$, when l and m are constants.

(iii) **Law of mass** : *For different strings of the same length and stretched by the same tension, the frequency of transverse vibration varies inversely as the square root of the mass per unit length of the strings.* Symbolically, $n \propto \frac{1}{\sqrt{\ }}$ when l and T are constants.

23·15. Modes of vibration of a stretched string.

Till now we have considered the simplest type of vibration executed by a string. Actually a string can vibrate in a number of ways with different frequencies. These are called the *modes of vibration.* The one with the lowest frequency is called the *fundamental* mode of vibration. In all modes of vibration, there must be a node at each end of the string.

In the simplest possible mode of vibration, in addition to these nodes at the ends, a single antinode is formed in the middle [Fig. 23·14 (*a*)]. The string then vibrates as a whole. This case was discussed earlier. The frequency of vibration n is the lowest and is given by the eqn. (23·10). This mode of vibration is the *fundamental.*

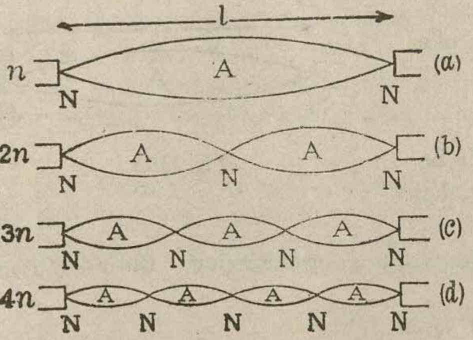


Fig. 23.14 : Modes of vibration of a streched string

In the next possible mode of vibration one node and two antinodes are formed in the intermediate region. The string then vibrates in two segments [Fig. 23·14 (*b*)]. If l be the length of the string, then in this case $l=\lambda$. Hence the frequency of vibration is given by

$$n_2=\frac{v}{\lambda}=\frac{1}{l}\sqrt{\frac{T}{m}}=2n$$

The frequency is thus twice that of the fundamental.

The next mode of vibration occurs when two nodes and three antinodes are formed in between the nodes at the ends. The string then vibrates in three segments [Fig. 23·14 (*c*)]. In this case $l=3\lambda/2$ and the corresponding frequency is given by

$$n_3=\frac{v}{2l/3}=\frac{3}{2l}\sqrt{\frac{T}{m}}=3n$$

Hence the frequency is thrice that of the fundamental. In this way the frequencies of the other modes of vibration can be obtained.

It is obvious that in, general, if the string vibrates in p segments, the frequency of vibration will be p times the frequency of the fundamental and is given by

$$n_p=\frac{p}{2l}\sqrt{\frac{T}{m}}$$

All the modes of vibrations, except the fundamental, are called **overtones**, *while those whose frequencies are exact multiples of the frequency of the fundamental are called* **harmonics**. Obviously, *all harmonics are overtones but all overtones are not harmonics.* The fundamental is the first harmonic. Each of the higher harmonics is named by the integer by which the frequency of the fundamental must be multiplied to give its frequency.

Thus the mode of vibration with the frequency $2n$ is the *second harmonic* or *first overtone*, that with the frequency $3n$ is the *third harmonic* or *second overtone* and so on. It follows that in the vibrations executed by a string, all the harmonics are present.

In general, when the string is set into vibration all possible modes of vibration are present at the same time. The resultant waveform is obtained by superposition of all these waves and is rather complicated.

We shall see later that the quality of a musical sound is determined by the number of the possible overtones which are actually present and their relative amplitudes (vide Art. 24·14). In the case of the sound emitted by a vibrating string, the possible overtones present is determined by the location of the point of excitation. For example, if a string is plucked at the middle, this point can not be a node. Hence the 2nd, 4th......etc. harmonics which have a node at the middle will become absent. In fact, all the even harmonics will remain absent. So in general, we can state that ***those overtones which have a node at the point of excitation cannot be present in the sound produced by a string.*** This principle in utilised by musicians to control the quality of the sound emitted by violin type instruments.

If the vibrating string is touched lightly at any point, a node will be produced at this point. As a result, all the overtones except those which have nodes at this point will be suppressed. For example, if a vibrating string is touched at one-third of its length, all the harmonics excepting the 3rd, 6th, 9th, 12th, 15th etc., which have a node at that point, will be suppressed. While playing the stringed instruments, musicians suppress the undesirable harmonics by touching the wire at suitable points with their fingers.

23·16. Stringed instruments.

Vibrations of stretched strings are employed to obtain musical sound. These musical instruments are called stringed instruments. Usually the strings are made of metal or of cat-gut. As the sound emitted by a string is too feeble, the strings are mounted on a sound chest (a hollow wooden box which contains air in its cavity). The vibration of the string then induces forced vibration of the sound chest and the air inside it, so that a much intense sound is produced. Depending upon the manner in which the string is set into vibration, these instruments are classified into three distinct categories viz. (1) the *plucked string* instruments in which the sound is produced by plucking the string, e.g., guitar, *sitar*, etc., (2) the *struck string* instruments in which the string is excited by striking with a hammer e.g., piano forte, (3) the ***bowed string*** instruments where the sound is produced by bowing the string, e.g., violin, *esraj* etc. The frequency of sound emitted by such instruments is determined by the tension, length and mass of the string. As mentioned earlier, the quality of the sound emitted by these instruments depends upon the position of the point of excitation of the string, whatever be such modes of excitation. In a piano, however, the hammers strike the strings at certain prearranged points. For this reason, a pianist has no control over the quality of sound emitted.

Example 23·8. *A steel string 40 cm. long and 0·2 sq. mm. cross-section is set into vibration so as to give its fundamental. If the tension in the string be 4 kg. wt. find the frequency of vibration of the string.* [*Density of steel=7·7 gm./c.c.* ; *g=980 cm./sec.*^{2}].

Solution : Mass per unit length of the string $m = 1 \times 0{\cdot}002 \times 7{\cdot}7$

$$= 0{\cdot}0154 \text{ gm./cm.}$$

∴ From eqn. (23·10) the frequency of the string

$$n = \frac{1}{80}\sqrt{\frac{4 \times 1000 \times 980}{0{\cdot}0154}} = 199{\cdot}5 \text{ c.p.s}$$

Example 23·9. *A length of wire on a sonometer vibrates in unison with a fork of frequency 320 when the tension is 3 kg. wt. If the wire is replaced by one of the same material but twice the diameter, what tension would be required so that the same length is in unison with a fork of frequency 256 ?*

Solution : Let l_1 and r_1 be the length and radius of the first wire. Then from eqn. (23·11),

$$320=\frac{1}{2l_1r_1}\sqrt{\frac{3\times10^3\times g}{\pi\rho}} \qquad \ldots \qquad \ldots \qquad \text{(i)}$$

For the second wire $l_2=l_1$ and $r_2=2r_1$. Hence

$$256=\frac{1}{2l_1\times2r_1}\sqrt{\frac{T\times10^3\times g}{\pi\rho}} \qquad \ldots \qquad \ldots \qquad \text{(ii)}$$

where T is the required tension in kg. wt. Dividing (ii) by (i)

$$\tfrac{256}{320}=\tfrac{1}{2}\sqrt{T/3}$$

$$\text{or,}\ T=3\times(\tfrac{256}{320}\times2)^2=7{\cdot}68\ \text{kg. wt.}$$

Example 23·10. *A sonometer wire emits a note of frequency 320. What will be the frequency of the note emitted by the same wire, if the tension is increased in the ratio 16 : 25 and the length is made 5 times the previous length.*

Solution : In the first case $n_1=\frac{1}{2l_1}\sqrt{\frac{T_1}{m_1}}$

In the second case $n_2=\frac{1}{2l_2}\sqrt{\frac{T_1}{m_2}}$

$$\therefore\quad \frac{n_2}{n_1}=\frac{l_1}{l_2}\sqrt{\frac{T_2}{T_1}\cdot\frac{m_1}{m_2}}$$

Here $n_1=320$; $l_2=5l_1$; $m_1=m_2$ (the wire being the same) and $T_2 : T_1=25 : 16$.

$$\therefore\quad n_2=320\times\tfrac{1}{5}\sqrt{\tfrac{25}{16}}=80\ \text{c.p.s.}$$

23·17. Sonometer.

The sonometer is an instrument which is frequently used to investigate the vibration of a stretched string. It consists essentially of a hollow wooden box S upon which a string is kept stretched [Fig. 23·15]. One end of the string is fixed to a peg at one edge of the box, while the other end passes over a smooth pulley at the other edge of the box. This end of the wire carries a hanger upon which known weights are placed. By varying the weights, the string may be stretched by a tension of any desired value. The string passes over three bridges, two of which A and C are fixed, while the third one B slides under the wire. The vibrating length BC of the string can be adjusted by sliding this bridge to any desired position.

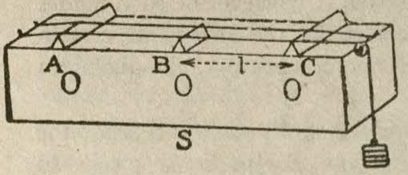


Fig. 23·15 : A Sonometer

In performing experiments with the sonometer, the string is adjusted to vibrate in unison with the source of sound, usually a tuning fork. This is usually done by keeping the tension in the string fixed at a desired value and altering its vibrating length by sliding the bridge B. The stem of the vibrating tuning fork is placed on the sounding box S and the unison is tested by placing a paper rider in the middle of the vibrating length of the string. When the frequency of vibration of the string becomes identical with the fork, resonance will be obtained. At this stage, as soon as the vibrating tuning fork is placed on the box S, the string vibrates with a large amplitude and the rider is thrown off. Alternatively, this unison can be tested by listening to the sounds produced by the fork and by the wire and examining whether they are in unison. But this requires the ear of a musician.

23·18. Verification of the laws of vibration of strings.

We shall now discuss in short the methods of verification of the law of vibration of strings with the help of a sonometer.

(i) *Law of length* : Several tuning forks of known frequencies are taken. A fixed load is placed on the hanger so that the string remains under a constant tension during the experiment. By sliding the movable bridge, a length l of the wire is tuned with a fork of frequency n. This is repeated with the other forks and it will be found that the product $n \times l$ is always constant. This shows that $n \propto 1/l$, when T and m are constants.

(ii) *Law of Tension* : Another wire, called the reference wire, is stretched on the sounding box at the side of the experimental wire. A known tension T_1 is then applied on the experimental wire. A fixed length l of it is selected and the vibrating length of the reference wire is adjusted in unison with it. Let this length be l_1.

The tension in the experimental wire is then changed to a value T_2 and a length l_2 of the reference wire is tuned with the same length l of it. It is found that $T_1/T_2 = l_2^2/l_1^2$.

In the two cases, if n_1 and n_2 be the frequencies of vibration of the reference wire and obviously of the experimental wire too, then according to the law of length, for the reference wire we have $l_1/l_2 = n_2/n_1$. Hence, for the experimental wire we get $n_1/n_2 = \sqrt{T_1/T_2}$. This shows that $n \propto \sqrt{T}$, when l and m are constants.

(iii) *Law of Mass* : To verify this, same procedure as in the last experiment is adopted with the only difference that instead of changing the tension in the experimental string, the experimental string itself is changed by another of different diameter or of different material. This is kept under the same tension as before. Taking a definite length of the various experimental wires and afterwards weighing a known length of the wires, the mass per unit length m is determined. It will be found that the product $l\sqrt{m}$ is always constant. As in the last experiment, this shows that $n \propto \sqrt{1/m}$ when l and T are constants.

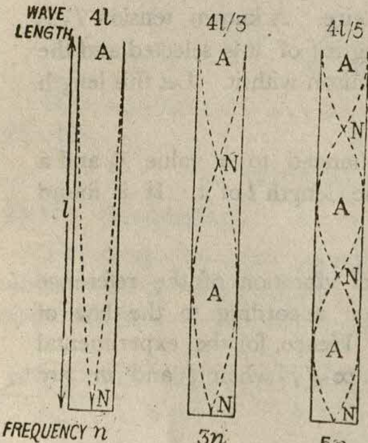
VIBRATIONS OF AIR COLUMNS

23·19. Longitudinal standing waves in air column.

In the preceding sections we discussed the formation of transverse stationary waves and the possible modes of vibration in a stretched string which acts as a source of sound. Another common source of sound is a vibrating air column enclosed in a pipe or tube. The pipe is called *closed* when it is open at one end and closed at the other or *open* when it is open at both ends. When the air column is set into vibration, stationary wave patterns are formed within the pipe just as in the case of a vibrating string. But in contrast to a vibrating string, the stationary waves formed in this case are *longitudinal* in nature. As these vibrations are set up within a pipe, a musical sound is produced. The particular class of musical instruments, called wind instruments e.g., flute, bugle, organ etc., utilise this principle. Due to the same reason, a pleasant sound is heard when we blow a conch or blow into an empty bottle.

23·20. Closed pipe.

If a vibrating body such as a tuning fork is held over a closed pipe, then the air near it gets alternately compressed or rarefied. As a result longitudinal progressive waves are generated which travel through the air column to the closed end of the pipe. On reaching there, the waves are reflected back towards the open end. Thus within the tube there are always two waves—one going from the open end to the closed end and the other—the reflected wave—going from the closed end to the open end. These two are identical waves travelling in opposite directions. They, therefore, give rise to longitudinal stationary waves. Since the closed end acts like a rigid barrier, the air in contact with it can not vibrate and so a node is formed at this end. On the contrary, the air at the open end is able to move most freely and thereby gets displaced by the maximum possible amount. Thus an antinode is formed at the open end.



(a) (b) (c)

Fig. 23·16. : Modes of vibration of air in a closed pipe

The several possible modes of vibration of air in a closed pipe are shown in the Fig. 23·16. Though the waves are longitudinal in nature in which the displacements are parallel to the axis of the tube, the figures are drawn, for convenience, as if they were transverse. The simplest possible mode of vibration is that in which there is a node at the closed end and an antinode at the open end [Fig. 23·16 (*a*)]. As the distance between two adjacent node and antinode is a quarter of the wavelength, we have,

$$l=\lambda/4$$

where l is the length of the pipe and λ is the wavelength of the constituent wave. If v be the velocity of sound, then the frequency n is given by

$$n=\frac{v}{\lambda}=\frac{v}{4l}$$

This is the *fundamental frequency* or *the first harmonic*.

The next possible mode of vibration occurs when in addition to the node at the closed end and the antinode at the open end, there are one more node and one more antinode in the intermediate region [Fig. 23·16 (*b*)]. Since the distance between a node and an antinode is $\lambda_1/4$, where λ_1 is the new wavelength, it is evident from the figure that the length of the pipe is given by $l=\frac{3}{4}\lambda_1$. Hence the corresponding frequency is given by

$$n_1=\frac{v}{\lambda_1}=\frac{3v}{4l}=3n.$$

The new frequency is, therefore, three times that of the fundamental. This is the *third harmonic* or the *first overtone*.

In the next possible mode of vibration, there will be a pair of nodes and a pair of antinodes placed evenly along the length of the tube in addition to the node at the closed end and the antinode at the open end [Fig. 23·16 (*c*)]. Obviously in this case the wavelength λ_2 is given by the relation $l=\frac{5}{4}\lambda_2$ and we have for the corresponding frequency $n_2=\frac{v}{\lambda_2}=\frac{5v}{4l}=5n.$

This is the *fifth harmonic* or the *second overtone*. Similarly, the frequencies of the higher harmonics can be calculated. It follows, therefore, that the possible frequencies of vibration for a closed pipe are in the ratio 1 : 3 : 5 : 7 :...etc. Hence, *only odd harmonics are present*.

From a close study of the above equations, we can conclude that in general if the wavelength of the p-th overtone be λ_p, then

$$l=\frac{(2p+1)}{4}\lambda_p \qquad \therefore\ \lambda_p=\frac{4l}{2p+1}$$

Hence the corresponding frequency is

$$n_p=\frac{v}{\lambda_p}=\frac{(2p+1)\,v}{4l}=(2p+1)\,n$$

As the quality of a sound depends upon the number and relative intensities of the overtones present, the quality of the sound emitted from a closed pipe lacks in fullness. When the air in a closed pipe is set into vibration, the first few overtones usually remain present, because higher overtones become gradually fainter and fainter. By blowing the pipe more and more powerfully, higher and higher overtones are produced and the quality of the sound emitted changes.

23·21. Open pipe.

If a vibrating body e.g., tuning fork, is held above an open pipe, a train of longitudinal waves travel through the air column to the other end where it is reflected back. Since the air outside the tube has more freedom of movement than that in its interior, we can treat them to be acoustically different media, the surface of separation lying at the open ends. This explains such reflection.

Within the tube, superposition of the exactly identical direct waves and reflected waves which are moving in opposite directions gives rise to longitudinal stationary waves. Since the air at the open ends possesses utmost freedom of movement, maximum possible displacement is produced at these points. Hence antinodos are always formed at the open ends. The various possible modes of vibration of air in an open pipe are shown in the Fig. 23·17. In the simplest possible mode of vibration shown in Fig. 23·17 (a), a node is formed in between the two antinodes at the open ends. As the distance between two adjacent antinodes is half a wavelength, we have for the length of the pipe $l=\frac{\lambda}{2}$.

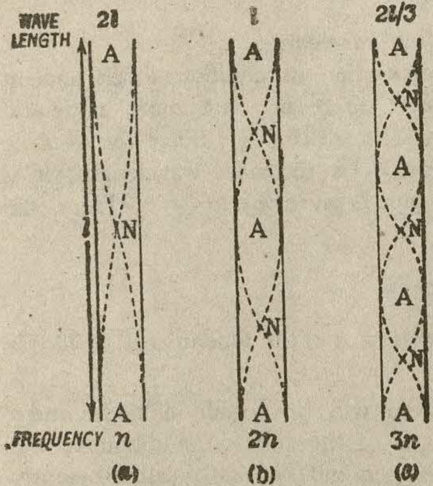


Fig. 23·17. : Modes of vibration of air in open pipe

If v be the velocity of sound, then the frequency n is given by

$$n=\frac{v}{\lambda}=\frac{v}{2l}$$

This is the *fundamental frequency* or the *first harmonic*. It follows that if the open pipe is of the same length as a closed one, the frequency of the fundamental in the open pipe is twice that in the closed pipe. The sound produced by an open pipe is, therefore, sharper.

In the next possible mode of vibration, there are two intermediate nodes and one intermediate antinode in between the antinodes at the ends [Fig. 23·17 (b)]. In this case the length of the pipe equals one wavelength, i.e., $l=\lambda_1$. The corresponding frequency is given by

$$n_1=\frac{v}{\lambda_1}=\frac{v}{l}=2n$$

Thus the frequency is twice that of the fundamental. This is the *second harmonic* or the *first overtone*.

The next higher harmonic is produced when in addition to the antinodes at the open ends, three nodes and two antinodes are formed in the intermediate region [Fig. 23·17 (c)]. In this case $l=\frac{3}{2}\lambda_2$ and the frequency $n_2=\frac{v}{\lambda_2}=\frac{3v}{2l}=3n$.

This is the *third harmonic* or the *second overtone*. Proceeding in this way the frequencies of the higher harmonics can be obtained.

In general, if the wave length of the p-th overtone be λ_p, then

$$l=2(p+1)\,\frac{\lambda_p}{4} \qquad \therefore\ \lambda_p=\frac{2l}{p+1}$$

So the corresponding frequency is

$$n_p=\frac{v}{\lambda_p}=\frac{(p+1)\,v}{2l}=(p+1)\,n$$

Thus it is seen that the frequencies are n, $2n$, $3n$, $4n$,...etc., i.e., all the harmonics are present. The contrast with a closed pipe should be noted, where only the odd harmonics are present. Due to this reason, the sound emitted by an open pipe is richer and sweeter (i.e. more musical) than that produced by a closed pipe. Open pipes are, therefore, most commonly employed in wind instruments. It should be noted that the frequency of the fundamental depends upon the length of the tube ; greater the length, lower is the frequency. This is utilised in most wind instruments, in which various notes are produced by varying the length of vibrating air column. This is achieved in a flute by boring a set of holes along the pipe, which can be opened or closed at will. At an open hole an antinode is produced and the vibrating air column is terminated there. Thus the note emitted by the pipe can be easily changed.

23·22. Comparison of vibrations of air columns in closed and open pipes.

The comparison between the vibrations of air columns in a closed and an open pipe of *equal length* is presented in a tabular form :—

Closed pipe	*Open pipe*
1. An antinode is always formed at the open end and a node at the closed end.	1. Antinodes are always formed at the open ends.
2. The wavelength of the fundamental is $\lambda=4l$ and its frequency is $n=v/4l$.	2. The wavelength of the fundamental is $\lambda=2l$ and its frequency is $n=v/2l$. Clearly the frequency of the fundamental is twice that in the closed pipe.
3. The wavelengths of the overtones present are $\lambda_p=\frac{4l}{2p+1}$ and their frequencies are $n_p=(2p+1)\,n$. Here p is a whole number i.e., $p=1$, 2, 3 etc.	3. The wavelengths of the overtones present are $\lambda_p=\frac{2l}{p+1}$ and their frequencies are $n_p=(p+1)\,n$. Here p is a whole number i.e., $p=1$, 2, 3 etc.
4. The possible frequencies of vibration are in the ratio 1 : 3 : 5 : 7 etc. Hence, only odd harmonics are present.	4. The possible frequencies of vibration are in the ratio 1 : 2 : 3 : 4 : 5 etc. Hence, all the harmonics are present.
5. The quality of the sound emitted lacks in fullness.	5. The sound emitted is richer and sweeter than that produced by a closed pipe.

23·23. Effect of temperature and humidity on the frequency of air column.

The frequency of the fundamental emitted by a closed pipe is $v/4l$ and that of an open pipe is $v/2l$. Hence in both the cases the frequency of the fundamental is directly proportional to the velocity of sound in air.

We shall see later that the velocity of sound in a gas is directly proportional to the square root of its absolute temperature [Art. 24·5]. Hence with increase in temperature, the velocity of sound in air increases and in consequence the frequency of the fundamental in both the pipes rises. The sound emitted, therefore, becomes sharper. With decrease in temperature, just the reverse phenomenon happens.

Similarly, with increase in humidity of air, the velocity of sound increases. Hence fundamentals of higher frequencies are emitted by both the pipes and the emitted sound become sharper. With the lowering of the humidity of air, just the reverse phenomenon takes place.

23·24. Determination of the velocity of sound or the frequency of a fork by resonance air column.

As the frequencies of the harmonics produced in a vibrating air column in a closed pipe depend on its length, a closed pipe may be adjusted to produce resonance with any source of sound by suitable alteration in its length. This provides a very simple method of determining the velocity of sound in air.

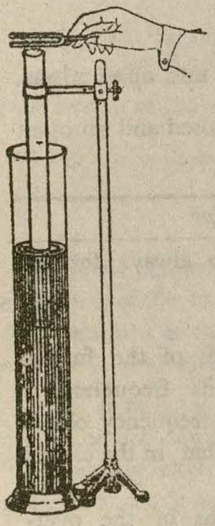


Fig. 23·18 : Resonance tube

In its simplest form, the apparatus consists of a long glass or metallic open pipe immersed in a glass jar filled with water [Fig. 23·18].

Initially the pipe is kept almost completely immersed in water so that the length of the air column is very short. A vibrating tuning fork is held near the open end and the tube is slowly pulled out of water till a loud note is heard due to resonance. This resonance corresponds to the fundamental mode of vibration of the air column and obviously at this stage the fundamental frequency must be the same as the frequency n of the tuning fork. The length l of the air column is then measured. Average of several readings are taken by slightly raising or lowering the tube. The velocity of sound v is then given by the relation,

$$v=4nl$$

End correction and its elimination : We have so far assumed that the antinode at the open end lies exactly at the mouth of the tube. Strictly speaking this is not so ; the antinode is situated at a small distance x beyond the end. x is called the *end correction*. Lord Rayleigh has shown that $x=0{\cdot}6r$, where r is the radius of the tube.

Hence,

The effective length of the vibrating air column $=(l+n)=(1+0{\cdot}6r)$.

Therefore, the expression for the velocity of sound becomes,

$$v=4n\,(l+0{\cdot}6r).$$

To eliminate the end correction, the tube is raised further out of water until a second resonance is obtained. This occurs when frequency of the first overtone produced by the new length of the tube equals that of the tuning fork. If this length be l', then, $l'+0{\cdot}6r=3\lambda/4$.

But for the first resonance $l+0{\cdot}6r=\lambda/4$.

Hence $l'-l=\lambda/2$.

Therefore, $v=n\lambda=2n(l'-l)$.

From this equation the velocity of sound can be determined by eliminating the end correction.

This gives the velocity of sound in moist air at room temperature. On applying the necessary correction for temperature and humidity, the velocity of sound in dry air at 0°C can be obtained.

The frequency of an unknown fork can be determined by this method from the above equation if the velocity of sound is known. In order to obtain the velocity of sound, the experiment is first performed with a fork of known frequency.

Example 23·11. *Calculate the shortest length of a tube open at both ends that will resound to a fork of frequency 480, if the velocity of sound in air is 332 m./sec.*

Solution : In this case, $v=2nl$

$$\therefore \quad l=\frac{v}{2n}=\frac{332\times 100}{2\times 480}=34{\cdot}6 \text{ cm.}$$

Example 23·12. *A tuning fork of frequency 512 is held over a glass tube of diameter 10 cm. standing in water. Find the length of the tube to be withdrawn from the water in order to obtain resonance with the fork.*

How much further out of the water must the tube be raised for resonance again to occur ? (velocity of sound=330 m./sec.)

Solution : In the first case, considering end correction,

$$330\times 100=4\times 512\ (l_1+0{\cdot}6\times 5) \quad \text{or,} \quad l_1=13{\cdot}11 \text{ cm.}$$

Hence, the tube should be withdrawn by 13·11 cm. to obtain first resonance. In the second case, $330\times 100=2\times 512\ (l_2-l_1)$

$$\therefore \quad (l_2-l_1)=32{\cdot}22 \text{ cm.}$$

Hence, the tube should be raised through 32·22 cm. more to get second resonance.

Example 23·13. *A closed pipe 100 cm. long and filled with a gas resounds to a given tuning fork. If an open pipe resounding to the same fork and containing air be 125 cm. long, what would be the velocity of sound in the gas. [Velocity of sound in air=330 m./sec.]*

Solution : Let the frequency of the fork be n. Let v be the velocity of sound in the gas.

In the case of closed pipe $n=\dfrac{v}{4\times 100}$

and in the case of open pipe $n=\dfrac{330\times 100}{2\times 125}$

$$\therefore \quad v=\frac{330\times 100\times 4\times 100}{2\times 125}=528 \text{ m./sec.}$$

Example 23·14. *When one of the ends of an open organ pipe emitting first overtone is suddenly closed, it also emits the first overtone. But the difference in pitch in the two cases is 350. Find the pitch of the note emitted ordinarily by the open pipe.*

Solution : Let the frequency of the first overtone of the open pipe of length l be n_1. Evidently, $n_1 = \frac{v}{l}$, where v is the velocity of sound. When the open end of the pipe is closed, it acts as a closed organ pipe of the same length l. Let the frequency of the first overtone emitted by such closed pipe be n_2 where,

$$n_2 = \frac{3v}{4l} = \frac{3}{4} n_1$$

Since $n_1 > n_2$ and $n_1 - n_2 = 350$, we have $n_1 = 350 + \frac{3}{4} n_1$, whence $n_1 = 1400$ c. p. s.

23·25. Interference.

When two sets of waves of the same frequency and same or nearly equal amplitudes travelling through the same medium overlap, then it is found that due to superposition, they wholly or partly nullify each other at certain times and places and reinforce each other to produce a greater effect at other times and places. This phenomenon is termed **interference** *and the two sets of waves are said to* **interfere.** We shall see later that for a successful observance of the phenomenon, *the two sets of waves must have a constant phase difference at any particular point.*

To get a clear insight into the phenomenon, let us consider the waves that spread out from two sources S_1 and S_2 which are vibrating in the same phase with the same amplitude and frequency. For simplicity we shall assume the waves to be transverse, it being understood that the following discussion holds equally well for longitudinal waves, too. We can represent the propagating waves by two sets of concentric circles with the sources S_1 and S_2 at their centres [Fig. 23·19]. Obviously these waves consist of alternate crests and troughs, half a wavelength apart. For the sake of convenience, we shall represent the crests by continuous and the troughs by dotted lines. As these two sets of waves overlap, the resulting wave pattern can be obtained from the principle of superposition. At some points the crest or trough of one wave coincides with the crest or trough of the other and the resulting amplitude will be twice as large as that of either of the waves. The waves are said to be in step or in phase at these points. In the figure, a crest meets a crest at the points A, B, C, etc. while a trough meets a trough at the points

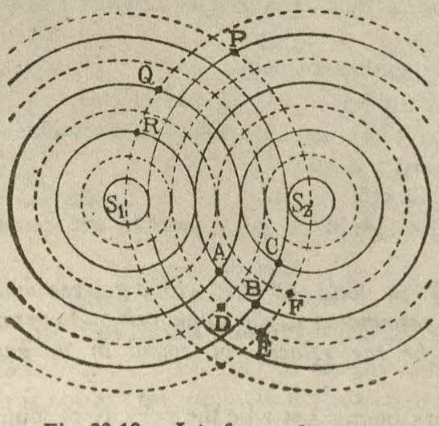


Fig. 23·19 : Interference between two sets of waves

D, E, F etc. These points are, therefore, points of maximum disturbance. If we remember that along a wave, phase becomes identical after the distance of one wavelength, we reach the conclusion that ***these points must be equidistant from the two sources or their distances from the two sources must differ by a whole number of wavelengths.*** In the figure, the points A and D are equidistant from S_1 and S_2, the distances of B or E from S_1 and S_2 differ by one wavelength and those of C or F by two wavelengths etc.

At some other points, the crest of one wave meets a trough of the other and the resulting amplitude will be a minimum, being equal to the difference in amplitude, of the two sets of waves. If these waves are of equal amplitude, they will cancel each other out and the resulting amplitude will be zero. The waves are said to be out of step, or out of phase at these points. In the figure, such a situation exists at the points P, Q, R etc. Arguing as above, we conclude that ***distances of these points from the two sources must differ by some odd number of half wavelengths.*** For example, the distances of P from S_1 and S_2 differ by half a wavelength, those of Q by three halves of a wavelength and so on.

It follows, therefore, that depending upon the fact that whether the two waves reinforce each other or cancel out each other, the amplitude of the resulting wave pattern will possess maximum amplitude at some points and minimum (usually zero) at some other points of the medium. This phenomenon is known as ***interference***. When the two waves being in phase reinforce each other, the interference is said to be ***constructive*** : if being out of phase, they annul the effect of each other, the interference is said to be ***destructive***. Such points of maximum and minimum amplitudes form a definite pattern called the ***interference pattern***. Fig. 23·20 shows a diagram of this type of pattern. The continuous thick black lines are drawn through the various series of points at which the amplitude is maximum. In between these lines are the dotted thick black lines which are drawn through the various series of points where the amplitude is minimum. These lines are hyperbolic in shape. At intermediate points, other amplitudes between the maximum and the minimum are possible.

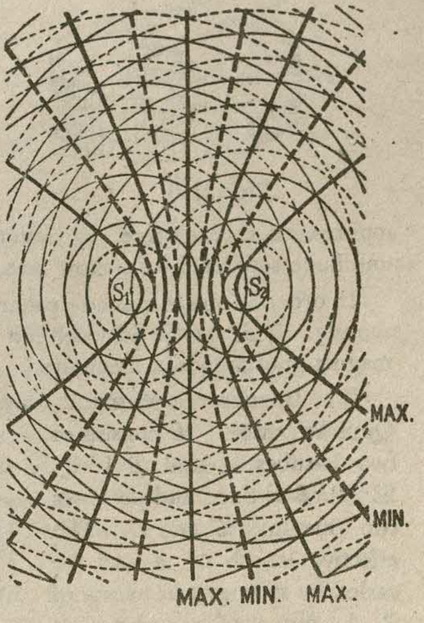


Fig. 23·20 : Interference pattern formed by two coherent sources

When the frequencies of the two waves are the same, the pattern remains stationary. To understand this, let us consider the points A in the Fig. 23·19, where at that particular instant, the crests of the two waves have met. Half a period later, troughs of both the waves will meet together at the same point which will therefore continue to be a point where constructive interference occurs. Similar arguments for the other points where the amplitudes are maximum or minimum show that they will always remain to be so.

In order to obtain a stationary pattern, it is also essential for the two sources to be always in phase with each other or to have a constant phase difference. Otherwise, the positions of the points at which the waves are in phase or out of phase, would be continually changing and thus no interference effects could be observed. If this condition be satisfied we say that the two sources are *coherent*. So *two sources are said to be* **coherent** *when they are always in phase or have a constant phase difference.*

Thus the conditions to be satisfied for obtaining stationary interference pattern may be summed up as follows : *the sources must have the same frequency and roughly the same amplitude and they must be coherent.*

The phenomenon of intererence is a typical property of waves and we shall see later that the existence of interference proves conclusively that light is a kind of wave.

Demonstration Experiments.

(1) Interference effects can be demonstrated beautifully with ripples on the surface of water. Two needles are attached to the end of a metal strip, the other end of which is clamped. A dish filled with water or mercury is taken and the strip is so arranged that the needles just touch the surface of the liquid [Fig. 23·21]. If the strip is set into transverse vibration, it will oscillate up and down so that identical sets of ripples will originate from the two needles. As they overlap, definite interference pattern like that shown in Fig. 23·20, will appear on the surface of water. Broad lanes consisting of waves of large amplitude separated by narrow lanes of undisturbed water will be clearly seen.

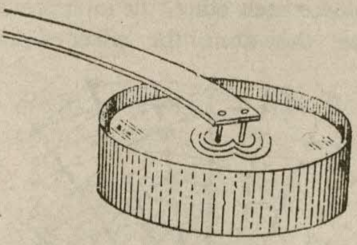


Fig. 23·21

In order to obtain a steady pattern, the strip must be kept into vibration continuously. This can be achieved if it is driven electrically by means of an electromagnet [not shown in the figure].

(2) Interference of sound waves may be demonstrated with the help of a Quincke's tube. It consists of two U-tubes *A* and *B* [Fig. 23·22]. *B* is fixed and possesses two openings *C* and *D*. The effective length of *A* can be varied by sliding it in or out of *B*. A source of sound e.g. a tuning fork, is placed at *C* and ta suitable detector is placed at *D*. The sound waves from *C* will arrive at *D* by ravelling along both the paths *CAD* and *CBD*. If the two paths are equal or

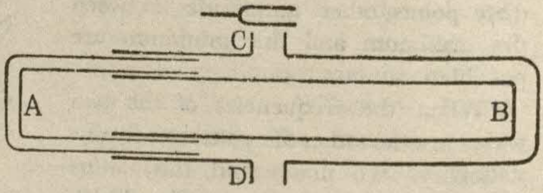


Fig. 23·22 : Quincke's tube

differ by a whole number of wavelengths, constructive interference will take place at D and the intensity of sound will be maximum. If the two paths, however, differ by an odd number of half wavelengths, there will be destructive interference at D and no sound will be heard. If the sliding tube A is slowly drawn out, the path difference changes and a series of maxima and minima may be observed.

23·26. Beats.

In the previous sections, we discussed the superposition of two sets of waves whose frequencies are exactly equal. We shall now discuss what happens if the frequencies of the two waves differ by a small amount. *When two sets of waves of nearly equal frequencies and amplitudes are superposed, it is found that the amplitude of the resultant wave fluctuates with time from maximum to minimum. This phenomenon of fluctuation of amplitude is known as* **beats**. A beat consists of one rise and one fall of the amplitude of the wave. It can be proved that the number of beats per second is equal to the difference of frequencies of the two component waves.

In the case of sound, these variations of amplitude give rise to variations of loudness because loudness of sound depends upon the amplitude of the wave and so alternate waxing and waning of sound will be heard. Usually this phenomenon of beats is referred to in the case of sound waves in particular.

Formation of beats : This can be studied both graphically and analytically.

(a) *Graphical method* : Let us consider a particular point in space through which the waves of frequencies n_1 and n_2 $(n_2>n_1)$ are travelling simultaneously. The displacement produced separately by the waves are plotted against time as shown in Fig. 23·23 (*a*) and Fig. 23·23 (*b*). In this figure, we have taken the length of the time axis to be equal to one second so that $n_1=10$ c.p.s. and $n_2=12$ c.p.s. When the two waves combine, the resultant wave can be obtained from the principle of superposition by algebraic addition of the ordinates of the two curves at the corresponding points and is shown in Fig. 23·23 (*c*). This shows that at particular instants (as at A, C and E), the waves are in phase and the amplitude of the resultant wave is maximum ; at other instants (as at B and D), the waves are in opposite phase and the amplitude of the resultant wave is minimum. This resultant wave of varying amplitude travels forward through the medium with the velocity of the component waves.

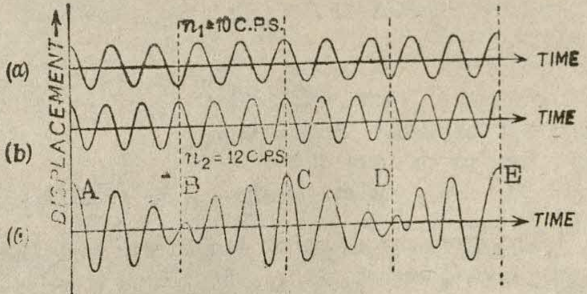


Fig. 23·23 : Formation of beats

Two successive maxima or two successive minima are said to constitute one beat. It is seen from the figure that in one second, two beats are produced. We can, therefore, conclude that the number of beats produced per sec. is equal to the difference of the frequencies of the component waves.

The phenomenon can be explained as follows. At the instant at A, the two waves are in phase. As time advances, they go out of step because their frequencies are different. The wave in Fig. 23·23 (b) possessing the higher frequency is constantly getting ahead of the wave in Fig. 23·23 (a). When it gains one complete vibration over the latter, the two waves will again come in phase and one beat is produced. When it gains two complete vibrations, the waves are again in phase and another beat is produced. This can be seen from the figure where between A and C, the wave in Fig. 23·23 (b) has made one more vibration than the wave in Fig. 23·23 (a). Between A and E, it has made two more vibrations. In general as the frequencies of the waves are n_1 and n_2, then in one second the number of vibrations which the second wave gains over the first is (n_2-n_1). So it gains a complete vibration over the first (n_2-n_1) times per second and the amplitude at a particular point reaches a maximum (n_2-n_1) times per second which obviously gives the number of beats produced per second. This is called the *beat frequency*.

(*b*) ***Analytical method*** **:** The above results can be obtained analytically as follows. For simplicity we assume that the two component waves of frequencies n_1 and n_2 $(n_2>n_1)$ have the same amplitude a. Then the equations of the waves are given by,

$$y_1=a \sin 2\pi n_1 t \text{ and } y_2=a \sin 2\pi n_2 t$$

From superposition principle, the resultant wave is given by

$$y=y_1+y_2=a \sin 2\pi n_1 t+a \sin 2\pi n_2 t$$

Since, $\sin C+\sin D=2 \sin \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$

$$\therefore \quad y=2a \sin 2\pi \left(\frac{n_1+n_2}{2}\right) t.\cos 2\pi \left(\frac{n_2-n_1}{2}\right) t$$
$$=A \sin 2\pi n t$$

where $A=2a \cos 2\pi \left(\dfrac{n_2-n_1}{2}\right) t$ and $n=\dfrac{n_1+n_2}{2}$

This represents a wave of frequency n and amplitude A. The amplitude of this wave, therefore, varies with time in a periodic manner. It becomes zero when

$$t=\frac{1}{2(n_2-n_1)}, \frac{3}{2(n_2-n_1)}, \frac{5}{2(n_2-n_1)} \text{ etc.}$$

and a maximum, when

$$t=\frac{1}{n_2-n_1}, \frac{2}{n_2-n_1}, \frac{3}{n_2-n_1}, \text{ etc.}$$

It follows immediately that the successive minima or maxima occur after the time of $1/(n_2-n_1)$ sec. Hence the number of beats per second which is given by the number of minima or maxima per second is n_2-n_1.

Conditions for perception of beats : In the case of sound waves, beats may be clearly heard if the frequencies of the two waves do not differ by more than about 10 c.p.s. If the frequency of the beats is too high, then due to persistence of hearing the ear fails to distinguish the fluctuations of loudness separately. A continuous note is then heard which is called the *beat tone*.

The amplitudes of the waves should not also differ by a large amount as otherwise the beats can not be clearly recognised. We have mentioned earlier that loudness of a sound is determined by the amplitude of the waves. If the amplitudes differ by a large amount i.e., if one of them be very much weaker than the other, then the variation in the amplitude of the resultant wave becomes comparatively smaller. Consequently loudness does not vary appreciably leading to difficulty in recognition.

Demonstation of beats : Beats can be easily demonstrated with the help of two tuning forks of the same frequency. If they are sounded, they will vibrate in unison and no beats will be heard. A prong of any one of them is then loaded with a drop of wax whence its frequency will be slightly reduced. If they are now sounded again, beats will be heard.

Practical Applications : (i) The phenomenon of beats is frequently used to determine whether two notes are in unison or not. Musicians apply this method for tuning their instruments. (ii) Beats are also widely used for *determining the unknown frequency* n_1 of a source of sound. By trial, a tuning fork of known frequency n is so chosen that when it is excited along with the source of sound, beats are produced In that case n and n_1 are nearly equal. The number of beats per second is now counted. If we call this m, then obviously $m=n_1 \sim n$. In order to decide whether n_1 is greater or smaller than n, the fork is loaded with a drop of wax thereby decreasing its frequency. If the number of beats increases, then obviously n_1 is greater than n and is given by $m=n_1-n$ or, $n_1=n+m$. On the other hand, if the number of beats decreases, then $m=n-n_1$ or, $n_1=n-m$. Hence n_1 can be determined. (iii) *The existence of any dangerous gas* inside a mine can be detected with the help of beats. For this, two identical organ pipes are taken ; one of them is filled with pure dry air and the other with the air from the mine. If this air be impure, then its density will be different ; hence the velocity of sound through it will change. Thus the frequencies of the fundamentals emitted from the two pipes will differ slightly. If they are sounded together, beats will be produced. If no beats are heard, then the air of the mine is considered as pure.

Example 23·15. *A tuning fork originally in unison with another tuning fork of frequency 512 produces 5 beats per second (a) when a little wax is attached to it (b) when it is filled. Calculate its frequency in both cases.*

Solution : (*a*) When loaded, the altered frequency n_2 will be less than the original frequency n_1.

$\therefore \; n_1-n_2=5$ or, $n_2=n_1-5=507$ c.p.s.

(*b*) When filled, the altered frequency n_2 will be greater than n_1.

$\therefore \; n_3-n_1=5$ or, $n_3=n_1+5=517$ c.p.s.

Example 23·16. *Two tuning forks 1 and 2, when sounded together produce 8 beats per sec. If the fork 1 is in resonance with a 32 cm. air column in a pipe closed at one end and the fork 2 is in resonance with a 33 cm. long similar air column, calculate the frequencies of the forks.*

Solution : Let n_1 and n_2 be the frequencies of the forks 1 and 2 respectively. Also let λ_1 and λ_2 be the corresponding wavelengths. Then if v be the velocity of sound

$$n_1=\frac{v}{\lambda_1}=\frac{v}{4\times 32} \text{ and } n_2=\frac{v}{\lambda_2}=\frac{v}{4\times 33}$$

$$\therefore \frac{n_1}{n_2}=\frac{33}{32} \quad \cdots \quad \cdots \quad \text{(i)}$$

$$\text{Now } n_1-n_2=8 \quad \cdots \quad \cdots \quad \text{(ii)}$$

$$\therefore \text{ From eqns. (i) and (ii), } \frac{n_2+8}{n_2}=\frac{33}{32}$$

$$\therefore n_2=256 \text{ c.p.s. and } n_1=264 \text{ c.p.s.}$$

23·27. Doppler effect.

In the foregoing discussions on wave motion, we have not considered any motion of the source of the waves or of the observer. We have tacitly assumed that both of them are at rest. *If, however, the source and the observer be in relative motion, the frequency of the waves noted by the observer appears to be different from the true frequency. This phenomenon is known as* **Doppler effect** after the name of the Austrian physicist C. J. Doppler who first discussed it in the case of light waves. ***When the source and the observer approach each other, the observed frequency is found to be greater than the true frequency. On the other hand, if they recede away from each other, the observed frequency appears to be less than the true one.***

The effect is exhibited by all wave motions in general. The effect is, however, most frequently experienced in the case of sound waves where a change in frequency is manifested by a change in the pitch. For example, if a fast moving railway locomotive approaches an observer with its whistle sounding and passes him, a noticeable change in pitch is observed. As it approaches the observer, the pitch appears to increase and then drops abruptly as it passes the observer. The pitch continues to decrease as the locomotive goes on receding from the observer. Similar effects are noticed by listening to the horn of a fast moving car or to the roar of the engine of a low-flying aircraft as it passes the observer.

We shall now calculate this apparent change in frequency. We shall discuss only two simple cases in which the motion of the source or of the observer takes place along the line joining them.

(i) *Source in motion ; observer stationary.*

Let us consider a source of waves moving with a velocity v towards a stationary observer. For simplicity's sake, we consider the waves to be a train of wavefronts, the distance between two consecutive wavefronts being equal to a wavelength. Let us suppose that the source emits a wavefront when it is at A [Fig. 23·24]. Let us also suppose that when the source emits the second wavefront the first wavefront has already moved to C. If the source was stationary, the second wavefront would also have been emitted at A. In that case the distance between the two wavefronts viz., AC would have been the unaltered wavelength λ. But since the source is in motion, let us suppose that when the second wavefront is emitted the source has come to B. In this case the distance between the two wavefronts becomes equal to BC. This is the altered wavelength λ_1. Thus $AC=\lambda$ and $BC=\lambda_1$. Also, in the time in which the first wavefront moves from A to C, the source moves from A to B. Hence $\frac{AC}{V}=\frac{AB}{v}$, where V is the velocity of the wavefront.

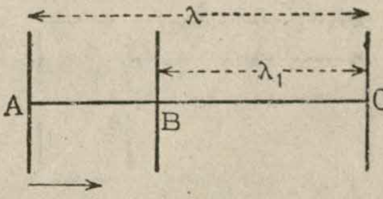


Fig. 23·24 : Doppler effect ; moving source

$$\text{or,}\quad \frac{\lambda}{V}=\frac{AC-BC}{v}=\frac{\lambda-\lambda_1}{v}=\frac{\lambda_1}{V-v}\quad \left[\because\ \frac{a}{b}=\frac{c}{d}=\frac{a-c}{b-d}\right]$$

$$\therefore\quad \lambda_1=\lambda\frac{V-v}{V}=\lambda\left(1-\frac{v}{V}\right)\qquad \ldots\qquad \ldots\qquad (23\cdot12)$$

But $n\lambda=n_1\lambda_1$, where n and n_1 are unaltered and the altered frequencies.

$$\text{Hence}\quad \frac{n_1}{n}=\frac{\lambda}{\lambda_1}=\frac{V}{V-v}\quad\therefore\quad n_1=n\frac{V}{V-v}=\frac{n}{1-\frac{v}{V}}\qquad \ldots\qquad (23\cdot13)$$

Instead of approaching the observer if the source recedes away from the observer, the velocity v should be regarded as negative ; in that case equations (23·12) and (23·13) become

$$\lambda_1=\lambda\frac{V+v}{V}=\lambda\left(1+\frac{v}{V}\right)\qquad \ldots\qquad \ldots\qquad (23\cdot14)$$

$$\text{and}\quad n_1=n\frac{V}{V+v}=\frac{n}{1+\frac{v}{V}}\qquad \ldots\qquad \ldots\qquad (23\cdot15)$$

The above equations show that when the source approaches a stationary observer, the observed frequency becomes greater than the true one ; on the other hand, if the source recedes away from a stationary observer, the observed frequency becomes less than the true one.

(ii) *Source stationary ; observer in motion.*

Let us consider an observer moving with a velocity v away from the source i.e,, in the direction of motion of the waves. As before, for simplicity's sake, we consider the waves as a train of wavefronts, the distance between two successive wavefronts being equal to one wavelength. Let us suppose that the observer is at A when he receives a wavefront [Fig. 23·25]. The next wavefront is at that instant some what behind ; let us suppose that it is at B. BA is, therefore, the unaltered wavelength λ. But the observer is in motion. As the second wavefront is coming towards him, he is also moving in the same direction. Let us suppose that the second wavefront overtakes him at C. Thus *for the observer* the distance BC is the altered wavelength λ_2. Hence $BA=\lambda$ and $BC=\lambda_2$. Also, in the time in which the wavefront moves from B to C the observer moves from A to C.

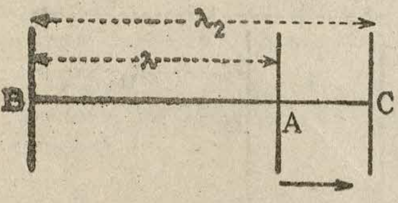


Fig. 23·25 Doppler effect ; moving observer

Hence, $\dfrac{BC}{V}=\dfrac{AC}{v}$, where V is the velocity of the wavefront.

$$\text{or,}\quad \frac{\lambda_2}{V}=\frac{BC-BA}{v}=\frac{\lambda_2-\lambda}{v}=\frac{\lambda}{V-v} \qquad \left[\because\ \frac{a}{b}=\frac{c}{d}=\frac{a-c}{b-d}\right]$$

$$\therefore\quad \lambda_2=\lambda\frac{V}{V-v}=\frac{\lambda}{1-\dfrac{v}{V}} \qquad \cdots \qquad \cdots \qquad (23{\cdot}16)$$

But $n\lambda=n_2\lambda_2$ where n and n_2 are the unaltered and the altered frequencies.

$$\therefore\quad \frac{n_2}{n}=\frac{\lambda}{\lambda_2}=\frac{V-v}{V} \quad \text{or,}\quad n_2=n\,\frac{V-v}{V}=n\left(1-\frac{v}{V}\right) \qquad \cdots \qquad (23{\cdot}17)$$

Instead of moving away from the source, if the observer moves towards the source, the velocity v should be regarded as negative. In that case eqn. (23·16) and (23·17) become,

$$\lambda_2=\lambda\frac{V}{V+v}=\frac{\lambda}{1+\dfrac{v}{V}} \qquad \cdots \qquad \cdots \qquad (23{\cdot}18)$$

$$\text{and}\quad n_2=n\frac{V+v}{V}=n\left(1+\frac{v}{V}\right) \qquad \cdots \qquad \cdots \qquad (23{\cdot}19)$$

These equations show that when the observer moves away from the source, the apparent frequency is less than the real one. On the other hand, if the observer moves towards the source, the apparent frequency becomes greater than the real one.

(iii) ***Both source and observer are moving in the same direction with velocities v_s and v_o.***

Due to the motion of the source, the modified frequency is given by

$$n_1=n\frac{V}{V-v_s} \qquad \text{[from eqn. 23·13]}$$

This frequency is again modified by the motion of the observer. The final modified frequency is given by

$$n_2=n_1\frac{V-v_o}{V} \qquad \text{[from eqn. 23·17]}$$

$$=n\frac{V}{V-v_s}\cdot\ \frac{V-v_o}{V}$$

$$\therefore\ n_2=n\frac{V-v_o}{V-v_s}=n\frac{1-v_o/V}{1-v_s/V} \qquad (23\cdot20)$$

The corresponding modified wavelength can similarly be found to be

$$\lambda_2=\lambda\frac{V-v_s}{V-v_o}=\lambda\frac{1-v_s/V}{1-v_o/V} \qquad \ldots \qquad \ldots \qquad (23\cdot21)$$

If either the source or the observer or both be moving in the opposite direction, the corresponding velocity or velocities should have the sign changed.

It is obvious from eqn. (23·20) that if the velocities v_s and v_o be equal i.e., if there be no relative velocity between the source and the observer, there is no change in the frequency.

In the most general cases, the motion of the source or the observer takes place in any direction. In such cases, for calculating the change in frequency, only the component of the velocities along the line joining the source and the observer should be taken into account. We have not also considered any motion of the medium through which the wave is travelling. If this is taken into account, an added term, the velocity of the medium, appears in the formula. This is not applicable in the case of light waves because the velocity of light does not depend upon the velocity of the medium.

We have mentioned earlier that Doppler effect applies to all types of wave motion and so the apparent shift in frequency can also be detected in the case of light.* The velocity of light is, however, so large that it is impossible to detect any change in frequency with terrestrial experiments. For celestial bodies the effect becomes detectable.

In the case of light waves, however, the frequencies are so larger that it is not possible to measure the changes in the frequencies. The changes in wavelengths are measured instead. The spectrum of light from any source shows a number of lines. Each of these lines corresponds to a definite wavelength. If the wavelength changes, the line shifts across the spectrum. When the source approaches the observer, the wavelength decreases and the line shifts to the violet side of the spectrum. Conversely, when the source recedes from the observer, the wavelength increases and the line shifts to the red side. By measuring the amount of shift, it is possible to measure the change in wavelength and hence it is possible to ascertain the velocity of the source of light. This has wide applications in astrophysics. For example, the effect leads to the detection of the rotation of the sun and also to the determination of its velocity of rotation. If the sun is rotating, its one edge would approach us while the opposite edge would recede from us. This is found to be so, because examination of the spectrums of the light coming from the two edges of the sun's disc shows that in one the lines have shifted to the

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*** However, for light, the eqns. (23·20) and (23·21) are to be modified. The modification requires essentially the concept of relativity which is beyond the scope of the text and is, therefore, not discussed here.**

violet end, while in the other they are shifted to the red end. The nature of the saturn's ring was also established with the help of this effect.

Nowadays, Doppler effect is used in radar for determining the velocity of a fast moving projectile.

Example 23·17. *A train is passing a railway station with a speed of 60 km./hr. and blowing continuously its whistle of frequency 320 c.p.s. What will be the apparent frequencies noted by a person waiting on the platform when the train is (a) approaching and (b) departing ? [Given, velocity of sound=340 m./sec.]*

Solution : Here, the actual frequency $n=320$ c.p.s. Speed of the train $v=60$ km./hr.$=1{\cdot}67\times10^3$ cm./sec. Velocity of sound $V=340$ m./sec.$=34\times10^3$ cm./sec.

(a) From eqn. (23·13), the altered frequency

$$n_1=320\times\frac{34\times10^3}{10^3\,(34-1{\cdot}67)}=337 \text{ c.p.s.}$$

(b) From eqn. (23·15) the altered frequency

$$n_1=320\times\frac{34\times10^3}{10^3\,(34+1{\cdot}67)}=305 \text{ c.p.s.}$$

Example 23·18. *A car moving with a speed of 40 km./hr. passes by a siren which is sounding at a frequency of 500 c.p.s. What does the frequency appears to be to the driver of the car before and after passing ? [Velocity of sound=340 m./sec.]*

Solution : Here, $n=500$ c.p.s. ; $v=40$ km./hr.$=1{\cdot}11\times10^3$ cm./sec. ; $V=$ 340 m./sec.$=34\times10^3$ cm./sec.

(a) When the car approaches the siren, the altered frequency n_2 is given by eqn. (23·19) as

$$n_2=500\times\frac{(34+1{\cdot}11)\,10^3}{34\times10^3}=516 \text{ c.p.s.}$$

(b) When the car moves away from the siren after passing it, the altered frequency n_2 is given by eqn. (23·17) as

$$n_2=500\times\frac{(34-1{\cdot}11)\,10^3}{34\times10^3}=484 \text{ c.p.s.}$$

Example 23·19. *Two trains move towards each other at speeds of 72 km./hr. and 54 km./hr. respectively. The first train whistles emitting a sound with a frequency of 800 c.p.s. Find the frequency of the sound which can be heard by passenger in the second train : (a) before the trains meet (b) after the trains meet. The velocity of sound is 340 m./sec.*

Solution : Here $v_s=72$ km./hr.$=20$ m./sec,
$v_0=54$ km./hr $=15$ m./sec,

(a) Before the trains meet, the modified frequency n_1 due to the motion of the source which is approaching is giving by

$$n_1=n.\frac{V}{V-v_s}$$

Since the listener moves towards the source with a velocity v_o, the final modified frequency is given by

$$n_2=n_1.\frac{V+v_o}{V}=n.\frac{V+v_o}{V-v_s}$$

$$=800\times\frac{340+15}{340-20}=887{\cdot}5 \text{ c.p.s.}$$

(b) After the trains meet, the modified frequency n_1 due to the motion of the source which is receding is given by

$$n_1=n.\frac{V}{V+v_s}$$

Since the listener also recedes away from the source with a velocity v_o, the final modified frequency is given by

$$n_2=n_1\frac{V-v_o}{V}=n.\frac{V-v_o}{V+v_s}$$

$$=800\times\frac{340-15}{340+20}=722{\cdot}2 \text{ c.p.s.}$$

Example 23·20. *Prove that according to Doppler's principle the change of pitch is greater when the source approaches the observer than when the observer approaches the source with the same speed.*

Solution : When the source approaches the observer with a velocity v, the altered frequency is

$$n_1=n\frac{V}{V-v}$$

When the observer approaches the source with the same velocity v, the altered frequency is

$$n_2=n\frac{V+v}{V}$$

$$\therefore\quad \frac{n_1}{n_2}=\frac{V^2}{V^2-v^2}$$

As $(V^2-v^2)<V^2$, therefore, $\frac{n_1}{n_2}>1$ or, $n_1>n_2$ Hence the prof.

23·28. Polarization.

The various properties of waves which we discussed in the foregoing sections are common to both longitudinal and transverse waves. These properties, therefore, can not distinguish one from the other. We shall now discuss a phenomenon known as **polarization** *which is a characteristic of transverse waves only, and not of longitudinal waves.* Hence if a wave can be polarized, we can certainly say that it is transverse. The phenomenon of polarization, therefore, helps us to distinguish a transverse wave from a longitudinal one.

The concept of polarization can be easily grasped with the help of the following demonstration experiment. A long rope *ABCD* is passed through two slits S_1 and

S_2 [Fig. 23·26]. The width of each slit is slightly greater than the diameter of the rope. One end D of the rope is kept fixed while the other end A is shaken back and forth arbitrarily making an angle with the slit S_1, but always in a plane perpendicular to the rope. The particles of the rope will, therefore, vibrate in any direction perpendicular to its length. As a result transverse wave will be generated and will travel along the rope. This transverse wave travelling along the portion AB of the rope is said to be ***unpolarized***. As this wave reaches the slit S_1, components of vibrations which are parallel to the slit get through it. So the particles in the portion BC of the rope beyond the slit S_1 begin to vibrate in a direction parallel to the slit. Therefore, the wave that emerges from the slit S_1 remains confined in a plane containing the slit. This wave is said to be ***polarized*** and the slit S_1 is called a ***polarizer***.

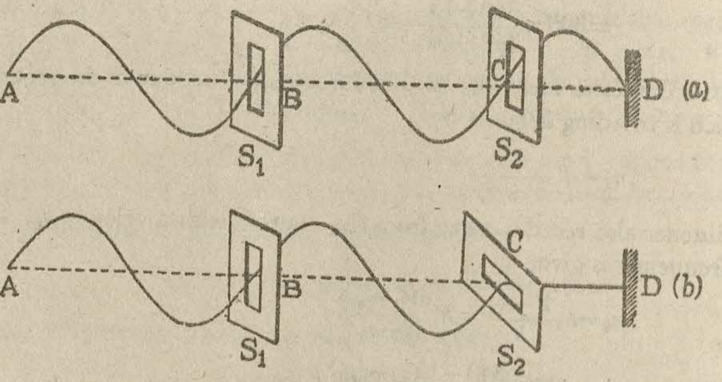


Fig. 23·26

This polarized wave travels through the portion BC of the rope and reaches the second slit S_2. If this slit is parallel to the first one, it is obvious that the wave passes through the second slit also and proceeds along the portion CD of the rope [Fig. 23·26 (*a*)]. If, however, the second slit is perpendicular to the first, it completely obstructs the wave so that the portion CD of the rope remains undisturbed [Fig. 23·26 (*b*)]. For any intermediate orientation of the slit S_2, the component of vibration parallel to it will be transmitted. Hence transverse wave of smaller amplitude and confined in a plane containing the slit S_2 will be generated in the portion CD of the rope. This slit S_2 is called the ***analyzer***. Obviously if the analyzer is rotated about in its own plane, the amplitude of the wave that passes through it varies from zero to a maximum.

If we now replace the rope by a spiral spring and generate longitudinal wave in it, we shall see that this wave passes through the slits irrespective of their orientations.

This behaviour of the transverse wave in the rope on passing through a slit is exhibited by all types of transverse waves. A longitudinal wave does not possess such characteristic because the motion of the particles takes place along the direction of propagation of the wave.

For a transverse wave, the motion of the particles may take place in any or all

possible directions but always perpendicular to the direction of propagation of the wave. We call such transverse wave **unpolarized**.

If the motion of all particles in the path of wave takes place in a definite plane, the transverse wave is said to be **polarized**.

The device which converts an unpolarized wave into a polarized one by suppressing the vibrations in every direction but one is called a **polarizer**. *Usually a similar device is employed for examining the nature of the polarized wave coming out of the polarizer. Such a device is called an* **analyzer**.

Phenomenon of polarization provides a surest check for ascertaining the nature of a wave. The discovery that light beams can be polarized led to the firm conclusion that light waves are transverse. We shall discuss this, in a detail, in a later section. Sound waves can never be polarized and hence they must be longitudinal in nature.

● EXERCISE ●

[A] Essay type Questions.

1. What do you mean by wave motion ? Draw a diagram representing a wave motion and indicate the amplitude, crest, trough and the wavelength.

2. What do you mean by a transverse wave and a longitudinal wave ? Explain with examples. Show how a longitudinal wave may be represented by a sine curve. State the difference between a transverse wave and a longitudinal wave.

3. Define amplitude, wavelength, period, frequency and velocity of a wave. Establish the relation among velocity, frequency and wavelength of a wave.

4. Define wavefront and a ray. When are the waves called spherical and plane ? How such waves are obtained in practice ?

5. State the characteristics of a progressive wave and establish its equation.

6. Discuss the phenomena of reflection and refraction of waves clearly stating the concerned laws.

7. State and explain the principle of superposition of waves. Mention some examples in support of this principle. When two waves are superposed, does one affect the progress of the other ? Explain.

8. What are stationary waves ? Explain their formation graphically. State the characteristics of such waves.

9. What do you mean by nodes and antinodes ? Explain with necessary figures.
Describe in each case one method for demonstation of transverse and longitudinal stationary waves.

10. Explain the formation of stationary waves analytically. Give one example of transverse and longitudinal waves respectively. Discuss about the distinction between progressive and stationary waves.

11. Enumerate the laws of vibration of a stretched string. Briefly discuss their experimental verification.

12. Define :—fundamental, overtone and harmonics.
Show by means of diagrams the possible modes of vibration of a stretched string and calculate the frequency in each case.

A string is struck at the middle point. Explain what harmonics will be present and what absent.

13. Briefly describe a sonometer and discuss its working principle.

14. Explain the formation of stationary waves in closed and open pipes.

Show that the frequency of the fundamental of an open pipe is twice that of a closed pipe of the same length.

15. Describe the possible modes of vibration of air column in closed and open pipes. Obtain the frequencies of their fundamentals and overtones.

16. Describe a method of determining the velocity of sound in air and the frequency of a tuning fork by resonance air column.

What is meant by end correction in this experiment ? How this is eliminated ?

17. Discuss what is meant by 'interference'. Explain in detail how interference pattern is formed.

18. What conditions must be satisfied for the formation of a stationary interference pattern ? Explain you answer.

Discuss how can you demonstrate the phenomenon of interference in the case of sound waves.

19. What are beats ? How are they produced ? Show both (i) graphically and (ii) analytically that the number of beats produced per second equals the difference between the frequencies of the superposed waves.

20. Discuss the conditions that must be satisfied for clear audibility of beats. How can you determine an unknown frequency with the help of beats.

21. What is Doppler effect ? Show how the frequency of sound is changed due to Doppler effect when the observer is stationary and the source is moving.

22. Give some common examples of the Doppler effect. Establish the expression for the modified frequency of the sound heard due to Doppler effect when the source is at rest and the observer is moving.

23. Calculate the modified frequency of the waves if both the source and the observer are moving. What happens when there is no relative velocity between the source and the observer.

Can Doppler effect be detected in the case of light ? Explain with examples. Mention some applications of Doppler effect in astrophysics.

24. Show directly without using the general formula and using neat diagrams that the frequency of sound vibrations emitted by a source will appear to be doubled to a listener fixed in space when the source approaches the listener with a speed equalling half the speed of sound.

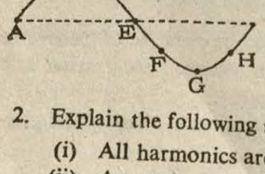
[*Jt. Entrance* '79]

25. What do you understand by polarization of a wave ? Explain how can you determine the nature of the wave from this phenomenon.

What do you mean by (i) an unpolarized wave, (ii) a polarized wave, (iii) polarizer and (iv) analyzer ?

[B] Short answer type questions

1. A transverse wave is travelling along a street from left to right. The figure represents the shape of the string at a given instant. *A, B, C, D, E, F, G* and *H* are points on the string. At ***this instant*** state :



(a) Which of the above points have an upward velocity ; (b) Which points will have a downward velocity ; (c) Which points will have zero velocity and (d) Which points have the maximum magnitude of velocity. [*I. I. T* '70]

2. Explain the following statements :—

(i) All harmonics are overtones but all overtones are not harmonics.

(ii) An open pipe emits a more musical sound than a closed pipe.

(iii) Longitudinal waves can travel through any medium but transverse waves can not travel through a gas .

(iv) The space-displacement curve and the time-displacement curve of a wave are of same form.

(v) In musical instruments, the strings are mounted on hollow wooden boxes.

(vi) A set of holes is bored along the length of flute.

(vii) By blowing a closed pipe more and more powerfully, the quality of the emitted sound changes.

3. Does the velocity of a wave also give the velocity of the particles of the medium? Explain.

4. Explain how by touching lightly at suitable points, the quality of the sound emitted by a vibrating string can be modified.

5. Can light waves be polarized? Sound waves? Water waves? Explain.

6. How the frequency of the fundamental emitted by a stretched string will be modified in the following cases :—

(i) When its length is doubled. (ii) When its tension is increased four times.

7. Explain with reasoning whether the following statements are true or false :—

(i) When the stem of a vibrating tuning fork is pressed against the box of a sonometer the sound emitted becomes louder. This occurs due to resonance.

(ii) In a closed pipe, the frequencies of the overtones are integral multiples of the frequency of the fundamental.

(iii) One characteristics of a stationary wave is the creation of wave crests and wave troughs.

(iv) Beats are produced when the frequencies of two sources differ by a large amount.

8. The frequencies of the fundamentals emitted from a closed pipe and an open pipe are identical. What is the ratio of their lengths?

9. Draw the wave patterns set up in a resonance tube open at both ends when the length of the tube is equal to the wave-length and also when the length is 3/2 times the wave-length. [*Jt. Entrance* '82]

[D] Simple Problems

1. Compare the wavelengths in air of the sound given by two tuning forks of frequencies 280 and 840 respectively. [*Ans.* 3 : 1]

2. Radio waves travel at a speed of 3×10^{10} cm./sec. What is the frequency of the waves broadcasted from a radiostation on the wavelength of 250 metre. [*Ans.* 1200 kilo cycles/sec.]

3. Sound waves with a frequency of 400 c. p. s. and amplitude 0·2 mm. propagate in air. The wavelength is 85 cm. (a) Find the velocity of the waves and (b) the maximum velocity of the particles of air. [*Ans.* (a) 340 m./sec. (b) 50·24 cm./sec.]

4. The frequency of a tuning fork is 480. The time in which the fork executes 20 complete vibrations, sound advances through a distance of 15 metre. Find the velocity and wavelength of sound. [*Ans.* 360 m./sec., 75 cm.]

5. The tip of a vertical stylus attached to a prong of a tuning fork just dips into water. When the fork is vibrating, waves generated in water have 21 crests in a distance of 10 cm. Find the velocity of the waves, given frequency of the fork=256 c.p.s. [*Ans.* 128 cm./sec.]

6. A sound produced in water has a wavelength of 3·7 metre. If the velocity of sound in water is 1480 metre/sec. and in air 350 metre/sec., find the frequency and the wavelength of the note heard by an observer in air. [*Ans.* 400 c.p.s. ; 87·5 cm.]

7. The equation of a transverse wave in a string is given by $y=5 \sin \pi\,(4t-0{\cdot}002x)$, where x and y are given in centimetre and t in second. Find the amplitude, frequency, speed and wavelength of the wave. [*Ans.* 5 cm. ; 2 c.p.s. ; 2000 cm./sec. ; 1000 cm.]

8. The equation of simple harmonic oscillation of a source is $y=6 \sin 300\,\pi t$ cm. Find the displacement from the position of equilibrium, the velocity and the acceleration of a point 150 cm. away from the source for a moment $t=0{\cdot}01$ sec. after the oscillations begin. The oscillations propagate with a velocity of 300 m./sec. [*Ans.* -6 cm. ; 0 ; $5{\cdot}324 \times 10^6$ cm./sec.^{2}]

9. What is the difference in phases of two points at a distance 10 m. and 15 m. respectively from the source of oscillations ? The period of oscillations is 0·02 sec. and the velocity of their propagation 400 m./sec. [*Ans.* $5\pi/4$ rad.]

10. The displacement from the position of equilibrium of a point 5 m. away from a source of oscillations is half the amplitude at the moment $t=\frac{T}{6}$. Find the wavelength. [*Ans.* 60 m.]

11. The equation of a stationary wave is $y=4 \cos \frac{\pi x}{5} \sin 60\ \pi t$ where x and y are in centimeter and t is in second. Find the amplitude and velocity of the component waves. What is the distance between nodes ? [*Ans.* 2 cm. ; 300 cm./sec. ; 5 cm.]

12. Find the frequency of the fundamental tone of a string tensioned by a force 8×10^3 newton. The length of the string 80 cm. and its weight is 40 gm. [*Ans.* 250 c.p.s.]

13. A steel string of radius 0·5 mm. produces a fundamental tone of frequency 320 c.p.s. when stretched by a force of 100 kg. wt. Find the length of the string. The density of steel= 7·7 gm./c.c. [*Ans.* 62·9 cm.]

14. Two violin strings, one of steel and the other of gut, possess the same length and cross-sectional area. If they are subjected to the same tension, compare the frequencies of the notes emitted by them. (Density of steel 7·8 gm./c.c., of gut 1·3 gm./c.c.) [*Ans.* $1 : \sqrt{6}$]

15. Two stretched strings *A* and *B* of the same material and same diameter emit, when excited, notes whose frequencies are in the ratio 1 : 8 respectively. If the length of *A* is four times that of *B*, compare the tensions in *A* and *B*. [*Ans.* 1 : 4]

16. When a stretched wire of length 60 cm. is set into vibration, it emits a note of frequency 280. Find the wavelength of the vibration (a) in the wire (b) in the air. (Velocity of sound in air=350 metre/sec.) [*Ans.* 120 cm. ; 125 cm.]

17. The tension in a wire 80 cm. long is 5 kg. wt. What change must be made in length in order to raise the pitch one octave, if the tension is increased to 7·2 kg. wt. (*London Univ.*) [*Ans.* decreased by 32 cm.]

18. Two wires emitting fundamental notes are in unison. If the length of the first wire is increased by 1%, what percentage change in the tension of the second wire will restore unison. [*Ans.* 1·97% decrease]

19. A sitar wire is 60 cm. long and emits a note of 320 c. p. s. How far from the top it may be pressed so that it may emit a note of frequency 384 c. p. s. ? [*Ans.* 10 cm.]

20. Two iron wires of equal lengths emit their fundamentals when stretched by loads of mass 144 gm. and 324 gm. If the frequencies of the fundamentals bear the ratio 5 : 3, calculate the ratio of the radii of the wires. [*Ans.* 2 : 5]

21. A wire of length 360 cm. is vibrating transversely in 6 segments under a tension of 2 kg. wt. If the mass of the wire is 1·6 gm., calculate the frequency of vibration. [*Ans.* 175 c. p. s.]

22. Calculate the frequency of (a) the fundamental (b) the first overtone of an organ pipe closed at one end and of length 125 cm. Assume the velocity of sound in air to be 350 m./sec.

Also solve the problem when the pipe is opened at both ends.

[*Ans.* 1st part : (a) 70 c.p.s. (b) 210 c.p.s. 2nd part : (a) 140 c.p.s. (b) 280 c.p.s.]

23. The frequencies of the fundamental emitted by a closed and an open pipe differ by 20. If the length of the open pipe be 170 cm., calculate the length of the closed pipe. Given, the velocity of sound in air=340 metre/sec. [*Ans.* 70·8 or 106·25 cm.]

24. An open organ pipe of length 50 cm. emits the fundamental note of the same pitch as that of a sonometer wire of length 75 cm. The wire has a mass of one gram per centimetre. Find the tension of the wire. (Velocity of sound=350 m./sec). [*Ans.* $27{\cdot}56\times10^8$ dyne]

25. A tuning fork is held over a glass tube standing in water. The first resonance is obtained when the length of the air column is 25 cm. Resonance is again produced when the length of the air column is 77·2 cm. Calculate the end correction. [*Ans.* 1·1 cm.]

26. A tuning fork held over a resonance tube produced resonance with air columns 16 cm. and 50·4 cm long. If the velocity of sound at room temperature be 352 metre/sec., calculate frequency of the fork and radius of the tube. [*Ans.* 511·6 c. p. s,, 2 cm.]

27. The bottom of an open organ pipe is suddenly closed. If it emits the first overtone in both the cases, the difference in frequency being 128, find the frequency of the tone emitted by the open pipe. [*Ans.* 512 c. p. s.]

28. The frequencies of the fundamentals of two pipes, one open and the other closed differ by 15 c. p. s. If the length of the closed pipe is 70 cm., find that of the other, given velocity of sound in air is 350 metre/sec. [*Ans.* 125 cm. or 159·1 cm]

29. A cylinder of length 100 cm and diameter 4 cm. is filled up with water. The cylinder is fitted with a tap at the bottom. A vibrating fork of frequency 480 c. p. s. is held above the mouth of the cylinder and water is slowly drained out by opening the tap Taking into account the end-correction, determine the lengths of the tube for which resonances will be observed, given the velocity of sound in air=352 m./sec. [*Ans.* 17·13 cm., 53·8 cm , 90·47 cm.]

30. A tuning fork *A* gives 5 beats per second when sounded with another tuning fork *B* of frequency 256. When a little wax is placed on a prong of the tuning fork *A*, there are more beats per second. What is the frequency of *A* ? [*Ans.* 251 c. p. s.]

31. A sonometer wire tensioned by 15 kg. wt. produces 8 beats per second, when sounded with a tuning fork. When the tension in the wire is 16 kg. wt., it becomes tuned in unison with the fork. Find the frequency of the fork. [*Ans.* 252 c. p. s.]

32. Two wires having same mass per unit length are mounted side by side on a sonometer and adjusted to have equal tensions. One wire is 65 cm. long and gives a fundamental tone of frequency 256 c. p. s. If the length of the other wire is 64 cm. how many beats per second will be heard when the two wires are sounded together. [*Ans.* 4 beats/sec.]

33. An open organ pipe produces 8 beats per sec. when sounded with a tuning fork of 256 vibrations per second, the fork giving the lower tone. How much the length of the pipe be altered to bring it in accord with the fork ? (*C. U.* '53)
[*Ans.* To be increased by 1/32th of the original length]

34. Two tuning forks *A* and *B* are sounded together and 5 beats per sec. are produced. If *A* and *B* are in resonance with 36 cm. and 37 cm. air column respectively both closed at one end, calculate the frequencies of the forks. [*Ans.* 185 c. p. s , 180. c p. s.]

35. A sonometer wire resonates with a given tuning fork. Keeping the tension same, if the length of the wire is increased by 2%, it produces 6 beats per sec. with the fork. Find the frequency of the fork. [*Ans.* 306 c. p. s.]

36. The movable bridge of a sonometer divides the wire into two parts whose lengths differ by 1 cm. The total length of the wire is 50 cm. When both parts of the wire are set into simultaneous vibration, 4 beats per sec. are produced. Find the frequency of vibration of each part. [*Ans.* 102 c.p.s., 98 c.p.s.]

37. Two tuning forks when excited simultaneously produces 6 beats/sec. The first one resonates with a closed column of air 15 cm. long and the second one with an open column of length 30·5 cm. Calculate the frequencies of the forks. [*Ans.* 366 and 360 c. p. s.]

38. A tuning fork produces 5 beats per sec., with the lengths 75 cm. and 80 cm. of a sonometer wire. Find the frequency of the fork. [*Ans*· 155 c. p. s.]

39. A closed pipe and an open pipe of lengths 15 cm. and 30·2 cm. respectively are sounded together and the number of beats produced is found to be 4 per second. Determine the velocity of sound in air. [*Ans.* 362·4 m./sec.]

40. A whistle of frequency 500 c. p. s. is blown at a station. What will be its apparent frequency to an observer on a train moving towards the station with a velocity of 36 km. per hour ? The velocity of sound is 350 metre per second. [*Ans.* 514·3 c. p. s.]

41. Calculate the velocity with which a source of frequency 10 thousand per second should approach the observer at rest in order to produce a Doppler effect of 200 per second.
(*C. U.* '54) [*Ans.* 21·96 ft./sec. assuming V=1120 ft/sec.]

42. A man stands on the sea shore and hears the hooting of a ship. When the ship stands still, the sound has a frequency of 380 c. p. s. When the ship moves towards the man, the frequency appears to be 390 c. p. s. When the ship moves away from the man, the frequency appears to be 375 c. p. s. Find the speed of the ship in both cases. The velocity of sound= 340 m./sec. [*Ans.* 31·38 km /hr., 16·32 km./hr.]

43. The ratio of the frequencies of a source of sound as received by a stationary observer when the source moves towards and away from the observer is 9 : 8. Determine the velocity of the source, given that velocity of sound in air is 340 m./sec, [*Ans.* 20 m./sec.]

44. Two engines pass each other in opposite directions. One of them is blowing a whistle, the frequency being 540. Calculate the frequency heard in the other engine before and after they have passed each other. Velocity of either engine 15 m./sec., velocity of sound 320 m./sec. [*Ans.* 593·1 and 491·6 c. p. s.]

45. A transverse wave is described by the equation $y=a \sin 2\pi\left(nt-\frac{x}{\lambda}\right)$, show that the maximum particle velocity is equal to four times the wave velocity if $\lambda=\pi a/2$.

46. A sonometer wire under a tension of 64 newton vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of the sonometer wire has a length of 10 cm. and a mass of 1 gm. The vibrating tuning fork is now moved away from the vibrating wire with a constant speed and an observer standing near the sonometer hears one beat per sec. Calculate the speed with which the tuning fork is moved if the speed of sound in air is 300 m./sec. (*I. I. T. '83*) [*Ans.* 75·19 cm./sec.]

47. A string 25 cm. long and having a mass of 2·5 gm. is under tension. A pipe closed at one end is 40 cm. long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m /sec, find the tension in the string. (*I. I. T. '82*) [*Ans.* 27·04 newton]

48. A metal wire of diameter 1 mm. is held on two knite edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire vibrating with its fundamental frequency, and a vibrating tuning fork produce 5 beats/sec. The tension in the wire is then reduced to 81N. When the two are excited, beats are heard again at the same rate. Calculate (i) the frequency of the fork, (ii) the density of the material of the wire. (*I. I. T '80*)
[*Ans.* (i) 95 c.p.s. (ii) 12·73 gm./sec.]

[D] Harder Prolebms.

1. A sonometer wire is stretched by a heavy brass cylinder of length 10·0 cm. whose axis is vertical. Its fundamental note is found to be in unison with a tuning fork of frequency 256 sec.$^{-1}$. On immersing part of the cylinder in water, 4 beats per second are heard when wire and fork are sounded simultaneously. Calculate the length of the cylinder immersed. [Density of brass=8·5 gm. cm.$^{-3}$] (*London Univ.*) [*Ans.* 2·636 cm.]

2. A wire of density 9 gm./c. c. is stretched between two clamps 100 cm. apart while subjected to an extension of 0·05 cm. What is the lowest frequency of transverse vibrations in the wire, assuming Young's modulus of the material to be 9×10^{11} dyne/cm.^{2} (*I. I. T. '75*) [*Ans.* 35·4 c. p. s.]

3. A tuning fork resonates with 50 cm. of a sonometer wire. When the stretching weights are completely immersed in water, the length of the wire that produces resonance with the same fork is 45 cm. Determine the density of the material of the weights. [*Ans.* 5·26 gm./c.c.]

4. A wire of mass per unit length 0·02 gm./cm. is stretched by a force of 8×10^6 dyne. The wire is found to resonate with a frequency of 480 c. p. s. The next higher frequency with which the wire resonates is 512 c. p. s. What is the length of the wire ? [*Ans.* 312·5 cm.]

5. (a) At what point should a stretched string be plucked to make its fundamental tone most prominent ? At what point should it be plucked and at what point touched to make (b) the second harmonic (c) the third harmonic most prominent ?

[*Ans.* (a) centre (b) plucked at $\frac{1}{4}$th of its length from one end then touched at centre (c) plucked and then touched respectively at $\frac{1}{6}$th and $\frac{1}{3}$rd of its length from one end.]

6. Two waves of the same type having frequencies 320 and 336 move out from the same point. Find their phase difference after 2·2 sec. [*Ans.* 0·4π rad.]

7. A vibrating tuning fork of frequency 256 is held above the mouth of a glass cylinder of length 50 cm. and capacity 400 c.c. Water is poured slowly until resonance is obtained. The volume of water poured in the cylinder is found to be 130 c.c. Find the velocity of sound in air. [*Ans.* 345·6 m./sec.]

8. Two wires are fixed on a sonometer. Their tensions are in the ratio 8 : 1, the lengths in ratio 36 : 35, the diameters in the ratio 4 : 1 and the densities are in the ratio 1 : 2. Find the frequency of the beats produced if the note of the higher pitch has frequency of 360 per second. (*I. I. T. '68*) [*Ans.* 10 beats/sec.]

9. A set of 24 tuning forks is arranged in a series of increasing frequency. If each fork produces 4 beats with the preceding one and the last fork sounds the octave of the first, calculate the frequencies of the first and last fork. (*Bombay '54*) [*Ans.* 92 and 184 c. p. s.]

10. A car travels perpendicular to a high wall with a speed of 36 km. p. h. The driver sounds a horn of frequency 200 c. p. s. What is the frequency of the sound heard by the driver coming (a) directly from the horn (b) reflected from the wall ? Velocity of sound=340 m./sec.
[*Ans.* (a) 200 c. p. s. (b) 212·1 c. p. s.]

11. Two observers *A* and *B* carry identical sound sources of frequency 1000 c. p. s. If *A* is stationary while *B* moves away from *A* at a speed of 2 metre/sec., how many beats per sec. are heard by *A* and *B* ? Given, velocity of sound=350 m./sec.
[*Ans.* 5·68 and 5·71 per sec.]

12. Two sources give out identical notes of 1360 c. p. s. An observer half-way between them moving from one to other hears 4 beats per sec. At what speed is he moving ? [Given velocity of sound=340 m./sec.] [*Ans.* 1·8 km./hr.]

13. A spectroscopic examination of a certain star shows that the apparent wavelength of a certain spectral line is 5001 A. U., whereas the observed wave-length of the same line produced by a terrestrial source is 5000 A. U. In what direction and at what speed do these figures suggest that the star is moving relative to the earth ? (*C. U. '47*)
[*Ans.* Moving away with a velocity of 6×10^6 cm./sec.]

14. A whistle is whirled in a circle of 100 cm. radius and traverses the circular path twice per second. An observer is situated outside the circle, but in its plane. What is the musical interval between the highest and lowest pitch observed if the velocity of sound is 332 metre per sec. ? (*London Univ.*) [*Ans.* 1·079]

[*Hint* : The interval between two musical notes is determined by the ratio of their frequencies]

15. Sound waves from a tuning fork *A* reach a point *B* by two different paths *A D B* and *A E B*. When *A E B* is greater than *A D B* by 17 cm., there is silence at *B*. When the difference is 34 cm. there is sound at *B* and when 51 cm. silence and so on. Explain the phenomenon and find the frequency of the fork if the velocity of sound is 340 m./sec.
[*Ans.* 1000 c. p. s]

16. If the fundamental frequency of a string 60 cm. long is *n*, where would you place a bridge under the string to produce notes of frequency n_1, for one part and n_2 for the other, so that the intervals from *n* to n_1 and from n_1 to n_2 shall be the same. (*London Univ.*)
[*Ans.* 37·1 cm. from one end]

17. A uniform rope of length 12 m. and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg. is attached to the free end of the rope. A transverse pulse of wavelength 0·06 m. is produced at the lower end of the rope. What is the wave length of the pulse when it reaches the top of the rope ? (*I. I. T. '84*) [*Ans.* 0·12 m.]

18. A steel wire of length 1 m., mass 0·1 kg. and uniform cross-sectional area 10^{-6} m² is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If the transverse

waves are set up by plucking the string in the middle, calculate the frequency of fundamental mode of vibration. (Given for steel, co-eff. of linear expansion$=1{\cdot}21\times10^{-5}/^\circ C$ and Young's modulus$=2\times10^{11}$ Newton/m^2.) (*I. I. T.* '84) [*Ans.* 11 c. p. s.]

19. A car travelling at a speed of 36 km./hr. sound its horn which has a frequency 500 Hz and this is heard in another car which is travelling behind the first car in the same direction with a velocity of 20 m./sec. The sound can also be heard in the second car by reflection from a bridge head. What frequencies will the driver of the second car hear ? Sound travels in air with a speed of 340 m./sec. (*Jt. Entrance '84*) [*Ans.* 514·3 Hz ; 545·5 Hz]

20. The ends of a string are rigidly fixed. If the temperature of the string falls by 10°C, calculate the change in its tension from the following data :

Area of corss-section of the string$=0{\cdot}01$ cm.^{2}
Co-efficient of linear expansion of the material of the string$=16\times10^{-6}/^\circ C$.
Young's modulus of the material of the string$=20\times10^{11}$ dyne/cm.^{2}

If this causes the frequency of transverse vibration of the string to increase to double its previous value, what was the original tension ? (*Jt. Entrance* '80)
[*Ans.* 32×10^5 dyne ; $10{\cdot}67\times10^5$ dyne]

24
CHAPTER

SOUND WAVES

24·1. Sound waves as elastic waves.

Sound is a form of energy. From the source of sound, this energy is propagated through the surrounding medium in the form of waves. When these waves reach our ear we have the sensation of sound. We have already mentioned that sound waves are longitudinal. This follows immediately from the fact that sound waves are transmitted through air or any other gas, for we know that a gas can transmit only longitudinal waves, not transverse waves.

A source of sound is always in a state of vibration. When a tuning fork is struck or a violin string is bowed, sound is produced. If we now lightly touch the fork or the violin string, the vibrations can be felt and there is immediate cessation of sound. The vibrating body produces a disturbance in the surrounding medium (usually air). Waves are, therefore, generated and these waves—called sound waves—travel outwards from the source.

We have already seen that the phenomena of interference, diffraction etc., which are characteristics of wavemotion are also shown by sound. This conclusively proves that sound is a kind of wave. These various evidences in favour may be put down in a compact manner as under :

(i) For the propagation of sound from one point to other, a medium is necessary. In such process, on bodily motion of the medium itself does take place. Such mode of propagation is a characteristic of wavemotion.

(ii) For generating wave, a disturbance must be created in the medium. This is also true in the case of sound. All sources of sound are vibrating bodies.

(iii) Like waves, sound travels through a medium with a particular velocity which is greater in solids and liquids than in gases.

(iv) Sound suffers reflection and refraction obeying the same laws as waves.

(v) The phenomenon of interference which is a characteristic of wavemotion only, is also shown by sound.

(vi) Diffraction is a distinguishing property of all wavemotions. Sound also exhibits diffraction.

(vii) Sound can travel through a gas. This property and the fact that sound can not be polarized show that sound waves are longitudinal.

(viii) Actual photograph of sound waves can be taken.

All these evidences clearly show that sound is a longitudinal wave.

Mechanism of propagation of sound waves.

We know that a vibrating body acts as a source of sound. The sound waves thus produced propagate through the surrounding medium as longitudinal waves.

To understand the mechanism of propagation of these waves, we shall take for simplicity a tuning fork as the source of sound, the surrounding medium being air. When a tuning fork is set into vibration, its prongs begin to vibrate to and fro about their mean positions. Let us consider the waves generated due to the vibration of one of its prongs only, it being understood that identical wavetrains proceeding in the opposite direction are also produced by vibration of the other prong.

As the prong of the tuning fork moves outwards towards b, it compresses the immediately adajcent layer of air [Fig. 24·1 (a)]. Due to elastic property, this compressed layer tries to come back to its normal condition. In such endeavour, it compresses the next layer and so on. All the layers therefore, get compressed in turn one after another. Thus a pulse of compression (C) advances through air with a definite velocity. This velocity is found to depend upon the elasticity and density of the medium.

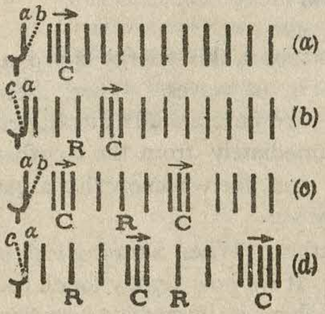


Fig. 24·1 : Propagation of sound waves

After completion of the forward movement, the prong begins to move inwards towards a, so that a partial vacuum is created in the region immediately behind it. The adjacent layer of air, therefore, expands and a rarefaction (R) is produced [Fig. 24·1 (b)]. As before, this layer causes a rarefaction of the air layer next to it and so on. In this way, every succeeding layer becomes rarefied one after another and a pulse of rarefaction is sent out. This pulse follows the pulse of compression and travels in the same direction with the same velocity. This goes on till the prong reaches the extreme end c of its backward journey.

So we conclude that during the forward journey of the prong, a pulse of compression is produced, while during its backward journey a pulse of rarefaction is created. But one forward and one backward movement constitute one complete vibration of the prong. Hence during one complete vibration of the prong, one compression and one rarefaction travel forward through air. As the durations of the forward and backward journeys of the prong are equal, it follows that the length of a compression must be the same as that of rarefaction.

The above processes will be repeated if the prong continues to vibrate and a succession of compressions and rarefactions will be created in the medium [Fig. 24·1 (c) and (d)]. These travel forward with the same velocity and a longitudinal wavemotion is, thereby, produced. The length of the wave is given by the total length of a compression and a rarefaction. When this longitudinal wave reaches our ear, it sets the Tympanic membrane within the ear into forced vibration in unison with it. Thus the sensation of sound is produced. The above discussion is valid for all types of sources and all kinds of media whether solid, liquid or gas.

24·2. Sources of sound.

We have already mentioned that a vibrating body acts as a source of sound. However, average normal ears respond only to those vibrations whose frequencies lie between 20 to about 20000 c. p. s.

These lowest and highest audible frequencies represent the *limits of audibility.* This range of frequency varies considerably from individual to individual. Generally the upper limit decreases with age of the individual. We can thus say that *any vibrating body whose frequency lies within the limits of audibility may be regarded as a source of sound.*

With proper excitation, all elastic substances may be set into such vibrations. We have already discussed two such common sources of sound, viz. a vibrating string and a vibrating air column. These are widely used in musical instruments. A few other most common sources of sound are vibrating rods, plates, diaphragms etc.

In the laboratory, a tuning fork is widely used as a source of sound. It consists of a steel bar bent in the shape of U and is provided with a stout stem at its bend. It is usually set into vibration by striking one of its prongs with a padded hammer. The importance of a tuning fork as a source of sound of single frequency.

24·3. Velocity of sound waves.

Sound waves propagate through a medium with a definite velocity, the value of which varies greatly from medium to medium. Sound travels faster in solids and liquids than in gases. That the velocity is not exceedingly large can be inferred from many common phenomena. For example, the sound of thunder is always heard after the lapse of a few seconds since the flash of lightning is actually seen.

Newton showed from theoretical considerations that the velocity of longitudinal waves in a homogeneous elastic medium is given by

$$V=\sqrt{E/\rho} \qquad \cdots \qquad \cdots \qquad (24{\cdot}1)$$

where E is the modulus of elasticity appropriate to the medium and ρ is the density of the medium. *For solids in the form of a narrow rod* or *wire*, the appropriate modulus of elasticity is Young's modulus because only one dimension is involved. Hence the velocity of sound is given by

$$V=\sqrt{Y/\rho} \qquad \cdots \qquad \cdots \qquad (24{\cdot}2)$$

In the case of a liquid or a gas, bulk modulus K must be used since a fluid does not possess Young's modulus or modulus of rigidity. Hence, for a fluid, the velocity of sound is given by

$$V=\sqrt{K/\rho} \qquad \cdots \qquad \cdots \qquad (24{\cdot}3)$$

24·4. Velocity of sound in gases.

(*i*) *Newton's formula* : When sound waves travel through a gas, a train of compressions and rarefactions follow each other. Compression of a layer produces an increase of pressure and consequently, the temperature of the layer rises. As the layer gets rarefied its pressure drops producing a corresponding fall in its temperature. Newton assumed that successive compressions and rarefactions occur at such a rate that enough time is left for the heat generated in a compression to be dissipated to the surrounding body of the gas or for the cold produced in a rarefaction to be fully compensated by heat flowing in from the surrounding volume of the gas. No change of temperature of the gas is, therefore, produced during the propagation of sound waves i.e., sound waves propagate through the gas under *isothermal conditions*. In that case Boyle's law is obeyed.

Let us consider a given mass of a gas under a pressure P and having a volume V. If the pressure is increased by a very small amount p, the volume decreases by a small amount v. From Boyle's law,

$$PV=(P+p)\ (V-v)=PV-Pv+pV-pv$$

The term pv being a product of two small quantities is negligibly small. Neglecting it, we have

$$v=pV \qquad \text{or,} \qquad P=\frac{p}{v/V}$$

But p represents the volume stress and v/V is the volume strain. Hence their ratio gives the bulk modulus K of the gas.

$$\therefore\ K=P$$

This result can be obtained alternatively by calculus. Boyle's law gives $PV=$ constant. Differentiating, we get

$$PdV+VdP=0$$

$$\text{or,} \qquad P=-\frac{dP}{dV/V}=\frac{\text{Volume stress}}{\text{Volume strain}}=\text{Bulk modulus}=K$$

The negative sign indicates that an increase of pressure produces a decrease in volume.

Hence from eqn. (24·3), we have for the velocity of sound through a gas

$$V=\sqrt{\frac{P}{\rho}} \qquad \cdots \qquad \cdots \qquad (24{\cdot}4)$$

For air at N.T.P., $P=76\times13{\cdot}6\times981$ dyne/cm.^{2} and $\rho=0{\cdot}001293$ gm./c.c. Substituting,

$$V=\sqrt{\frac{76\times13{\cdot}6\times981}{0{\cdot}001293}}=280 \text{ metre/sec. (approx.)}$$

But the experimentally determined value of V is 332 metre/sec. So the theoretical value is too low and for many years, the reason of such discrepancy remained a mystery.

(ii) Laplace's correction.

Over 100 years later, the source of discrepancy was pointed out by Laplace. He argued that the compressions and rarefactions taking place during the propagation of sound occur so rapidly that the rise or fall of temperature during compression or rarefaction have not enough time to be dissipated away by thermal contact with the surrounding layers of air. The poor radiating and conducting powers of a gas lend support to this argument. The condition is, therefore, not isothermal as suggested by Newton but *adiabatic*. Hence, instead of Boyle's law, the adiabatic equation PV^{γ}=constant, should be applied where γ is the ratio of specific heat of the gas at constant pressure to that at constant volume.

Now if the pressure be increased adiabatically by a small amount p producing a small diminution v in volume, we have

$$PV^{\gamma}=(P+p)(V-v)^{\gamma}=V^{\gamma}(P+p)\left(1-\frac{v}{V}\right)^{\gamma}$$

Expanding by Binomial theorem, and neglecting the higher powers of v/V, which are very small quantities, we get

$$P=(P+p)\left(1-\gamma\frac{v}{V}\right)=P-\gamma\frac{Pv}{V}+p-\gamma\frac{pv}{V}$$

Both p and v being small quantities, the term involving their product viz. pv, can be neglected.

$$\therefore \quad p=\gamma\frac{Pv}{V}$$

$$\text{or,} \quad \gamma P=\frac{p}{v/V}$$

$\therefore$ Bulk modulus of elasticity $K=\gamma P$.

This result can be derived in a simpler manner by using calculus. From the adiabatic equation PV^{γ}=constant, we have on differentiation,

$$V^{\gamma}dP+P\gamma V^{\gamma-1}dV=0 \qquad \text{or,} \quad VdP+\gamma PdV=0$$

$$\text{or,} \quad \gamma P=-\frac{dP}{dV/V}=\text{Bulk modulus}=K$$

Substituting in eqn. (24·3), the velocity of sound in a gas is given by,

$$V=\sqrt{\frac{\gamma P}{\rho}} \qquad \dots \quad \dots \quad \dots \qquad (24{\cdot}5)$$

For air $\gamma=1{\cdot}41$; taking the values of P and ρ as before, we get $V=332{\cdot}5$ metre/sec., which is in good agreement with the experimental result.

This equation shows that the velocity is independent of frequency and amplitude. For very intense sounds, however, the velocity is found to increase with the amplitude.

24·5. Effect of various factors on the velocity of sound.

(i) **Effect of pressure** : *Temperature remaining unchanged, a change of pressure does not alter the velocity of sound through a gas.*

If the temperature remains constant, then for a given mass of gas, we have from Boyle's law PV=constant. But for a given mass $V \propto 1/\rho$ where ρ denotes the density of the gas. $\therefore$ P/ρ=constant. Since the velocity of sound is given by $V=\sqrt{\gamma P/\rho}$, it follows that the ***velocity of sound in any gas is independent of the pressure, so long as the temperature of the gas remains constant.***

(ii) **Effect of temperature** : If the temperature of the gas is changed, its density changes producing a consequent alteration in the velocity of sound through the gas.

From the ideal gas equation, we have

$$PV=nRT$$

where n is the number of moles of the gas, T its temperature in absolute scale and R is the universal gas constant.

If the corresponding mass of the gas be m and its molecular weight be M, then $n=m/M$

$$\therefore\quad PV=\frac{m}{M}RT$$

$$\text{or,}\quad \frac{P}{\rho}=\frac{P}{m/V}=\frac{RT}{M}$$

Combining with the eqn. (24·5) for the velocity of sound through the gas, we get

$$T=\sqrt{\frac{\gamma RT}{M}} \qquad \cdots \quad \cdots \quad \cdots \qquad (24{\cdot}6)$$

$$\therefore\quad V\propto\sqrt{T}$$

This shows that the *velocity of sound in a gas is directly proportional to the square root of its absolute temperature.*

If V_t and V_0 be the velocities of sound in a gas at the temperature t°C. and 0°C. respectively and if T and T_0 be the corresponding absolute temperatures, then

$$\frac{V_t}{V_0}=\sqrt{\frac{T}{T_0}}=\sqrt{\frac{t+273}{273}}=\left(1+\frac{t}{273}\right)^{\frac{1}{2}}$$

Expanding the r.h.s. by Binomial theorem and neglecting the higher order terms of $t/273$, we get the approximate equation

$$V_t=V_0=\left(1+\tfrac{1}{2}\times\frac{t}{273}\right) \qquad \cdots \quad \cdots \qquad (24{\cdot}7)$$

Putting $V_0=332$ m./sec., $V_t=332\left(1+\frac{1}{2}\times\frac{t}{273}\right)=(332+0{\cdot}61t)$ m./sec.

Hence *the velocity of sound in air increases by about 0·61 metre/sec. or 61 cm./sec. for each degree centigrade rise of temperature.*

(iii) **Effect of humidity** : Water vapour remains present to some extent in the atmosphere. As the density of water vapour is less than that of dry air at the same temperature and pressure, the presence of water vapour lowers the density of air. Since the velocity of sound in a gas varies inversely as the square root of the density, therefore, *the velocity of sound in moist air is always greater than that in dry air at the same temperature.*

It can be shown that, if V_m represents the velocity of sound in moist air at a temperature t°C. and pressure P and V_d, the velocity in dry air under the same condition of temperature and pressure, then

$$V_d=V_m\sqrt{\frac{P-0{\cdot}378f}{P}}$$

where f is the saturation pressure of water vapour at t°C.

24·6. Velocity of sound in different gases.

Let us consider two gases which possess the densities ρ_1 and ρ_2 under identical conditions of temperature and pressure. Then if the velocities of sound through them be V_1 and V_2 respectively, we have

$$V_1=\sqrt{\frac{\gamma P}{\rho_1}} \text{ and } V_2=\sqrt{\frac{\gamma P}{\rho_2}}$$

$$\therefore\quad \frac{V_1}{V_2}=\sqrt{\frac{\rho_2}{\rho_1}} \qquad \cdots \qquad \cdots \qquad (24\cdot8)$$

So the velocities of sound in different gases are inversely proportional to the square roots of their densities. For example, if we want to compare the velocities of sound in oxygen and hydrogen, then since the density of oxygen is 16 times that of hydrogen, we have

$$\frac{V_1}{V_2}=\sqrt{\frac{16}{1}}=4$$

Hence under identical conditions of temperature and pressure, sound travels four times faster in hydrogen than in oxygen.

It should be mentioned that in the above discussion we have tacitly assumed that the value of γ remains the same for all the gases considered. But the value of γ depends upon the number of atoms in the molecule of a gas. The above assumption is justified in the sense that all ordinary gases are diatomic and for all of them $\gamma=1\cdot41$. In general, however, we must take into account the nature of the gas and put the proper value of γ.

Example 24·1. *Steel has Young's modulus 21×10^{11} dyne/cm². and density 7·8 gm./c.c. Find the velocity of sound through steel.*

Solution : $V=\sqrt{\frac{E}{\rho}}=\sqrt{\frac{21\times10^{11}}{7\cdot8}}=5\cdot189\times10^5$ cm./sec.

$=5189$ m./sec.

Example 24·2. *Sound propagates in kerosene with a velocity of 1330 m./sec. Find the compressibility of kerosene. Given, density of kerosene=0·8 gm./c.c.*

Solution : $V=\sqrt{\frac{K}{\rho}}$, K being the Bulk modulus of kerosene.

$$\therefore\quad K=V^2\rho$$
$$=(1\cdot330\times10^5)^2\times0\cdot8 \text{ dyne/cm.}^2$$

$$\therefore \text{ Compressibility}=\frac{1}{K}$$
$$=7\cdot065\times10^{-11} \text{ cm.}^2\text{/dyne.}$$

Example 24·3. *Find the velocity of sound in metre per second in carbon dioxide gas at 0°C., given that the pressure is 10^6 dyne per sq. cm., and the density 0·00198 gm. per c.c. (γ for carbon dioxide is 1·34).*

Solution :

We have $V_0=\sqrt{\frac{\gamma P}{\rho}}=\sqrt{\frac{1\cdot34\times10^6}{0\cdot00198}}=260$ m./sec.

Example 24·4. *Calculate the velocity of sound in air at 20°C. when the atmospheric pressure is 76 cm. of mercury. Given that γ is 1·41 and density of air is 0·001293 gm. per c.c. at 0°C.*

Solution : The velocity of sound in air at 0°C. is given by,

$$V_0=\sqrt{\frac{\gamma P}{\rho}}=\sqrt{\frac{1{\cdot}41\times 76\times 13{\cdot}6\times 981}{0{\cdot}001293}}=332{\cdot}5 \text{ m./sec.}$$

∴ From eqn. (24·7), the velocity of sound at 20°C.

$$=332{\cdot}5\left(1+\frac{1}{2}\times\frac{20}{273}\right)=344{\cdot}7 \text{ m./sec.}$$

Example 24·5. *A person hears a sound of thunder just 4 sec. after he received the flash of lightning. If the temperature of the air be 20°C., calculate his distance from the place where the sound originates. [Given, velocity of sound at 0°C.=332·5 m./sec.]*

Solution : From the preceding example, we find that the velocity of sound in air at 20°C. is 344·7 m./sec. Now, if the distance required be x, we have $V_{20}=\frac{x}{t}$, where $V_{20}=344{\cdot}7$ m./sec.

Hence, $$x=V_{20}\times t=344{\cdot}7\times 4=1378{\cdot}8 \text{ m.}$$

Example 24·6. *Calculate the velocity of sound in hydrogen assuming the velocity of sound in air to be 330 m./sec. and the density of air to be 14·3 times that of hydrogen.*

Solution : Let V_H and V_A be the velocities of sound in hydrogen and air respectively. Now, we have from eqn. (24·8),

$$V_H=V_A\times\sqrt{14{\cdot}3}=330\times 3{\cdot}78=1248 \text{ m./sec.}$$

Example 24·7. *A narrow test tube, 16·5 cm. long emits a note of frequency 512 c.p.s. when a current of air is blown across its mouth, the temperature being 10°C. Determine the velocity of sound at 0°C.*

Determine the pitch of the note (a) if the temperature is raised to 20°C., (b) if the air is replaced by hydrogen at 10°C. Density at N.T.P. of air=0·001293 gm./cc., of hydrogen=0·000089 gm./c.c.

Solution : From the relation $V=4nl$ for closed pipe, we have

$$V_{10}=4\times 512\times 16{\cdot}5 \text{ cm./sec. in air.}$$

$$\therefore\ V_0=\frac{V_{10}}{1+\frac{1}{2}\cdot\frac{t}{273}}=\frac{4\times 512\times 16{\cdot}5}{1+\frac{1}{2}\cdot\frac{10}{273}}$$

$$=33190 \text{ cm./sec.}$$

(a) $$V_{20}=V_0\left(1+\frac{1}{2}\cdot\frac{20}{273}\right)$$

$$=33190\times\frac{566}{546} \text{ cm./sec.}$$

If n_1 be the altered frequency, we have

$$n_1=\frac{V_{20}}{4l}=\frac{33190\times566}{546\times4\times16\cdot5}=521\cdot3 \text{ c.p.s.}$$

(b) If n_2 be the required frequency

$$n_2=\frac{V_{10}\text{ (hydrogen)}}{4l}$$

$$\text{Since } n=\frac{V_{10}\text{ (air)}}{4l}$$

$$\therefore \text{ we have, } \frac{n_2}{n}=\frac{V_{10}\text{ (hydrogen)}}{V_{10}\text{ (air)}}=\frac{V_o\text{ (hydrogen)}}{V_0\text{ (air)}}=\sqrt{\frac{\rho\text{ (air)}}{\rho\text{ (hydrogen)}}}$$

$$\therefore\ n_2=512\sqrt{\frac{0\cdot001293}{0\cdot000089}}=1952 \text{ c.p.s.}$$

Example 24·8. *A tuning fork of frequency 512 c.p.s. emits a note of wavelength 67·5 cm. in air at 17°C. If the density of air at N.T.P. be 1·293 gm. per lt. calculate the ratio of the two principal specific heats of air. Assume that the density of mercury is 13·6 gm. per c.c.*

Solution : Velocity of sound in air at 17°C.

$$V_{17}=n\lambda=512\times67\cdot5 \text{ cm./sec.} \qquad \ldots \quad \ldots \text{ (i)}$$

If ρ_o and ρ_{17} be the densities of air at 0°C. and 17°C. respectively, we have,

$$\frac{\rho_0}{\rho_{17}}=\frac{273+17}{273}=\frac{300}{273}$$

$$\rho_{17}=\rho_o\frac{273}{300}=0\cdot001293\times\frac{273}{300}$$

$$\therefore\ V_{17}=\sqrt{\frac{\gamma P}{\rho_{17}}}=\sqrt{\frac{\gamma\times76\times13\cdot6\times980\times300}{0\cdot001293\times273}} \qquad \ldots \quad \ldots \text{ (ii)}$$

From (i) and (ii) we get

$$\gamma=1\cdot39.$$

Example 24·9. *At what temperature sound travels in nitrogen with the same velocity as in oxygen at 15°C., given that the atomic weight of the gases are 14 and 16 respectively.*

Solution : Let the required temperature be T°K. Velocity of sound in nitrogen at T°K.

$$V_N=\sqrt{\frac{\gamma RT}{2\times14}} \qquad \ldots \quad \ldots \text{ (i)}$$

Also, velocity of sound in oxygen at 15°C.

$$V_o=\sqrt{\frac{\gamma R\,(273+15)}{2\times16}} \qquad \ldots \quad \ldots \text{ (ii)}$$

Since $V_N=V_o$, from (i) and (ii) we get,

$$T=\frac{14\times288}{16}=252°\text{K}.$$

∴ The required temperature is −21°C.

24·7. Determination of the velocity of sound.

(i) **In air** : In the year 1829, Arago carried out the measurement of the velocity of sound in air. In the experiments, two observers were stationed on the peaks of two hills several miles apart. One of the observers fired a gun. With the help of a stop watch, the other observer noted the interval between seeing the flash and hearing the report. A large number of observations were made and the mean time interval (t sec.) was calculated. If the distance between the two stations be x, then the velocity of sound in air is given by

$$V=\frac{x}{t} \qquad \dots \qquad \dots \qquad (24{\cdot}9)$$

In the above determination, it was tacitly assumed that light travels with such high velocity that the flash was seen by the second observer the moment the gun was fired by the first.

Though the method is a simple one, it is liable to a number of errors which are discussed below :—

(*a*) ***Effect of wind*** : The velocity of sound is affected by the velocity of wind. The sound travels more rapidly with the wind than against it. Let the velocity of the wind be v. If the true velocity of sound (in still air) be V, then with the wind, the velocity of sound will be $V+v$, while against the wind, it will be $V-v$.

To eliminate this error, observations are taken in both directions. Both the observers are provided with a gun and a stop watch. When one fires the gun, the other measures the time interval, say t_1. Then the second observer fires the gun and the first notes the interval, say t_2. If the wind is supposed to be blowing from the first station towards the second, then

$$V+v=\frac{x}{t_1} \qquad \text{and} \qquad V-v=\frac{x}{t_2}$$

$$\therefore \quad V=\frac{x}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}\right) \qquad \dots \qquad \dots \qquad \dots \qquad (24{\cdot}10)$$

Hence the error due to wind is eliminated.

If the wind is blowing obliquely across the line joining the two observers, the calculation is a bit complicated ; the above value, however, approaches very near the true value, except when the velocity of wind is too large.

(*b*) ***Personal equation*** : In performing any type of observation an appreciable interval elapses between the actual event taking place and the observer realising that it has occurred. The interval between an impression being received and the actual recording being done by the observer is called his *personal equation.*

The personal equation differs for different observers. For a well trained observer, it is usually of the order of 1/10 second.

This error can be eliminated only by automatic recording of the interval by electrical means.

(*c*) ***Effect of temperature and humidity of air*** : As we have seen earlier (vide Art. 24·5) the velocity of sound depends upon the temperature and the humidity of air. The mean temperature and the relative humidity of air during the experiment were measured and the necessary corrections were employed to find the velocity of sound in dry air at 0°C.

(ii) **Velocity of sound in water :** In the year 1825, Colladon and Sturm determined the velocity of sound in water by carrying out the experiments in Lake Geneva.

Two boats were anchored at a considerable distance apart. A bell hung under water from one of these boats ; in the other, a large trumpet shaped receiver dipping well below the water surface was arranged. An arrangement was made whereby a small quantity of gunpowder was ignited when the bell was struck. The observer in the other boat measures the interval between seeing the flash and receiving the sound of the bell in the under-water receiver. Hence the velocity of sound in water can be found. The value obtained was 1435 metre per sec. at 8·1°C., i.e. about four times the velocity in air.

(iii) **Velocity of sound in solids :** Biot was the first to carry out the determination of the velocity of sound in cast iron. He constructed a very long pipe of cast iron (about 1000 metre) by joining end to end a large number of pipes. On striking a hammer at one end, two reports were heard at the other ; the first was due to sound travelling through the iron pipe and the second due to sound passing through air. The interval, t, between the reports was measured.

If V_1 be the velocity of sound in iron and V be that in air under the experimental conditions, then

$$t=\frac{L}{V}-\frac{L}{V_1} \quad \ldots \quad \ldots \quad \ldots \quad (24\cdot11)$$

where L is the length of the pipe.

$$\therefore \quad V_1=\frac{LV}{L-Vt}$$

Hence V_1 can be determined if V is known.

Example 24·10. *A person X on the top of a mountain fires a gun. Another person Y on the top of another mountain heard the sound 5 sec. after seeing the flash. If the distance between X and Y be 1750 m. find the velocity of sound in air.*

Solution : Velocity $V=\frac{\text{distance}}{\text{time}}=\frac{1750}{5}=350$ m./sec.

Example 24·11. *Two persons A and B are stationed at the top of two mountains distant 5·5 km. from one another. A heard the report of B's gun 16·5 sec. after seeing the flash. Later, A fires his gun and B heard the report 15·5 sec. after seeing the flash. Determine the velocity of sound and the component of the speed of the wind along AB.*

Solution : From eqn. (24·10), the velocity of sound

$$V=\tfrac{1}{2}\times5\cdot5\times10^3\left(\frac{1}{16\cdot5}+\frac{1}{15\cdot5}\right)=344\cdot1 \text{ m./sec.}$$

The component of the velocity of wind along AB is given by

$$v=\frac{x}{t_1}-V=\frac{5\cdot5\times10^3}{15\cdot5}-344\cdot1$$
$$=10\cdot8 \text{ m./sec.}$$

24·8. Reflection and Refraction of sound waves.

In the previous chapter we have seen that when waves are incident on the boundary separating two homogeneous media, reflection and refraction of waves take place according to definite laws known as the laws of reflection and laws of refraction. These laws hold true for all types of waves. Hence sound waves also exhibit the phenomena of reflection and refraction which can be demonstrated simply by the following experiments :

Demonstration Experiments.

(*a*) *Reflection at a plane surface* : A large plane wooden board *W* is mounted vertically on a table. This board acts as the reflecting surface. Two hollow tubes *A* and *B* are placed horizontally in front of the board [Fig 24·2]. Their axes meet at some common point, say *O*, on the board. The tube *B* can be turned horizontally about the point *O*. A wooden screen *S* is placed in between the tubes.

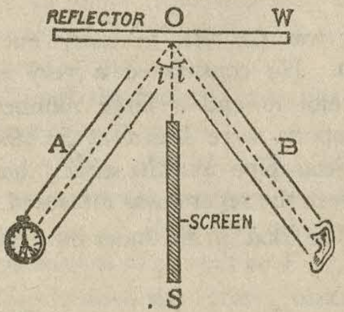


Fig. 24·2 : Reflection of sound at a plane surface

A small watch is held close to the end of the tube *A* as shown in the figure. The observer's ear is pressed to the end of *B* and the tube is slowly turned to determine the position at which the ticking sound of the watch is heard most clearly and loudly. It will be found that at this position, the tubes subtend equal angles with the board.

Hence if we draw normal to the board at the point *O*, the angle between the normal and the axis of the tube *A* (angle of incidence) will be equal to that between the normal and the axis of the tube *B* (angle of reflection). If the tube *B* is displaced in either direction from this position, no sound will be heard. This demonstrates that sound waves obey the law of reflection. The screen *S* cuts off the direct sound of the watch from the ear.

(*b*) *Reflection at a curved surface* : Two large concave reflectors. M_1 and M_2, mounted on stands are placed facing each other on a table [Fig. 24·3]. The axis of one is made coincident with that of the other. A small watch is kept at the focus of one of the reflectors, say M_1. A funnel *F* is placed at the focus of M_2. A rubber tube is fastend to the end of *F*. On placing the ear to the other end of the tube at *E*, the ticking sound of the watch can be heard distinctly. If either the funnel *F* or the mirror M_2 is displaced slightly, the sound becomes inaudible.

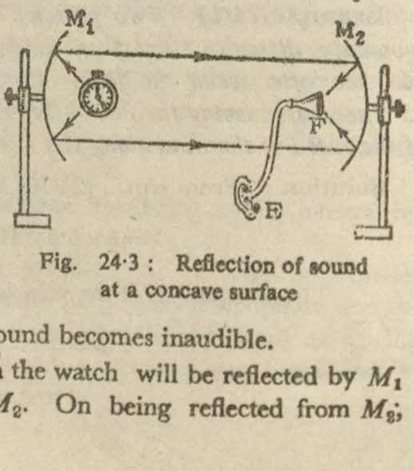


Fig. 24·3 : Reflection of sound at a concave surface

This happens because the sound waves from the watch will be reflected by M_1 and will proceed as a parallel beam towards M_2. On being reflected from M_2, the sound waves will converge towards the focus of M_2. Hence the ticking sound can be heard distinctly only at the focus of M_2.

(*c*) *Refraction by a 'lens'* : A large 'lens' about 6 inches or more in diameter is made by cementing two discs of oiled silk or cellophane together round their edges and filling the space between them with a gas which is denser than air, e.g. CO_2. It will act like a converging lens.

A watch is placed on one side of this lens at a comparatively large distance from it [Fig. 24·4]. The ticking of the watch can be heard distinctly by placing the ear at some particular point *E* on the other side of the lens. The position of this point can be found by trial. At other points, nearer or further from the lens, the ticking is not heard.

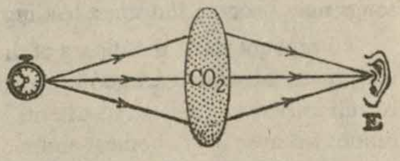


Fig. 24·4 : Refraction of sound

This shows that sound waves from the watch are refracted by the 'lens' and converge at a particular point on the other side of it.

If the 'lens' is made of a gas which is less dense than air, e.g. hydrogen, it will act like a diverging one and no such point of convergence of sound can be found.

An important discussion : When a wave is incident on a reflecting surface, reflection occurs when the size of the reflector is fairly large in comparison with the length of the said wave. For regular reflection, the irregularities on the reflecting surface should be of dimensions much smaller than wavelength of the incident wave.

Wavelength of sound is much longer than that of visible light. The range of wavelength of audible sound extends from about 1·5 cm. to about 16 metre whereas that of visible light from about 4×10^{-5} cm. to about 8×10^{-5} cm. Hence for the reflection of sound waves, much larger reflectors are required than those necessary for visible light. However, for visible light, the reflector should be very smooth whereas this is quite unnecessary in the case of sound waves. Thus buildings, brick walls, a row of trees, mountains, clouds etc. serve as good reflectors of sound but not of light. On the other hand, a small mirror reflects light but not sound.

24·9. Practical applications of reflection of sound.

(i) *Speaking tube* : This is simply a metal tube [Fig. 24·5] When words are spoken at one end of the tube, the sound waves proceed through the tube by suffering repeated reflections from its sides. The energy of the wave, therefore, remains confined within the limits of the tube. When the ear is pressed to the other end of the tube, the words can be heard clearly and distinctly.

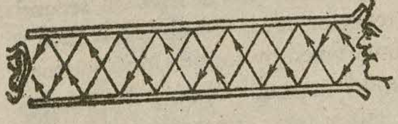


Fig.24.5 : Speaking tube

A doctor's stethoscopes or the horns of a gramophone makes use of this principle.

(ii) When one speaks inside a room, the sound gets reflected from the walls, floor and ceiling. The intensity (*i.e.* loudness) of the sound, thereby, increases. This advantage is not available in open air. Hence the difficulty of speaking or singing in the open air compared to that in a small room.

(iii) In large rooms or in public halls multiple reflections of sound from the walls, the floor and ceiling gives rise to a sensation of the prolongation of the sound. This phenomenon is known as *reverberation*. As a result, speeches or songs may become indistinct leading to difficulty to the audience.

To prevent this, the floors of the halls are carpeted and the walls are covered with materials which are good absorbers of sound so that multiple reflection of sound is minimised to the desired extent. Presence of audience also minimises reverberation because our bodies and clothes act as good absorbers of sound. For this reason, it is often easier to speak before a large audience than in an empty hall.

(iv) The rolling of thunder is also due to multiple reflections of sound from different layers of clouds and land surfaces.

(v) In churches or in auditoriums, a concave reflecting surface is placed behind the seat of the orator. The seat is arranged to be at the focus of the reflector. The reflected sound proceeds as a parallel beam and thereby enables the audience seated at the other end of the hall to hear the voice of the speaker clearly. This design has been rendered obsolete after the invention of amplifiers.

(vi) To hear a distant feeble sound we usually place the palm at the back of the ear in a curved way. The sound waves being reflected from the curved face of the palm converge towards the ear. This increases the intensity of the sound which becomes audible.

24·10. Echo.

The most familiar example of the reflection of sound is the phenomenon of echoes. ***If a sound is reflected back from an obstacle to the observer in such a way that he hears it distinctly as a repetition of the original sound, an echo is said to be produced.*** Walls, large buildings, cliffs, hillsides, woods etc. act as good reflectors for the production of echoes.

Minimum distance of the reflector : (i) *Sound of short duration* : When we hear a sound, its impression persists in our hearing mechanism for about 1/10 second. This is known as *persistance of hearing*. For this reason, we can distinctly hear two sounds as separate only if they reach our ears at least 0·1 second apart ; otherwise we get the impression of a single sound. Hence we can say that an echo is heard only when the reflected sound reaches our ears at least 0·1 second after the arrival of the original sound. The reflector must be placed at such a minimum distance from the observer that the time taken by sound in its journey to the reflector and back to the observer is at least 0·1 second. Taking the velocity of sound in air under ordinary conditions to be 340 m./sec. approximately, sound travels 34 m. in 0·1 sec. Hence, in order that an echo may occur, the reflecting surface must be at least 34/2=17 m. away from the observer.

(ii) ***Articulate sound*** : To hear the echo of an articulate sound, the reflecting surface must be situated at a still greater distance. This happens due to the fact that a man cannot distinctly pronounce more than 5 syllables in one second : our ears also can not recognise as separate more than 5 syllables in one second ; in 1/5 second, sound travels 68 m. If a man utters a single syllable, say *a*, he will hear the echo distinctly if the reflected sound arrives after 1/5 second. Hence the reflecting surface must be situated at the minimum distance of 68/2=34 m. from him.

If the man pronounces three syllables, *a*, *b*, *c* for example, the reflector being situated at the same distance i. e. 34 m. from him, the echo of the first syllable ***a*** will reach him at the moment he is about to pronounce the second syllable ***b***. So he can not hear the echo of *a* distinctly. Arguing similarly, we reach the conclusion that he will be able to recognise only the echo of the last pronounced syllable, viz. *c*, distinctly. Such an echo is called a ***monosyllabic echo***. If the man stands at a distance of 2×34 m.=68 m. from the reflecting surface, he will be able to hear the echo of the last two pronounced syllables distinctly. This is called a ***disyllabic echo***. So we can conclude that if the reflecting surface be ***n***×34 m. from the observer, the echo of the last pronounced ***n*** syllables can be recognised distinctly. Echoes in which two or more syllables can be distinguished clearly are generally called *poly-syllabic echoes*.

Multiple echoes : If an observer stands between two reflecting surfaces separated by a distance of at least 34 m., he will be able to hear a series of echoes. This happens because sound reflected from one of the reflectors is again reflected back by of the other and so on. The observer thus gains the impression of multiple echoes. However due to absorption of sound at each reflection, the successive echoes will be more and more feeble. As mentioned earlier, the rolling sound of thunder occurs due to multiple reflections from different layers of clouds and land surfaces.

Practical applications. (i) *Determination of the velocity of sound* : The velocity of sound can be determined roughly by making use of the phenomenon of echo. Let us suppose that the observer is standing at a known distance, say x, from a suitable reflecting surface. x must be greater than the minimum distance required for the hearing of a distinct echo. The observer makes a sharp sound e.g. by firing a pistol, and records the time t after which the echo is received. If the velocity of sound under the experimental conditions be V, then

$$Vt=2x$$

$$\therefore \quad V=\frac{2x}{t} \qquad \ldots \qquad (24{\cdot}12)$$

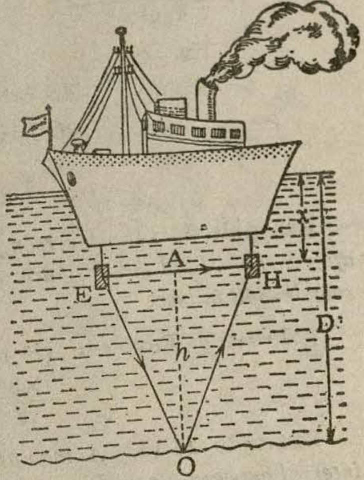


Fig. 24·6 : Determination of the depth of an ocean

(ii) ***Echo depth sounding*** : The depth of an ocean or the height of a flying aeroplane can be measured with the help of echo.

To find the depth of an ocean, a hydrophone *H* (an under-water microphone) is placed below water at one end of a ship [Fig. 24·6]. A small amount of explosive material E is kept under water at the same level from the other end of the ship. When the charge is exploded, the hydrophone receives two sounds : one that travels directly along the path EH and the other that travels along the path EO to the bed of the ocean, is reflected and travels back along OH. The intervals t_1 and t_2 which elapsed from the explosion to the actual reception of the two sounds are noted electrically. If V_w be the velocity of sound in sea water under the given conditions of the experiment,

$$\text{then } EH=V_w t_1$$

$$\therefore \quad EA=AH=V_w t_1/2$$

$$\text{and } EO+OH=V_w t_2 \qquad \text{But } EO=OH$$

$$\therefore \quad EO=OH=V_w t_2/2$$

$$\therefore \quad h=OA=\sqrt{EO^2-EA^2}=\frac{V_w}{2}\sqrt{t_2^2-t_1^2}$$

If the hydrophone or the explosive lies at the depth x, then depth of the ocean is given by

$$D=x+h=x+\frac{V_w}{2}\sqrt{t_2^2-t_1^2}$$

The height of a flying aerophane can be determined similarly, but the calculation is somewhat different. Let us suppose that an aeroplane is flying horizontally at a height H along the path AB with a velocity v [Fig. 24·7]. At A a sound is produced and the echo is received after a time t when the plane is at B. The path of sound is AOB, where obviously $AO=OB$; $OC=H$. Then if the velocity of sound in air under the given conditions be V, then

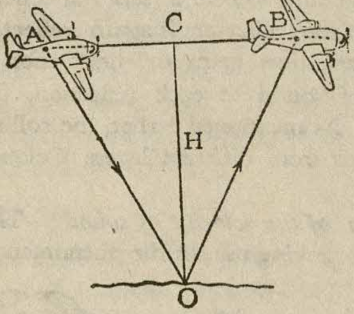


Fig. 24·7

$$AO+OB=Vt \quad \therefore \quad AO=OB=Vt/2$$

$$\text{Also,} \quad AB=vt \qquad \therefore \quad AC=CB=AB/2=vt/2$$

$$\therefore \quad H=OC=\sqrt{AO^2-AC^2}$$

$$=\frac{t}{2}\sqrt{V^2-v^2}$$

Example 24·12. *A person fires a gun and hears the echo from a distant cliff after 5 sec. Find the distance of the cliff from the person. Given, velocity of sound=340 m./sec.*

Solution : From eqn. (24·12), the distance is

$$x=\frac{5\times 340}{2}=850 \text{ m.}$$

Example 24·13. *A man at a certain distance from a cliff notes that the interval between a sound he makes and its echo is 4 sec. He then approaches the cliff by 170 m. and finds the corresponding interval to be 3 sec. Calculate (a) the velocity of sound and (b) the man's original distance from the cliff.*

Solution : Let d be the original distance of the man from the cliff. Then from eqn. (24·12), we get,

$$d=\frac{4V}{2}=2V \qquad \ldots \qquad \ldots \qquad (1)$$

$$\text{and } d-170=\frac{3V}{2} \qquad \ldots \qquad \ldots \qquad (2)$$

Solving (1) and (2) we get,

$V=340$ m./sec.

and $d=680$ m.

Example 24·14. *Find the minimum distance of a reflecting surface which produces a clear echo of a sound consisting of 3 syllables. Given the velocity of sound=1120 ft./sec.*

Solution : Since one fifth of a second is required to pronounce and hear one syllable distinctly, hence the minimum interval between the sound of 3 syllables and its echo should be $\frac{3}{5}$ sec.

$\therefore$ The required minimum distance

$$d=\frac{\frac{3}{5}\times 1120}{2}=336 \text{ ft.}$$

24·11. Some phenomena due to refraction of sound.

As mentioned below, sound wave in its passage through the atmosphere sometimes gets refracted and gives rise to the following interesting phenomena :

(*i*) **Effect of wind :** Presence of wind gives rise to a kind of refraction of sound waves. Fig. 24·8 (*a*) shows the plane wavefronts of sound travelling parallel to the ground in still air. Usually the velocity of wind, if present, increases with distance above the ground. When the wind blows in the same direction as that of sound waves, then the upper part of the wavefronts travels faster than the lower. As a result, the wavefronts bend towards the ground [Fig. 24·8 (*b*)]. Audibility along the ground is, therefore, enhanced.

If, on the other hand, wind blows against the direction of propagation of sound, then the upper portion of the wavefronts travel slower than the lower portion. Consequently, the wavefronts bend upwards and the audibility along the ground gets diminished [Fig. 24·8 (*c*)]. This explains why sound is more clearly heard with the wind than against it.

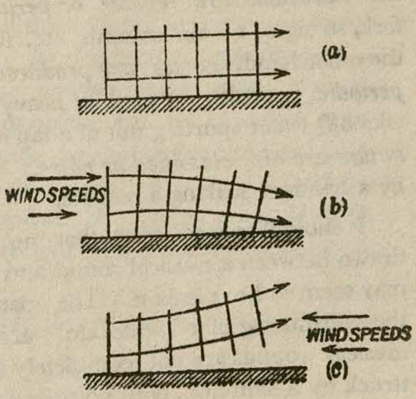


Fig. 24·8 : Refraction of sound due to wind

(*ii*) **Effect of variation of temperature :** The velocity of sound through a medium depends upon the density of the medium ; smaller the density, greater will be the velocity and vice versa. On a hot sunny day, the air near the ground is warmer than that at higher levels so that the density of air gradually increases with height. Consequently, sound wave travels faster near the ground than it does at upper levels. If we consider plane wavefronts of sound moving parallel to the ground, then due to this cause the wavefronts will bend away from the ground as shown in Fig. 24·9 (*a*). Thus the audibility will fall off rapidly with distance along the ground.

On the other hand, during the night time the air near the ground becomes colder than that at higher levels. Due to the consequent upward decrease in density, the velocity of sound increases as it goes upwards. As a result, the wave fronts will bend downwards as shown in Fig. 24·9 (*b*) and the audibility at ground level gets enhanced. Hence distant sounds are heard more clearly at night than during day time.

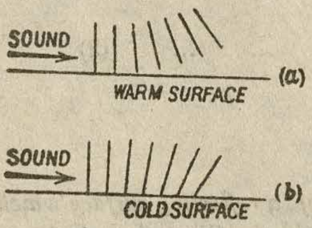


Fig. 42·9 : Refraction of sound due to variation of temperature

24·12. Musical sound and noise.

Sound may be divided roughly into two categories : (i) Musical sound and (ii) Noise. *Any sound that seems pleasant to hear is said to be musical ; if it appears unpleasant, it is called a noise.* This distinction is given from the physiological point of view.

The classification can also be made from the physical point of view. We know that sound is produced by a vibrating body. *A musical sound is produced when the vibrations are regular or periodic.* Sounds emitted by a vibrating tuning fork, string or an air column etc. furnish some examples of musical sound. On the other hand, *noises are produced when the vibrations are irregular or non-periodic.* Sounds produced by heavy traffic, running machines, an aircraft about to take off, water spurting out of a tap etc. are termed as noise. *Sounds of an abrupt nature are also classified as noise.* Examples are provided by the firing of a gun, by a hammer striking a nail etc.

It should be mentioned that no hard and fast line of demarkation may be drawn between a musical sound and a noise. Noises repeated at regular intervals may seem to be pleasant. The pattering sound of rain falling on a tinned roof, the murmuring of a rivulet etc. are some examples. On the other hand, many musical sounds are not completely free of noise. For example, when a bell is struck by a hammer, both noise and musical sound are produced. Moreover, the distinction between the two is rather subjective in character. The same sound which seems to be musical to one person may appear to be noisy to other persons and under different conditions.

24·13. Note and Tone.

A musical sound having a single frequency is called a tone. Obviously a tone is produced when the source of sound executes S.H.M. The prongs of a tuning fork execute S.H.M. Hence the sound emitted by a tuning fork is a tone.

A musical sound, however, generally consists of number of components having different frequencies. Such a sound is called a note. In this case the source also vibrates periodically in a complex manner. Examples are provided by the musical sounds emitted by a vibrating, air colcumn etc.

Any periodic motion can be obtained by the superposition of a number of S.H.M.s. It follows, therefore, that *a note is a combination of several tones.* An optical analogy is provided by white light which is a combination of light of seven different colours. In a note, the *fundamental tone* or *prime tone* and those possessing higher frequencies are called the *overtones* or upper practical tones. Those overtones whose frequencies are integral multiples of the frequency of the fundamental are called ***harmonics***. The particular harmonic whose frequency is double the fundamental frequency is called the second harmonic or *octave* of the fundamental. Clearly, *all the* ***harmonics*** *are overtones but all overtones are not* ***harmonics***. For example, if the note emitted by a musical instrument consists of the tones of frequencies 320, 480, 640, 800, 960, then the tone of frequency 320 is the fundamental and the others are overtones. Out of these overtones, those having the frequencies 480 and 960 are the octave, while the remaining two having frequencies 480 and 800 are not harmonics.

24·14. Characteristics of a musical sound.

One musical sound differs from the other in three fundamental particulars ; (*a*) *Intensity* (*b*) *Pitch* (*c*) *Quality*.

(*a*) **Intensity** ; It is defined as *the rate of flow of sound energy through unit area placed normal to the direction of propagation of sound.*

The **loudness** of a sound is the magnitude of the auditory sensation produced by the sound. *Intensity of a sound determines its loudness.* With increasing intensity, the loudness of a sound increases. The converse is also true. Clearly, the concept of loudness is subjective ; it depends on the ear and judgment of the individual observer. Two sounds of equal intensity may not appear to be equally loud to different observers if the sounds differ greatly in frequency. So loudness of sound depends also upon frequency. It is evident that intensity and loudness are not the same. Intensity is a physical property of sound whereas loudness is a physiological property.

It may be mentioned that *intensity is a characteristic of every sound which may be musical or not.*

Intensity depends on the following factors :

(*i*) *Amplitude of vibration of the source* : Intensity increases if the source of sound vibrates with a larger amplitude. The converse is also true. In fact, *the intensity of sound is proportional to the square of the amplitude.*

The effect can be easily understood because if the source vibrates with a greater amplitude, is imparts large amount of energy to the surrounding medium and hence the amount of energy reaching our ear also increases producing the sensation of a louder sound.

(*ii*) *Size of the source* : A vibrating body of larger area sets into vibration a greater volume of the medium and thus sends out a greater amount of energy. Hence *larger the size of the source, louder will be the sound produced by it.* For example, a kettledrum produces a much louder sound than a *tabla.*

(*iii*) *Distance from the source* : *Intensity of a sound falls off inversely as the square of the distance of the listener from the source,* producing a corresponding diminution in its loudness. This occurs because as the sound energy travels further from the source, it spreads out over a larger area and hence the rate of flow of energy per unit area decreases. The intensity is, therefore, lowered. The above law is known as inverse square law.

(*iv*) *Density of the medium* : Density of the medium affects the intensity of the sound. *Greater the density, louder will be the sound.*

(*v*) *Presence of other bodies* : Suitable bodies of larger surface area present near the source may be thrown into forced vibration whence the intensity of the sound will be enhanced. This is utilised in intensifying the sounds emitted by a tuning fork, or a vibrating string with the help of a sounding box. If the neighbouring bodies be set into resonant vibration, intensity increases by a much more marked degree. The case of resonant air column used for determination of velocity of sound is an example.

Presence of suitable reflectors also increases the intensity of sound.

(*b*) **Pitch :** *The characteristic of a musical sound by which a sharp or a shrill sound is distinguished from a dull, flat sound of the same intensity is called its pitch.* Shriller sounds are said to have higher pitch. If we press the keys of a harmonium one after another, different notes are heard due to the difference in pitch.

The pitch of a note is usually expressed by its frequency. *Greater the frequency, higher will be the pitch.* The converse is also true. This dependence of pitch on frequency can be illustrated by loading a tuning fork with a drop of wax. This will reduce the frequency of the fork and it will emit a flatter sound.

We know that an increase in the tension of a string raises its frequency of vibration. It will be found that the string then emits a shriller sound i.e., the pitch becomes higher.

Wavelength being inversely proportional to the frequency, sound of shorter wavelength is of higher pitch ; conversely sound of longer wavelength has lower pitch.

It should be noted that *pitch is a particular property of musical sound ; a noise has no definite pitch.* The optical analogue of pitch is the colour of light. Hence it is often called musical colour.

Although pitch is determined mainly by frequency, it had been found that pitch is also influenced by intensity and waveform. Increase in intensity by keeping the frequency constant raises the pitch of a tone ; however, for frequencies greater than 1000 Hz, the change of pitch with intensity is negligible. Moreover, tones of very short duration have a lower pitch than those of longer duration, for the same frequency.

Clearly, pitch and frequency are not identical. Pitch is a physical property of sound, but frequency is a physical property. Frequency of a sound does not depend upon other factors like intensity etc.

(c) Quality : *The characteristic of a musical sound which enables us to distinguish between notes of the same pitch and intensity produced by different sources is called its quality or timbre.* Due to this feature of a musical sound, we can distinctly recognise the sounds produced by different instruments in an orchestra although all of them are playing the same note.

Generally the sound produced by an instrument is not a pure tone but a note consisting of a blending of several simple tones. The frequency of the fundamental which is relatively most intense determines the pitch of the note. The presence of the overtones determines the quality of the note. The qualities of two notes are distinguished by the number of overtones present and their relative amplitudes. As the resultant wave curve is obtained by the superposition of all the components present, any change in these two factors will bring about a change in the waveform of the sound produced. Hence we may simply say that *the qualities of two notes will differ when their waveforms are different.*

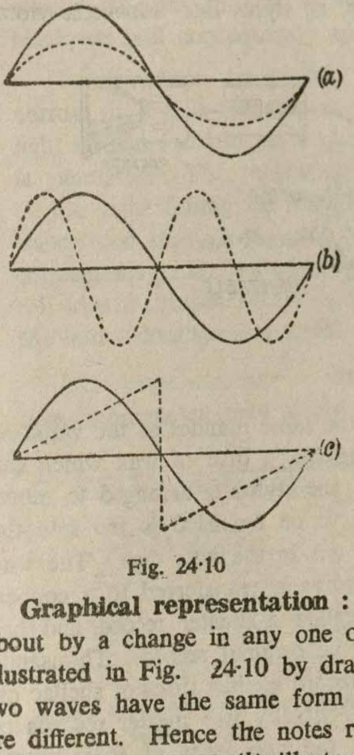


Fig. 24·10

The quality of a musical sound does not depend upon the relative phase of the tones present.

Graphical representation : The distinction between two notes brought about by a change in any one of the three characteristics mentioned above is illustrated in Fig. 24·10 by drawing the wave curves. In Fig. 24·10 (*a*) the two waves have the same form and the same frequency, but their amplitudes are different. Hence the notes represented by these wave curves will differ in loudness. Fig. 24·10 (*b*) illustrates two waves of the same form and of the same amplitude. But their frequencies differ. Consequently the notes corresponding to the two waves will differ in pitch. The waves shown in Fig. 24·10 (*c*) possess the same amplitude and frequency but their wave-forms are different. Hence the qualities of the corresponding notes will be different.

24·15. Recording and reproduction of sound.

Everyone of us is now familiar with various devices e.g. record player, tape recorder etc., which can reproduce faithfully the sound recorded earlier. In films also recording and reproduction of sound play an important role. Generally speaking, sound is recorded by first converting the sound energy into electrical energy, the electric current fluctuating in an identical manner as the waveform of the sound. With a proper arrangement, these fluctuations are imprinted on a suitable surface which thereby stores the waveform for subsequent reproduction. In the process of reproduction, a suitable device follows the fluctuations of the imprint and produces an electric current varying in the same manner. When this is reconverted into sound, we get back the original sound.

(i) *Record player* : In the modern method of recording, sound waves are converted into a fluctuating electric current with the help of a microphone. This current is then properly amplified with an electronic amplifier [Fig. 24·11 (*a*)]. The amplified fluctuating current sets a cutting stylus into transverse vibration, the amplitude of such vibration varying in the same manner as the variation in the current. The cutting stylus is pressed against a disc of wax which can rotate at a constant speed. As the disc rotates, the stylus is arranged to move slowly and radially inwards. When the sound to be recorded is fed into the microphone, a wavy spiral groove is, therefore, cut in the wax disc. The wax record is too soft for general use and the recording is transferred to a copper disc by an electroplating technique. This is called a master record and has tiny ridges which correspond to the grooves of the original record. Thousands of copies are then made by pressing this master record against discs of shellac or plastic which are softened by slight warming. Generally two master records of different sounds are pressed simultaneously on the two faces of these discs. This leaves on their surfaces grooves identical to those cut in the original recording. If a record is viewed under a microscope, this wavy spiral groove can be easily noticed.

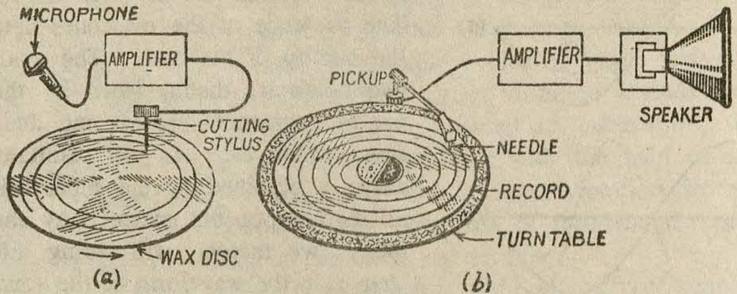


Fig. 24·11

To reproduce the sound, this record is placed on a turntable and rotated uniformly with the same speed as that employed in recording. A device, known as 'pick-up', presses a needle against the record, the needle being such that it fits into the groove [Fig. 24·11 (b)]. As the record rotates, the needle follows the groove whose wavy irregularities thereby causes the needle to vibrate from side to side. The pickup converts back these vibrations of the needle to a fluctuating electric current. This current is fed to an electronic amplifier. This amplified current from the amplifier is then led to a loudspeaker and the recorded sound is reproduced.

(ii) *Sound film* : Two methods are in use for recording sound on films, viz., the variable density method and the variable width method. We shall discuss in short only the former. The sound to be recorded is converted into a fluctuating electric current by a microphone. This is then amplified and allowed to pass through a special form of lamp. In accordance with the variation in the strength of the current, the intensity of the lamp also varies. The light from the lamp is focussed on a fine slit whose image is cast on the edge of the cinema film. The film moves past the slit at a steady rate. Due to the variation in intensity of the incident light, different parts of the film will receive different amount of light. On developing the film, a band of varying blackness will consequently appear along the edge of the film.

To reproduce the sound, light from a bright source is passed through a narrow slit and is focussed on the sound-track part of the film. The transmitted light is then incident on a photo-electric cell whereupon a small electric current is generated. The magnitude of this current is proportional to the intensity of the light falling on the photo-electric cell. As the intensity of the light transmitted through the film varies according to its degree of blackness which corresponds to the waveform of the original sound, the current generated by the cell also fluctuates accordingly. This varying electric current is amplified and fed into a loudspeaker whence the original sound is reproduced.

(iii) *Tape recorder* : In tape recorders, sounds are recorded on a plastic tape impregnated with finely powdered iron oxide. The tape is drawn past a recording head with a uniform speed. The recording head consists of an electromagnet with a very narrow gap between its poles [Fig. 24·12]. The amplified fluctuating current from the microphone is passed through this electromagnet, the field of which, therefore, varies according to the waveform of the sound to be recorded. As the tape moves past the recording head, it becomes magnetized by this field, the degree of magnetization varying from place to place in accordance with the variation in the field. In this way the original sound is recorded on the tape. When the tape is played back, it moves past a similar head, called the reproducing head, at the same speed used in recording. Since the tape is magnetized, it produces a weak magnetic field in the reproducing head, the strength of which varies with the fluctuations in the degree of magnetization of the moving tape. Consequently a varying electric current in induced in the coil. This is amplified and passed into a loudspeaker when the original sound is reproduced.

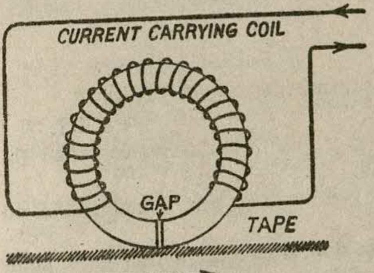


Fig. 24·12

● EXERCISE ●

[A] Essay type questions.

1. Mention the various evidences in favour of the fact that sound is a kind of wave. Explain the mechanism of propagation of sound through air.

2. What is the difference between sound wave and light wave ? Discuss the nature of sound wave with illustrations. [*H. S.* '79]

3. State Newton's equation for velocity of sound in a medium. Deduce with proper reasoning Newton's expression for the velocity of sound in a gas. Obtain Laplace's correction to the above formula.

4. What are the effects of (a) pressure (b) temperature (c) density and (d) humidity on the velocity of sound in air ? Discuss in detail.

5. Discuss how the velocity of sound in air can be measured. Discuss also how the various sources of error can be eliminated.

6. Discuss about the determination of the velocity of sound in water and in solid.

7. What is an echo ? How is it produced ? Calculate the minimum distance of the reflector for a sound of short duration and articulate sound.

8. Briefly explain how depth of ocean at different points may be measured by using sound waves. Also explain the kind of sound waves necessary for revealing minute details of the ocean bed (i. e. ocean floor) by this method. [*Jt. Entrance* '75]

9. What is the difference between a musical sound and a noise ?
What do you mean by a tone and a note ? Bring out clearly the difference between the two.

10. Discuss the characteristics of musical sound. On what factors do they depend ? [*H. S* '80]

11. Describe two experiments for demonstrating reflection and refraction of sound. Describe the working principle of an instrument which provides a practical application of the reflection of sound.

[B] Short answer type questions.

1. The velocity of sound is generally greater in solids than in gases at N.T.P. why ? [*I. I. T.* '77]

2. Explain why sound is heard at greater distances (a) at night than during day time (b) in the direction of the wind than in the opposite direction.

3. Light is reflected by a small mirror but not sound ; on the other hand sound is reflected from a long wall but not light—Why ?

4. In a sports meet the timing of a 200 metres straight dash is recorded at the finish point by starting an accurate stop watch on hearing the sound of the starting gun fired at the starting point. Will the time recorded be more accurate in summer than in winter ? [*I. I. T.* '73]

5. Explain what will happen to the pitch of the fundamental note emitted by an open organ pipe when—

   (i) its mouth is suddenly closed by hand
   (ii) air in it is replaced by carbon-dioxide gas
   (iii) hot air is blown in the pipe, and
   (iv) greater amount of moisture is introduced in the pipe. [*Jt. Entrance* '73]

6. How do the same notes (of the same fundamental frequency) from different musical instruments differ ? [*I. I. T.* '70]

7. No echo is heard when a sound is produced very near a wall or a cliff. Why ?

8. Explain with reasons whether the following statements are true or false :—

   (i) Louder sound is emitted from a hollow vessel.
   (ii) Any vibrating body can act as a source of sound.
   (iii) The elasticity of the medium plays a definite role in the propagation of sound waves.
   (iv) Interference and polarization of sound wave is possible.
   (v) Both pressure and temperature have no effect on the velocity of sound in a gaseous medium.

(vi) The velocity of sound is completely independent of its frequency and amplitude.
(vii) In a rainy day sound travels faster than in a cold, dry day.
(viii) A tuning fork emits a musical sound.
(ix) A musical sound can not always be distinguished clearly from a noise.
(x) Sound from a source may be intensified by the presence of suitable neighbouring bodies.
(ix) Intensity of sound does not depend upon the nature of the medium.
(xii) A thicker string emits a sharper sound.
(xiii) If a tuning fork is filed, it will emit a flatter sound.
(xiv) Frequency of the octave determines the pitch of a note.
(xv) A musical sound is produced only when the source is vibrating in S.H.M.

9. Each property of sound listed in column *A* below depends upon one quantity listed in column *B*. Write down the suitable pairs from the two columns :

Column A	*Column B*
Pitch	Waveform
Quality	Frequency
Loudness	Intensity

(*I. I. T. '80*)

10. How can you increase the pitch of a sound emitted from a vibrating string. [*Jt. Entrance '83*]

11. A plane wavefront of sound travelling through air is incident on a plane surface of water. The angle of incidence is 60° : Assuming that Snell's law holds true in the case of sound waves, it follows that on refraction in water, the sound wave will move away from the normal. State with reasons whether the statement is true or false. [*I. I. T. '84*]

12. What are the factors that make the voice of your one friend from that of the other different ? How can you explain the roar of a lion and buzzing of a mosquito from the point of view of the charcteristics sound ? [*Jt. Entrance '72*]

[C] Simple Problems.

1. Copper has Young's modulus $11{\cdot}8\times10^{11}$ dyne/cm². and density 8·6 gm./c.c. Find the velocity of sound in copper. [*Ans.* 3704 m./sec]

2. The sound produced at the surface of a sea reflects back from the bottom in 4 sec. What is the depth of the sea ? Given, the compressibility of sea water$=4{\cdot}6\times10^{-11}$ cm.²/dyne and its density=1·03 gm./c.c. [*Ans.* 2906 m.]

3. Assuming that the velocity of sound in air at N. T. P. is 1090 ft./sec., find the value at 50°C. and 70 cm. pressure. (*C. U. '58*) [*Ans.* 1186 ft./sec.]

4. The velocity of sound in hydrogen at N. T. P. is $1{\cdot}26\times10^{5}$ cm./sec. If its density is 9×10^{-5} gm./c.c. calculate the value of γ. [*Ans.* 1·41]

5. At what temperature will the velocity of sound in air be twice as great as it is at 0°C. ? (*C. U. '31*) [*Ans.* 819°C]

6. Find the wavelength in air at 30°C. of a note of frequency 320 c. p. s. Given, the velocity of sound in air at 0°C.=332 m./sec. [*Ans.* 109·2 cm.]

7. A vibrating tuning fork is held over a resonance tube. If the difference between two successive resonant lengths be 34·4 cm. at 25°C., what will it be at 100°C. ? [*Ans.* 38·5 cm.]

8. An open organ pipe is tuned to a frequency of 384 in a concert hall at 12°C. What will be the frequency of the note emitted by this pipe when the temperature rises to 25°C. ? [*Ans.* 392·6 c.p.s.]

9. Calculate the effective length of an open organ pipe tuned to a note whose frequency is 435, at a temperature of 20°C. when the velocity of sound is 342·5 m./sec. What will be the frequency of the note emitted by the pipe when the temperature falls to 10°C ? (*Oxford*) [*Ans.* 39·4 cm. ; 427·6 c. p. s.]

10. Find the length of a pipe closed at one end that will give a fundamental note of frequency 256 c.p.s. in air at a temperature of 20°C., the end-correction being 4·5 cm. (velocity of sound at 0°C.=331 m./sec.). [*Ans.* 29 cm.

11. If the pitch of a note from a closed organ pipe of metal is to be independent of temperature, show that the ratio of the cubical expansion of air to that of the metal is 2/3.

12. The velocity of sound in oxygen at N. T. P. is 315 m./sec. Find the velocity in (a) hydrogen at N. T. P. (b) in oxygen at 30°C. and 78 cm. of Hg. pressure. Given, density of oxygen at N.T.P.$=1{\cdot}44$ gm./lt. and that of hydrogen at N. T. P.$=0{\cdot}09$ gm./lt.

[*Ans.* (a) 1260 m./sec. (b) 331·8 m./sec.]

13. Calculate the velocity of sound in CO_2 at 30°C. and 760 mm. of Hg. pressure, given that 22·2 lt. of CO_2 at N.T.P. weighs 44 gm. and γ for $CO_2=1{\cdot}26$. [*Ans.* 267·34 m./sec.]

14. Find out the molecular weight of a gas for which γ is 1·4 and in which the speed of sound at 20°C. is 1305 m./sec. Given $R=8{\cdot}31\times10^7$ erg./(°K mole). [*Ans.* 2]

15. Calculate the velocity of sound in dry hydrogen at 90°C. assuming that the density of hydrogen at N. T. P.$=0{\cdot}089$ gm./lt. and $\gamma=1{\cdot}41$. (*A.M.I.E.*) [*Ans.* 1461 m./sec.]

16. A sound note of frequency 200 Hz. is produced in hydrogen. Find its wavelength assuming that velocity of sound in air is 1100 ft./sec. and density of air is 14·4 times that of hydrogen. (*A. M. I. E.*) [*Ans.* 20·87 ft]

17. On a windless day a person fires his gun and another person at a distance of 1552·5 m. from the first person hears the sound after 4·5 sec. of the flash. Find the velocity of sound.

[*Ans.* 345 m./sec.]

18. *A* and *B* are two stations 10 km. apart. Wind is flowing from *A* to *B*. A sound produced at *A* is found to reach *B* in 29 second, whereas a sound produced at *B* takes 30 second to reach *A*. Find the speed of the sound and that of wind.

[*Ans.* 339·1 m./sec. ; 5·75 m./sec.]

19. A thunder is heard 6 second after a lightning flash. Calculate the distance at which it occurred if the mean temperature of the air be 25°C. Given, the velocity of sound at 0°C= 332 m./sec. (*Calcutta Univ.* '75.) [*Ans.* 2081 m.]

20. A stone is dropped into a well 44·1 m. deep. Find after how much time will the sound of the splash be heard, given that the velocity of sound in air is 340 m./sec. [*Ans.* 3·13 sec.]

21. An observer sets his watch by the sound of a signal gun fired at a distant tower. He finds that his watch is slow by two second. Find the distance of the tower from the observer. Temperature of air during observation is 15°C. and the velocity of sound in air at 0°C. is 332 metre/sec.

[*Pat.* '39] [*Ans.* 682 metre]

22. An aeroplane is flying parallel to the ground with a speed v. A sound produced from the aeroplane is reflected back from the ground in t sec. If the speed of sound be V show that the height of the aeroplane from the ground is $\frac{t}{2}\sqrt{V^2-v^2}$.

23. While a wood-cutter strikes a tree by his axe, sound produced is reflected back from the nearby hill within 5 second. If the velocity of sound in air is 1120 ft./sec., how far is the hill away from the wood-cutter ?

[*Ans.* 2800 ft.]

24. The sound of a gun known to be 6 mile away was heard 30 sec. after the flash was seen. At the same time the man firing the gun heard an echo of the report from a cliff 20 sec. after the gun was fired. How far from the gun was the cliff ?

[*Ans.* 2 mile]

25. A man stationed between two cliffs fires a gun. He hears the first echo after 2 sec. and the next after 5 sec. What is his position between the two cliffs ? When will he hear the third echo ? Given, the velocity of sound=1120 ft./sec.

[*Ans.* 1120 ft. from one cliff and 2800 ft. from the other ; 7 sec.]

26. While approaching a cliff in a car, a man fires his gun, the hill being 3 km. away. If the velocity of the car be 5 m./sec. and that of the sound be 340 m./sec. find when and where the echo will be heard by the man.

[*Ans.* 17·4 sec., 2·913 km.]

27. An engine is approaching a tunnel surmounted by a cliff and emits a short whistle when 900 m. away. The echo reaches the engine after 5 sec. Calculate the speed of the engine assuming that of sound to be 340 m./sec.

[*Ans.* 72 km./hr.]

28. An echo repeats 4 syllables. Find the distance of the reflecting surface if it takes 1/5 of a second to pronounce and hear one syllable distinctly. The velocity of sound is 1120 ft./sec.

[*Ans.* 448 ft.]

29. A man stood facing a high brick wall 280 ft. away from him, clapping regularly. He noticed that when he was just clapping 12 times in 6 second, each clap coincided with the echo of the previous one. What value did he deduce for the velocity of sound in air ? [*Ans.* 1120 ft./sec.]

30. The velocities of sound in air and in sea water are 340 m./sec. and 1435 m./sec. A signal is sent from a floating ship. Two signals are received through the medium of air and water by a second floating ship. If the time interval between the two signals is 5 sec. find the distance between the ships. (*Jt. Entrance '80*) [*Ans.* 2227·9 m.]

[D] Harder Problems.

1. An organ pipe open at both ends sounds in unison with a tuning fork at 20°C. When the fork and the pipe are sounded together at 30°C., 5 beats per sec. are heard. Determine the frequency of the fork assuming that it is not affected by the temperature change. [296·2 c.p.s.]

2. The distance in miles of a lightning flash from an observer may be found approximately by counting the number of pulse beats between the lightning flash and the thunder clap and dividing the number by 6. Justify this rule, assuming that the human pulse beats 75 times per minute and the velocity of sound is 1080 ft. per sec. (*Cambridge*)

3. At the upper layer of the atmosphere a themometer does not get into thermal equilibrium with the environment owing to the low density of the gas. Hence temperature there cannot be measured with a thermometer. For this purpose, rockets with grenades are used which explode at a certain altitude. Find the temperature at an altitude of 25 km. from the earth's surface, if the sound produced by an explosion at an altitude of 26 km. is detected 7 sec. after that produced by an explosion at an altitude of 24 km. Velocity of sound at 0°C.=330 m./sec. [*Ans.* −68·4°C.]

4. A whistle gives a certain note when sounded, the temperature being 20°C. To what must the temperature be raised for it to give a note $\frac{6}{5}$ the frequency of that at 20°C. [*Ans.* 149°C.]

5. When a tuning fork and a column of air are sounded together they produce 4 beats per second, the fork giving the lower note and the temperature of air being 15°C. They produce 3 beats per second when the temperature of air falls to 10°C. Find the frequency of the fork assuming it to be independent of temperature. [*Ans.* 108·1 c. p. s.]

6. A certain organ pipe at 15°C. is in unison with a steel wire of diameter 0·50 mm. vibrating transversely in its fundamental mode under a tension of 5·00 kg. wt., the vibrating length being 34·6 cm. Describe and explain what will be heard if the temperature of the air in the pipe rises 2°C., other conditions remaining constant, and the pipe and the wire are sounded together. [Density of steel=7·80 gm./c.c.] (*London*) [*Ans.* 1 beat/sec.]

7. A pop-gun consists of a cylindrical barrel 3 sq. cm. in cross-section closed at one end by a cork and having a well-fitting piston at the other. If the piston is pushed slowly in, the cork is finally ejected, giving a "pop" the frequency of which is found to be 512. Assuming that the initial distance between the cork and the piston was 25 cm. and that there is no leakage of air, calculate the force required to eject the cork. Atmospheric pressure=1 kgm. wt./sq. cm. Velocity of sound=340 metre/sec. (*Cambridge*) [*Ans.* 1·5 kg. wt.]

8. Air in a tube closed at one end vibrates in resonance with tuning forks whose frequencies are 210 and 350 vibrations per sec. when the temperature is 20°C. Explain how this is possible and find the effective length of the tube. Assume that the velocity of sound in air at 0°C. is 33,150 cm. per sec. (*London*) [*Ans.* 122·6 cm]

9. If the velocity of sound in hydrogen at 0°C. is 4200 ft./sec., what will be the velocity of sound (at the same temperature) in a mixture of two parts by vol. of hydrogen to one of oxygen (Density of oxygen is 16 times that of hydrogen) ? (*Ranchi '66*) [*Ans.* 1714·3 ft./sec.]

10. An engine approaching a tunnel whistles and the driver hears an echo after an interval of 20 second. Ten minutes later the echo is heard after an interval of 16 second. How far is the engine now from the tunnel and what is its speed ? (Velocity of sound in air=332 m./s). [*Ans.* 1·11 m./s., 2665 m.]

11. A gun is fired on a seashore in front of a line of cliffs. A man standing 300 ft. away from the gun and equidistant from the cliffs notices that the echo takes twice as long to reach him as does the direct report. Find the distance of the gun from the cliffs. [*Ans.* 225 ft.]

25 CHAPTER

LIGHT WAVES

25·1. Wave nature of light.

Light is the radiant energy emitted from luminous sources and on entering the eye produces the sensation of sight. As to the nature and mode of propagation of light energy, Newton in 1675 proposed *the corpuscular theory*, which assumes that light consists of a stream of corpuscles (i.e., particles) emitted from luminous sources with great speed. These corpuscles behave largely in accordance with the laws of mechanics obeyed by other bodies. They are imagined to possess no weight and to be perfectly elastic. This theory explained the rectilinear propagation and reflection of light well enough, but explanation of the bending of light path by refraction led Newton to assume further the existence of a short range attractive force between the corpuscles and the surface. This force accelerates the corpuscles towards the surface and hence causes their paths to change. But this assumption immediately demanded that light should travel faster in a denser refracting medium than in air. This prediction of the theory was found to be quite inaccurate when Foucault in 1850 by an ingenious experiment showed conclusively that the velocity of light in water is less than that in air. This result dealt a death blow to the corpuscular theory which was then finally abandoned.

On the other hand, the *wave theory* first proposed by Huygens in 1678, assumes that light energy is transmitted as a wavemotion. According to this theory a source of light acts as a generator of waves ; from the source wavefronts of light move outwards through space as spheres of constantly increasing radii in a similar manner as when a stone is dropped into a pond, circular ripples move radially outwards over the water surface. The wavelengths of light are very small and for convenience are usually expressed in terms of a very small unit of length, called the *Angstrom* (Å). One angstrom unit is 10^{-8} cm. The wavelengths of light lie approximately within the range of 3900 A to 7600 A

The wave theory successfully explained the phenomenon of reflection and refraetion and in contradiction to the corpuscular theory, predicted that velocity of light in a denser refracting medium should be less than that in air. This prediction was corroborated by the result of Foucault's experiment. Discovery of the phenomenon of interference of light led to the final acceptance of wave theory.

An immediate objection to this theory is that waves invariably need a medium for transmission. Since light comes from far-off heavenly bodies, practically through vacuum, Huygens postulated an all pervading hypothetical medium, called *ether*, which was assumed to fill 'empty' space as well as the interior of matter. But all attempts to prove experimentally the existance of such an hypothetical medium were unsuccessful. Yet this theory was accepted for its ability to explain not only rectilinear propagation, reflection and reffraction but also other phenomena associated with light namely, *diffraction*, *interference*, *polarization* etc., which can never be explained with corpuscular theory.

In the year 1865, Maxwell developed the theory of electromagnetic waves. These waves consist of periodic variation of electric and magnetic fields. The electric and magnetic intensities are at right angles to each other and are also normal to the direction of propagation of the wave. Since electric and magnetic fields may exist in empty space, no material medium is necessary for the propagation of such waves. One of the brilliant successes of the theory was the prediction that light waves are electromagnetic waves which had been found to be true.

25·2. Finite velocity of light.

Until the middle of the 17th century, it was generally believed that light travels with an infinite velocity. Galileo was the first who proposed that light takes a finite time, however small it might be, to travel from one point to other. But his attempt to verify this experimentally failed due to inadequacy of the experiment to cope up with the tremendous velocity of light. The first successful attempt was made by the Danish astronomer Romer in 1675. Based on astronomical observations made on one of the satellites of the planet jupiter, he obtained definite evidence that light propagates with a finite velocity. The velocity of light (c) determined from such observations was about 220,000 km./sec. which is of the same order of magnitude as the *best present value* of $c=299{,}792$ km./sec. in vacuum.

Fizeau in 1849 developed a method for the determination of c entirely on the surface of earth. Later his method was modified by Foucault in 1850 and he was the first to prove that velocity of light in water is considerably less than that in air. This was a decisive experiment, because as stated earlier, this led to the rejection of the corpuscular theory. The most precise measurement of c in air was carried out by Michelson during 1926—1929 by using a modified version of Foucault's method. After correction for the refractive index of air to give the velocity in vacuum, the average result of his experiments was $c=299{,}796 \pm 4$ km./sec. Later methods like Kerr cell method provided even higher accuracy than Michelson's experiment.

The velocity of light and, in fact, of all radiant energy, in vacuum is one of the fundamental constants in Physics. It plays a central role in Einstein's theory of relativity. According to this theory, no object can move with a velocity greater than that of light in vacuum. Thus nature has imposed a limit to the speed of all material bodies. The fastest moving particles, still known are found is cosmic rays ; their velocities fall short of the velocity of light by about 1 percent. The theory of relativity also predicts that conversion of mass into energy and vice versa is possible. The conversion relation $E=mc^2$ involves the speed of light. Hence it is quite natural that measurements still continue to be made of the speed of light for obtaining more accurate values.

25·3. Interference of light.

The wave theory of light was first expounded by Huygens in 1678. More than a hundred years passed before the strongest and most convincing evidence in favour of it was obtained from an experiment performed by Young in 1803. Young's experiment proved conclusively that light showed the property of *interference*, which is a characteristic of wavemotion [vide Art. 23·22]. On the other hand, Newton's corpuscular theory completely failed to account for such observations.

A modified form of Young's experimental arrangement is shown schematically in Fig. 25·1. A narrow slit S_1 is illuminated by monochromatic light (i.e., light of single wavelength) from a suitable source. The slit S_1, therefore, acts as a source ; light from this slit is again incident on an opaque barrier which contains two more narrow slits S_2 and S_3 parallel to S_1. These slits are very close together and are so placed as to be equidistant from the first slit S_1. A screen is placed at a convenient distance behind the pair of slits. It is found that a pattern of alternative bright and dark lines parallel to the slits appear on the screen. These are called *interference fringes*. A bright line is formed at the centre of the pattern. If one of the slits S_2 and S_3 is closed, the pattern disappears and the screen appears faintly and almost uniformly illuminated. Obviously the formation of such pattern suggests that light emerging from the pair of slits S_2 and S_3, is reinforced at certain places producing brightness and is cancelled at other places making them dark.

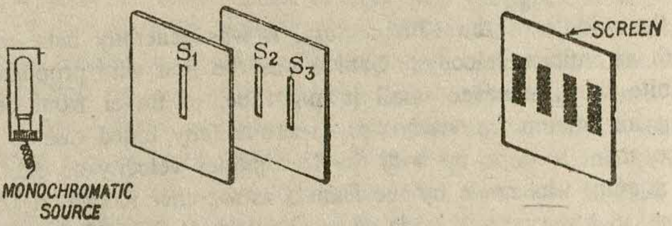


Fig. 25·1 : Interference experiment

Formation of such fringes cannot be explained on the basis of rectilinear propagation of light because this predicts that on the screen we shall see two bright lines, one lying in the line $S_1 S_2$ and other in the line $S_1 S_3$.

The explanation follows if we assume that light is a kind of wave. Each of the slits S_2 and S_3 acts as a single source of light and so a series of waves is emitted from each of the two. This is shown in Fig. 25·2. As the two slits share the light from the same source S_1, the waves emanating from them possess identical amplitude and frequency. Also because the slits are placed at equal distances from S_1, the wavefront of light originating from S_1 at any moment passes through both the slits at the same instant. Hence the waves emanating from them are always in the same phase. The two slits, therefore, behave as coherent sources of light waves.

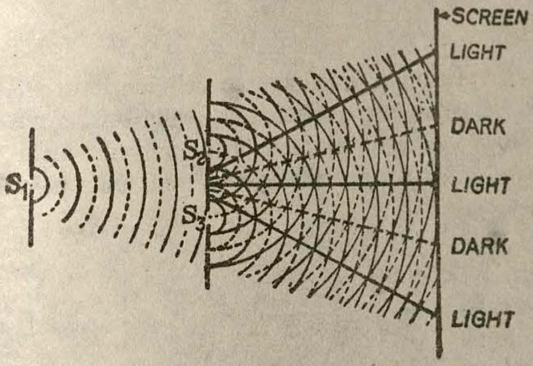


Fig. 25·2 : Formation of fringes

Hence, as the light waves originating from these slits overlap, they will interfere with each other and give rise to a stationary interference pattern as pointed out in Art. 23·22. At those points where the two waves arrive in phase, *constructive interference* takes place and the waves reinforce each other. With light waves, this means that maximum brightness is produced at these points. This occurs when the paths traversed by the waves from S_2 and S_3 are either equal or differ by a whole number of wavelengths. In the Fig. 25·2, these points lie on the heavy solid lines. About halfway between these lines lie other points where the waves from S_2 and S_3 arrive in opposite phase. So at these points *destructive interference* takes place and the waves annul each other producing darkness. For Such points the paths covered by the waves from S_2 and S_3 differ by an odd number of half wavelengths. In the Fig. 25·2, the heavy dotted lines are drawn through these points. It is obvious now that if we place a screen in the region beyond the pair of slits S_2, S_3, alternate bright and dark fringes will be seen. For the point on the screen which is equidistant from the slits, the paths traversed by the waves are equal and henee a bright fringe is formed at this point. This point lies at the centre and the fringe pattern extends equally on both sides of it. As we go further and further from this point, the illumination of the fringes falls and ultimately they become too faint to be visible.

It may be mentioned that the wavelength of the light can be calculated by measuring the separation between the pair of slits S_2 and S_3, the distance of the screen from them and the distance of the middle of any fringe from that of the central. The result so obtained closely agrees with those from other optical experiments.

If instead of monocromatic light we use white light, then each component colour having different wavelengths will produce its own interference pattern. So a coloured fringe pattern is obtained.

It is not possible to obtain such fringes with light from two separate sources, even if they emit light of same wavelength and amplitude i.e., of same colour and intensity. This is due to the fact that the phase difference between light waves emitted from separate sources changes continually in a rapid manner.

Consequently, the fringe pattern on the screen must shift position rapidly. As our eyes can not follow such rapid shifts, the screen will appear uniformly bright.

25·4. Polarization of light.

We have seen in Art. 23·25, that the important difference between longitudinal and transverse waves is that the latter can be polarized, while this is not possible for the former. The discovery of polarization in light proved conclusively that *light waves are transverse.* So in light waves, vibrations take place at right angles to the direction of propagation i.e., to the ray. A beam of ordinary light consists of an innumerable number of such waves. Ether particles in the path of these waves vibrate in all possible directions, with equal probabilities but always perpendicular to the direction of propagation of the beam. *Ordinary light is, therefore, unpolarized.* The end-on view of the vibrations in such a beam may be diagrammatically represented as in Fig. 25·3 (*a*). If by some means, described later, all ether particles in the path of the beam of light are made to vibrate in one direction, then the light is said to be *plane polarized.* The diagrammatic representation of such light is given in Fig. 25·3 (*b*).

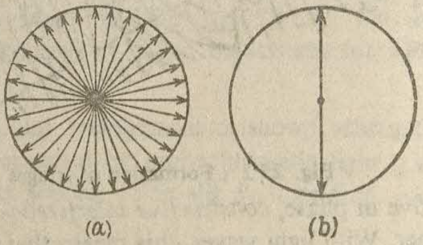


Fig. 25·3 : Vibrations in (a) unpolarized (b) polarized light

Among the various methods of production of polarized light we shall describe only the action of *tourmaline* crystals. When ordinary light enters such a crystal, it absorbs vibrations in all directions, except one i. e., it transmits vibrations in one direction only [Fig 25·4]. This phenomenon is known as *selective absorption.* The emergent light is, therefore, plane polarized.

Since our eyes can not distinguish polarized light from an unpolarized one, the phenomenon of polarization of light can be experimentally verified by placing another tourmaline crystal in the path of the emergent light from the first as shown in Fig. 25·5. In this arrangement, the tourmaline *A*, whose purpose is to produce polarized light is called the *polarizer.* The tourmaline *B* is called the *analyzer* whose purpose, as we shall see now, is to analyze the polarized light coming from *A*. Keeping *A* fixed, if *B* is rotated slowly in its own plane with the incident ray as axis, there will be variation in the intensity of light transmitted through *B* as judged by the eye at *E*. The maximum intensity is

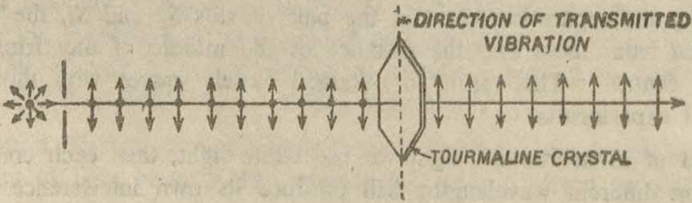


Fig. 25·4 Production of polarized light

observed when both the crystals transmit vibrations in the same direction [Fig. 25·5 (*a*)]. No light is seen when the vibrations transmitted by the former become perpendicular to the direction of transmission of vibration by the latter [Fig. 25·5 (*b*)]. In the former case the tourmalines are said to be *parallel*, while in the latter case they are said to be *crossed*. In the first case, the vibrations transmitted by the first crystal go on uninterrupted through the second ; in the second case the vibrations through the first crystal get completely absorbed by the second, so that no light is received by the eye. This proves that light waves can be polarized and hence they are transverse in nature.

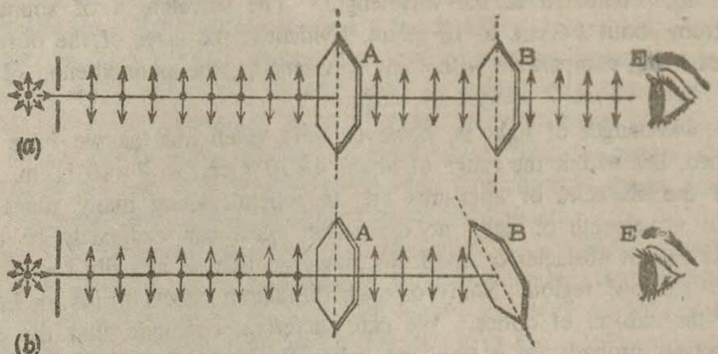


Fig. 25·5 : Analysis of polarized light

25·5. Validity of Geometrical Optics as an approximation.

The branch of Physics which studies light and its various phenomena is called *optics*. Geometrical optics is that branch of optics which deals with only properties of light and their consequences.

One such property is that *in a homogeneous medium, the path of light is rectilinear*. Formation of a sharply defined shadow of an opaque obstacle placed in the path of light provides a unique evidence of this property.

Now, in a homogeneous medium waves travel in straight lines. However, if the waves meet any obstacle in their paths or pass through an aperture, they instead of travelling straight ahead bend to some extent round the edge of the obstacle or aperture into the 'shadow' region. This departure from rectilinear propagation when waves meet an obstacle or pass through an aperture is known as *diffraction*. This is exhibited by all wave-motions. For sound waves, the phenomenon is very much in evidence. A sound made on one side of a partition is quite audible to a person standing on the other side, showing that sound waves can bend round an obstacle. The phenomenon of diffraction can also be readily demonstrated with ripples on the surface of water.

We know that light is a kind of wave and hence it is expected that light should also exhibit the phenomenon of diffraction. This is directly in contradiction with the property that light propagates in straight lines. We shall now discuss how we can reconcile these two apparently contradictory properties of light.

Investigations with all types of waves have shown that the phenomenon of diffraction becomes prominent only when the size of the obstacle or the aperture becomes comparable with the wavelength. Diffraction effects become practically insignificant when the size of the obstacle or the aperture is large compared to the wavelength. The wavelength of sound waves varies from about 1·6 cm. to 16 metre. Evidently, the sizes of the obstacles or apertures being comparable with their wavelengths, the phenomenon of diffraction of sound waves is easily observable.

The wavelength of light is, however, very small and as we have already mentioned, lies within the range of about 4×10^{-5} cm. to 7×10^{-5} cm. As the sizes of the obstacles or apertures are, in general, many many times greater than the wavelength of light, no diffraction effect can ordinarily be detected. With very small obstacles or apertures, however, light is actually found to bend into the 'shadow' region. Study of such diffraction effects forms an important part of the subject of optics. We can, therefore, conclude that the property of rectilinear propagation is only true when the sizes of obstacles or the apertures are very large compared to the wavelength of light. Thus the subject of geometrical optics based on this property represents an approximate picture. The approximation, however, makes no appreciable error for all practical purposes. The error is so small that it is not ordinarily discernible.

● EXERCISE ●

[A] Essay type questions.

1. Discuss the evidences that light is a wave phenomenon and that the waves are transverse.

2. What is ether ? Discuss its importance in relation to the wave theory of light. Why is ether called an hypothetical medium ?

3. Discuss about the velocity of light and its importance in physics.

4. What do you mean by interference ? State the conditions to be fulfilled for the effect. Describe an experiment to demonstrate the effect in the case of light and show how the conditions are satisfied.

5. What is plane polarized light ? Discuss a method of producing it. What is an analyzer ? How is it used to distinguish between polarized and unpolarized light ?

6. Explain what you know about polarisation. How does it help to determine the character of a wave ? Can you obtain polarisation of sound waves ? How can light waves be polarized ? A plane polarized light is seen through an analyzer. If the analyzer is rotated, what shall you see ? [*H. S. '80*]

7. What is meant by diffraction ? Under what conditions it becomes prominent ? Give an idea about the sizes of obstacles or apertures for observing the diffraction of light.

8. 'Geometrical optics is valid only as an approximation'—Discuss.

[B] Short answer type questions.

1. What is meant by the statement that the wavelength of yellow light is 5893Å ?

2. "Nature has imposed a limit to the speed of all material bodies".—What is the said limit ?

3. When are two sources of light said to be coherent ? Why is interference of light waves from two different lamps never be observed ?

4. What information does the phenomenon of polarization provide regarding the nature of light ?

5. If light can bend around an obstacle due to diffraction, why can't we see around a wooden partition ?

6. Light from a sodium flame passes through a narrow slit and then through a pair of very close narrow slits and falls on a screen. Describe and explain (a) the appearance of the screen, (b) the effect of closing one slit of the pair, (c) the effect of replacing sodium flame by a source of white light.

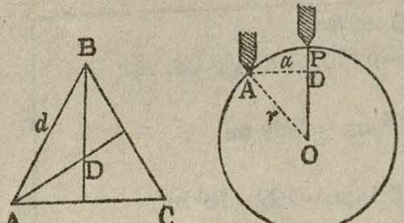
APPENDIX

1. S. I. system of units. The International System of Units, abbreviated, S.I. (from the French name *Systeme International d'* (*Unite's*) was adopted in 1960. It is a modernized version of the metric system and is expected to replace the C.G.S. system throughout the world in near future. In this system, the fundamental units corresponding to the six fundamental quantities viz., length, mass, time, electric current, temperature and luminous intensity are *metre*, *kilogram*, *second*, *ampere*, *Kelvin* and *candela* respectively. Derived units in this system for some important quantities are given below: Force—*newton*, Energy—*Joule*, Power—*watt*, Torque—*newton metre*, Electric charge—*coulumb*, Electric potential—*volt*, Electric resistance—*ohm*, capacitance—*farad*, Inductance —*henry*, Magnetic flux—*weber*, Magnetic Intensity—*tesla* etc.

Besides the fundamental units, the complete S.I. system contains other recommendations also. For example, it is recommended that multiples and submultiples of the fundamental units be stated in steps of 10^3 or 10^{-3}. So length should be preferably expressed in kilometre (1 km.$=10^3$ m.) or millimetre (1 m.m.$=10^{-3}$ m.) but not in centimetre (1 cm.$=10^{-2}$ m.)

2. Proof of the relation $r=\frac{d^2}{6h}+\frac{h}{2}$

In Fig. (*a*), let *ABC* represent the equilateral triangle of side *d*, formed by the tips of the three fixed legs of the spherometer.



(a) (b)

Let *D* be the circumcentre of this triangle; at *D* the screw tip touches the plane of the triangle. Now, from the properties of an equilateral triangle,

$$AD=\tfrac{2}{3}\times\text{length of the median}$$

$$\tfrac{2}{3}=\left(\frac{\sqrt{3}}{2}\times\text{length of a side of the triangle}\right)$$

i.e. $AD=\frac{d}{\sqrt{3}}=a$ (say) ... ... (i)

Fig. (*b*) represents a vertical section of the assembly of the spherometer and the spherical surface whose radius of curvature is to be measured. Here *O* is the centre of the sphere of which the said surface is a part. Let *A* and *P* be the respective points at which one of the fixed legs and the screw tip of the spherometer touch the surface. Then $OA=OP=r$. Had the surface been a plane one, the screw tip would have been touched it at *D*. Then $PD=h$, $AD=a$ and $OD=OP-PD=r-h$.

Now, from the geometey of the Fig. (*b*),

$OA^2=AD^2+OD^2$ or, $r^2=a^2+(r-h)^2$

or, $2rh=a^2+h^2$ ... ... (ii)

Combining eqns. (i) and (ii) we get $r=\frac{d^2}{6h}+\frac{h}{2}$

3. Some useful conversion factors.

1 inch=2·54 cm.
1 metre=39·4 in.
1 pound=454 gm.
1 kilogram=2·20 lb.
1 gm-wt.=980 dyne.
1 lb-wt.=32 poundal.
1 litre=1,000 ml.
1 B.Th.U.=252 cal.
1 B.Th.U.=778 ft.-lb.
1 Horse Power=746 watt.
1 Kilowatt=1·34 H.P.
1 calorie=4·18 joule.
1 joule=10^7 erg.
1 atmosphere=$1·013\times10^6$ dyne/cm.^{2}
1 angstrom=10^{-8} cm.

4. Some physical constants.

Acceleration due to gravity=980 cm./sec^2.
Universal gravitational constant=$6·670\times10^{-8}$ C.G.S. unit.
Mass of earth=$5·975\times10^{27}$ gm.
Mean radius of earth=$6·371\times10^8$ cm.=3959 mi.
Mean distance from earth
to the sun=$1·49\times10^{18}$ cm.=$9·29\times10^7$ mi.
Mean distance from earth
to moon=$3·84\times10^{10}$ cm.=60 earth radii.
Mass of the sun=$1·99\times10^{33}$ gm.=333,000 × mass of the earth.
Velocity of light in vacuum=$2·9979\times10^{18}$ cm./sec.
Charge of electron=$-1·602\times10^{-19}$ coul.
=$-4·803\times10^{-10}$ statcoul.
Ratio of charge to mass
of electron=$1·759\times10^{11}$ coul./kg.
Plank's constant=$6·625\times10^{-34}$ j-sec.
Boltzman's constant=$1·380\times10^{-23}$ j/°K.
Avogadro's number=$6·023\times10^{23}$ molecule/mole.
Universal gas constant=8·314 j/mole. °K.
Volume of ideal gas at S.T.P.=22·4 litre/mole.
Absolute zero of temp.=−273·16°C.
Latent heat of fusion of ice=80 cal./gm.=144 B.Th.U./lb.
Latent heat of vapor. of water=540 cal./gm.=972 B.Th.U./lb.

5. Some Greek Alphabet.

Α	α	Alpha	Ν	ν	Nu
Β	β	Beta	≡	ζ	Xi
Γ	γ	Gamma	π	π	Pi
Δ	δ or ∂	Delta	Ρ	ρ	Rho
Ε	ε	Epsilon	Σ	σ	Sigma
z	ζ	Zeta	Τ	τ	Tau
Η	η	Eta	Φ	φ	Phi
Θ	θ	Theta	Ψ	ψ	Psi
Λ	λ	Lambda	Ω	ω	Omega
Μ	μ	Mu			

QUESTIONS OF THE COUNCIL OF HIGHER SECONDARY EDUCATION

PHYSICS : 1986

Group A

Answer any two questions

1. (*a*) Establish graphically the equation $S=ut+\frac{1}{2}ft^2$, where $u, f, S,$ and t have their usual meaning. 4

(*b*) A particle moving along a straight line with uniform acceleration describes 145 cm and 185 cm in the sixth and the tenth second respectively. What distance will it cover in the sixteenth second ? 3

(*c*) A particle moves '*a*' metres along a straight line and then it moves '*b*' metres along a direction making angle θ with the original direction. Obtain the resultant displacement of the particle. 3

2. (*a*) Define centripetal force and establish an expression for the same. Why is centrifugal force called a pseudo force ? 5

(*b*) Explain the effect of diurnal motion of the earth on the weight of a body on the surface of the earth. 2

(*c*) Establish the relation between linear velocity and angular velocity. 3

3. (*a*) What is work ? Define joule, erg and watt. 4

(*b*) What is meant by potential energy and kinetic energy ? 2

(*c*) Deduce an expression for the kinetic energy of a body moving with linear velocity u. 4

Group B

Answer any two questions

4. (*a*) Write down the laws of simple pendulum. 4

(*b*) Explain how the height of a hill can be determined by determining the time periods of a pendulum at the foot and top of the hill. 3

(*c*) The length of a simple pendulum is four times the length of another simple pendulum. If the time period of the pendulum of longer length is 6 secs, calculate the time period of the other pendulum. 3

5. (*a*) Define Young's modulus of elasticity. Same weight is suspended at the lower ends of two wires of same length but of different material. The diameters of the wires are 0·4 mm and 0·6 mm respectively. If the elongation of the first wire is three times the elongation of the second wire, compare the Young's modulii of materials of the two wires. 1+3

(*b*) What should be the velocity of an artificial satellite of the earth so that it may revolve round the earth along a circular orbit 300 km above the earth's surface ? Explain weightlessness of a body on an artificial satellite. What is meant by escape-velocity ? 3+2+1

$g=980$ cm/sec^2 ; Radius of the earth=6400 km.

6. (*a*) State Archimedes' Principle. How would you verify the same experimentally ? 4

(*b*) A body is floating on water with a portion of the body immersed. Will the body float up or down if the entire air above the body be removed ? Give reasons for your answer. 3

(*c*) The internal and external diameter of a hollow sphere is 6 cm and 8 cm respectively. If it floats just fully immersed in a liquid of sp. gr. 1·2, find out the density of the material of the sphere. 3

7. (*a*) What do you mean by the statement that normal atmospheric pressure is 76 cm of mercury column ? 2

(*b*) What is a siphon ? Explain its principle of action. Mention the conditions for the working of a siphon. 6

(*c*) Calculate the limiting height of obstacle for siphoning kerosene oil of sp. gr. 0·80 from one vessel to another when the atmospheric pressure 76 cm of mercury. Density of Hg=13·6 gm/cc. 2

Group C

Answer any Two questions

8. (*a*) Define coefficients of linear expansion, surface expansion and volume expansion of a solid. Establish the relation between them. 6

(*b*) A metre scale, made of steel, is correct at 10°C. What would be the correct distance between two consecutive cm-markings on this scale at 30°C and 60°F ? 1+2

A brass rod measured with the help of the above scale at 30°C is found to be 5 metre long. What would be the correct length of the rod at 10°C ? Coeff. of linear expansion of steel=12×10^{6}/°C. Coeff. of linear expansion of brass =18×10^{-6}/°C. 3

9. (*a*) State Boyle's law and Charles' law. Establish the equation of state for a perfect gas. How can you obtain the absolute scale of temperature from Charles' law ? 7

(*b*) Deduce the relation between the volume and pressure coefficient of an ideal gas. 5

10. (*a*) Define water-equivalent and thermal capacity. How do they differ ? A block of platinum of mass 200 gms is heated in a furnace and then dropped into 650 gms of water at 10°C, kept in a vessel of water equivalent 50 gm. If the final temperature of the mixture be 25°C, calculate the temperature of the furnace. Sp. heat of platinum=0·030. 2+1+3

(*b*) Distinguish between vapour and gas. Define dew point and relative humidity.

The dew point and the temperature on a certain day were 10°C and 16·5°C respectively. The saturation vapour pressures at 10°C, 16°C and 17°C are respectively 9·10 mm, 13·50 mm and 14·40 mm of mercury column. Calculate the relative humidity of the day. 2+2+2

11. Write notes on : (*a*) Effect of pressure on melting point and boiling point. (*b*) First law of thermodynamics. 6+6

Group D

Answer any two questions

12. (*a*) What is simple harmonic motion ? Mention its characteristics. Show that the principle of conservation of mechanical energy holds good in such motion. 1+3+2

(*b*) The amplitude of a particle of mass 4 gm executing simple harmonic motion is 5 cm. The acceleration of the particle is 12 cm/sec^2 when it is 3 cm away from the mean position. Calculate the velocity of the particle and the force acting on the particle when it is 4 cm away from the mean position. 2

13. (*a*) What are forced vibration and resonance ? 4

(*b*) A vibrating tuning fork of frequency 256 vibrations per sec is held near the upper open end of a vertical tube 200 cm long filled with water and water is allowed to run out slowly from the lower end. Neglecting end correction, calculate the positions of water surface in the tube for first and second resonances. Velocity of sound in air=320 m/sec. 4

14. (*a*) Write down Newton's formula for the velocity of sound in air. What is its defect ? How was the formula modified by Laplace ? 1+1+2

(*b*) Write down the difference between longitudinal and transverse wave. How would you establish that light is a transverse wave ? 2+2

PHYSICS : 1987

Group A

Answer any Two questions

1. (*a*) Draw and explain the velocity—time graph of a particle moving with uniform acceleration. Explain the nature of the graph when the acceleration decreases with time. 2+2

(*b*) A train begins to move with an acceleration of 2 metre/sec^2 when a man is 9 metres away from a door of the train. The man begins to run and catches it in 3 sec. What is his acceleration ? 3

(*c*) Two particles are dropped from the top of a tower at an interval of 2 seconds. Find their relative velocity and relative acceleration at a moment when both of them are falling. 3

2. (*a*) State Newton's first and second laws of motion, and define force from the first law. 3+2

(*b*) Mass is the measure of inertia—explain. 3

(*c*) What do you mean by static friction and kinetic friction ? 2

3. (*a*) Determine the value of potential energy of a body of mass m, situated at a height h. 3

(*b*) Show that the law of conservation of energy is obeyed by a body falling freely under gravity. 5

(*c*) Show that the momentum of a body having mass m and kinetic energy E is $\sqrt{2mE}$. 2

Group B

Answer any two questions

4. (*a*) State Newton's law of gravitation. What is universal gravitational constant ? Find the relation between the said constant and the acceleration due to gravity. 2+1+2

(*b*) The mass and the radius of the earth are respectively 81 times and 4 times the values for the moon. Compare the accelerations on their surfaces. 3

(*c*) Why does a body feel weightless in an artificial satellite ? 2

5. (*a*) State and explain Hooke's law. 2

(*b*) Draw and explain the load-elongation characteristic of an elastic solid material. 1+2

(*c*) If the strain be 1% of 0·1, find the change in length of a wire 5 metre long. If the cross-section is 1 sq. mm. and load 10 kg. wt., what is the ratio of stress and strain ? 1+2

(*d*) Define Poisson's ratio and bulk modulus. 2

6. (*a*) Compare specific gravity and density. 2

(*b*) A body of density d_1 is placed slowly on the surface of a liquid of density d_2 and depth h. After time t, the body reaches the bottom of the liquid. Determine the value of t. 3

(*c*) State Pascal's law and with its help, explain the principle of operation of a hydraulic press. 5

7. (*a*) Explain with the help of diagram the working principle of a common pump used for lifting of water. Discuss if there is any upper limit of the height to which water can be raised by such a pump. 8

(*b*) If water is used in place of mercury in a barometer, what will be the height of the water column at normal atmospheric pressure ? Density of mercury 13·6 gm/c.c. 2

Group C

Answer any two questions

8. (*a*) Define the real and apparent coefficients of expansion of a liquid and establish the relation between them. 2+2

(*b*) What do you understand by anomalous expansion of water ? Explain with the help of a diagram the variation of volume of a certain amount of water, if temperature changes from 0°C to 10°C. 2+3

(*c*) Give three practical examples where expansion of solids has been made use of. 3

9. (*a*) Define latent heat of ice, dew point and relative humidity. 3

(*b*) What is the difference between evaporation and boiling ? 2

(*c*) When two pieces of ice are pressed together, they form a single piece. Explain why. 3

(*d*) If 64,800 calorie of heat is extracted from 100 gm of steam at 100°C, what will be the result ? Latent heat of steam=540 cal/gm ; latent heat of ice=80 cal/gm. 4

10. (*a*) Write down the postulates of the kinetic theory. 3

(*b*) What is mechanical qeuivalent of heat ? 2

(*c*) Explain isothermal and adiabatic processes. 3

(*d*) A meteorite of mass 42 kg has its velocity reduced from 15 km/sec to 5 km/sec while crossing the atmosphere of the earth. What will be the amount of heat produced for this change of velocity ? $J=4{\cdot}2\times10^7$ ergs/cal. 4

11. (*a*) Define thermal conductivity. 2

(*b*) Write the differences between conduction and convection of heat. 3

(*c*) Wind blows from sea to land during daytime but from land to sea at night.—Explain. 3

(*d*) A metallic vessel having surface area of 1 sq. metre and thickness 0·5 cm is filled with ice and then placed inside water at 100°C. How much ice will melt in 36 seconds ? Thermal conductivity of metal=0·02 c.g.s units, latent heat of ice=80 cal/gm. 4

Group D

Answer any two questions

12. (*a*) What do you mean by time period ? 1

(*b*) If the angular amplitude and the length of a simple pendulum are respectively θ and l, find the velocity of the bob at the lowest position. 4

(*c*) Show graphically the resultant motion of a particle on which two simple harmonic motions of the same frequency are superposed (i) in phase and (ii) out of phase. 3

13. (*a*) Explain the difference between progressive and stationary waves. 4

(*b*) An aeroplane is moving parallel to the ground with a velocity *v*. A sound from the plane reaches it again by reflection from the ground after *t* sec. Find the height of the plane, if the velocity of sound be *V*. 3

(*c*) What are beats ? 1

14. (*a*) State the laws of transverse vibration of a string. 3

(*b*) Determine the relation between the length of a closed pipe and the wavelength of the vibrating air column inside the pipe. 2

(*c*) What are the characteristics of musical sound ? 3

H. S. PHYSICS : 1988

Group—A

Answer any *two* questions.

1. (a) Show the subtraction of two vectors by a figure. What do you mean by the resolution of a vector ? How do you find out the rectangular components of a vector ? 4

(b) A man can reach just the opposite point of the bank by swimming in time t_1 and can swim the same distance down the current in time t_2. If the speed of the man in still water be u and that of current be v, then find the ratio of t_1 and t_2. 3

(c) A man is walking at the rate of 2 km per hour. The rain appears to him to fall vertically with a speed of 2 km/hour. Find out the real value and direction of velocity of rainfall. 3

2. (a) State Newton's second law of motion. How can you establish the first law from his second law ? 1+2

(b) Show that if a weight is hung from the midpoint of a wire, it cannot be kept horizontal. 2

(c) Action and reaction are equal and opposite—describe an easy experiment to show this. 2

(d) A mass of 2·0 kg is hanging from the hook of a spring balance fixed to the roof of a lift. What will be the reading of spring balance when the lift is (i) ascending with an acceleration of 0·2 m/s, (ii) descending with an acceleration of 0·1 m/s, (iii) moving up uniformly with a velocity of 0·15 m/s ? 3

3. (a) Find the relation between angular velocity and linear velocity. Obtain the expression for centripetal acceleration. 1+3

(b) The weight of a body on the surface is less due to diurnal motion of the earth—explain. 2

(c) Define Joule and Horse-power. 2

(d) A cycle rider is moving with a speed of 18 km / hour along a curved path of radius 20 metre. Find the angle of his inclination with the vertical. (g=980 cm/sec^2). 2

Group—B

Answer any *two* questions

4. (a) Find out how acceleration due to gravity at a place varies with altitude. 3

(b) What do you mean by a geo-stationary satellite ? 2

(c) Derive an expression for the velocity of an artificial satellite moving in a circular orbit. 2

(d) A satellite is moving round the earth in a circular path of 300 km from the surface. Find out its velocity. Radius of the earth=6400 km ; g=980 cm/sec^2. 3

5. (a) Define Young's modulus and Poisson's ratio. 2

(b) Find out the work done per unit volume in case of extension of a metal wire. 2

(c) What is a spring balance ? 2

(d) Two ends of a steel wire of diameter 4 mm are fixed by supports at 300° C. What will be the force exerted on the supports if temperature falls down to 30°C ? Coefficient of linear expansion of steel$=10\times10^{-4}$ / °C ; Young's modulus of steel$=1{\cdot}1\times10^{12}$ dynes/cm^2. 4

6. (a) Explain the principle of action of a siphon, mentioning clearly the conditions under which it will be in action. 4

(b) Explain whether the rate of flow of liquid through a siphon will change if the atmospheric pressure changes. 2

(c) How can you determine the height of a mountain with the help of a barometer, if the average density of air is known ? 2

(d) Kerosene oil has to be brought over an obstacle by the action of a siphon. What should be the maximum height of the obstacle for this purpose ? Sp. gr. of kerosene=0·8 ; atmospheric pressure=76 cm of Hg. 2

7. (a) State and explain Archimedes' principle. 3

(b) What is buoyancy ? Show how and on what factors its value depends. 3

(c) A floating body loses its weight—explain. 1

(d) The weight of a body in air is 0·4 gm. Its weight along with an immerser in water is 3·37 gm. The weight of immerser in air is 4·0 gm. Find the sp. gravity of the body. Sp. gr. of immerser=8·0. 3

Group–C

Answer any *two* questions

8. (a) Derive the relation between the coefficient of cubical expansion and coefficient of linear expansion of a solid. 3

(b) A metal scale does not give correct value at all temperatures–explain. 2

(c) The density of glass at 10°C and 60°C is respectively 2·6 gm/cc and 2·596 gm/cc. Find out the average value of the coefficient of linear expansion of glass in this temperature range. 4

(d) Explain whether the base angle of an isosceles triangle made by copper rods will change by heating. 3

9. (a) State Boyle's law and Charles' law. Derive the ideal gas equation from the same. What is the universal gas constant ? 6

(b) How can you get Boyle's law and Avogadro's law from the kinetic theory of gases ? 3

(c) How do you get the idea of the absolute zero of temperature from Charles' law ? 3

10. (a) What are the factors on which the rate of evaporation depends ? 3

(b) State and explain the fundamental principle of calorimetry ; mention the conditions under which it is applicable. 3

(c) State the influence of pressure on the melting point of a substance. 2

(d) A piece of metal of mass 10 gm was heated to 100°C and then dropped in a mixture of water and ice. The volume of mixture decreased by 0·1 cc though the temperature remained the same. Find the specific heat of the metal. The latent heat of ice=80 cal/gm. Densities of ice and water at 0°C are 0·916 and 1·0 gm/cc respectively. 4

11. (a) State and explain the first law of thermodynamics. 2

(b) Describe a method for determining the mechanical equivalent of heat. 4

(c) At which temperature a piece of wood and a piece of metal will appear equally hot on touching and why ? 2

(d) The velocity of a meteorite of mass 42 kg changed from 15 km/sec to 5 km/sec while passing through the atmosphere. Calculate the amount of heat produced during the process. $J=4\,2\times10^7$ ergs/cal. 4

Group—D

Answer any *two* questions

12. (a) What is resonance ? Explain with an example. 3

(b) Show that the equation $x=a \sin wt$ represents a simple harmonic motion. 2

(c) Explain the terms amplitude, phase and time period of a simple harmonic motion. 3

13. (a) Discuss the correction made by Laplace to Newton's formula for the velocity of sound in gas. 3

(b) Explain what information is obtained about its nature from polarisation of light. Why there is no polarisation of sound waves ? 2

(c) Find out the velocity of sound in hydrogen gas at N.T.P. The density of hydrogen at N.T.P. $=0{\cdot}090$ gm/litre ; $\gamma=1{\cdot}4$. 3

14. (a) What is interference of waves ? Describe an experiment to show the interference of sound wave. 3

(b) Derive the relation between the fundamentals produced in two tubes of same length—one open, the other is closed. 2

(c) 5 beats are produced when two tuning forks *A* and *B* are sounded together. Find out the frequencies of the forks, if they produce resonance with 36 cm and 37 cm respectively of air column of a tube closed at one end. 3

H. S. PHYSICS : 1989

Group—A

Answer *any two* questions

1. (a) What is the parallelogram law in case of two vectors ? With the help of this law, find analytically the magnitude and direction of the resultant. Discuss whether the resultant of two unequal inclined vectors can be zero or not. 6

(b) A body moving with a constant retardation traverses 5·7 m and 3·9 m in the 6th and 9th second respectively. When will the body stop moving ? 3

(c) The speed of a body remaining same discuss whether its velocity can change. 1

2. (a) State Newton's third law of motion and explain the same with examples. What is the difference between Impulse and Impulsive force ? 4

(b) Find a relation between angle of repose and coefficient of friction. What are the conditions of equilibrium due to several coplanar forces ? 4

(c) What do you understand by centripetal force and centrifugal force ? 2

3. (a) Find the kinetic energy of a body having mass m and moving with velocity v. 2

(b) Prove the law of conservation of energy for a body falling freely under gravity. 4

(c) Derive the relation between Horse power and Kilo Watt. 2

(d) A man of 100 lb weight takes 5 minutes to climb a tower 300 ft high. What is the Horse power of the man ? 2

Group—B

Answer *any two* questions

4. (a) State Newton's law of gravitation. Why is the gravitational constant called universal ? State the difference between mass and weight of body. 2+1+2

(b) Find a relation between gravitational constant and mean density of Earth. 3

(c) What will be the acceleration due to gravity at a height of 2 miles above the surface of the earth ? (Diameter of earth is 8000 miles and g on the earth's surface is 980 cm/sec^2). 2

5. (a) State Hooke's law. Define 'longitudinal stress' and 'longitudinal strain'. What are their units in the C.G.S system ? 2+2+1

(b) Define 'Poisson's ratio'. Discuss whether it is an elastic modulus. 2

(c) Two wires of different material having diameters 2 mm and 3 mm respectively are each 4 metres long. By hanging a weight of 5 kg in each wire, it is found that elongation of one is twice that of the other. Find the ratio of the Young's modulus of the material of the two wires. 3

6. (a) Find the pressure at a point inside a liquid. 3

(b) State Pascal's law. How is the law of multiplication of force obtained from this law ? 4

(c) A liquid of mass m_1 and density ρ_1 is mixed with another liquid of mass m_2 and density ρ_2. If the volume of the mixture does not change then what will be the density of the mixture ? 3

7. (a) Describe a simple vacuum pump and explain its principle of action. 5

(b) What do you understand by normal atmospheric pressure ? What are the conditions of equilibrium of a floating body ? 1+2

(c) A piece of wood of volume 20·5 c.c. is tied with a piece of lead having a volume of 1 c.c. Show, whether the combination will sink or float in water. [Sp. gravity of wood and lead are respectively 0·5 and 11·4] 2

Group—C

Answer *any two* questions

8. (a) Show how the density of a liquid changes with temperature. 2

(b) A seconds pendulum indicates correct time at 20°C. At what temperature will it go slow by 30 seconds per day ? α for the material of the pendulum is 18×10^{-6}/°C. 4

(c) Explain with the help of graph the anomalous expansion of water. What is its advantage in nature ? 4

(d) Bimetallic strip can be used as an indicating instrument—explain. 2

9. (a) Define : Calorie, Specific heat, Latent heat of fusion of ice. 3

(b) Is it possible to measure the temperature of a furnace (temperature range from 800°C to 1000°C) with the help of a 50°C thermometer ? Discuss briefly. 3

(c) What is the difference between evaporation and boiling ? 2

(d) On a certain day the temperature of air is 26·6°C and the dew point is 9·5°C. If the maximum vapour pressures at 9°C, 10°C, 26°C and 27°C are 8·6 mm, 9·2 mm, 25·2 mm and 26·7 mm respectively, find the relative humidity on the aforesaid day. 4

10. (a) State the basic assumptions of the Kinetic theory of ideal gases. According to this theory what interpretation of temperature is obtained ? 5

(b) What is mechanical equivalent of heat ? 2

(c) From what minimum height a piece of ice has to fall so as to melt completely ? [Assume that 20% of the energy is lost ; given $J=4·2\times10^{7}$ ergs/cal., and Latent heat of fusion of ice=80 cal./gram] 3

(d) Why do the two specific heats of gas differ ? 2

11. (a) Define thermal conductivity. 2

(b) When a metallic piece cooled by liquid air touches the body a burning sensation is felt—why ? 2

(c) What are the properties of thermal radiation ? 4

(d) A room has two glass windows, each of area 2 square metres and thickness 3 mm. The inside and outside temperature of the room are 25°C and −5°C respectively. How much coal is to be burnt per day to replenish the heat that is conducted away through the two windows ? [Heat due to burning of coal = 8000 cals/gram and thermal conductivity of glass = $2·4\times10^{-3}$ CGS unit.] 4

Group—D

Answer *any two* questions

12. (a) What is simple harmonic motion ? 1

(b) How do the velocity and acceleration of a body executing simple harmonic motion change ? 4

(c) Find the time period of a simple pendulum when the angular amplitude is small. 3

13. (a) What are the characteristics of progressive waves ? 3

(b) What is an echo ? Light can be reflected by a small mirror but an extended area is required for reflection of sound—why ? 1+1

(c) A gun is fired from a plane moving horizontally at 120 miles/hour and an echo from the ground is heard after 3 seconds. What is the height of the plane ? [Velocity of sound=1120 ft/sec.] 3

14. (a) Write down the laws of transverse vibration of a string. 3

(b) Discuss the characteristics of musical sound. 3

(c) What will be the rotation per minute of a siren disc having 32 holes so that the emitting frequency is in unison with a tuning fork of frequency 256 ? 2

Jt. ENTRANCE—PHYSICS : 1989

Attempt *all* questions

The figures in the margin indicate marks for each question.

1. A body is moving in a straight line with a constant acceleration starting from rest. Discuss about the nature of the curve of the time dependance of the length of the path described by the body. 2

2. Two identical loads W suspended each from the middle of a string. The strings sag due to the loads and make angles of α_1 and α_2 with the horizontal ($\alpha_1 > \alpha_2$). Which of the strings is subject to greater tension ?—Explain. 2

3. If a car is run along a straight track by an engine working at a constant power, how will the distance traversed depend on time ? Deduce the relation. 2

4. A water filled cone of height 50 cms and base area 20 sq. cms. is placed on a table with the base on the table. What is the thrust offered by the water on the table ? 2

5. A 50 kg man stands on a weighing machine in a lift. When the lift starts moving with uniform acceleration, the machine shows his weight as 45 kg. Was the lift moving upwards or downwards ? Give reasons for your answer. What was the acceleration of the lift ? [$g=9{\cdot}8$ m/s^2.] 2

6. Why does 1 lb of iron at 100°C melt much more ice than 1 lb of lead, also at 100°C ? What is the unit of specific heat capacity in the M.K.S. system ? 2

7. A gas is expanded from a state 1 to another state 2 (higher pressure state) by two different processes : (*a*) first by keeping the volume constant and then, by keeping the pressure constant, (*b*) first by keeping the pressure constant and then by keeping the volume constant. In which process more heat will be supplied to the gas ? Answer with reasons. 2

8. Two bars of the same cross-sectional area and the same length but made of different materials are clamped lengthwise one after another between two undeformable walls. The materials of the bars differ in mechanical and thermal properties. What must be the relationship between Young's moduli and the linear co-efficients of thermal expansion so that heating the bars does not change the position of the boundary between them ? 2

9. Two balloons of the same volume are filled with two gases at the same pressure, one with hydrogen and the other with helium. Which of the two has greater lifting force ?—Explain. 2

10. A man fills tires ot his car to a gauge pressure of 30 lb/in^2 at 27°C when atmospheric pressure of 15 lb/in^2. By afternoon (temperature 12°C) he reached a town where atmospheric pressure is 10 lb/in^2. What will be the gauge pressure of tires of his car ? Volume of tires may be taken as constant. 2

11. A beam of parallel rays of width $b=20$ cm propagates in glass at an angle $\phi=60°$ to its plane face. Find the beam width after it goes over to air through this face. The refractive index of glass is $n=1{\cdot}8$. 2

12. How can you identify in a simple way whether a mirror is plane, concave, or convex ? Which type of mirror is used in cars and other vehicles as view-finder and why ? 2

13. When a small lamp is held 1·5 metres above the surface of a tank, the image of the lamp seen by reflection in the surface appears to coincide with image of the bottom. Find the depth of the tank. Refractive index of water is $\frac{4}{3}$. 2

14. At what distance f_1 from a biconvex lens must we place an object for the distance between the object and the real image to be minimal ? 2

15. A ray is incident at an angle of incidence i_1 on a prism of angle A and refractive index μ and it is found to emerge at an angle of emergence i_2. Then, find the deviation of the ray when it is incident on the prism at an angle i_2. 2

16. A ring of circumferential length of 10 cm. is made of a wire having a resistance $R_0=10\Omega$. Find the points at which current-carrying conductors should be connected so that the resistance R of the subcircuit between these points is equal to 1Ω. 2

17. A parallel plate air capacitor has capacitance C. What will be the capacitance of the capacitor if it is half-immersed in an oil of dielectric constant 1·6 with the plates remaining perpendicular to the surface of the oil ? 2

18. A cell of e.m.f. V volts and negligible internal resistance is connected across a potentiometer whose sliding contact is made exactly at the middle. A voltmeter is connected between the sliding contact and one fixed end of the potentiometer. If it is assumed that the resistance of the voltmeter is not very high compared with the resistance of the potentiometer, what voltage will the voltmeter show : higher than or less than $V/2$? Justify your answer. 2

19. What are the distinguishing factors between a primary cell and secondary cell ? Where is the seat of e.m.f. of a cell ? 2

20. The burner of an electric stove is made up of three sections of equal resistance. If the three sections are connected in parallel, water in a tea-kettle begins to boil in 6 minutes. When will the same mass of water in the tea-kettle begin to boil if the sections are connected in series ? 2

21. A conductor in series with an ammeter and a semiconductor in series with another ammeter are connected in parallel. At a certain voltage both ammeters register the same current. Will this condition remain as such if the voltage of the *DC* source is increased ? Explain your answer. 2

22. A sound wave travels from air to water. The angle of incidence at the interface between air and water is $\angle_1$ and the angle of refraction in water is $\angle_2$. Assuming Snell's law to be valid, explain whether $\angle_1$ is greater or less than $\angle_2$. 2

23. A rectangular bar of a magnetic substance is placed in a uniform magnetic field. Show a few lines of force and lines of magnetisation in a neat diagram. The field intensity inside and outside the specimen are different.—Why ? 2

24. A solenoid carrying a current supplied by a *DC* source with constant e.m.f. contains an iron core inside it. How will the current change during the removal of the core from the solenoid ?—Explain. 2

25. How many quanta of different energy can be emitted by a hydrogen atom if the electron is in the third orbit ?—Justify. 2

26. A train is crossing an observer standing on a platform. The first compartment of the train takes 2 seconds to cross the observer while the second compartment takes 2·5 seconds to cross him. The train is moving with uniform acceleration. Find, (*i*) the velocity of the train when the front of the first compartment crosses the observer, (*ii*) the acceleration of the train. Given the length of each compartment is 15 metres. 5

27. A man is standing on the top of a 40 lb. 13 ft uniform ladder with its base 5 ft out from a smooth vertical wall. The ladder is not slipping. Find the minimum co-eff. of friction between the foot of the ladder and the ground. The man weighs 180 lb. 5

28. A mercury barometer gives false indications because of a bubble of air that got into the space over the mercury. At a pressure of 760 mm of Hg. column the barometer reads 740 mm and at 727·5 mm of Hg column the reading is 710 mm. Find the length of the barometer tube (from the upper end of the mercury level in the cup). 5

29. Water flows at the rate of 0·1500 kg/min through a tube and is heated by a heater dissipating 25·2 *W*. The inflow and outflow water temperatures are 15·2°C and 17·4°C respectively. When the rate of flow is increased to 0·2318 kg/min and the rate of heating to 37·8 *W*, the inflow and outflow temperatures are unaltered. Find (*i*) the specific heat capacity of water, (*ii*) the rate of loss of heat from the tube. 5

30. A mirror 0·5 metre high hangs on a wall. A man stands at a distance of 2 metres away from the mirror. What is the height of the portion of the opposite wall in the room that can be seen by the man in the mirror ? The opposite wall is 6 metres away from the mirror. 5

31. An inch cube is constructed of a material whose refractive index is 1·65. Calculate the least radius of the opaque circular disc, which must be placed centrally over each face of the cube, so that a small air-bubble at its centre shall be invisible from an external point. 5

32. Three resistors of 4, 5 and 12 ohms respectively, are connected in parallel and to the terminals of a 10-volt battery whose internal resistance is 0·5 ohm. What current flows in each resistor and in the battery ? 5

33 Two very long conductors are in vacuum at a separation of 4 cms. The current through one of these is 25 amperes and through the other 5 amperes. Calculate the length of a segment of the conductor subjected to a force of 125 dynes. 5

34. Two wires are fixed on a sonometer. The tensions are in the ratio 8 : 1, the lengths in the ratio 36 : 35, the diameters in the ratio 4 : 1 and the densities are in the ratio 1 : 2. Find the frequency of beats produced if the note of the higher pitch has a frequency of 360 *Hz*. 5

35. Suppose an electron of charge $-e$ and mass m is revolving in an orbit of radius r round a nucleus of charge $+E$. Show that the energy of the electron is $-me^2E^2/2L^2$, where L is the angular momentum of the electron$=mvr$. 5

INDEX

A

Absolute
    humidity 404
    scale of temperature 350
Acceleration 24
    angular 100
    centrifugal 106
    centripetal 104
    due to gravity 181
    in S.H.M. 478
    instantaneous 25
    negative 25
    relative 34
    uniform 24
Action and reaction 55
Adiabatic change 429
Adiathermaneous 465
Air column,
    vibration of 524
Air pressure 256
    measurement of 257
Analyzer 543
Aneroid barometer 258
Angle
    of contact, 280
    of friction 86
    limiting 87
    of repose 86
Angular
    acceleration 100
    displacement 97
    velocity 98
    momentum 119
        conservation of 120
Antinode 514
Archimedes' principles 247
Atmospheric pressure 256
    magnitude of 256
    standard 257
    measurement of 257
Avogadro number 444

B

Banking of roads 108
Barometer,
    Aneroid 258
    Fortin's 257
Beats 533
Black body radiation 466
Boiling 390
    condition for 401
    laws of 392
    point 390
Bolometer 463
Boltzman's constant 444
Boyle's law 341
Breaking stress 224
British Thermal Unit (B. Th. U.) 366
Brawnian motion 438
Bulk modulus 219
Buoyancy 245

C

Calorie, 366
    kilogram 366
Calorimeter 370
Calorimetry 365
    fundamental principle of 370
Capillarity 281
Celsius scale 263
Centigrade heat unit 366
Centre
    of buoyancy 246
    of gravity 138
    of mass 143
Centigrade scale 293
Centrifuge 110
Centrifugal force 106
    as a pseudo force 106
    reactions 106
Centripetal
    acceleration 104
    force 105
Change of state 381
Charle's law 345
Coefficient
    of gas 341
    of liquid 321
    of solid 301
    of friction 85
    of viscosity 286
Coherent 532
Collision
    perfectly elastic 64
    perfectly inelastic 64
Component of a vector 9
Composition of vectors 11
Compressibility 219
Condensation 388
Conduction, thermal 450
Conductivity,
    thermal 450
    thermometric 455
Conservation
    of angular momentum 120
    of energy 165
    of mechanical energy 161
    of linear momentum 61
Conservative force 165
Convection 459
Cooling
    Newton's law of 468
    corpuscular theory of 578
Couple 117
    work done by 154

Critical
  pressure 402
  temperature 402
  velocity 283
Curvilinear motion 18

D

Dalton's law of partial pressure 446
Damped vibration 493
Decceleration 25
Density 231
Dew-point 403
Diathermaneous 465
Diffraction 583
Diffusion 437
Diffusivity 454
Displacement 19
  angular 97
Dissipative forces 165
Doppler effect 536
Dyne 49

E

Earth,
  mass of 185
  satellites of 199
Echo 564
Elasticity 213
Elastic
  collision 64
  constants 223
  fatigue 224
  limit 223
Energy 155
  conservation 165
  heat 291
  kinetic 154
  potential 156
  radiant 461
  in S.H.M. 484
Equilibrium 129
  of floating bodies 249
  of a system of particles 129
  neutral 140
  stable 140
  thermal 370
  unstable 140
Equilibrant 131
Erg 152
Escape velocity 203
Ether 579
Evaporation 389
Exchanges, Pre'vost's
  theory of 491
Expansion coefficient,
  cubical 304
  linear 301
  superficial 303
  real and apparent 321
  of gases 341
  volume 347
  pressure 347
  water, anomalous 333

F

Fahrenheit scale 293
Falling bodies 186
Fixed point 293
Fluid flow, 282
  streamline 282
  turbulent 282
Foot-poundal 152
Force 47
  centrifugal 106
  centripetal 105
  conservative 165
  dissipative 165
  of friction 59
  gravitational 48
  impulsive 54
  moment of 114
  measurement of 47
  no-work 151
  of reaction 55
Force-constant 214
Forced vibration 494
Freely falling body 186
Freezing point 382
Frequency
  beat 534
  fundamental 520
  resonant 495
  of S. H. M. 484
  of waves 507
Friction 82
  angle of 86
  coefficient of 86
  in fluid flow 83
  kinetic 82
  laws of 85
  limiting 84
  rolling 83
  sliding 83
  static 83
Fundamental
  frequency 519
  mode 520

G

Gas,
  constant 346
  expansion of 341
  external work by 424
  ideal 354
  internal energy of 423
  kinetic theory of 436
  pressure of 442
  real 442
  specific heats of 425
Gravitation 177
Gravitational,
  acceleration 181
  attraction of extended bodies 179
  attraction of the earth 181
  constant 178

Gravitational,
    potential energy 156
    system of units 48
Gravity, 181
    acceleration due to 181
    centre of 138
    specific 232

H

Harmonics 520
Heat,
    as a form of energy 417
    capacity 368
    condition of 450
    exchange of 466
    of fusion 382
    latent 381
    mechanical equivalent of 418
    specific 367
    transmission of 450
    of vaporisation 388
Hooke's law 215
Horse power 167
Humidity,
    absolute 404
    relative 404
Hydraulic break 244
Hydrostatic paradox 238
Hygrometer 406

I

Ideal gas law 354
Impulse 52
Impulsive force 54
Inelastic collision 64
Inertia, moment of 118
Ingenhausz's experiment 454
Intensity of sound 569
Interference,
    of light waves 580
    of sound waves 530
Isothermal change 428

J

Jets 68
Joule (unit) 418
Joule's law 418

K

Kelvin temperature scale 293
Kepler's laws 197
Kinetic,
    energy 154
    friction 83
    theory of gases 441

L

Lami's theorem 132
Latent heat 381
    of fusion 382
    of vaporisation 390
Law of fusion 388
Light,
    corpuscular theory of 578
    Doppler effect 536
    interference of 580
    polarisation of 582
    velocity of 579
    wave nature of 578
Limiting angle 87
Linear expansion,
    coefficient of 301
Longitudinal,
    strain 216
    stress 216
    wave 504

M

Manometer 259
Mass, centre of 143
    and inertia 45
    and weight 182
Mechanical,
    energy 154
    equivalent of heat 417
Melting point 382
Modes of vibration 519
Modulus, of elasticity 216
    bulk 219
    shear 221
    Young's 217
Molar,
    gas constant 425
    specific heat 425
Molecular theory of matter 436
Moment,
    of force 114
    of inertia 118
    of momentum 119
Momentum, 47
    angular 119
    conservation of linear 61
Motion,
    accelerated, uniformly 26
    angular 97
    Brownian 438
    circular, uniformly 104
    curvilinear 18
    in fluid 283
    laws of 43
    oscillatory 477
    periodic 478
    planetary 197
    rectilinear 19
    rotational 19, 97
    of satellites 200
    simple harmonic 478
Musical sound 568

N

Newton (unit)'s 49
    law of cooling 468
    law of gravitation 177
    laws of motion 43

Nodes 514
Note 568
Noise 568

O

Optics, geometrical as
  an approximation 583
  Organ pipes, 524
  closed 524
  open 525
  overtones 526

P

Pascal's law 241
Pendulum,
  compensated 308
  simple 190
Period 478
Periodic motion 478
Phase 483
Pipe,
  closed 524
  open 525
Pitch 570
Planetory motion, 197
  laws of 198
Plastic material 214
Poise 286
Poisson's ratio 222
Polarisation.
  of light 582
  of waves 541
Polariser 542
Polygon rule 6
Potential energy 156, 214
Poundal 49
Power 166
  horse power 166
Pressure,
  atmospheric 257
  of fluid 234
  of ideal gas 442
Pre'vost's theory 466
Pump,
  compression 263
  lift 261
  vacuum 264
Pyrometer 468

Q

Quality, tone 570
Quincke's tube 532

R

Radiation, 461
  black body 466
  detection 462
  emission and absorption of 464
  as a form of energy 461
  properties of thermal 463
Recording and reproduction
  of sound 571
Reference frame 17
Reflection, of waves 511
Refraction, of waves 511
Relative,
  acceleration 34
  humidity 403
  velocity 32
Regelation 387
Resolution, of vector 9
Resonance air columns 528
Resultant,
  of vectors 3
  of parallel forces 136
Retardation 25
Rigid body 213
Rocket 69
Rolling friction 90
Root-mean-square velocity 442
Rotational motion, 19, 97
  Similarity with
    translatory motion 121

S

Satellites 200
Saturated vapour 397
Scalar quantities 1
Shear 220
Simple harmonic motion, 478
  acceleration in 482
  amplitude in 478
  energy in 484
  frequency of 482
  period of 482
  reference circle 479
  velocity in 481
Simple pendulum 190
Siphon 260
Snell's law 512
Solar constant 468
Sonometer 522
Sound,
  as elastic waves 551
  intensity of 569
  interference of 530
  musical 568
  pitch of 570
  quality of 571
  recording and reproduction of 571
  reflection of 511
  refraction of 511
  sources of 568
  velocity of 560
  Doppler effect of 536
Specific,
  gravity 232
  heat of a gas 425
  heat of a solid 367
Speed 20
State, change of 381
Stationary waves 513
Stefan-Boltzman's law 467
Strain, 216
  longitudinal 216
  shearing 220
  volume 219

Stream lines 282
Stress 215
    longitudinal 217
    shearing 220
    volume 219
Stress-strain diagram 223
String, vibration of 518
Superposition, principle of 513
Surface tension 277

T

Temperature,
    absolute (kelvin) 350
    celsius (centigrade) 293
    critical 402
    Fahrenheit 293
    interpretation from
        kinetic theory 443
Tesile strain & stress 220
Tension, surface 277
Thermal,
    capacity 368
    conductivity 452
    expansion, 299, 439
        of gas 341
        of liquid 321
        of solid 299
Thermocouple 462
Thermodynamics,
    first law of 422
Thermometer,
    bimetallic 311
Thermostat 311
Thrust 234
Tone 568
Torque 114
Torsional deformation 221
Transformation of energy 159
Translational motion 19
Transverse wave 502

U

Uniform circular motion 104
Uniformly accelerated motion 26
Universal gas constant 356

V

Vapour pressure 397
    saturated 397
    unsaturated 397
Vaporisation,
    latent heat 388

Vectors, addition of 2
    components of 10
    multiplication of, by a number
    resolution of 9
    subtraction of 7
    representation of 1, 13
Velocity, angular 98
    critical 283
    escape 203
    instantaneous 23
    relative 33
    r.m.s. 442
    uniform 20
Vibration, of air columns 524
    damped 493
    forced 494
    resonance of string 494
Viscosity 285
Volume elasticity 219

W

Watt 166
Water, anomalous expansion 333
Wave, amplitude 506
    equation 508
    frequency of 507
    front 507
    interference of 530
    length 507
    longitudinal 504
    polarisation of 541
    progressive 501
    reflection of 511
    refraction of 511
    stationary 513
    superposition 513
    transverse 502
Weight 182
Weightlessness 203
Work done against friction 162
    done along curved path 151
    done by and against force 153
    done by couple 154
    done by gases 424
Work-energy principle 163

Y

Young's modulus 217

Z

Zero, absolute 351